

# 9th AIMS International Conference on



## Dynamical Systems, Differential Equations And Applications

# 9th

Sunday, July 1 -- Thursday, July 5, 2012  
Hyatt Regency Grand Cypress  
Orlando, Florida, USA

# Abstracts



American Institute of  
Mathematical Sciences

Department of Mathematics & Statistics  
University of North Carolina Wilmington





# The 9th AIMS Conference on Dynamical Systems, Differential Equations and Applications

July 1 – July 5, 2012

Orlando, Florida, USA

## ABSTRACTS

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The American Institute of Mathematical Sciences

Department of Mathematics and Statistics

The University of North Carolina Wilmington

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# Table of Contents

<b>Plenary Lectures</b>		1
<b>Special Sessions</b>		4
Special Session 01	Qualitative Studies of PDEs: Entire Solutions and Asymptotic Behavior	4
Special Session 02	Nonlinear Evolution PDEs and Interfaces in Applied Sciences	7
Special Session 03	Mathematics of Social Systems	12
Special Session 04	Nonlinear PDEs and Control Theory with Applications	17
Special Session 05	Hybrid Monte Carlo	24
Special Session 06	Dispersal in Heterogeneous Landscape	28
Special Session 07	Recent Progress in the Mathematical Theory of Compressible and Incompressible Fluid Flows	32
Special Session 08	Propagation Phenomena Appearing in Reaction-Diffusion Systems	35
Special Session 09	Mathematics for Information Processing and Management	39
Special Session 10	Computational and Nonautonomous Dynamics	43
Special Session 11	Advances in Classical and Geophysical Fluid Dynamics	48
Special Session 12	Singular Perturbations and Boundary Layer Theory	52
Special Session 13	Global Dynamics in Hamiltonian Systems	55
Special Session 14	Mathematical Models in Biology and Medicine	59
Special Session 15	Nonlinear Evolution Equations, Inclusions and Related Topics	66
Special Session 16	Reaction Diffusion Equations and Applications	73
Special Session 17	Singular Perturbations	78
Special Session 18	Qualitative Theory of Evolutionary Equation and its Application	81
Special Session 19	Waves and Convection	84
Special Session 20	Stochastic-Statistical Modeling of Climate	88
Special Session 21	Dynamical Systems and Spectral Theory	92
Special Session 22	Topological and Variational Methods for Boundary Value Problems	95
Special Session 23	Topological and Combinatorial Dynamics	100
Special Session 24	Geometric Mechanics	106
Special Session 25	Dynamics in Complex Biological Systems	111
Special Session 26	Qualitative Aspects of Nonlinear Boundary Value Problems	117
Special Session 27	Transport Barriers in Dynamical Systems	120
Special Session 28	Analysis and Numerics of Differential Equations and Dynamical Systems Mathematical Fluid Mechanics	125
Special Session 29	Self-organized Behavior of Nonlinear Elliptic Equations and Pattern Formation of Strongly Interacting Systems	129
Special Session 30	Recent Developments on Turbulence	133
Special Session 31	Mathematical Models of Cancer and Cancer Therapy	137
Special Session 32	Existence and Multiplicity Results in Elliptic Variational Problems	142
Special Session 33	Nonlinear Elliptic and Parabolic Problems in Mathematical Sciences	147
Special Session 34	Multi-phase Flows in Porous Media and Related Systems	152
Special Session 35	Qualitative Theory of Nonlinear ODEs and Applications	156
Special Session 36	Stochastic Partial Differential Equations and their Optimal Control	160
Special Session 37	Mathematical Models and Computations in Cell and Developmental Biology	164
Special Session 38	Bifurcations and Asymptotic Analysis of Solutions of Nonlinear Models	166
Special Session 39	Polynomial Methods for Differential Equations and Dynamical Systems	170
Special Session 41	New Developments in Qualitative Behavior of Evolutionary PDEs	173
Special Session 42	Global or/and Blowup Solutions for Nonlinear Evolution Equations and Their Applications	177
Special Session 43	Stochastic Networks with Applications to Neuroscience	183
Special Session 44	Applications of Chaotic and Stochastic Multiscale Dynamics	186

Special Session 45	Stochastic and Deterministic Dynamical Systems, and Applications	191
Special Session 46	Discrete/Continuous and Nonstandard Analysis	195
Special Session 47	Dynamics and Games	198
Special Session 48	Nonlinear Evolution Equations	202
Special Session 49	Growth Models and Interface Dynamics	206
Special Session 50	Mathematical Novelties in Inverse Problems in Imaging Sciences	210
Special Session 51	Ordinal Symbolic Dynamics and Applications	213
Special Session 52	Fractional Differential and Integral Equations, Theory and Applications	217
Special Session 53	Greedy Algorithms and Tensor Product Representations for High-dimensional Problems	220
Special Session 54	Dynamics in Complex Networks	222
Special Session 55	Nonlinear Elliptic and Parabolic Problems	226
Special Session 57	Nonlinear and Dispersive Partial Differential Equations	232
Special Session 58	Variational Analysis and Equilibrium Problems	235
Special Session 61	PDE Models for Biological Pattern Formation	238
Special Session 62	PDEs and Dynamical Systems, and Their Applications	241
Special Session 63	Infinite Dimensional Dynamics and Applications	246
Special Session 64	Analysis of PDEs and Particle Systems: From Life Sciences, Economics and Materials Science	250
Special Session 67	Applied Analysis and Dynamics in Engineering and Sciences	254
Special Session 68	Analysis and Simulations of Nonlinear Systems	259
Special Session 69	Dissipative Systems and Applications	264
Special Session 70	Modeling and Dynamics of Infectious Diseases	267
Special Session 72	Special Methods for Solving Systems of Non-linear Differential Equations and their Applications to Sciences and Engineering	272
Special Session 73	Mathematical Models for Upwelling Ocean Currents and Related Phenomena	275
Special Session 75	Heteroclinic Cycles: Theory and Applications	279
Special Session 76	On PDEs from Biology	283
Special Session 77	The Navier-Stokes Equations and Related Problems	286
Special Session 78	Multiple Time Scale Dynamics with a View Towards Biological Applications	291
Special Session 79	Numerical Methods based on Homogenization and on Two-Scale Convergence	294
Special Session 80	Advances in the Numerical Solution of Nonlinear Evolution Equations	299
Special Session 81	Analysis and Simulation of Multi-scale Problems	302
Special Session 82	Multi-component Integrable Systems, Solitons, and Nonlinear Waves	305
Special Session 84	Theory, Numerics and Applications of Quasi-Periodic and Almost Periodic Schrödinger Operators	308
<b>Contributed Sessions</b>		<b>310</b>
Contributed Session 01	Equations and Qualitative Analysis	310
Contributed Session 02	ODEs and Applications	313
Contributed Session 04	Modelling and Math Biology	317
Contributed Session 06	Control and Optimization	322
Contributed Session 07	Scientific Computation and Numerical Algorithms	323
Contributed Session 09	PDEs and Applications	326
Contributed Session 10	Bifurcation and Chaotic Dynamics	332
<b>Poster Sessions</b>		<b>334</b>
<b>List of Contributors</b>		<b>339</b>

# Plenary Lectures

## Mathematics of Crime

### Andrea Bertozzi

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There is an extensive applied mathematics literature developed for problems in the biological and physical sciences. Our understanding of social science problems from a mathematical standpoint is less developed, but also presents some very interesting problems. This lecture uses crime as a case study for using applied mathematical techniques in a social science application and covers a variety of mathematical methods that are applicable to such problems. We will review recent work on agent based models, methods in linear and nonlinear partial differential equations, variational methods for inverse problems and statistical point process models. From an application standpoint we will look at problems in residential burglaries and gang crimes. Examples will consider both “bottom up” and “top down” approaches to understanding the mathematics of crime, and how the two approaches could converge to a unifying theory.



