Contributed Session 9: PDEs and Applications

Spiral wave patterns in the complex Ginzburg-Landau equation

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The complex Ginzburg-Landau (CGL) equation is the amplitude equation that describes many excitable systems in the vicinity of a Hopf bifurcation. The general form of this equation is given by

$$u_t = u - (1+ia)u|u|^2 + (1+ib)\nabla^2 u \tag{1}$$

where $a, b \in \mathbb{R}$, and $u \in \mathbb{C}$.

It is well known that this equation admits plane wave solutions as well as what are known as two-dimensional spiral waves or vortices. These are coherent structures that are characterized by having a non-zero degree or winding number. In this work we deal with a specific type of solutions of the CGL equation in two spatial dimensions comprising multiple spiral waves with well separated centers.

The aim of the work to be presented here is to describe the pattern dynamics through a law of motion of the centers of the spirals. We use singular perturbation methods to show that the interaction between spirals changes not only in intensity, but also in direction, as the spirals separate. We relate the distance between the centers of spirals with the parameters involved in the equation, and we use this concept to determine the natural separation at which bound states are reached. We also show that for large distances the interaction becomes attractive, but exponentially small. Finally, we show that the frequency of each spiral is related to the separation of the spirals, and therefore it changes as they move. We give an asymptotic expression for this frequency.

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Determination of Thermophysical properties in 2D Nonstationay Heat Problems

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The nonstationary two-dimentional heat conduction problems in a cylindrical domains will be considered and solved in a way so that we can determine the thermophysical properties of the solid(such as: conductivity, diffusivity, capacity, activity)

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Sumudu Transform Applications to the Cosner Conjecture

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In this paper we show how to use the Sumudu Transform, whose deeper properties were extensively studied recently by the author and others, to shed more light on the outcome of the Cosner Conjecture. This conjecture announced first in 1994, and partially solved by the Author, and Cosner - Lou, establishes the control effect of drift on the dispersal and survival of populations in habitats subject the no-flux conditions. Being unit and scale preserving, the Sumudu transform is used to establish new insights, and confirm previous results surrounding the conjecture.

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Fractional Fourier Transform of Tempered Distribution

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In this Paper, we introduced the fractional Fourier transform as an extension of the ordinary Fourier transform and extended it to the Tempered distributions using Adjoint method. The fractional Fourier analysis is used for investigation of fractal structures; which in turn are used to analyze different physical phenomena. For instance, it has got the applications in optical engineering, in quantum mechanics and intensity distribution in optics and signal processing. as has been discussed by Alieva and Barbe [3]. It is proved that the fractional FT and its inverse, is a continuous isomorphism from S (IRn) onto S (IRn). Obtained the convolution theorem and in the end illustrated the application of fractional FT in solving Convolution equations and Initial value problems.

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Semi-classical states for nonlinear Schrödinger equations with potentials vanishing at infinity

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We discuss the existence of positive bound states for a class of nonlinear Schrödinger equations with potentials vanishing at infinity. In the semi-classical limit, those solutions concentrate around a local minimum of the potential.

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On a free boundary problem

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We will be concerned with the existence of positive solutions for an elliptic problem with discontinuous nonlinearity. The problem is written in an equivalent integral equation. The shape of the free boundary is studied.

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Numerical Stability of Solitarywave-like Solutions in a Two Layer Fluid over a Bump

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We study the layer incomressible fluid passing over a bump. When the theory of KdV equation fails, a forced modified KdV equation is derived. We find four different types of symmetric solitarywave-like solutions of time independent forced modified KdV equation and study the numerical stability of the solutions of the solutions as time evolutes.

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The Effect of An Inert Material on the Stability of Propagating Polymer Fronts

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Frontal Polymerization (FP) is the chemical process whereby monomer is converted to polymer via a localized reaction zone. The front propagates via the coupling of thermal diffusion and exothermic Arrhenius reaction kinetics. The interest in FP stems from its untapped potential for the production of novel materials. We utilize a sandwich-type two-layer model to investigate the stability of propagating one-dimensional polymerization fronts in the presence of an inert material. One layer is reactive and supports a frontal polymerization reaction, while the second is inert. Heat exchange is allowed between layers, and as a result of this, the inert layer may significantly affect the propagation of the polymerization wave. An analysis is carried out which demonstrates the possibility of muliple propagating fronts in the system. We perform a linear stability analysis of the propagating fronts, and determine the conditions necessary for the onset of oscillatory modes in the system.

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Numerical Simulations of FitzHugh-Nagumo Equations in Two-dimensional Heterogeneous Medium

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The FHN system with initial conditions and Dirichlet and no-flux boundary conditions in two-dimensional space is studied numerically. The parameters of the system vary in the space (hetergeneity). For this purpose we developed an GUI (graphical user interface), written in Matlab, which allows a user to change all model parameters easily using slides and a computer mouse clicking. We solve initial-boundary problem for the FHN system numerically using an algorithm that automatically adjusts the time step to achieve an efficient simulation while controlling the error in the solution. We use the moving-grid interface method and its FORTRAN implementation which is a generalization of the method of lines . The Matlab engine operates by running in the background as a separate process from the FORTRAN simulation programs. All data is sent from the GUI to the FORTRAN simulation programs through the Matlab engine . The current simulation incorporated Matlab commands in a driver Fortan program. A user can change the model parameters and functions on any stage of calculations. It is shown that numerical simulations of FHN system in heterogeneous medium exhibit a variety of different types of solutions such as spiral, target patterns and self-sustained waves.

