Contributed Session 2: ODEs and Applications

Homoclinic Solutions to the damped Duffing's equation

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In this paper, homoclinic solution of the damped Duffing's equation is studied. First, existence of the homoclinic solution is obtained by using the Schauder-Tikhonov theorem. Then, the totally analytic homoclinic solution of the above problem is obtained using the homotopy analysis method (HAM) .Finally, the obtained results are illustrated graphically to show salient features of the solutions. Key words: Damped Duffing's equation , nonlinear differential equations, homoclinic orbit.

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A Necessary and Sufficient Condition for the Existence of Periodic Solutions of Linear Impulsive Differential Systems with Distributed Delay

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A necessary and sufficient condition is established for the existence of periodic solutions of linear impulsive differential systems with distributed delay of the form

$$x'(t) = \int_{-\tau}^{0} \mathrm{d}_s \eta(t,s) x(t+s) + f(t), \quad t \neq \theta_i,$$
$$\Delta x(\theta_i) = A_{i0} x(\theta_i) + \sum_{k=-j}^{-1} A_{ik} x(\theta_{i+k}) + f_i, \quad i \in \mathbb{Z}.$$

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Limit cycles of Liénard systems

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We study the existence of limit cycles of the Liénard equation $d^2x/dt^2 + ef(x)dx/dt + g(x) = 0$ (1) where e is a parameter. In particular,we study the relaxational regimes of (1) when f is even and g(x)=x,in the Liénard plane.We give the shape of the limit cycles.We illustrate this study by different examples.We study the bifurcation curves of some examples of systems (1).

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On The Growth Of Meromorphic Solutions Of Complex Linear Differential Equations With Meromorphic Coefficients

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In this paper, we investigate the growth of meromorphic solutions of higher order linear differential equation $f^{(k)} + A_1(z) e^{P_1(z)} f' + A_0(z) e^{P_0(z)} f = 0 \ (k \geq 2)$, where $P_1(z)$, $P_0(z)$ are nonconstant polynomials such that $\deg P_1 = \deg P_0 = n$ and $A_j(z) \ (\not\equiv 0) \ (j = 0,1)$ are meromorphic functions with order $\rho(A_j) < n \ (j = 0,1)$. We obtain two results which greatly extend the result of Z. X. Chen and K. H. Shon.

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Spectral Properties of p-Biharmonic Problems

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We are interested in spectral properties of p-biharmonic problems for nonlinear equations of the type

$$(|u''|^{p-2}u'')'' = \lambda |u|^{p-2} u \qquad \text{on} \quad [0,1],$$

where $\lambda \in \mathbb{R}$ is the spectral parameter.

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On a modified version of ILDM method and its asymptotics

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It is known that processes which take place in complex chemical kinetics and combustion systems have very different time scales. It is often desirable to decouple such

systems into fast and slow sub-systems for reduction of their complexity. One of such reduction methods is based on Intrinsic Low-Dimensional Manifolds (ILDM). This method successfully locates slow manifolds of considered system but also has a number of problems. Firstly, it cannot treat some zones on the phase plane ("turning zones", i.e. zones where stability of the manifold changes). Secondly, the numerical algorithm of the method produces additional solutions that do not have any connection to the dynamics of the system. In this paper the modified ILDM method is suggested (so called TILDM), that is partially free of the disadvantages of the original technique. The new version is based on the geometrical properties of fast-slow systems. The asymptotic analysis of the TILDM method is performed.

Key-Words: multi-scale systems, reduction methods, slow manifolds, intrinsic low-dimensional manifolds (ILDM), method of invariant manifolds (MIM).

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Asynchronous methods for nonlinear Differential Algebraic Equations

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In this paper, We propose an Asynchronous Multisplitting Waveform Relaxation method for Differential Algebraic Equations of the form

$$\begin{cases} x' = f(x,y,t) \\ 0 = g(x,y,t) \\ x(t_0) = x_0 \end{cases} \tag{1}$$

We assume that the system is of index one that $\frac{\partial g}{\partial y}$ is assumed to be invertible along the exact solution.
Waveform relaxation (WR) is a technique to solve large system of initial-value problems consisting of ODE's in parallel. The right hand side of the system is split into independent subsystems. One then solves iteratively all the subsystems in parallel and exchanges information.
Multisplitting Waveform Relaxation (MWR) method is a generalization of WR by using techniques of multisplitting in the sense that the multisplitting allows overlap between subsystems. This technique was introduced in [1] for linear ODEs. We generalize this formulation to nonlinear differential algebraic case in asynchronous mode and we prove that MWR method converges faster than WR. Numerical results by using parallel MPI library will be given.
References:
[1] Frommer, A., Pohl,B. (1995) A Comparison Result for Multisplittings and Waveform Relaxation Methods. Numer. Linear Algebra Appli. 2:335–346.