## The Computation of Invariant Sets via Newton’s Method

### Michael Dellnitz

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In connection with the understanding of complex dynamics it is crucial to have reliable numerical tools at hand, which allow for the robust computation of invariant sets or invariant manifolds. In this talk we will give an overview about related so-called set oriented methods, which have been developed over the last years. In the main part we will particularly focus on the presentation of a novel numerical approach that enables the computation of general invariant sets by Newton’s method. By construction, this technique is applicable even to (unstable) invariant sets related to non-stationary or aperiodic behavior. Additionally it will be shown how to utilize these set oriented methods in the context of global (multiobjective) optimization.

## Dynamical systems in fluid mechanics

### Eduard Feireisl

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We discuss the infinite-dimensional dynamical systems arising in the theory of the motion of compressible, viscous, and heat conducting fluids. Our approach is based on the theory of weak solutions to the full Navier-Stokes-Fourier system and the related concept of relative entropies. Weak solutions are known to exist for any finite energy initial data and globally-in-time.

The following topics will be discussed: the relative entropies and their role in the study of stabilization and compactness of solutions to the full Navier-Stokes-Fourier system; the entropy inequality and dissipation inequality - mathematical interpretation of Second law of thermodynamics; stabilization to equilibrium solutions; systems driven by time-dependent external forces, oscillations, absorbing sets, and attractors.





### Conservation laws in mathematical biology

**Avner Friedman**

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Many mathematical models in biology can be described by a system of hyperbolic conservation laws with nonlinear and nonlocal coefficients. In order to determine these coefficients one needs to solve auxiliary systems of equations, for example elliptic or parabolic PDEs, which are coupled to the hyperbolic equations. In this talk we give several examples: The growth of tumors, the shrinking of dermal wounds, T cell differentiation, the growth of drug resistant bacteria in hospitals, and the transport of molecules along microtubules in axon. In these examples, the auxiliary systems range from elliptic-parabolic free boundary problems to nonlocal ODEs. Motivated by biological questions, we shall describe mathematical results regarding properties of the solutions of the conservation laws. For

example, we shall determine the stability of spherical tumors and the growth of fingers; we shall discuss conditions for shrinking of the wound; suggest how to reduce the growth of drug resistant bacteria, and derive biologically motivated asymptotic behavior of solutions.

### Dispersal in Heterogeneous Landscapes

**Yuan Lou**

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From habitat degradation and climate change to spatial spread of invasive species, dispersal plays a central role in determining how organisms cope with a changing environment. How should organisms disperse “optimally” in heterogeneous environments? The dispersal of many organisms depends upon local biotic and abiotic factors and as such is often biased. In contrast with unbiased dispersal which is better understood from theoretical perspectives, we have fairly limited knowledge of the consequences of biased dispersal, especially in the context of the spatial dynamics of interacting species. In this talk I will discuss some recent development on the evolution of biased dispersal via Lotka-Volterra two species competition models.



### Lessons in Uncertainty Quantification for Turbulent Dynamical Systems

**Andrew Majda**

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The modus operandi of modern applied mathematics in developing very recent mathematical strategies for uncertainty quantification in partially observed high-dimensional turbulent dynamical systems is emphasized here. The approach involves the synergy of rigorous mathematical guidelines with a suite of physically relevant and progressively more complex test models which are mathematically tractable while possessing such important features as the two-way coupling between the resolved dynamics and the turbulent fluxes, intermittency and positive Lyapunov exponents, eddy diffusivity parameterization and turbulent spectra. A large number of new theoretical and computational phenomena which arise in the emerg-

ing statistical-stochastic framework for quantifying and mitigating model error in imperfect predictions, such as the existence of information barriers to model improvement, are developed and reviewed here with the intention to introduce mathematicians, applied mathematicians, and scientists to these remarkable emerging topics with increasing practical importance.



### Higher-order averaging and formal series for numerical integrators

**J M Sanz-Serna**

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We shall show how the formal power series used nowadays to analyze numerical integrators may be applied to perform in an explicit way high-order averaging on oscillatory (periodic or quasiperiodic) differential systems. The averaged systems obtained by applying this procedure are written in terms of (i) coefficients that are universal (i.e. independent of the system being averaged) and which therefore may be computed once and for all and (ii) basis functions whose structure is known a priori. We shall also illustrate the application of this technique to the derivation of exponentially small error bounds, as those first considered by Neishtadt. This is joint work with Ph. Chartier and A. Murua.



### Symbolic dynamics of the N-centre problem at negative energies

**Susanna Terracini**

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It is part of the mathematical folklore that dynamical systems featuring many nonlinear interactions should be subject to chaotic behavior and possess complex dynamics. On the other hand, for systems appearing in nature, this is far to be a rigorous statement and, even more, lacks a rigorous proof, specially when we leave behind the perturbative setting. We consider the planar N-centre problem, with a family of homogeneous potentials including the gravitational Kepler potential. Our strategy is to attack the problem of constructing complex trajectories problem by the use of global variational methods. We prove the existence of infinitely many collision-free periodic solutions with negative and small energy, for

any distribution of the centres inside a compact set. The proof is based upon topological, variational and geometric arguments. The existence result allows to characterize the associated dynamical system with a symbolic dynamics, where the symbols are the partitions of the N centres in two non-empty sets.

### Adaptive delay and its implication for pattern storage and recognition in nonlinear biological systems

**Jianhong Wu**

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Time delay has been regarded as a destabilizing factor, causing oscillation and complicated behaviors, in many nonlinear feedback systems. Increasing evidence however shows that delay adaptation (state-dependent delay) in feedback is part of the evolutionary process of a biological system, and the evolutionary success drives the delay to a suitable value that enables the feedback system to exhibit desired dynamics more efficiently and accurately. This talk presents a few recent studies about how to design and compute a nonlinear feedback system with adaptive delay that performs complex cognitive tasks such as memory storage, and pattern recognition in high dimensional spaces.





# Special Sessions

## Special Session 1: Qualitative Studies of PDEs: Entire Solutions and Asymptotic Behavior

Peter Polacik, University of Minnesota, USA  
Eiji Yanagida, Tokyo Institute of Technology, Japan

The aim of this session is to discuss properties of entire solutions and asymptotic behavior of global solutions in various types of PDEs. Topics will include Liouville-type theorems, blow-up, traveling waves, convergence, concentration phenomena, etc.

