Contributed Session 4: Modelling and Math Biology

Turing Bifurcation in a Ratio-Dependent Predator-Prey Model with Diffusion

Shaban Aly
Al-Azhar University, Egypt
Shhaly12@yahoo.com

Ratio-dependentpredator-prey models are increasingly favored by field ecologists as an alternative or more suitable ones for predator-prey interactions where predation involves searching process. In this paper we study the conditions of the existence and stability properties of the equilibria in a reaction diffusion ratio-dependent predator-prey model in which predator mortality is neither a constant nor an unbounded function, still, it is increasing with the pre dator abundance. We show that at a certain critical value a diffusion driven (Turing type) instability occurs, i.e. the stationary solution stays stable with respect to the kinetic system (the system without diffusion) but becomes unstable with respect to the system with diffusion and that Turing bifurcation takes place: a spatially non-homogenous (non-constant) solution (structure or pattern) arises.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Numerical integration of a hierarchically size-structured population model with contest competition

Oscar Angulo
Universidad de Valladolid, Spain
oscar@mat.uva.es
Juan Carlos Lopez Marcos

We will study the numerical integration of nonlinear models which describe the dynamics of a hierarchical size-structured population. More precisely, we consider a model with contest competition which has the form

$$u_t + (g(x,B(x,t))\,u)_x = -\mu(x,B(x,t))\,u, \qquad (1)$$
$$x_m < x < x_M,\ t > 0$$
$$g(x_m,B(x_m,t))\,u(x_m,t) = C(t) + \int_{x_m}^{x_M} \alpha(x,B(x,t))\,u(x,t)\,dx\,, \quad t > 0, \qquad (2)$$

$$u(x,0) = \phi(x), \quad x_m \le x \le x_M, \qquad (3)$$
$$B(x,t) = \int_x^{x_M} w(\sigma)\,u(\sigma,t)\,d\sigma,$$
$$x_m < x < x_M, \quad t > 0. \qquad (4)$$

The independent variables x and t represent, respectively, size and time, where x_m and x_M are, respectively, the minimum and maximum value reached by a given population. The function $u(x,t)$ is the population density with size x at time t. The population dynamics is determined by the growth rate g, the mortality rate μ, the reproduction rate α and the external inflow C. The vital functions (growth, mortality and reproduction rates) depend on the structuring variable and on the functional $B(x,t)$, used to describe the competition among individuals for available resources. In this case, contest competition, no individual in a class of smaller size can affect the amount of resource available to an individual of greater size. Therefore, we have a nonlinear partial differential equation which has a functional dependence on nonlocal terms, with a nonlocal and nonlinear boundary condition with functional dependence on nonlocal terms. Existence and uniqueness of solutions for this model has been studied by Kraev.

In the present work, we carry out the numerical integration of equations (1)-(4) by means of a method that integrates along the characteristic curves and uses a constant number of grid nodes. The integral terms are approximated by means of the composite trapezoidal quadrature rule. We will analyse the consistency, stability and convergence properties of the numerical scheme and optimal rate of convergence is derived. We also provide numerical experiments to show the accuracy of the scheme.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A Qualitative Mathematical Analysis of a Class of Dynamical Models in Biochemistry

Oanh Chau
university of La Réunion, Reunion
oanh.chau@univ-reunion.fr
K. Addi and D. Goeleven

In this paper, we develop a mathematical approach which can be used to display, in a systematic, rigorous and efficient way, various qualitative properties of a large class of dynamical and quasi-steady biochemical models. The applicability of the methodology has been examined for various biochemical reactions, enzyme kinetics and multi enzyme systems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Modelling seasonal effects on the West Nile Virus Infection

Gustavo Cruz-pacheco
Institute of Applied Mathematics, National University of Mexico, Mexico

cruz@mym.iimas.unam.mx
Lourdes Esteva and Cristobal Vargas

West Nile Virus (WNV) Infection is an arboviral infection which is endemic in West Africa, West Asia and parts of Europe. In 1999 it was detected for the first time in North America and since then it has traveled rapidly across the continent causing mortality in humans, horses and birds, although only birds transmit the desease. In this talk we formulate and analize a mathematical model of this infection. We find the Basic Reproductive Number Ro in terms of measurable epidemiological and demographic parameters. Using experimental and field data we estimate Ro for several species of birds. Numerical simulations of the temporal course of the infection show that for some parameters new outbreaks can appear from the endemic state due to the coupling between the seasonal oscillations and the natural oscillations of the system through a mechanism of parametric resonance.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Pattern formation in age-structured populations

Caterina Cusulin
Department of Mathematics, University of Trento, Italy
cusulin@science.unitn.it

In this talk we discuss models for the spatial spread of a population, assuming that the diffusion depends on the age of individuals. We show that the presence of both age structure and spatial diffusion leads to the destabilization of spatial homogeneous steady states and consequently to spatial pattern formation, following Turing's reaction-diffusion mechanism.