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New discrete analogue of neural networks with nonlinear amplification function and it's periodic dynamic analysis

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In this paper, new discrete analogue of neural networks with nonlinear amplification function is obtained by analysis and approximation techniques. Using continuation theorem of coincidence degree theory, periodic solution for discrete model is studied, and sufficient condition is given to guarantee the existence of periodic solution. Moreover, global stability on periodic solution is investigated by Lyapunov method. At last, one example is given to show the effectiveness of results in this paper

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Fuchik spectrum for the Sturm-Liouville boundary conditions

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Consider the equation

$$x'' + \mu^2 x^+ - \lambda^2 x^- = 0,$$

together with the boundary conditions

$$\begin{aligned} x(0)\cos\alpha - x'(0)\sin\alpha = 0, \\ x(\pi)\cos\beta - x'(\pi)\sin\beta = 0. \end{aligned} \tag{1}$$

Fuchik spectrum for $\alpha = 0$, $\beta = \pi$ (the Dirichlet problem) is known. We obtain the expressions for the Fuchik spectrum in the case of boundary conditions of the Sturm-Liouville type (1).

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Similarity solutions of degenerate boundary layer equations

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Abstract An analysis is made for a steady-state laminar boundary layer flow, governed by the Ostwald-de Waele power-law model of an incompressible non-Newtonian fluid past a semi-infinite power-law stretched plate subject to a suction or injection with uniforme free-stream velocity. A generalization of the usual Blasius similarity transformation is used to find Similarity solutions. Under appropriate assumptions, partial differential equations are transformed into an autonomous third-order nonlinear degenerate ordinary differential equation with boundary conditions. By means of a shooting method, we establish the existence of infinitely many unbounded global solutions. The asymptotic behavior is also discussed. Some properties of those solutions depend on the power-law index.

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The infinite product representation of solutions of indefinite Sturm-Liouville problems with two turning points

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We study the infinite product representation of solutions of second order differential equation of Sturm-Liouville type on a finite interval having two turning points under the assumption that one of the turning points is of odd order while the other is of even order. Such representations are useful in the associated studies of inverse spectral problems for such equations.

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On (non)chaotic behaviour in homogeneous quadratic systems of ODEs in $\mathbb{R}^3$

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Many authors were trying to answer the question: how complicated must an ODE be in order to exhibit chaos (see [1]). For 3D-dissipative four-terms-systems it is proven that they are nonchaotic. We will consider a special class (generally not dissipative, nor conservative 9-terms-systems) of quadratic homogeneous systems of ODEs in $\mathbb{R}^3$ (in short, QHS), which was studied by the author [2]. It was proved that these systems which are the 'natural candidate' for the simplest chaotic QHS do not admit chaotic behavior. The presentation of the last result will be the goal of my report.

I will also mention some relations between QHS $\vec{x}' = \vec{x} * \vec{x}$ and the corresponding algebras $A = (R^n, *)$ (in particular: the existence of some special algebraic elements, subalgebras and ideals, and the rank of algebra). These properties might be corresponding for (non)chaotic behavior in $\vec{x}' = \vec{x} * \vec{x}$.

Finally, note, that it would be extremely hard to deduce that there is no chaos here directly from the general (9-parameter) form, so algebraic approach plays here the crucial role.

References:
[1] J. Heidel, F. Zhang, Nonchaotic Behavior in the Three-dimensional Quadratic Systems. Nonlinearity. 10, 1289-1303 (1997).
[2] M. Mencinger, On nonchaotic behavior in quadratic systems, to appear in Nonlinear Phenom. Complex Syst.

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Asymptotic Equivalance of Dynamic Equations on Time Scales

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A time scale T is an arbitrary nonempty closed subset of the real numbers R. The most well-known examples are T=R and T=Z. Time scale approach allows one to treat the continuous, discrete as well as more general systems simultaneously. In this paper we establish sufficient conditions for asymptotic equivalance of dynamic equations on time scales which are well-known in the case T=R.

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Rigorous asymptotic expansions for critical wave speeds in a family of scalar reaction-diffusion equations

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We investigate traveling waves in the family of reaction-diffusion equations given by

$$u_t = u_{xx} - 2u^m(1-u).$$

For $m \geq 1$ real, there is a critical wave speed $c_{\text{crit}}(m)$ which separates waves of exponential structure from those that decay only algebraically. We derive rigorous asymptotic expansions for c_{crit} by perturbing off two solvable cases, the classical Fisher-Kolmogorov-Petrowskii-Piscounov equation ($m = 1$) and a degenerate cubic equation ($m = 2$). In addition, we study the asymptotic critical speed in the limit as $m \to \infty$. Our approach uses geometric singular perturbation theory, as well as the blow-up technique, and confirms results previously obtained through asymptotic analysis.

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Existence of Flames in Combustion

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The existence of flames in combustion is an important problem and has a broad set of applications. This kind of problem is studied by many authors such as Buckmaster, Ludford, Marion and etc. From mathematical point of view, the existence of travelling wave solutions is proved for a system of ordinary differential equations. This system has a non continuous term which is called the reaction rate function and this occurs because of cold boundary difficulty. In order to prove the existence of travelling waves for this system, some general topology arguments will be applied.