### Stability analysis of asymptotic profiles for fast diffusion equations

Goro Akagi  
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Ryuji Kajikiya

Let  $\Omega$  be a bounded domain of  $\mathbb{R}^N$  with smooth boundary  $\partial\Omega$ . We are concerned with the Cauchy-Dirichlet problem for fast diffusion equations of the form

$$\partial_t (|u|^{m-2}u) = \Delta u \quad \text{in } \Omega \times (0, \infty), \quad (1)$$

$$u = 0 \quad \text{on } \partial\Omega \times (0, \infty), \quad (2)$$

$$u(\cdot, 0) = u_0 \quad \text{in } \Omega, \quad (3)$$

where  $\partial_t = \partial/\partial t$ ,  $m \in (2, 2^*)$  with  $2^* := 2N/(N-2)_+$  and  $u_0$  might be sign-changing. Every solution  $u = u(x, t)$  of (1)–(3) vanishes in finite time at a power rate. This talk is concerned with asymptotic profiles of vanishing solutions. We first introduce the notions of stability and instability of (possibly sign-changing) asymptotic profiles and present some stability criteria. Moreover, we also discuss annular domain cases, which do not fall within the criteria, by developing some perturbation method for radial symmetric functions.

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### Finite Morse index solutions and asymptotics of weighted nonlinear elliptic equations

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We study the behavior of finite Morse index solutions of the equation

$$-\operatorname{div}(|x|^\theta \nabla v) = |x|^l |v|^{p-1} v \quad \text{in } \Omega \subset \mathbb{R}^N \quad (N \geq 2), \quad (1)$$

where  $p > 1$ ,  $\theta, l \in \mathbb{R}^1$ , and  $\Omega$  is a bounded or unbounded domain. Through a suitable transformation of the form  $v(x) = |x|^\sigma u(x)$ , the following nonlinear Schrödinger equation with Hardy potential

$$-\Delta u = |x|^\alpha |u|^{p-1} u + \frac{\ell}{|x|^2} u \quad \text{in } \Omega \subset \mathbb{R}^N \quad (N \geq 3), \quad (2)$$

where  $p > 1$ ,  $\alpha \in (-\infty, \infty)$  and  $\ell \in (-\infty, (N-2)^2/4)$ , reduces to a special case of (1).

We demonstrate that the general form (1) is more natural to use even if one's main interest is in (2).

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### Sharp decay estimates of $L^q$ -norms for non-negative Schrödinger heat semigroups

Kazuhiro Ishige  
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Norisuke Ioku, Eiji Yanagida

Let  $H = -\Delta + V$  be a nonnegative Schrödinger operator on  $L^2(\mathbb{R}^N)$ , where  $N \geq 3$  and  $V$  is a radially symmetric nonpositive function in  $\mathbb{R}^N$  decaying quadratically at the space infinity. For any  $1 \leq p \leq q \leq \infty$ , we denote by  $\|e^{-tH}\|_{q,p}$  the operator norm of the Schrödinger heat semigroup  $e^{-tH}$  from  $L^p(\mathbb{R}^N)$  to  $L^q(\mathbb{R}^N)$ . In this paper, under suitable conditions on  $V$ , we give the exact and optimal decay rates of  $\|e^{-tH}\|_{q,p}$  as  $t \rightarrow \infty$  for all  $1 \leq p \leq q \leq \infty$ . The decay rates of  $\|e^{-tH}\|_{q,p}$  depend whether the operator  $H$  is subcritical or critical via the behavior of the positive harmonic function for the operator  $H$ .

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### On the asymptotic behavior of solutions for semilinear parabolic equations involving critical Sobolev exponent

Michinori Ishiwata  
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In this talk, we are concerned with the asymptotic behavior of solutions for semilinear parabolic equations involving critical Sobolev exponent. Particularly, we are interested in the behavior of threshold solutions and we will discuss the blow up rate, blow-up limit of such solutions.

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### Existence and asymptotic stability of periodically growing solutions of nonlinear parabolic equations

**Ken-ichi Nakamura**

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**Toshiko Ogiwara**

We study the behavior of unbounded global orbits in a class of strongly monotone semiflows and give a criterion for the existence of orbits with periodic growth. We also prove the uniqueness and asymptotic stability of such orbits. We apply our results to a certain class of nonlinear parabolic equations and show the convergence of the solutions to a periodically growing solution which grows up in infinite time with time-periodic profile.

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### Convergence for asymptotically autonomous parabolic equations on $R^N$

**Peter Polacik**

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**J. Foldes**

We consider parabolic equation on  $R^N$  whose nonlinearities are asymptotically autonomous, in both space and time, as time approaches infinity. We will present a convergence result for positive solutions of such equations and discuss the key tools of its proof.

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### Ground state solutions of the nonlinear Schroedinger equation with interface

**Wolfgang Reichel**

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**Tomas Dohnal, Kaori Nagatou, Michael Plum, Wolfgang Reichel**

We consider the nonlinear Schroedinger equation  $-\Delta u + V(x)u = \Gamma(x)|u|^{p-1}u$  in  $R^n$  with  $V(x) = V_1(x)$ ,  $\Gamma(x) = \Gamma_1(x)$  where  $x_1 \geq 0$  and  $V(x) = V_2(x)$ ,  $\Gamma(x) = \Gamma_2(x)$  where  $x_1 \leq 0$  with functions  $V_1, V_2, \Gamma_1, \Gamma_2$  which are periodic in each coordinate direction. This problem describes the interface of two periodic media, e.g. photonic crystals. We study the existence of ground state solutions and provide an existence criterion. Examples of interfaces satisfying these criteria are provided. In 1D it is shown that the criteria can be reduced to conditions on the linear Bloch waves of the operators  $-\frac{d^2}{dx^2} + V_1(x)$  and  $-\frac{d^2}{dx^2} + V_2(x)$ .

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### On the number of maximum points of least energy solutions to a two-dimensional Hénon equation with large exponent

**Futoshi Takahashi**

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In this talk, we prove that least energy solutions of the two-dimensional Hénon equation

$$-\Delta u = |x|^{2\alpha} u^p \quad (x \in \Omega), \quad u > 0 \quad (x \in \Omega), \\ u = 0 \quad (x \in \partial\Omega),$$

where  $\Omega$  is a smooth bounded domain in  $\mathbb{R}^2$  with  $0 \in \Omega$ ,  $\alpha \geq 0$  is a constant and  $p > 1$ , have only one global maximum point when  $\alpha > e - 1$  and the nonlinear exponent  $p$  is sufficiently large. This answers positively to a recent conjecture by C. Zhao (preprint, 2011).

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### On Toda system: classification and applications

**Juncheng Wei**

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**Chang-Shou Lin, Dong Ye**

We give a complete classification of the solutions to the following  $SU(N + 1)$  Toda system with a single source:

$$\Delta u_i + \sum_{j=1}^N a_{ij} e^{u_j} = 4\pi\gamma_i \delta_0$$

in  $R^2$ . here the matrix  $(a_{ij})$  is the  $SU(N + 1)$  cartan matrix. Then we apply these classification and non-degeneracy result to give a rigorous proof of the existence of non-topological solutions for  $SU(3)$  Chern-Simons system.

