Contributed Session 3: Delay and Difference Equations

A Stochastic-difference-equation-model of moving equilibria in the public health care sector: a low quality-low performance trap and a resolution

Ahmet Kara
Fatih University, Turkey
econs@fatih.edu.tr

This paper develops a stochastic model of the public healthcare sectors in developing countries and demonstrates the existence, in a particular subset of the Turkish public healthcare sector, of stochastically-evolving equilibria moving towards a low quality-low performance trap over time. The dynamics of the movement in question hinges, in part, on the socially necessary but demographically asymmetric burden, on some public health institutions, of providing affordable health care to certain sections of the population. The paper formulates various stochastically-driven policy options that could help the sector to escape the trap, moving the sector towards high quality-high welfare equilibria.

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Positivity and stability for Partial Neutral Differential Equations

Soumia Lalaoui rhali
Sidi Mohamed Ben Abdellah University, Faculty of Taza, Morocco
slalaoui@ucam.ac.ma
K. Ezzinbi

Consider the neutral partial differential equation

$$\begin{cases} \dfrac{d}{dt}Dx_t = ADx_t + Lx_t, \; t \geq 0 \\ x_0 = \varphi \in C \end{cases} \tag{1}$$

where
X is a Banach space, $C = C([-r,0],X)$, $r > 0$,
$A : D(A) \subset X \to X$ is a Hille Yosida linear operator
$D : C \to X$ is a continuous linear operator defined by

$$D\varphi = \varphi(0) - D_0\varphi, \quad \varphi \in C$$

D_0, L are bounded linear operators from C to X.
The aim of this work is to study the positivity and stability of the solution semigroup $(\mathcal{U}(t))_{t\geq 0}$ of Eq.(1) defined for all $\varphi \in C_0 = \left\{\varphi \in C, \, D\varphi \in \overline{D(A)}\right\}$ by

$$\mathcal{U}(t)\varphi = x_t(.,\varphi)$$

where $x_t(.,\varphi)$ denotes the integral solution of Eq.(1) associated to the initial condition φ.

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Approximation of solutions to a class of second order history-valued delay differential equations

M Muslim
Indian Institute of Technology Kanpur, India
muslim@iitk.ac.in
Dhirendra Bahuguna

In this paper we study the approximations of solutions to a class of second order history-valued delay differential equations in a separable Hilbert space. Using a pair of associated nonlinear integral equations and projection operators we consider a pair of approximate nonlinear integral equations. We first show the existence and uniqueness of solutions to this pair of approximate integral equations. We then establish the convergence of the sequences of the approximate solutions and the pair of approximate integral equations to the solution and the pair of associated integral equations, respectively. We finally consider the Faedo-Galerkin approximations of the olution and prove some convergence results.

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Reduction principle in the theory of stability of difference equations

Andrejs Reinfelds
Institute of Mathematics, Latvia
reinf@latnet.lv
K. Janglajew

Consider the following difference equations in Banach space $\mathbf{X} \times \mathbf{Y}$

$$\begin{cases} x(t+1) &= A(t)x(t) + f(t,x(t),y(t)), \\ y(t+1) &= B(t)y(t) + g(t,x(t),y(t)) \end{cases} \tag{1}$$

where the maps f and g are ε-uniform Lipschitzian with respect to second and third variable and vanishes at the origin. Let $X(t,s)$ and $Y(t,s)$ be the Cauchy evolution map of the corresponding linear system

$$\begin{cases} x(t+1) &= A(t)x(t) \\ y(t+1) &= B(t)y(t), \end{cases}$$

respectively. Assume that operators satisfy the estimates

$$\nu = \max\left\{ \sup_{t\in\mathbb{Z}} \left(\sum_{s=-\infty}^{t-1} |Y(t,s+1)||X(s,t)| \right), \sup_{\tau\in\mathbb{Z}} \left(\sum_{s=t}^{+\infty} |X(t,s+1)||Y(s,t)| \right) \right\}$$

and by

$$\mu = \sup_{t\in\mathbb{Z}} \left(\sum_{s=t}^{+\infty} |Y(t,s)| \right).$$

Let $4\varepsilon\nu < 1$ and $2\varepsilon\mu < 1 + \sqrt{1-4\varepsilon\nu}$. Then there exists a continuous map $u\colon \mathbb{Z}\times\mathbf{X} \to Y$, which is uniform Lipschitzian with respect to the second variable, such that the trivial solution of difference equation

$$x(t+1) = A(t)x(t) + f(t,x(t),u(t,x(t)))$$

is stable, asymptotically stable or nonstable if and only if the trivial solution of difference equation (1) is stable, asymptotically stable or nonstable.

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Geometry of the Stability Regions of a Closed Loop Dynamics in Time Delay vs. PID Gains

Rifat Sipahi
Universite de Technologie de Compiegne, France
rsipahi@hds.utc.fr
Silviu-Iulian Niculescu

Proportional-Integral-Derivative feedback (PID) is widely utilized for the controller design of variety of applications, like in high-precision position control, auto-focusing control in laser probes, smart actuator control. In general, PID controller gains are designed to achieve “desired criteria” based on performance and stability-robustness. However, in many cases, when communication/propagation delays are present, “standard” design techniques fall short and lead to poor performance and even instability for the corresponding closed loop schemes. The complications are mainly due to the way in which the parameters of the controller affect the root distribution of the corresponding characteristic equation. Hence, PID controller design becomes non-trivial and existing techniques do not give satisfactory answers.

The main purpose of this talk is to focus on the behavior of the spectrum of the characteristic equation with respect to the PID controller gains. Our approach enables a simple mechanism to characterize the geometry of the crossing hypersurfaces (corresponding to the existence of the roots on the imaginary axis) of a PID-controlled dynamics under time delay influence, and ultimately it becomes possible to design appropriate PID controller gains guaranteeing the closed loop stability.

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