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On some stochastic fractional integro-differential equations

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The purpose of this paper is to study the integro-partial differential equation of fractional order:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \sum_{|q|\le 2m} a_q(x) D^q u(x,t) = f_1(u(x,t)) + \int_0^t f_2(u(x,s))dW(s),$$

with the nonlocal condition

$$u(x,0) = \varphi(x) + \sum_{k=1}^{p} c_k u(x,t_k),$$

where $\quad 0 \le t_1 < t_2 < ... < t_p$,
x is an element of the n-dimensional Euclidean space $R^n, D^q = D_1^{q_1} ... D_n^{q_n}$,

$$D_j = \frac{\partial}{\partial x_j}, 0 < \alpha \le 1,$$

$q = (q_1, ..., q_n)$ is an n dimensional multi-index, $|q| = q_1 + ... + q_n$ and $W(t)$ is standard Wiener process over the filtered probability space Ω, F, F_t, P.
It is supposed that $\sum_{|q|=2m} a_q(x) D^q$ is uniformly elliptic on R^n . The existence of solutions of the considered Cauchy problem and some properties are studied under suitable conditions on φ , the constants $c_1, ..., c_p$, the functions a_q, f_1 and f_2.
Keywords: Nonlocal initial condition, stochastic integro-partial differential equations, fractional order.
AMS subject classifications: 34K30, 34K05, 26A33, 35A05.

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Mathematical analysis of the peridynamic model in non-local elasticity theory

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This talk addresses the well-posedness of the linearised peridynamic model which is governed by a partial integro-differential equation. The so-called peridynamic modelling is a non-local elasticity theory that is free of any spatial derivative but relies upon differences of the displacement. It accounts for effects of long-range interactions and has recently been applied to problems in which discontinuities such as fracture or damaging arise. We also discuss the convergence towards the classical Navier-Lamé equation as the parameter describing the non-locality tends to zero.

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Positive solutions of the fully nonlinear cooperative system of parabolic equations with Dirichet boundary conditions.

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We will analyze positive solutions of the fully nonlinear cooperative system of parabolic equations with Dirichet boundary conditions. Namely we will explore how the symmetry properties of the domain and equation influence the shape of the functions in the ω-limit set. We will also discuss the main differences in the proof between the system of equations and the problem for the single equation. The main tools will be maximum principle, Harnack inequality and method of moving hyperplanes.

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A survey of mathematical models for fixed bed adsorption of gases

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We present different models arising in chemical engineering and related to gas chromatography. These models describe an isotherm adsorption process of separation of a gaseous mixture: each component can exist either in a mobile phase or a solid (and static) one, with a finite or instantaneous exchange kinetic. Many authors, in the fields of chemical engineering and mathematics, have investigated these models under various assumptions, from a theoretical or numerical point of view.

We explain first the relations between these approaches. Next, we present some new results in the case of a monovariant system with one or two active components for the Cauchy problem using the Godunov type scheme: existence, uniqueness in the piecewise-smooth class of functions. Lastly, we give some open problems (relaxation, ...)

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The thermistor problem with degenerate thermal conductivity and metallic conduction

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The heat produced by an electrical current passing through a conductor device is governed by the so–called thermistor problem. This problem consists in a coupled system of parabolic–elliptic equations, whose unknowns are the temperature inside the conductor and the electrical potential.

The aim of this work is to establish the existence of a capacity solution to the thermistor problem supposing that the thermal and the electrical conductivities are not bounded below by a positive constant value. Furthermore, the thermal conductivity vanishes at points where the temperature is null. These assumptions on data include the case of practical interest of the Wiedemann–Franz law with metallic conduction and lead us to very complex mathematical situations.

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Fast diffusion equations on negatively curved manifolds

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We first discuss asymptotics of solutions to the fast diffusion equation $\dot{u} = \triangle u^m$ ($m \in (0,1)$) on manifolds M of nonpositive sectional curvature. Two different behaviours may occur, depending on whether the minimum of the L^2–spectrum of $-\triangle_M$ is zero or strictly positive, the second case occurring when (sectional curvature)$\leq K < 0$ ($\triangle$ is the Laplace–Beltrami operator). In the first case the asymptotics are similar to what happens in $\mathbb{R}^n$, while in the second any solution corresponding to L^q initial data vanishes in finite time.

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Traveling front solutions in nonlinear diffusion degenerate Fisher-KPP and Nagumo equations via Conley index

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Existence of one dimensional traveling wave solutions $u(x,t) := \phi(x-ct)$ at the stationary equilibriums, for the nonlinear degenerate reaction-diffusion equation $u_t = [K(u)\,u_x]_x + F(u)$ is studied, where K is the density coefficient and F is the reactive part. We use the Conley index theory to show that there is a traveling front solutions connecting the critical points of the reaction diffusion equations. We consider the nonlinear degenerate generalized Fisher-KPP and Nagumo equations.