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### Slowly traveling waves, blow-up at spatial infinity and homoclinic orbits in nonlinear parabolic equations of fast diffusion type

**Michael Winkler**

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We consider the spatially one-dimensional nonlinear diffusion equation  $u_t = u^p u_{xx}$  for  $p > 0$ . We construct positive classical solutions, defined for all negative times, which are of the form  $u(x, t) = (-t)^{-\frac{1}{p}} F\left(x + \frac{1}{p\alpha} \ln(-t)\right)$ , with arbitrary  $\alpha > 0$ , by solving an associated ODE for  $F$ . These ‘ancient

slowly traveling wave solutions' have the following properties: 1.) If  $p \leq 2$  then  $u$  blows up at time  $t = 0$  with empty blow-up set. 2.) If  $p > 2$  then  $u$  can be extended so as to become an entire positive classical solution  $\bar{u}$ , defined on  $\mathbb{R} \times \mathbb{R}$ , such that  $\bar{u}_x > 0$  on  $\mathbb{R}$ , but such that  $\bar{u}$  connects the trivial equilibrium to itself in the sense that  $\bar{u}(x, t) \rightarrow 0$  as  $t \rightarrow \pm\infty$ , locally uniformly with respect to  $x \in \mathbb{R}$ .

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**Asymptotic behavior of singular solutions for a semilinear parabolic equation**

**Eiji Yanagida**

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**Masaki Hoshino, Shota Sato**

We consider singular solutions of a semilinear parabolic equation with a power nonlinearity. It is known that in some range of parameters, this equation has a family of singular steady states with a separation property. In this case, we show the existence of time-dependent singular solutions and study their asymptotic behavior. In particular, we prove the convergence of solutions to the singular steady states and give exact convergence rates.

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## Special Session 2: Nonlinear Evolution PDEs and Interfaces in Applied Sciences

**Gunduz Caginalp**, University of Pittsburgh, USA  
**Maurizio Grasselli**, Politecnico di Milano, Italy  
**Alain Miranville**, Universite de Poitiers, France

Applied sciences often pose problems in terms of interfaces that can be studied through nonlinear evolution equations. Among these are phase field or diffusive interface problems, where Allen-Cahn and Cahn-Hilliard type equations play a basic role. This session will mainly focus on such models and their mathematical properties, namely, well-posedness, regularity, and stability and asymptotic behavior (considered here in a broad sense). Their implications for applications will also be an issue.

### Stationary free surface flows in three dimensions

**Frederic Abergel**  
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**Jacques-Herbert Bailly**

We consider an incompressible, viscous, free surface flow down a three dimensional channel. In the absence of surface tension, we prove the existence of a unique stationary solution in weighted Sobolev spaces. The result is based on a thorough study of the pseudodifferential operator relating the normal velocity of the fluid and the normal component of the associated stress tensor along the free surface, and requires the use of the Nash-Moser Implicit Function Theorem. Extensions to other geometries are discussed.

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### Diffuse interface Cahn-Hilliard-Ladyzhenskaya models with singular potentials

**Stefano Bosia**  
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In order to describe the evolution of a mixture of two incompressible and (partially) immiscible fluids, a diffuse interface model consists of the Ladyzhenskaya-Navier-Stokes type equations for the (average) fluid velocity coupled with a convective Cahn-Hilliard equation with a singular (e.g., logarithmic) potential. In this talk we will present some recent results concerning such systems endowed with no-slip boundary conditions for the velocity and no-flux boundary conditions on the order parameter field. We shall discuss existence of weak solutions in dimension three, some regularity results and the existence of a trajectory attractor both in a weak as well in a strong topologies. Further issues and open questions, concerning e.g. uniqueness, will also be addressed.

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### Consistent hierarchy of Cahn-Hilliard systems

**Franck Boyer**  
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**S. Minjeaud**

In this work, we investigate n-component Cahn-Hilliard systems. Our goal is to derive models in a consistent way: we require that a solution of the (n-1)-component model is a solution to the n-component model. Properties of such models are given in some cases and numerical illustrations are proposed. We also show that this consistency property is important in applications, in particular in the framework of phase-field modeling.

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### Phase field models and the connection between microscopic and macroscopic anisotropy

**Gunduz Caginalp**  
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Anisotropy at the microscopic level is manifested at the macroscopic level that is on a vastly larger scale. The phase field approach can facilitate our understanding of this complex process. We use a formulation in which we work directly with an integral equation for the phase variable. The technique allows one to derive macroscopic conditions at the interface from the microscopic potentials. Differential geometry and asymptotic analysis yield interface conditions, in arbitrary spatial dimension, for interactions that include anisotropy as well as non-local potentials. The interface condition can be expressed in various mathematical formulations, e.g., in terms of the principal curvature directions of the interface, or the second order directional derivatives of the interface and the Hessian of the surface tension. This work is in collaboration with X. Chen and E. Esenturk.

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### Long-time behavior of the Caginalp system with singular potentials and dynamic boundary conditions

**Laurence Cherfils**

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**S. Gatti, A. Miranville**

This talk deals with the Caginalp phase-field model equipped with both singular potentials and dynamic boundary conditions. Classical solutions (in the sense of distributions) are not assured in this case. Thanks to a suitable definition of solutions, we are able to get dissipative estimates, leading to the existence of the global attractor with finite fractal dimension, as well as of an exponential attractor.

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### Approximation of multivalued attractors and applications to the Navier-Stokes equations

**Michele Coti Zelati**

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**Florentina Tone**

We present the theory of multivalued dynamical systems, with particular emphasis on the approximation of global attractors of continuous-time semigroups by discrete ones. As an application, we focus on a fully implicit time-discretization of the two-dimensional Navier-Stokes equations, establishing new uniform bounds on the time-step parameter. As a byproduct, we obtain a general long-time stability result and we prove the convergence of the discrete attractors to the continuous one as the time-step approaches zero.

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### Congestion in cell migration models

**Julien Dambrine**

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**B. Maury, N. Meunier, A. Roudneff-Chupin**

This work concerns a class of macroscopic models for cell migration in a saturated medium for two-species mixtures. Those species tend to achieve some motion according to a desired velocity, and congestion forces them to adapt their velocity. This adaptation is modelled by a correction velocity which is chosen minimal in a least-square sense. We are especially interested in two situations: a single active species moves in a passive matrix (cell migration) with a given desired velocity, and a closed-loop Keller-Segel type model, where the desired velocity is the gradient of a self-emitted chemoattractant. We propose a numerical strategy to discretize the model, and

illustrate its behaviour in the case of a prescribed velocity, and for the saturated Keller-Segel model.

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### Long-term analysis of strongly damped nonlinear wave equations

**Filippo Dell’Oro**

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**Vittorino Pata**

We consider the strongly damped nonlinear wave equation

$$u_{tt} - \Delta u_t - \Delta u + f(u_t) + g(u) = h$$

with Dirichlet boundary conditions, which serves as a model in the description of thermal evolution within the theory of type III heat conduction. In particular, the nonlinearity  $f$  acting on  $u_t$  is allowed to be nonmonotone and to exhibit a critical growth of polynomial order 5. The main focus is the longterm analysis of the related solution semigroup, which is shown to possess the global attractor in the natural weak energy space.

