Plenary Lectures

Mathematics of Crime

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There is an extensive applied mathematics literature developed for problems in the biological and physical sciences. Our understanding of social science problems from a mathematical standpoint is less developed, but also presents some very interesting problems. This lecture uses crime as a case study for using applied mathematical techniques in a social science application and covers a variety of mathematical methods that are applicable to such problems. We will review recent work on agent based models, methods in linear and nonlinear partial differential equations, variational methods for inverse problems and statistical point process models. From an application standpoint we will look at problems in residential burglaries and gang crimes. Examples will consider both "bottom up" and "top down" approaches to understanding the mathematics of crime, and how the two approaches could converge to a unifying theory.

The Computation of Invariant Sets via Newton's Method

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In connection with the understanding of complex dynamics it is crucial to have reliable numerical tools at hand, which allow for the robust computation of invariant sets or invariant manifolds. In this talk we will give an overview about related so-called set oriented methods, which have been developed over the last years. In the main part we will particularly focus on the presentation of a novel numerical approach that enables the computation of general invariant sets by Newton's method. By construction, this technique is applicable even to (unstable) invariant sets related to non-stationary or aperiodic behavior. Additionally it will be shown how to utilize these set oriented methods in the context of global (multiobjective) optimization.

Dynamical systems in fluid mechanics

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We discuss the infinite-dimensional dynamical systems arising in the theory of the motion of compressible, viscous, and heat conducting fluids. Our approach is based on the theory of weak solutions to the full Navier-Stokes-Fourier system and the related concept of relative entropies. Weak solutions are known to exist for any finite energy initial data and globally-in-time.

The following topics will be discussed: the relative entropies and their role in the study of stabilization and compactness of solutions to the full Navier-Stokes-Fourier system; the entropy inequality and dissipation inequality - mathematical interpretation of Second law of thermodynamics; stabilization to equilibirum solutions; systems driven by time-dependent external forces, oscillations, absorbing sets, and attractors.

Conservation laws in mathematical biology

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Many mathematical models in biology can be described by a system of hyperbolic conservation laws with nonlinear and nonlocal coefficients. In order to determine these coefficients one needs to solve auxiliary systems of equations, for example elliptic or parabolic PDEs, which are coupled to the hyperbolic equations. In this talk we give several examples: The growth of tumors, the shrinking of dermal wounds, T cell differentiation, the growth of drug resistant bacteria in hospitals, and the transport of molecules along microtubules in axon. In these examples, the auxiliary systems range from elliptic-parabolic free boundary problems to nonlocal ODEs. Motivated by biological questions, we shall describe mathematical results regarding properties of the solutions of the conservation laws. For example, we shall determine the stability of spherical tumors and the growth of fingers; we shall discuss conditions for shrinking of the wound; suggest how to reduce the growth of drug resistant bacteria, and derive biologically motivated asymptotic behavior of solutions.

Dispersal in Heterogeneous Landscapes

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From habitat degradation and climate change to spatial spread of invasive species, dispersal plays a central role in determining how organisms cope with a changing environment. How should organisms disperse "optimally" in heterogeneous environments? The dispersal of many organisms depends upon local biotic and abiotic factors and as such is often biased. In contrast with unbiased dispersal which is better understood from theoretical perspectives, we have fairly limited knowledge of the consequences of biased dispersal, especially in the context of the spatial dynamics of interacting species. In this talk I will discuss some recent development on the evolution of biased dispersal via Lotka-Volterra two species competition models.

Lessons in Uncertainty Quantification for Turbulent Dynamical Systems

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The modus operandi of modern applied mathematics in developing very recent mathematical strategies for uncertainty quantification in partially observed high-dimensional turbulent dynamical systems is emphasized here. The approach involves the synergy of rigorous mathematical guidelines with a suite of physically relevant and progressively more complex test models which are mathematically tractable while possessing such important features as the two-way coupling between the resolved dynamics and the turbulent fuxes, intermittency and positive Lyapunov exponents, eddy diffusivity parameterization and turbulent spectra. A large number of new theoretical and computational phenomena which arise in the emerging statistical-stochastic framework for quantifying and mitigating model error in imperfect predictions, such as the existence of information barriers to model improvement, are developed and reviewed here with the intention to introduce mathematicians, applied mathematicians, and scientists to these remarkable emerging topics with increasing practical importance.

Higher-order averaging and formal series for numerical integrators

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We shall show how the formal power series used nowadays to analyze numerical integrators may be applied to perform in an explicit way high-order averaging on oscillatory (periodic or quasiperiodic) differential systems. The averaged systems obtained by applying this procedure are written in terms of (i) coefficients that are universal (i.e. independent of the system being averaged) and which therefore may be computed once and for all and (ii) basis functions whose structure is known a priori. We shall also illustrate the application of this technique to the derivation of exponentially small error bounds, as those first considered by Neishtadt. This is joint work with Ph. Chartier and A. Murua.

Symbolic dynamics of the N-centre problem at negative energies

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It is part of the mathematical folklore that dynamical systems featuring many nonlinear interactions should be subject to chaotic behavior and possess complex dynamics. On the other hand, for systems appearing in nature, this is far to be a rigorous statement and, even more, lacks a rigorous proof, specially when we leave behind the perturbative setting. We consider the planar N-centre problem, with a family of homogeneous potentials including the gravitational Kepler potential. Our strategy is to attack the problem of constructing complex trajectories problem by the use of global variational methods. We prove the existence of infinitely many collision-free periodic solutions with negative and small energy, for any distribution of the centres inside a compact set. The proof is based upon topological, variational and geometric arguments. The existence result allows to characterize the associated dynamical system with a symbolic dynamics, where the symbols are the partitions of the N centres in two non-empty sets.

Adaptive delay and its implication for pattern storage and recognition in nonlinear biological systems

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Time delay has been regarded as a destabilizing factor, causing oscillation and complicated behaviors, in many nonlinear feedback systems. Increasing evidence however shows that delay adaptation (state-dependent delay) in feedback is part of the evolutionary process of a biological system, and the evolutionary success drives the delay to a suitable value that enables the feedback system to exhibit desired dynamics more efficiently and accurately. This talk presents a few recent studies about how to design and compute a nonlinear feedback system with adaptive delay that performs complex cognitive tasks such as memory storage, and pattern recognition in high dimensional spaces.