In particular we consider a population divided into two separated classes, namely juveniles and adults and, under the hypoteses that juveniles and adults diffuse with different velocities, we look for conditions on the functions describing the dynamics of the population and on the diffusion coefficients that lead to diffusion driven instability and then to pattern formation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Modelling and simulating the aggregative behavior in phytoplankton cells

Nadjia El saadi
The National Institute of Planning and Statistics INPS, Algiers, Algeria
enadjia@yahoo.fr
Alassane Bah

This work deals with the use of an Individual-Based Model (IBM) to study the aggregation of phytoplankton cells. This model describes the movements of the cells and also their duplication and their death. The movements of phytoplankton cells are based on a system of It?? stochastic differential equations with drift coefficients responsible for attraction of cells and independent diffusion terms. A cell can divide or die according to a standard branching process with the same probability. We present some numerical simulations which show the influence of two parameters (the coefficient of diffusion and the maximal length under which a cell may detect another one by the mean of chemical processes) on the asymptotic spatial distribution of the phytoplankton. The analysis is made in two situations: with and without branching processes in order to quantify the role of spatial interactions on appearing of aggregation patterns.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Dynamics of a 3-species ecological system with delay effects

Wei Feng
UNCW, USA
fengw@uncw.edu
Xin Lu

In this paper, we study reaction-diffusion equations modeling the dynamics of a 3-species ecological system with delay effects. Various comparison arguments are used to determine the stability of the steady state solutions and the existence of a positive global attractor which ensures the long-tern co-existence of all species. The results in the time-delay system are compared with those in the system without delays, and the effects of delay terms are discussed. Numerical simulations of the models are also demonstrated.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Some features of the steady state of a SIR model with age structure and immigration.

Andrea M. Franceschetti
University of Trento, Italy
francesc@science.unitn.it

We consider a SIR model for an epidemic spreading in an age-structured population subject to immigration and at the demographic equilibrium thank to immigration. We show the existence of a threshold-like limiting behaviour for the steady states as the number of infective immigrants goes to zero characterized in terms of the spectral radius of a linear operator. Then we prove uniqueness for the

endemic equilibrium in case of a transmission coefficient given in form of a 2×2 WAIFW matrix.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Dynamics and Hopf Bifurcation Analysis In a White Blood cell Production Model

Talibi Alaoui Hamad
Université Chouaib Doukkali, Morocco
talibi@math.net
Abdelhadi Abta and Abdelilah Kaddar

We are interested in the dynamics of a Haematopoetic Stem cells model with two delays introduced by S. Bernard, J. Belair and M.C. Mackey in 2003 and 2004, having two stationary equilibrium . One of them is trivial, the second $E^*(\tau, r)$, depending on the delays, may be non-trivial . We show that if the rate of apoptosis γ and the factor of amplification A are close enough to zero, the stability of the non trivial state as well as the occurrence of a Hopf bifurcation depend only on the time delay τ and there exist a unique critical values τ_0 and $\overline{\tau}$ such that $E^*(\tau, r)$ is asymptotically stable for $\tau < \tau_0$ and unstable for $\tau_0 < \tau < \overline{\tau}$. The main result of this paper is to establish an explicit algorithm for determining direction of the Hopf bifurcation and the stability or instability of the bifurcating branch of periodic solutions, using the methods presented by O. Diekmann et al. (1995).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Model of neurotransmitters transport dynamics in axon terminal of presynaptic neuron

Piotr Kalita
Jagiellonian University, Poland
kalita@softlab.ii.uj.edu.pl
Andrzej Bielecki and Marian Lewandowski

In this paper certain processes taking place in a terminal button of presynaptic neuron are modelled. The model includes fast transport of neurotransmitter, slow transport of neuropeptide, their exocytosis as well as transport of calcium ions and influence of action potential on a cell membrane on above processes.

Starting from analysis of biological phenomena undergoing inside the terminal button and on the cell membrane we formulate the system of partial differential equations modelling the considered processes. Initial and boundary value problems for the model are stated as well.