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### Attractors for a Caginalp phase-field model type on the whole space $R^3$

**Brice Doumbe**

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**Brice Doumbé**

We consider in this paper a generalization of Caginalp phase-field system derived from a generalization of the Maxwell-Cattaneo law in an unbounded domain namely  $R^3$  in our case; which make the analysis challenging. We prove the well-posedness of the problem and the dissipativity of the associated semigroup. Finally, we study the long time behavior of solutions in terms of attractors.

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### Vlasov-Poisson system with diffuse boundary conditions

**Emre Esenturk**

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**Walter Strauss, Hyungju Hwang**

Kinetic models play an important role in many areas of mathematical physics. One of the branches of the theory deals with systems whose elements interact with each other in a conservative way through a self-consistent field. A typical example of such systems is the hot plasma which is governed by a collisionless

kinetic equation (Vlasov) coupled with electrostatic equation (Poisson).

In the last decades many important results have been obtained for the Vlasov-Poisson system. In most of the works the theories were developed in the whole space. Much less is known on systems with boundary conditions. In this talk we will present results for the nonlinear Vlasov-Poisson system with the absorbing and non-absorbing diffuse boundary conditions. Existence and uniqueness of solutions will be addressed in one dimension for the finite interval and the half line and in higher dimensions for particular geometries. Some exact solutions will be discussed for simple cases.

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**Global solutions for the 2D NS-CH model for a two-phase flow of viscous, incompressible fluids with mixed partial viscosity and mobility**

**Ciprian Gal**

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Whether or not global solutions of the 2D Navier-Stokes-Cahn-Hilliard system without full viscosity and mobility can develop finite time singularities is a difficult issue. A major part of this talk deals with global regularity of strong solutions for the NS-CH system with mixed partial viscosity and mobility. In addition, we will also discuss the 2D NS-CH system without viscosity but with full mobility. We prove the global existence and uniqueness of classical solutions.

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**Stability analysis and travelling fronts in the phase field crystal model**

**Peter Galenko**

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**Ken Elder**

The phase field crystal (PFC) model was proposed to incorporate the physics naturally embedded on atomic length scales for processes that occur on diffusive time scales. The model describes a field that is spatially periodic in a solid and constant in metastable or unstable (liquid) phases. Using the PFC-model, we provide a stability analysis to predict the velocity and periodicity of a crystal front invading a uniform unstable state. Amplitude expansions of the hyperbolic and parabolic PFC models are used to examine a periodic solid invading a metastable liquid in both slow and rapid front propagation regimes. The analysis is applicable to systems with long-range interactions, such as block copolymers (second order

transformation) as well as the solidification of under-cooled liquids (first order phase transformation).

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**Attractors for weakly damped wave equations**

**Olivier Goubet**

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This talk is devoted to the issue of the existence and regularity for low regular solutions to weakly damped forced equations, as Korteweg de Vries and nonlinear Schrodinger equations.

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**On thermodynamically consistent schemes for two-phase flow with mass density contrast and species transport**

**Guenther Gruen**

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**Eduard Campillo-Funollet, Fabian Klingbeil**

Recently, Abels, Garcke, and Grün proposed a class of diffuse interface models for two-phase flow with different mass densities which allow both for energy estimates and for solenoidal velocity fields. It differs from earlier approaches – apparently not consistent with thermodynamics or not frame-indifferent – by a convective coupling of velocity field and Cahn-Hilliard flux. We present a subtle discretization of these terms which entails energy estimates and existence results in the discrete setting. Numerical simulations in various settings – Rayleigh-Taylor instability, rising droplets, species transport and Henry’s law – are compared with those obtained by other models. Finally, we suggest extensions of the model to electrowetting with electrolyte solutions in the case of general mass densities.

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**A fourth order Cahn-Hilliard type equation**

**Haydi Israel**

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Our aim in this talk is to discuss the well-posedness (existence and uniqueness of solutions) and the asymptotic behavior, in terms of finite-dimensional attractors (global and exponential attractors) for an equation of Cahn-Hilliard type.

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### Well-posedness of finite energy solutions to supersonic flow structure interactions

**Irena Lasiecka**

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**J. Webster**

We shall consider a model of flow-structure interaction which consists of perturbed wave equation coupled with a nonlinear plate. The interaction between two media takes place on the edge of the plate. We shall consider supersonic case where it is known that the static problem loses ellipticity. Existence and uniqueness of finite energy solutions will be addressed. By using methods of microlocal analysis Hadamard wellposedness of finite energy solutions will be shown.

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### The thin film equation with backwards second order diffusion

**Amy Novick-Cohen**

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**A. Shishkov**

We focus on the thin film equation with lower order “backwards” diffusion which can describe, for example, structure formation in biofilms and the evolution of thin viscous films in the presence of gravity and thermo-capillary effects. We treat in detail the equation

$$u_t + \{u^n(u_{xxx} + \neq u^{m-n}u_x - Au^{M-n}u_x)\}_x = 0,$$

where  $\neq = \pm 1, 0 < n, m < M$ , and  $0 \leq A$ .

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### A thermodynamic format for phase-field modeling

**Paolo Podio-Guidugli**

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We discuss the thermodynamic issues underlying the phase-field modeling of phase segregation and phase transitions by atom rearrangement in crystalline materials, both in the absence and in the presence of diffusion. In particular, we contrast the standard derivation of the classic mathematical models of Allen-Cahn (or Ginzburg-Landau, according to [2]) and Cahn-Hilliard with the versions given by Friedand Gurtin [1], Gurtin [2], and Podio-Guidugli [3].

[1] E. Fried and M.E. Gurtin, Continuum theory of thermally induced phase transition based on an order parameter. *Physica D* 68(1993).

[2] M.E. Gurtin, Generalized Ginzburg-Landau and Cahn-Hilliard equations based on a microforce balance. *Physica D* 92 (1996).

[3] P. Podio-Guidugli, Models of phase segregation and diffusion of atomic species on a lattice. *Ric. Mat.*, 55 (1), 105-118 (2006).

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### Time regularity and uniqueness of non-Newtonian binary fluid mixtures

**Dalibor Prazak**

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**M. Grasselli**

We consider binary mixture of incompressible non-Newtonian fluid with diffused interfaces. More precisely, the fluid is governed by Ladyzhenskaya rate-type model, where stress tensor is a polynomial function of  $|Du|$  and also a smooth function of the order parameter, the latter being driven by convected Cahn-Hilliard equation. We consider the 3D setting with Dirichlet/Neumann boundary conditions. Assuming  $p$ -coercivity of stress tensor, with  $p > 11/5$ , we employ fractional Nikolskii spaces to establish higher time integrability of (arbitrary) weak solution. This implies uniqueness, but also existence of finite-dimensional attractor.

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### Degenerating PDE system for phase transitions and damage.

**Elisabetta Rocca**

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**Riccarda Rossi**

In the talk we propose a suitable weak notion of solution for a PDE system coupling a weak heat and momentum equation with a generalized principle of virtual powers for the phase/damage parameter, partially motivated for the theory of rate-independent damage process. In order to prove an existence result for this weak formulation, an approximating problem is introduced, where the elliptic degeneracy is ruled out. First, a global-in-time existence result for the full system consisting of the heat, displacement, and phase/damage parameter equations is established and well-posedness is proved. Then, the passage to the limit to the degenerate system is performed via suitable variational techniques.