Key words: Traveling waves, degenerate reaction-diffusion, Conley index, connected simple systems

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Scalar conservation law with discontinuous flux in a bounded domain

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We consider the Dirichlet problem for a first order hyperbolic equation with a convection term discontinuous with respect to the space variable. So, let Ω be a unidimensional bounded domain and T a positive real, the problem can be read as follows: find a measurable function u on $Q =]0,T[\times\Omega$ such that:

$$\begin{aligned}\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(k(x)g(u)) = {} & 0 && \text{in } Q,\\ u(0,.) = {} & u_0 && \text{on } \Omega,\\ u = {} & 0 && \text{on }]0,T[\times\partial\Omega.\end{aligned}$$

We suppose the function k is bounded, discontinuous at a point x_0 of Ω and has bounded variations.

We introduce a definition of a weak entropy solution to the above problem and then we prove existence and uniqueness of the entropy solution for a class of functions g. Existence property is obtained by regularization of the function k. For the uniqueness result we use the method of doubling variables and a Rankine-Hugoniot condition along the line of discontinuity.

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Asymptotic behavior for small width of interface in phase transitions

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In this paper we consider a generalization of the semilinear phase field model from [AJR], for a more general entalphy function.

More interesting, this generalization allows to study more general couplings between a diffusion field and a phase-field. For instance, the phase-field can be seen as the density of bacterial collony or the mass of growing tumor.

In this work, we study the asymptotic behaviour of the solutions of this phase model, when the "width of interface", $\xi \mapsto 0$, which is known as thin-interface limit. We will show that the solutions associated to the initial conditions "closed to" the structure of phase transitions,preserve this structure for a large finite time which tends to ∞ as ξ goes to zero.

[AJR] A. Jiménez-Casas, A. Rodriguez-Bernal, "Linear stabilility analysis and metastable solutions for a phase-field model," Proceeding of the Royal Society of Edimburgh, 129A, 571-600, (1999).

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Sturm-Liouville operators with indefinite weights and parabolic equations

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Consider a non-self-adjoint differential operator of form

$$(Ay)(x) = \frac{1}{r(x)}(Ly)(x) \qquad (1)$$

and the corresponding forward-backward parabolic equation

$$r(x)\frac{\partial u(x,t)}{\partial t} = -Lu(x,t), \quad x \in \mathbb{R}, \quad 0 < t < +\infty .$$

Here the weight function $r(x)$ changes sign, $L = -d^2/dx^2 + q(x) = L^*$ is a Sturm-Liouville operator in $L^2(\mathbb{R}, dx)$.

Such problems arise in transport theory, statistical physics and hydrodynamics. They leads to the similarity problem for the non-self-adjoint operator $\frac{1}{r(x)}L$ with an indefinite weight $r(x)$. Operators with a discrete spectrum have been considered by Beals, Kaper *et al* and Pyatkov. For definitisable operators Ćurgus and Langer have developed another method.

We present the new approach to spectral analysis of the operator A, which allows to remove assumptions mentioned above. It is based on Naboko-Malamud similarity criterion. Several effective conditions of similarity of A to a self-adjoint operator are obtained. We construct an example of a nondefinitizable operator of type (1) that is similar to a normal operator. Eigenvalues in the essential spectrum of A are studied. Geometric and algebraic multiplicities of eigenvalues are given.

As an application we prove that if L is a nonnegative Sturm-Liouville operator with a finite-zone or fast-decreasing potential and $r(x) = \operatorname{sgn} x$, then the corresponding forward-backward parabolic problems have a unique solution.

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Exact Solution of an Axisymmetric Deformation of a Double-Layered Elastic Cylinder in AxialCompression

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The problem of an axisymmetric deformation of a double-layered isotropic elastic cylinder of finite length is solved. The cylinder whose lateral surface is delimited by rigidly wall is subjected to an axial compression.

Using a finite Hankel integral transforms of a displacement vector according to a radial variable we reduce the coupled Lam?? equilibrium system to an ordinary differential one. The transformed displacements are calculated by a characteristic polynomial method after extracting a fourth ordinary differential equation of the axial displacement.

The stresses and the displacements in the elastic cylinder are then given in a closed form by the Hankel inverse transformation and the Hooke law. Some graphs are also given for different elastic materials.

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Finite Time Blow-up For The Nonlocal Gelfand Problem

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We study the unbounded solutions for the nonlocal Gelfand problem $u_t = \Delta u + \lambda \frac{e^u}{\int_\Omega e^u dx}$ with initial and boundary conditions for $\lambda \geq \lambda^*$, where λ^* is the critical value of parameter λ, above which the steady-state problem has no solution.