The fast transmission is described by the linear parabolic equation whereas the system of equations for the slow transmission contains the nonlinear part which reflects the interaction between calcium ions and neuropeptide molecules. For the fast transmission we take into account the neurotransmitter syntesis, its diffusion in the vesicles and transport through the cell membrane triggered by the action potential. For the slow transmission we consider the endocytosis of calcium through the voltage gated ion channels, their diffusion and activation of neuropeptide molecules.

As the parameters of the model we assume, among others: areas, number and capacity of channels, distribution and capacitance of vesicle docking sites and diffusion parameters of the cytoplasm.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Propagating bursts in a model of the subthalamo-pallidal loop

Abdoul Kane
University of Toronto, Canada
kane.90@osu.edu

We use a biophysically constrained model to study the propagation of waves in the subthalamo-pallidal network, a neuronal pacemaker involved in the planning and control of movement. Within that network the membrane potential of subthalamic and pallidal cells display activity over many time scales, which makes it difficult to carry a straightforward computational or analytical investigation. By combining perturbative and asymptotic methods we describe a general technique for reducing such complex network equations to simpler yet biophysically relevant systems. We then use the reduced model to formulate conditions for structured self sustained propagation and compute the functional dependance of the propagation speed on network parameters.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Construction of Dengue Virus Force of Infection with Radial Basis Functions

Chulin Likasiri
Chiang Mai University, Thailand
julin.likasiri@gmail.com
Radom Pongvuthithum

Dengue fever is an important endemic disease in Thailand and other tropical countries in the world. It is transmitted to human by mosquitoes. Nevertheless, the density of mosquitoes is not the only main factor increasing the transmission rate of the dengue fever. The other factors including human host (age and immunity), weather and time are also main uncertainty causes having influence

in dengue prevalence. The relationship between Dengue virus infection and its factors and parameters are very complicated. One of the most important parameters is the force of infection, λ, which has been widely studied. However, it has never been fully regarded as a function. Using basic model for single strain with age and time effects, this study develops a systematic way to approximate the force of infection, λ, by Radial Basis Functions (RBF). This will lead to a function for the basic reproduction number, R_0, an essential number to understand the behavior of the disease. Data gathered from Tak province, Thailand, from the year 2000 to 2004 reveal the epidemiological pattern for the disease. The comparison between the proposed force of infection function and the real data has been done in order to evaluate the proposed method.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Coexistence of solutions of chemotactic diffusion systems on Food Chain in a Flow Reactor

Xiaodong Liu
Dalian Maritime University, Peoples Rep of China
xdliuros@hotmail.com
Jing Liu and Yanping Lin

A reaction-diffusion system modelling single food-chain of m predator-prey micro-organisms populations in mixed culture contained in a flow reactor is considered. We investigate mathematically the effects of diffusivity and chemotaxis on the ability of existence for predator-prey micro-organisms populations. We will use the of *a prior estimates* obtaining a priori bounds of the population u_i in the uniform norm for m populations food-chain model and the index theory to establish sufficient conditions for coexistence of the predator-prey populations for two populations food-chain model without the uniqueness assumptions on semi-trivial solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Dynamics of a Ratio-Dependent Predator-Prey Model with Harvesting of predators

Priscilla S. Macansantos
Univ. of the Philippines, Philippines
xdliuros@hotmail.com

We consider the dynamical properties of the ratio-dependent model with constant rate harvesting of predators. We take off from an earlier paper by Xiao and Jennings, and consider bifurcations occuring for the model.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Using Worldwide ReefCheck Monitoring Data to Develop Coran Reef Index of Biological Integrity

Hai yen Nguyen
University of Yamanashi , Japan
haiyen@ccn.yamanashi.ac.jp
Kou Ikejima, Ole Pedersen, Kengo Sunada and Satoru Oishi

The Coral Reef Index of Biological Integrity (CR-IBI) were constituted based on ReefCheck monitoring data and bioindicators' ecological functions on reefs. 76 "reference" and 72 "degraded" sites in shallow water and 39 "reference" and 37 "degraded" sites in deep water were classified based on the criteria: more than 35

$\longrightarrow \infty \diamond \infty \longleftarrow$

Role of nutrient and toxin producing phytoplankton in marine ecosystem: a mathematical study supported by experimental findings

Samares Pal
Ramakrishna Mission Vivekananda Centenary College, India
samaresp@yahoo.co.in