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**Efficient numerical methods for the Cahn-Hilliard-Brinkman equation****Steven Wise**University of Tennessee, USA  
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I will describe some first and second-order (in time) convex splitting schemes for the Cahn-Hilliard-Brinkman equation, which models two-phase flows in porous media and related phenomena. I will present some simulation results in 2 and 3D and discuss some applications in modeling tumor growth, bio-films, and lava lamps.

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**Global dynamics of a diffuse interface model for solid tumor growth****Kun Zhao**University of Iowa, USA  
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In this talk I will report recent progress on a diffuse interface model (referred as the Cahn-Hilliard-Hele-Shaw system) which arises in modeling of spinodal decomposition in binary fluid in a Hele-Shaw cell, tumor growth and cell sorting, and two phase flows in porous media. Previous numerical simulations showed that the model is capable of modeling all the stages of a solid tumor growth - avascular, vascular and metastasis. In this work, wellposedness, regularity and long-time asymptotic behavior of solutions to an initial-boundary value problem of the model are rigorously justified.

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## Special Session 3: Mathematics of Social Systems

Andrea Bertozzi, UCLA, USA

This session concerns mathematical modeling of social systems - including problems in biology involving social interaction and problems in social science modeling. Is it the intention of the organizer to form a broad based session that will be useful for young mathematical scientists interested in broadening their research. The session will include work on agent based models, methods in linear and nonlinear partial differential equations, variational methods for inverse problems, and statistical and stochastic models. Examples of applications range from crime modeling including gang interactions and patterns in property crime to models for social interactions of swarms in biology.

### Stationary states for the aggregation equation with power law attractive-repulsive potentials

Daniel Balagué

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José A. Carrillo, Thomas Laurent, Gael Raoul

We analyze the stability of a uniform distribution on a sphere as stationary solution of the aggregation equation with power law attractive-repulsive potentials. We give a sharp condition that establishes its stability under radial perturbations.

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### Modeling social dynamics

Alethea Barbaro

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In recent years, agent-based models have become very popular for modeling social dynamics. This technique has been used to model organisms as diverse as fish, insects, birds, and even people. Studying these models at a kinetic level opens new mathematical perspectives into the dynamics of such systems, raising new and interesting mathematical questions. Here, we will present some agent-based models for social systems, and examine the PDEs arising from these models.

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### An adversarial evolutionary game for criminal behavior

Maria D'Orsogna

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Traditional models of human cooperation are usually cast in the form of a prisoner's dilemma, where although cooperation may be beneficial, players may choose to "defect" and pursue selfish goals. In this talk we consider an adversarial evolutionary game developed for criminal activity where players not only choose whether or not to cooperate for the common good but also whether or not to actively harm

others by committing crimes. This new choice gives rise to four possible strategies among which that of the so called "informant", a player who cooperates while still committing crimes. We find two possible equilibration regimes, a defection-dominated and an ideal, cooperation-dominated one and show that the number of informants is crucial in determining which of these two regimes is achieved. Since large numbers of informants lead to the ideal cooperative society we also study their active recruitment from the overall society, by considering differential recruitment costs and benefits, via an optimal control problem where finite resources are included. We discuss our results in the context of extreme adversarial societies, such as those marred by wars, insurgencies and organized crime.

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### A mathematical model for flight guidance in honeybee swarms

Razvan Fetecau

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Angela Guo

When a colony of honeybees relocates to a new nest site, less than 5% of the bees (the scout bees) know the location of the new nest. Nevertheless, the small minority of informed bees manages to provide guidance to the rest and the entire swarm is able to fly to the new nest intact. The stalker bee hypothesis, one of the several theories proposed to explain the guidance mechanism in bee swarms, seems to be supported by recent experimental observations. Originally proposed by Lindauer in 1955, the theory suggests that the informed bees make high-speed flights through the swarm in the direction of the new nest, hence conspicuously pointing to the desired direction of travel. Once they reach the front of the swarm, they return at low speeds to the back, by flying along the edges of the swarm, where they are less visible to the rest of the bees. This work presents a mathematical model of flight guidance in bee swarms based on the stalker bee hypothesis. Numerical experiments, parameter studies and comparison with experimental data will be presented.

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**Swarming on random graphs****Theodore Kolokolnikov**Dalhousie, Canada  
tkolokol@gmail.com**James von Brecht, David Uminsky, Andrea Bertozzi**

We study a standard aggregation model where instead of the usual assumption of all-to-all interaction, we assume that two given particles interact with probability  $p$ . We are particularly interested in the following question: How does the connectivity of the underlying graph affect the confinement property of the swarm? Under certain generic assumptions on the interaction kernel, we show that the confinement is preserved up to  $p=O((\log n)/n)$ , where  $n$  is the number of particles. In other words, very few edges are needed to preserve the confinement. The results rely on basic probability arguments and eigenvalues of random matrices.

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**Analysis of a new PDE based model for ant foraging****Manish Kumar**University of Cincinnati, USA  
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The dynamics of ant foraging is of interest in understanding how local interactions lead to global emergent behaviors in complex systems. We present a new mathematical model, inspired by the Keller Segel model of bacterial chemotaxis, for ant foraging that accounts for different behaviors exhibited by foragers in search of food and food carrying ants. The model distinguishes between the dynamics of food searching (foraging) ants and food carrying ants. Numerical simulations based on the model show trail formation in foraging ant colonies to be an emergent phenomenon and also account for adaptive behavior observed in recent experiments. The model has interesting characteristics related to the absence of finite time singularities that emerge in the Keller-Segel model of chemotaxis. We also discuss preliminary steady state stability results.

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**Aggregation via Newtonian potential and aggregation patches****Thomas Laurent**UCR, USA  
laurent@ucr.edu**Bertozzi, Leger**

We consider the motion of a density of particles  $\rho(x,t)$  by a velocity field  $v(x,t)$  obtained by convolving the density of particles with the gradient of the Newtonian potential, that is  $v = -\text{grad } N^* \rho$ . An important class of solutions are the ones where the particles are uniformly distributed on a time evolving domain. We refer to these solutions as aggregation patches, by analogy to the vortex patch solutions of the 2D incompressible Euler equations. Numerical simulations as well as some exact solutions show that the time evolving domain on which the patch is supported typically collapses on a complex skeleton of codimension one. We also show that going backward in time, any bounded compactly supported solution converges as  $t$  goes to minus infinity toward a spreading circular patch. We provide a rate of convergence which is sharp in 2D. This is a joint work with Bertozzi and Leger.

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**Rethinking network analysis: topology, dynamics and network structure****Kristina Lerman**USC Information Sciences Institute, USA  
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Many popular metrics used in social network analysis are based on the random walk. However, the random walk may not be appropriate for modeling and analyzing many social phenomena, including epidemics and information diffusion, in which one node may interact with many others at the same time, for example, by broadcasting the virus or information to its neighbors. To produce meaningful results, social network analysis algorithms have to take into account the nature of interactions between the nodes. We classify interactions as conservative (informally, one-to-one) and non-conservative (one-to-many) and show that while the former describes random walk dynamics, the latter is mathematically equivalent to epidemic dynamics. We then relate these to well-known metrics used in network analysis: PageRank and Alpha-Centrality. We demonstrate by ranking nodes in an online social network used for broadcasting information, that non-conservative Alpha-Centrality leads to a better agreement with an empirical ranking scheme than the conservative PageRank. Our work unifies two areas of network analysis — centrality and epidemic models — and leads to insights into the relationship between dynamic processes and network structure, specifically, the existence of an epidemic threshold for non-conservative processes.