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Coupling of Scalar Conservation Laws in Stratified Porous Media

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We carry out the mathematical analysis of a quasilinear parabolic-hyperbolic problem in a multidimensional bounded domain Ω. In a region Ω_p a diffusion-advection-reaction type equation is set while in the complementary $\Omega_h \equiv \Omega\backslash\Omega_p$, only advection-reaction terms are taken into account. So the problem in hands may be read as follows for any positive and finite T: find a measurable and bounded function u on $]0,T[\times\Omega$ such that

$$\begin{cases} \partial_t u + \sum_{i=1}^n \partial_{x_i}(G_h(u)B_{i,h}) = 0 \text{ in }]0,T[\times\Omega_h, \\ \partial_t u + \sum_{i=1}^n \partial_{x_i}(G_p(u)B_{i,p}) = \Delta\phi(u) \text{ in }]0,T[\times\Omega_p, \\ u(0,.) = u_0 \text{ and } u = 0 \text{ on }]0,T[\times\partial\Omega, \end{cases}$$

associated with *suitable* transmissions conditions along the interface $\Gamma_{hp} = \partial\Omega_p \cap \partial\Omega_h$, supposed to be an hyperplane of $\mathbb{R}^n$ of the form

$$\Gamma_{hp} = \{(x_1,..,x_n) \in \Omega, x_{i_0} = \textit{constant} \text{ for a fixed } i_0 \text{ of } \{1,..,n\}\}.$$

We assume that ϕ is an increasing function, G_p and G_h are Lipschitz continuous on $\mathbb{R}$ such that G_h **is nondecreasing**. Besides $B_{i,p}$ and $B_{i,h}$ for $i \in \{1,..,n\}$ are $W^{1,+\infty}$-class functions respectively on Ω_p and Ω_h and such that

$$\Gamma_{hp} \subset \{\sigma \in \Sigma_{hp}, 0 \leq \mathbf{B}_h(\sigma).\nu_h\}.$$

We start by providing the definition of a weak solution through an entropy inequality on the whole domain. Since Γ_{hp} contains the outward characteristics for the first-order operator set in Ω_h, the uniqueness proof begins by focusing first on the layer Ω_h, and then on the layer Ω_p. The existence property uses a discontinuous vanishing viscosity method in accordance with the layer. On the hyperbolic zone, we pass to the limit in L^∞-weak $\star$, by referring to the notion of an entropy process solution.

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Blowup in a shadow system

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Show the existence of blow-up solutions in a shadow system of a nonlinear parabolic system.

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Oscillation of nonlinear impulsive hyperbolic equations with several delays

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In this paper, oscillatory properties of solutions for certain nonlinear impulsive hyperbolic equations with several delays are investigated and a series of new sufficient conditions and a necessary and sufficient condition for oscillation of the solutions are established. The equation here we discuss is nonlinear and the boundary condition is also nonlinear. The results fully indicate that the oscillation are caused by delays,impulse. And hence reveal the varied difference between these equations and those without delay and impulse.

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Generalization of Lions Theorems for First-Order Differential-Operator Equations with Variable Domains of Operator Coefficients

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It is proved the existence and uniqueness theorem for the weak solutions of Cauchy problem

$$du(t)/dt + A(t)u(t) = f(t),\ t \in]0,T[;\ u(0) = u_0.$$

1. Here the linear unbounded operators $A(t)$ in a Hilbert space H with t - dependent domains $D(A(t))$ and their adjoint operators $A^*(t)$ with domains $D(A^*(t))$ satisfy

$$[u]^2_{(t)} \equiv Re(A(t)u + c_0 u, u)_H \geq c_1 |u|^2_H \ \forall\ u \in D(A(t)),$$

$$Re(A^*(t)v + c_0 v, v)_H \geq c_1 |v|^2_H$$

$\forall\ v \in D(A^*(t)),\ c_0 \geq 0,\ c_1 > 0.$
2. For the bounded inverses $A_0^{-1}(t) \in L_\infty(]0,T[,\mathfrak{L}(H))$ of the operators $A(t) + c_0 I$

$$c_0 |(A_0^{-1}(t)g,h)_H| \ \leq\ c_2 [A_0^{-1}(t)g]_{(t)} |h|_H$$

$\forall\ g,\ h \in H,\ c_2 \geq 0.$
3. The $A_0^{-1}(t)$ have the weak derivative $dA_0^{-1}(t)/dt \in L_\infty(]0,T[,\mathfrak{L}(H))$ in H and

$$|((dA_0^{-1}(t)/dt)g,h)_H| \ \leq\ c_3 [A_0^{-1}(t)g]_{(t)} |h|_H$$

$\forall\ g,\ h \in H,\ c_3 \geq 0.$

For the case of maximal accretive $A(t)$ this theorem generalizes theorem 1.1 of [1, P. 129]. Using the new theorem, the correct solvability of the mixed problems for the equations

$$\partial u(t,x)/\partial t + \sum_{|\alpha|\le 2m+1} a_\alpha(t,x) D_x^\alpha u(t,x) = f(t,x),\ t\in]0,T[,\ x\in\Omega,$$

with the initial condition $u(0,x) = u_0(x)$, $x\in\Omega$, and boundary conditions

$$\frac{\partial^j u(t,x')}{\partial\nu^j} - \sum_{i\in J_{-m}}^{i<j} a_{i,j}(t)\frac{\partial^i u(t,x')}{\partial\nu^i} = 0,$$
$$t\in[0,T],\ x'\in S,\ \ j\in J_m;$$
$$\frac{\partial^j u(t,x')}{\partial\nu^j} - \sum_{i\in J_{-m}}^{i<j} a_{i,j}(t)\frac{\partial^i u(t,x')}{\partial\nu^i} = 0,$$
$$t\in[0,T],\ x'\in S_k^-,\ k=\overline{1,n},\ j\in J_m^-,\ m=0,\ 1,\ldots,$$