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Periodic solutions for a DC-DC switching converter

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The switched model of a DC-DC converter controlled by constant-frequency pulse width modulation can be described by a couple of different first order differential systems for each switch position. Although these systems are piecewise linear, the presence of the switching discontinuity in he model leads to genuinely nonlinear phenomena.

Our aim is to give sufficient conditions for existence of one-periodic solutions which cross the voltage ramp once per cycle. In addition, some properties about these solutions will be found.

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Interval oscillation criteria for second order nonlinear differential equations with damping

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Numerous results regarding oscillation of nonlinear differential equations with damping rely on so-called integral averaging technique which requires information on the behavior of coefficients of the given equation on the entire positive semi-axis. Unfortunately, many theorems fail to apply to equations were the damping coefficient is oscillatory or changes sign. On the other hand, even most efficient oscillation criteria fail to apply in cases when, for instance, the integral of coefficient approaches minus infinity. This shortcoming of integral averaging approach called for development of a new technique, referred to as an interval oscillation method, which uses information on the behavior of coefficients of a given differential equation only on an infinite sequence of intervals.

In this talk, exploiting generalized Riccati-type transformations, we obtain a number of efficient interval oscillation criteria which establish oscillatory nature of two classes of nonlinear differential equations with damping. Our theorems extend and complement known oscillation results and can be used to prove oscillation of a given differential equation in situations where none of the results known in the literature apply. Several illustrative examples are considered in the final part of the talk to demonstrate the efficiency of new theorems.

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Types of solutions and multiplicity results for two-point nonlinear boundary value problems

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We consider nonlinear boundary value problems (BVP) of the form $x'' = f(t,x,x')$, $x(a) = 0$, $x(b) = 0$, **(i)**. By the type of a solution $z(t)$ to the BVP is meant the number of zeros in the interval (a,b) of the difference $x(t) - z(t)$, where $x(t)$ is a solution of the Cauchy problem $x'' = f(t,x,x')$, $x(a) = 0$, $x'(a) \sim z'(a)$. Conditions for the existence of a solution $z(t)$ of a definite type are given, as well as a number of multiplicity results. A sample of the existence results follows.

Theorem. The BVP $x'' + p(t)x' + q(t)x = F(t,x,x')$, $x(a) = 0$, $x(b) = 0$, (**ii**) where F is bounded, has an i-type solution if a solution of the Cauchy problem $x'' + p(t)x' + q(t)x = 0$, $x(a) = 0$, $x'(a) = 1$ has exactly i zeros in (a,b) and $t = b$ is not a zero. The coefficients p, q, as well as the functions F and F_x, $F_{x'}$ are continuous.

We discuss also the possibility of reduction the problem **(i)** to the form **(ii)**.

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On the unusual Fučik spectrum

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We construct the Fučik spectrum for the third order nonlinear boundary value problem

$$x''' = -\mu^2 x'^{+} + \lambda^2 x'^{-},\ \mu, \lambda > 0, \tag{1}$$

$$x(0) = x'(0) = 0 = x(1). \tag{2}$$

($x'^{\pm} = \max\{\pm x', 0\}$). By the Fučik spectrum we mean the set of all (λ, μ) such that the problem (1), (2) has a nontrivial solution. We investigate also properties of solutions to the problem (1), (2) and made comparison with some known Fučik type problems. Let us mention that the Fučik spectrum for the problem under consideration significantly differs from the known Fučik spectra.

We consider also some fourth order problems and investigate the Fučik spectra.

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Existence of positive decaying solutions for nonlinear singular second order equations

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We consider the singular boundary value problem

$$\begin{cases} (r(t)u')' + f(t,u) = 0, \quad t \in [0, \infty), \\ u(t) > 0, \\ \lim_{t\to+\infty} u(t) = 0 \end{cases}$$

where $r(0) = 0, r(t) > 0$ for $t > 0; f(t,0) \equiv 0, f(t,u) > 0$ for $(t,u) \in (0,\infty) \times (0, \alpha^*)$ for some $\alpha^* > 0$. Our general model includes the ordinary differential equation associated to the semilinear elliptic partial differential equation

$$\Delta u + K(|x|)g(u) = 0,$$

when looking for radial solutions. Under quite general assumptions on r and f, we give multiplicity results for positive decaying to zero solutions of our equation, obtaining the existence of a family of solutions depending on the initial value $u(0) > 0$ and the initial quasiderivative $\lim_{t\to 0^+} r(t)u'(t) \le 0$. The key ingredients of the proof are Schauder fixed point theorem and an innovative modification of Pohozaev identity. The conditions guaranteeing the solvability yield some of the existing results when the equation reduces to the semilinear elliptic form. Our results can be viewed as an extension of the known ones to equation with more general coefficients and nonlinearities.

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