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### Moderation, as an escape from a persistent cycle of ideological revolutions

**Seth Marvel**

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**Seth Marvel, Hyunsuk Hong, Anna Papush, Steve Strogatz**

In many interconnected social communities—for example a well unified political party, a network of credit-lending institutions, or a nationally bound community of academic linguists—we often observe a sort of punctuated equilibrium of ideas: The community as a whole focuses on a certain topic or methodological approach for a while, then rapidly converts to a new, seemingly more reasonable alternative under sustained pressure from a small group of committed adherents. In many cases, the new approach has as many shortcomings as the old, but this is not fully appreciated until it has been widely adopted. (Once it is appreciated however, the community is ripe for a new shift of thought, and the cycle continues.) As a way out of these swings between ideological extremes, we might seek to encourage a more balanced or qualified approach. Here, however, we show that (in the context of a simplified model) doing so successfully is surprisingly difficult—perhaps explaining the persistence of the dramatic ideological shifts in the first place. In particular, only one out of the seven strategies that we analyze succeeds in expanding the subcommunity of moderates without risking its extinction.

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### Paladins as predators: invasive waves in a spatial evolutionary adversarial game

**Scott McCalla**

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Invasive waves are found in a novel variant of a reaction-diffusion system used to extend an evolutionary adversarial game into space wherein the influence of various strategies is allowed to diffuse. The waves are driven by a nonlinear instability that enables an otherwise unstable informant state to travel through an initially uncooperative state leaving a cooperative state behind. The wave speed's dependence on the various diffusion parameters is examined in one and two dimensions. Various other phenomena, such as pinning near a diffusive inhomogeneity, are also explored.

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### Filtering and estimation of self-exciting Cox processes with applications to social systems

**George Mohler**

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Self-exciting point processes have shown promise for modeling social event patterns where the occurrence of an event increases the likelihood of subsequent events. However, standard models assume a Poisson background rate for spontaneous events, an unrealistic assumption in many social systems. We introduce a self-exciting Cox process model where the background rate is driven by an Ornstein-Uhlenbeck stochastic differential equation. We then develop a methodology for simultaneous filtering and estimation of the intensity. Application to crime and security data sets are investigated.

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### Urban social dynamics: “Don’t buy the house, buy the neighborhood”

**Jean-Pierre Nadal**

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**Laetitia Gauvin, Annick Vignes**

“Don’t buy the house, buy the neighborhood” (Russian proverb). This talk is concerned with issues related to urban social segregation. Going beyond the simplest models introduced by T. C. Schelling in the 70’s, we introduce a model of housing transactions between agents heterogeneous in their income, hence in their willingness to pay. The goal is to see how the spatial income segregation depends on both economic constraints and social interactions. A key feature of the model is that agents preferences for a place are expressed in terms of a local attractiveness, which depends on both an intrinsic attractiveness of the location and the social characteristics of its neighborhood. The demand for an asset thus depends on its attractiveness, which in turn evolves according to the social characteristics of the newcomers. The resulting stationary space distribution of income is analytically characterized and illustrated by agent-based simulations. The main results are that, (1), socio-spatial segregation occurs if, and only if, the social influence is strong enough, and, (2), even so, some social diversity is preserved at most locations. Comparing with the Parisian housing market, the results reproduce general trends concerning the price distribution and the income spatial segregation.

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**Multi-Stage complex contagions****Mason Porter**University of Oxford, England  
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The spread of ideas across a social network can be studied using complex contagion models, in which agents are activated by contact with multiple activated neighbors. The investigation of complex contagions can provide crucial insights into social influence and behavior-adoption cascades on networks. In this paper, we introduce a model of a multi-stage complex contagion on networks. Agents at different stages—which could, for example, represent differing levels of support for a social movement or differing levels of commitment to a certain product or idea—exert different amounts of influence on their neighbors. We demonstrate that the presence of even one additional stage introduces novel dynamical behavior, including interplay between multiple cascades, that cannot occur in single-stage contagion models. We find, for example, that cascades—and hence collective action—can be driven not only by higher-stage influencers but also by lower-stage influencers.

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**Hotspot invasion: traveling wave solutions to a reaction-diffusion model for criminal behavior****Nancy Rodriguez**Stanford University, USA  
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We study a class of reaction-diffusion models that can be taken as basic models for the dynamics of criminal activity. In this talk I will first discuss the existence of steady-state solutions and discuss the condition that determine whether there are one, two, or three steady-states. The latter case corresponds to a bi-stable system and in this case we prove the existence of traveling wave solutions in one dimension. Physically, this correspond to the invasion of 'hotspots' into areas that have naturally low crime rates. I will also discuss some numerical results on obstruction of wave propagation.

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**Social interactions on networks: self-excitation, third-party inhibition, and the link with game theory****Martin Short**UCLA, USA  
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We introduce a point process model for social interactions on a network, including self-excitation and third-party inhibition. Here, a coupled system of state-dependent jump stochastic differential equations is used to model the conditional intensities of the directed network of interactions. The model produces a wide variety of transient or stationary weighted network configurations and we investigate under what conditions each type of network forms in the continuum limit. We also explore the link between this model and recent work on repeated games.

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**Locust dynamics: behavioral phase change, swarming, and nonlocal models****Chad Topaz**Macalester College, USA  
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Locusts exhibit two interconvertible phases, solitary and gregarious. Solitary (gregarious) individuals are repelled from (attracted to) others, and crowding biases conversion towards the gregarious form. We construct a nonlinear partial integrodifferential equation model of the interplay between phase change and spatial dynamics leading to the formation of locust swarms. Analysis of our model reveals conditions for the onset of a locust plague, characterized by a large scale transition to the gregarious phase. A model reduction to ordinary differential equations describing the bulk dynamics of the two phases enables quantification of the proportion of the population that will gregarize, and of the time scale for this to occur. Numerical simulations provide descriptions of the swarm structure and reveal transiently traveling clumps of gregarious insects.

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### Pattern formation under nonlocal social interaction

**David Uminsky**

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**James H. von Brecht, Theodore Kolokolnikov, Andrea Bertozzi**

Pairwise particle interactions arise in diverse biological systems ranging from insect swarms and flocking, to self-assembly of nanoparticles. Particle systems which communicate through kernels with long-range attraction and short-range repulsion may exhibit rich patterns in their bound states. In this talk we present a theory to classify the morphology of various patterns in  $N$  dimensions from a given social interaction force. We also present a method to solve the inverse problem: Given an observed pattern, can we construct a social interaction potential which exhibits that pattern.