is established. Here $J_m = \{j_s\in[0,\ldots,2m] : s=\overline{1,q}\}$, $J_m^- = \{j_s\in([0,\ldots,2m]\backslash J_m) : s = \overline{q+1,m+1}\}$, $J_{-m} = [0,\ldots,2m]\backslash(J_m\cup J_m^-)$ and S_k^- are sets of all points $x'\in S$ with negative cosines of the angles between an external normal ν to the boundary S of $\Omega\subset\mathbb{R}^n$ and axes Ox_k, $k=\overline{1,n}$. For the case of odd-order partial differential operators $A(t)$ the last result generalizes theorem 6.1 of [1, P. 142].

References:

[1] Lions J.-L. Equations differentielles operationnelles et problemes aux limites. Berlin, 1961.

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On a Class of Elliptic Free Boundary Problems

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We consider a class of elliptic free boundary problems of type $div(a(x)\nabla u) = -(\chi(u)h(x))_{x_n}$ in a bounded domain Ω of $\mathbb{R}^n$, where $n\ge 3$, $a(x)$ is an elliptic $n\times n$ matrix, and h is a non-negative function satisfying $h_{x_n}\ge 0$. Under suitable conditions, we prove that the free boundary $\partial[u>0]\cap\Omega$ is represented by a continuous function $x_n = \Phi(x_1,\ldots,x_{n-1})$.

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Characteristics method for a transient viscoelastic flow of Oldroyd model

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We formulate and analyze a characteristics finite element approximation of a class of flows in transient viscoelastic fluids described by the Oldroyd model. We propose a characteristics-based method to treat the transport part of the equations. The stress, velocity and pressure approximations are P_1 discontinuous, P_2 continuous and P_1 continuous finite element, respectively.
By assuming that the continuous problem admits a sufficiently smooth and sufficiently small solution, and using a fixed point method, we show existence of solution to the approximate problem. We also give an error bound for the numerical solution.

References:

[1] A. Machmoum, D. Esselaoui. *Finite element approximation of viscoelastic fluid flow using characteristics method*, Comput. Methods Appl. Mech. Engrg. 190 (2001) 5603-5618.
[1] J. Baranger, S. Wardi. *Numerical analysis of finite element method for a transient viscoelastic flow*, Comput. Methods Appl. Mech. Engrg. 125, 171-185, (1995).

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Finite speed of propagation in degenerate reaction-diffusion-convection processes

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The talk deals with the non-linear Fisher-KPP reaction-diffusion-convection equation

$$u_t + h(u)u_x = (D(u)u_x)_x + g(u), \qquad t\ge 0,\ x\in R$$

which models various biological phenomena; in particular population migration and dispersal processes. The discussion is mainly centered on the existence of front-type solutions which play a relevant role in describing the asymptotic behavior of general classes of this dynamic. We show the appearance of new types of sharp profiles due to the contemporary presence of a doubly-degenerate diffusivity $D(u)$ and of a convective effect $h(u)$ and we provide their classification. The results enables to discuss the property of finite speed of propagation of this model. An application is also presented, concerning the evolution of a bacterial colony. Finally, we prove the existence of similar sharp profiles in the case when $D(u)$ is an aggregative term, describing the behavior of a population which reacts against a menace of extinction.

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On Pfaff systems with L^p coefficients

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Pfaff systems arise in some classical problems of differential geometry such as the isometric immersions of a Riemannian space or the fundamental theorem of surface theory. Under classical assumptions on the regularity of the coefficients, local existence of a nontrivial solution is granted provided that the coefficients of the system satisfy some compatibility conditions. Recent developments in domains like mathematical elasticity lead us to generalize these results in the case of Sobolev spaces. While the Pfaff systems are linear, the compatibility conditions are nonlinear and this makes difficult an approach of the problem by regularization.

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Global attractor for a lattice dynamical system without uniqueness

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A great number of processes coming from Physics,Chemistry, Biology, Economics and other sciences can be described by reaction-diffusion equations. In this paper we study the asymptotic behaviour of a lattice dynamical system obtained from the discretization of a reaction-diffusion equation in an unbounded domain. The study of such systems is thus important due to the necessity of numerical resolution of PDE by physicist and engineers. In particular, the discretizated system that we study has the form

$$\left\{ \begin{array}{c} \dot{u}_i = \frac{a}{h^2}\left(u_{i-1} - 2u_i + u_{i+1}\right) - f_i\left(u\right), \\ u\left(0\right) = u_0 \in l^2, \end{array} \right. \tag{2}$$

where $a, h > 0$, $f_i : [0,T] \times \mathbb{R} \to \mathbb{R}$ are continuous functions for every i, and we assume de followings conditions:

(H1) $f_i\left(s\right)s \geq \alpha\left|s\right|^2 - c_{0,i},\ c_0 \in l^1, \alpha > 0.$

(H2) $\left|f_i\left(u_i\right)\right| \leq C\left(\left|u_i\right|\right)\left|u_i\right| + c_{2,i}$, where $c_{2,i} \in l^2$ and $C\left(\cdot\right)$ is a continuous and increasing function.