Plankton is the basis of all aquatic food chain. Phytoplankton, in particular, occupies the first trophic level. Plankton do huge services for our earth, food for marine life, oxygen for human life and also absorbs half of the carbon-di oxide from the earth atmosphere. In this paper nutrient, toxin producing phytoplankton and zooplankton (Nutrient-TPP-Zooplankton) system is proposed and analyzed. It is assumed that the grazing pressure of zooplankton reduce due to presence of toxin producing phytoplankton. Our analysis leads to different thresholds which are expressible in terms of model parameters and determine the existence and stability of various states of the system. The analytical findings will be verified through our field observations which were carried on the eastern part of Bay of Bengal for the last three years.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Adaptive dynamics of a function-valued trait describing the transition in a sequential hermaphrodite population

Jordi Ripoll
Universita degli Studi di Trento, Italy
jripoll@science.unitn.it

Considering phenotypic evolution in the context of

diploid population models, we study the evolutionary/adaptive dynamics of the age at sex-reversal in a sequential hermaphrodite population. This critical age can be considered as a random variable with a given probability law (function-valued evolutionary trait). We assume a resident population at stable equilibrium and consider a small invading/mutant population made up of heterozygotes and homozygotes. We have used convex analysis in order to show that an unbeatable strategy or evolutionarily stable strategy (ESS; Maynard-Smith 1976) is a Heaviside step function, i.e. all individuals of the population change sex at the same age. More precisely, the computation of such an (infinite dimensional) strategy is based on linear/affine optimization on compact convex sets. Our result is a generalization of the one obtained by E.L. Charnov in the sense that the trait is more general. We have found that the adaptive value of the sex-ratio (proportion between females and males) of the population at equilibrium is in general different from one. However, if fertilities are age independent then an equal sex-ratio occurs. In addition, assuming also an age independent mortality, one has that the transition from female to male takes place at 69.3% of the life expectancy of the population. Our analytical computations are in accordance with recent empirical studies.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Impact of environmental stressors on population dynamics: mathematical modeling.

Irina Tikhonova
University of California at Davis, USA
itikhonova@ucdavis.edu
Frank Loge and Tim Ginn

Applied population dynamics is relied upon to answer questions about how human activity affects non-human populations with increasing frequency. Our analysis focuses on chronic population health effects due to life history of exposure to anthropogenic stressors. We propose a new category of population dynamics, termed dose-structured population dynamics, incorporating effective dose of stressor chemical as a structural variable in predator-prey and disease transmission models.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Suppression of spatio-temporal chaos in the model of fibrillation in an excitable medium

Semen Vysotskiy
Moscow State University - Physics faculty, Russia
saimon@polly.phys.msu.ru
A. Loskutov and R. Cheremin

We study the model of excitable tissue, that describes the development of fibrillation (spatio-temporal chaos) via multiplication and coexistence of a large number of spiral waves. We show that certain weak point external forcing on such a system is able to eliminate all spiral waves and thus stabilize system dynamics. After that the only excitation source left is the external pacemaker. We find the frequences and amplitudes on which such stabilization is possible for different impulse profiles. Also the dependence of the suppression efficiency on the external pacemakers number and the distance between them is studied. For the system dynamics analysis we use the algorithm proposed by Bray et. al. - the count of spiral waves in the medium as a function of time.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stability and dynamics in the nonlinear discrete-time model of competition between two age-stage structured populations

Nikolay N. Zavalishin
A. M. Obukhov Institute of atmospheric physics RAS, Russia
nickolos@ifaran.ru
D.O. Logofet and N.G. Ulanova

Populations of raspberry (Rubus idaeus) and aspen (Populus tremula) often occupying felled forest areas can be described by a set of age-stage groups of individuals differing both in the ontogenetic stage and chronological age. For each population a linear discrete time matrix model can be designed and calibrated on the basis of two reproductive phases representation: seed-origin and vegetative. During first years after felling shrubs of the seed-origin type determines the population dynamics which demonstrates an exponential growth reflected by the linear model. Further succession is connected mainly with a vegetative type, and such strong factors as competition for natural resources with other species start play a leading role that can be simulated only by means of nonlinear model. Strongly aggregated minimal model available for mathematical analysis is designed on the basis of modifying the linear one by multiplying each of its equations on nonlinear term reflecting the competition pressure of aggregated groups from one population on recessive groups of the other. For equilibria stability and permanence conditions are obtained together with cyclic and chaotic dynamics. Since plant populations are usually inertial, conditions of absence for chaos are considered. Calibration of the aggregated system by real data from field measurements as well as its aggregation to the min-

imal one initiates additional mathematical optimization problems. This work is supported by the grant 05-04-49291 of the Russian Foundation for Basic Research.

$\longrightarrow \infty \diamond \infty \longleftarrow$