This discretization comes from the problem

$$\left\{ \begin{array}{c} \dfrac{\partial u}{\partial t} = a\Delta u - f(x,u), x \in \mathbb{R}^N, t > 0, \\ u(0) = u_0 \in L^2\left(\mathbb{R}^N\right), \end{array} \right.$$

which was studied in [1] from the point of view of global attractors.

The main result that we have obtained in this work is the existence of a compact global attractor for the lattice system (/[1]). Comparing this work with preceding results, our conditions on the functions f_i are weaker, since we do not assume that they are Lipschitz. Hence, the unicity of solutions of the Cauchy problem is lost. To avoid this problem we define a multivalued semiflow instead of a semigroup of operators and use the appropiate theory of global attractors. Finally, we prove the upper semicontinuity of the attractor with respect to the attractors obtained by the finite-dimensional approximations of the system.

These results can be extended easily to the case of the discretization of a system of reaction-diffusion equations (instead of a scalar one). Then, one of the applications in which we can use this model is the Fitz–Hugh–Nagumo equation, a well known model of transmission of signal across axons.

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Weak solutions to a Stefan problem

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I will investigate the quasistationary Stefan problem with Gibbs-Thomson correction. The main subject will be the issue of existence of unique weak solutions for arbitrary initial surface. The system is equivalent to a parabolic problem of the third order, hence a suitable approach is required to obtain the optimal result from the regularity point of view. The considerations will be done in the L_p-spaces. And the initial surface is required to belong to the $W_p^{2-3/p}$-class, only.

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Estimating Heat Source in Two-Dimensional Problem

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This paper considers, a two-dimensional inverse heat conduction problem. The direct problem will be solved by an application of the heat fundamental solution, and the heat source to be estimated by using least square method.

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On positive solutions for a certain class of elliptic BVPs

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We discuss the existence of solutions for the BVP problem associated with a certain class of elliptic PDEs and their continuous dependence on functional parameters. Since we shall propose an approach based on variational methods, we treat our equation as the Euler-Lagrange equation for a certain integral functional J. We develop a duality theory which relates the infimum on a special set X of the energy functional associated with the problem, to the infimum of the dual functional on a corresponding set X^d. The links between minimizers of both functionals give a variational principle and, in consequence, their relation to our boundary value problem. Our approach also enables a numerical characterization of solutions of our problem and give, also in the superlinear case, a measure of a duality gap between the primal and dual functional for minimizing sequences.

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An Ocean Turbulence Model with a Fairly General Seabottom

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We consider the hydrostatic approximation of the Navier-Stokes equations. This is a general model arising in oceanography for the description of the circulation of water in oceans and lakes. In this model, the vertical dimension of the domain (maximum depth of a large portion of the ocean or lake) is very small in front of its horizontal dimensions. Usually, the domain Ω is described through a depth function $D\colon \bar{\omega} \subset \mathbb{R}^2 \mapsto \mathbb{R}$, where ω represents the sea surface, in the following way $\Omega = \{(x,y,z) \in \mathbb{R}^3 \,/\, (x,y) \in \omega,\, -D(x,y) < z < 0\}$.

We show how we must reformulate the hydrostatic approximation with an arbitrary seabottom, that is, where Ω cannot be described through a depth function.

Finally, we analyze a one-equation turbulence model based on this new formulation and give an existence result.

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On a 2D free–boundary problem modelling the action of a limiter in the magnetic confinement of a plasma in a Stellarator

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One of the main difficulties of the magnetically controlled plasma fusion (in axisymmetric geometric devices as Tokamaks or non axisymmetric geometric ones as Stellarators), is to determinate the conditions on the magnetic field and on the current density in order to maintain the plasma far from the camera walls. A way to prevent mechanically this is to introduce a *limiter*: a solid object which determines the boundary of the plasma since it plays the role of a *thin obstacle* for it. The influence of limiters on plasma confinement has been investigated, from the experimental point of view, in the TJ-II Stellarator (CIEMAT, Madrid) and some evidences have been found about how they improve the confinement [Hidalgo2004].

We consider the associated theoretical aspects by studying a 2D free–boundary problem arising in the magnetic confinement of a plasma in a Stellarator device. Inspired in [Díaz-Padial-Rakotoson1998], our model can be expressed an elliptic *inverse thin obstacle problem.* The action of the limiter is modeled by the multivalued maximal monotone graph β and thus there is a double free-boundary: the boundary of the plasma and the part of it which is in contact with the limiter.

We prove the existence of a solution by means of a Galerkin argument for a new family of elliptic problems associated to an equivalent *direct* (but *non-local*) formulation of problem.

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Limit ODE and invariant manifolds in a nonlinear wave equation

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In this work we consider the following nonlinear wave

equation as a model for a controlled spring-mass-damper system:

$$\begin{cases} u_{tt}(x,t) - u_{xx}(x,t) - \alpha u_{txx}(x,t) + \\ \qquad \varepsilon f\left(u(1,t), \frac{u_t(1,t)}{\sqrt{\varepsilon}}\right) = 0 \\ u(0,t) = 0 \\ u_{tt}(1,t) = -\varepsilon \left[u_x + \alpha u_{tx} + r u_t\right](1,t) - \\ \qquad \varepsilon f\left(u(1,t), \frac{u_t(1,t)}{\sqrt{\varepsilon}}\right) \end{cases}$$

where $u = u(x,t)$, $x \in (0,1)$, $t > 0$ and the parameters $\alpha, r > 0$ and $\varepsilon \geq 0$. This is a strongly damped wave equation with dynamical boundary conditions and a nonlocal nonlinearity f. This nonlinearity can be thought as an imposed control acceleration to the system.

Inspired in the results obtained for the lineal model ($f = 0$) when ε is small, our objective is now to see in a rigorous way that the PDE model admits a nonlinear ODE as a limit when $t \to \infty$ in a certain sense. This will be done using the theory of finite-dimensional attracting invariant manifolds. Under certain hypothesis, this theory allows us to prove the existence of an exponentially attracting invariant manifold of dimension 2 for small ε, where the dynamics correspond to that of an ODE that can be obtained explicitly.

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Second order estimates for boundary blow-up solutions of elliptic equations

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We investigate blow-up solutions of the equation $\Delta u = f(u)$ in a bounded smooth domain $\Omega \subset R^N$. Under appropriate growth conditions on $f(t)$ as t goes to infinity we show how the mean curvature of the boundary $\partial\Omega$ enters in the second order term of the asymptotic expansion of the solution $u(x)$ as x goes to $\partial\Omega$.

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Combined effects of singular nonlinearities and convection terms in the generalized Lane-Emden-Fowler equation

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We establishes several existence, nonexistence, and bifurcation results for the generalized Lane-Emden-Fowler equation with convection term and singular nonlinearities. We are also concerned with the asymptotic behaviour of solutions. Our analysis is based on the maximum principle and adequate comparison techniques for nonlinear elliptic equations.

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Dissipative quasi-geostrophic equationandmild solution

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This paper studies the well posedness of the initial value problem for the quasi-geostrophic type equations

$$\partial_t\theta + u\nabla\theta + (-\Delta)^\gamma\theta = 0 \quad \text{on } \mathbb{R}^d \times]0,+\infty[$$
$$\theta(x,0) = \theta_0(x), \quad x \in \mathbb{R}^d$$

where $0 < \gamma \leq 1$ is a fixed parameter and the velocity field $u = (u_1, u_2, \ldots, u_d)$ is divergence free; i.e., $\nabla u = 0$). The initial data θ_0 is taken in Banach spaces of local measures (see text for the definition), such as Multipliers, Lorentz and Morrey-Campanato spaces. We will focus on the subcritical case $1/2 < \gamma \leq 1$ and we analyse the well-posedness of the system in three basic spaces: $L^{d/r,\infty}$, $\dot{X}_r$ and $\dot{M}^{p,d/r}$, when the solution is global for sufficiently small initial data. Furthermore, we prove that the solution is actually smooth. Mild solutions are obtained in several spaces with the right homogeneity to allow the existence of self-similar solutions.

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Convergence of scattering operators for the Klein-Gordon equation with a nonlocal nonlinearity

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We consider the scattering problems for two types of nonlinear Klein-Gordon equations. One is the equation of the Hartree type, and the other one is the equation with power nonlinearity. We show that the scattering operator for the equation of the Hartree type converges to that for the one with power nonlinearity in some sense. Our proof is based on some inequalities in the Lorentz space, and a strong limit of Riesz potentials.

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Ground State Energy of the Polaron Model in Relativistic Quantum Electrodynamics

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We consider the model of the Dirac operator coupled with the quantized radiation field without external potential. The Hamiltonian of the model strongly commutes with the total momentum operator. Hence the Hamiltonian has a direct integral decomposition with respect to the total momentum $\mathbf{p}$. We study the each fibre of the total Hamiltonian $H(\mathbf{p})$ — the polaron model of the relativistic quantum electrodynamics. We show that the Hamiltonian $H(\mathbf{p})$ is bounded from below, and give some properties of the ground state energy $E(\mathbf{p})$. In particular, we give the paramagnetic-type inequality $E(\mathbf{p}) \leq E(0)$.

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Separation of variables for nonlinear equations

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The three-dimensional nonlinear wave equation for a potential ϕ

$$-\varkappa^2\phi_{tt} + \triangle\phi + \frac{1}{2}\alpha\left(\nabla\phi\cdot\nabla\phi\right)_t + \frac{1}{2}\beta\left(\phi_t^2\right)_t = 0 \qquad (1)$$

describes, in particular, the waves in an isentropic gas flow for the non-dissipative case. In two-dimensional case, this is the shallow water equation. Separation of variables method for the nonlinear equation in polar (and cylinder) coordinates developed in [5-6] allows to construct nonlinear corrections (see [1-4]) to a series of classic linear solutions used and studied in books of Rayleigh and Lamb. The aim this talk is to extend the method to vector versions of the nonlinear equation and to apply it to the nonlinear Navier-Stokes equation. Possibility to apply this approach to the cases of coordinates of elliptic and parabolic cylinder, where Mathieu functions and Weber functions are involved, will be also considered.

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Blow-up for the Euler-Bernoulli problem with a fractional boundary dissipation

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We consider a beam problem with a polynomial source and a boundary damping of order between 0 and 1. Sufficient conditions on the initial data are established to have blow up of solutions in finite time.

Key Words: Blow-up, fractional derivative, boundary feedback, Euler-Bernoulli beam.

AMS Subject Classification: 45K05, 35B40, 35B45, 35B37, 93B52, 26A33

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Attractors in nonlinear diffusion involving coupled agents and application in turbulence modelling

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Expansion of a turbulent jet generated in motionless fluid by a short narrow pulse is driven by the turbulent diffusion fed by the velocity shear. We analyze this process using the K-epsilon model which essentially involves the nonlinear diffusion and coupling between the turbulent energy, its dissipation rate and momentum. Solutions are sought in the form of power series in spatial coordinate applicable within the jet. A dynamical system is derived with respect to time-dependent series coefficients. Due to the many variables involved it is difficult to investigate the full system, therefore we consider its simplified version with reduced number of variables. Attractors (centre manifolds) are found in spaces of the series coefficients.

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Nonlinear dynamics on centre manifolds describing turbulent floods: k-omega model

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In shallow turbulent flows such as floods vertical mixing tends to smooth out the flow characteristics in cross-sectional direction. The evolution of the cross-flow characteristics presents considerable interest. We model a narrow turbulent flow using the k-omega model of turbulence in the framework of the centre manifold theory. As a first step, the flow is assumed homogeneous along the substrate. The centre manifold theory reveals that the evolution of the average characteristics controls the evolution of the vertical structure of the flow. We focus on the influence of key physical factors affecting the dynamics: turbulent mixing, energy production due to the shear, volume energy dissipation and gravity (for inclined substrates). Two cases are analyzed: the horizontal flow and the flow on an inclined plane without wind forcing. In the first case the turbulence decays, and in the second case the turbulent energy and flow momentum settle on some equilibrium non-zero levels. We derive the systems of ODEs governing the time evolution of the average turbulent energy, dissipation and momentum and analyze their asymptotic behaviour.

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Existence, blow up and local exponential stability for non-linear strongly damped wave equations of Kirchhoff type

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In this talk we discuss the existence, finite time blow up, local exponential stability of solutions to strongly damped wave equations of Kirchhoff type with damping and source terms.

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Dead core for nonlinear parabolic problems under Robin boundary conditions

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Let $u(x,t)$ be a solution of the following problem

$$\begin{aligned} &u_t - div(g(|\nabla u|^2)\nabla u) = -f(u), && in\ D\times(0,T),\\ &g(|\nabla u|^2)\frac{\partial u}{\partial n} + \alpha u = \chi, && on\ \partial D\times(0,T),\\ &u(\mathbf{x},0) = u_0(\mathbf{x}), && \mathbf{x}\in D, \end{aligned}$$

where D is a bounded domain in R^N with Lipschitz boundary ∂D, f, g are suitable nonnegative functions, $\frac{\partial}{\partial n}$ indicates the normal derivative directed outward from ∂D. Conditions are formulated for the existence of a dead core, a region where the solution u(x,t) vanishes identically.

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New Exact Solutions to the Systems of Nonlinear Diffusion-Kinetic Equations

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Exact solutions to nonlinear partial differential equations play an important role in gaining a correct understanding of the qualitative features of many phenomena and processes. Exact solutions to nonlinear equations make it possible to understand the mechanisms of complex nonlinear effects, such as the spatial localization of transfer processes, the multiplicity or lack of steady states, the existence of blowup modes, the occurrence of periodic regimes, etc. Of special interest are exact solutions to nonlinear equations depending on arbitrary functions (such solutions possess great generality and are very convenient for testing numerical and approximate methods).

Until now the group methods have been the most commonly used for the construction of exact solutions. But the use of new methods of general and functional separation of variables allows obtaining many unknown solutions of nonlinear heat- and mass transfer equations; even then the complex kinetics of conversion is considered.

This method can be applied for the solution to nonlinear unsteady diffusion equations there the transfer coefficient and the bulk reaction are arbitrary function of concentration even in the case complex rheology medium. Some new exact solutions to systems of diffusion equations are found in explicit form.

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A Linearly Implicit Finite Difference Method for a Klein-Gordon-Schrödinger-type System Modeling Dissipative Electron-Ion Plasma Waves

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We consider a Klein-Gordon-Schrödinger-type system of equations in one space dimension with Dirichlet boundary conditions, which describes the nonlinear interaction between high frequency electron waves and low frequency ion plasma waves in a homogeneous magnetic field [N.Karachalios et al., ZAMP, Vol.56(2005), pp.218-238]. In the first part, we derive some a priori bounds for the solution pair of the continuous problem. Subsequently, we propose and analyse a linearly implicit finite difference method for the construction of numerical approximations to the solution pair of the problem. In the second part, we implement the numerical scheme by means of a f90 code and derive quantitative conclusions for the behaviour of the physical system. It turns out that the results of the calculations successfully reveal the features predicted by the theoretical treatment.

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A Uniqueness Result for a Problem involving Stress-driven Diffusion

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A result of uniqueness is presented for a system of partial differential equation with mixed types involving stress-driven diffusion. The behavior of the solution is discussed.

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