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### Community detection among street gangs and gamma-convergence on graphs

**Yves Van Gennip**

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The goal of data clustering and community detection is to cluster nodes of a weighted graph into groups such that the edge weights between vertices in the same group is high and the outgroup weights are low. Linear methods based on graph Laplacians, such as spectral clustering, are popular tools to accomplish this goal. In this talk we investigate both linear methods and nonlinear methods based on the Ginzburg-Landau functional to detect social structures among street gangs. We use Gamma-convergence to connect the nonlinear method with the nonlocal means method (well known in image processing) as well as with classical continuum Ginzburg-Landau results.

This work has been done in collaboration with Andrea Bertozzi, Jeff Brantingham, Blake Hunter and others.

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## Special Session 4: Nonlinear PDEs and Control Theory with Applications

**Barbara Kaltenbacher**, University of Alpen-Adria, Klagenfurt, Austria

**Irena Lasiecka**, University of Virginia, USA

**Petronela Radu**, University of Nebraska-Lincoln, USA

**Lorena Bociu**, NC State University, United States

In the large context of nonlinear evolution equations we will focus on systems of PDEs which exhibit a hyperbolic or parabolic-hyperbolic structure. The topics of this special session will revolve around qualitative and quantitative properties of solutions to these equations, such as existence and uniqueness, regularity of solutions, and long time asymptotic behavior. Associated control theoretic questions such as stabilization, controllability and optimal control will be addressed as well. Both bounded and unbounded domain problems will be under considerations. Of special interest in our discussions are interactions involving nonlinearity and geometry. Several methods available for investigation of such problems will be presented. We anticipate also to discuss specific problems that arise in applications such as nonlinear acoustics, traveling waves in elasticity and viscoelasticity, plasma dynamics, and semiconductors.

### Rational decay of structural acoustic dynamics

**George Avalos**

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A rate of rational decay is obtained for solutions of a PDE model which has been used in the literature to describe structural acoustic flows. This structural acoustics PDE consists partly of a wave equation which is invoked to model the interior acoustic flow within a given cavity. Moreover, a structurally damped elastic equation is invoked to describe time-evolving displacements along the flexible portion of the cavity walls. The coupling between these two distinct dynamics will occur across a boundary interface. We obtain the uniform decay rate of this structural acoustic PDE without incorporating any additional boundary dissipative feedback mechanisms. By way of deriving this stability result, necessary a priori inequalities for a certain static structural acoustics PDE model are generated, thereby allowing for an application of a recently derived resolvent criterion for rational decay.

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### Weak and regular solutions for nonlinear waves with super-critical sources and nonlinear dissipations

**Lorena Bociu**

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We consider nonlinear wave equations characterized by energy building, super-critical sources and nonlinear dissipation terms. We provide sharp results on the range of parameters for damping terms and sources that determine the exact regions for existence and uniqueness for both weak and regular solutions, and finite time blow-up.

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### Algebraic Riccati equations with unbounded coefficients lacking analyticity of the free dynamics semigroup

**Francesca Bucci**

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The talk will report on the recent advances in the study of the Linear-Quadratic problem on an infinite time horizon for composite systems of evolutionary Partial Differential Equations which comprise both parabolic and hyperbolic components, such as models for thermoelastic, acoustic-structure and fluid-solid interactions. Emphasis will be placed on the far-reaching role of functional analytic methods at a theoretical level, as well as of parabolic regularity and microlocal analysis in order to establish the appropriate regularity of boundary traces which ultimately allows to ensure well-posedness of the corresponding algebraic Riccati equations. Most results presented in the talk are obtained jointly with Irena Lasiecka (Virginia) and Paolo Acquistapace (Pisa).

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### Uniform decay rates for the wave equation with locally distributed nonlinear damping in unbounded domains with finite measure

**Marcelo Cavalcanti**

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**Valeria Domingos Cavalcanti, Flavio R. Dias Silva**

This talk is concerned with the study of the uniform decay rates of the energy associated with the wave equation with locally distributed nonlinear damping

$$u_{tt} - \Delta u + a(x)g(u_t) = 0 \quad \text{in } \Omega \times (0, \infty)$$

subject to Dirichlet boundary conditions, where  $\Omega \subset \mathbb{R}^n$   $n \geq 2$ , is an *unbounded* open set with *finite measure* and *unbounded* smooth boundary  $\partial\Omega = \Gamma$ . The

function  $a(x)$ , responsible for the localized effect of the dissipative mechanism, is assumed to be nonnegative, essentially bounded and, in addition,

$$a(x) \geq a_0 > 0 \text{ a.e. in } \omega,$$

where  $\omega = \omega' \cup \{x \in \Omega; \|x\| > R\}$  ( $R > 0$ ) and  $\omega'$  is a neighbourhood in  $\Omega$  of the closure of  $\partial\Omega \cap B_R$ , where  $B_R = \{x \in \Omega; \|x\| < R\}$ .

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### Dissipative acoustic solitons

**Ivan Christov**

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Lagrangian-averaged models for compressible flow have recently been proposed [Bhat & Fetecau, DCDS-B 6 (2006) 979–1000]. Like their counterparts in turbulence, these models introduce a minimal (“cut-off”) length scale beyond which energy dissipation cannot occur. When applied to weakly-nonlinear acoustic phenomena in inviscid, lossless single-phase fluids, the Lagrangian-averaged model represents a higher-order dispersive regularization of the governing equation, which exhibits nonlinear dissipation as well [Keiffer et al., Wave Motion 48 (2011) 782–790]. Kink-type solitary wave solutions are derived analytically, and an implicit finite-difference scheme with internal iterations is constructed in order to study their collisions. It is shown that two kinks can interact and retain their identity after a collision, meaning that these waves represent dissipative acoustic solitons. For a different choice of parameters, finite-time blow-up can be observed numerically. Finally, while the classical equations of nonlinear acoustics can be reduced to Burgers’ equation, we show that the present model reduces to the Korteweg–de Vries equation.

This work is, in part, a collaboration with R.S. Keiffer, R. McNorton and P.M. Jordan from the Naval Research Laboratory, Stennis Space Center.

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### A unified theory for damped evolutionary equations

**Valéria Domingos Cavalcanti**

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**Marcelo Cavalcanti, Vilmos Komornik, Jose Henrique Rodrigues**

The main goal of this talk is to establish an abstract theory which allows us to determine the exponential decay in a certain Hilbert space  $H$  for locally damped evolution equations. Our approach can be used for a wide assortment of equations as, for instance, the

heat equation, Airy-Burgers equation, Schrödinger equation, among others.

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### Partial continuity for parabolic systems

**Mikil Foss**

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Consider the parabolic system

$$u_t - \operatorname{div}[a(x, t, u, Du)] = 0 \quad \text{in } \Omega_T := \Omega \times (-T, 0),$$

where  $\Omega \subset \mathbb{R}^n$  is a bounded domain and  $T > 0$ . The vector field  $a : \Omega_T \times \mathbb{R}^N \times \mathbb{R}^{N \times n} \rightarrow \mathbb{R}^N$  satisfies natural  $p$ -growth and ellipticity assumptions with  $p > 2n/(n+2)$ . I will present partial continuity results for weak solutions to the above problem. More precisely, the results establish that there is an open set of full measure in  $\Omega_T$  in which the solution is Hölder continuous. The key assumption for the problem being considered is that the vector field  $a$  is continuous with respect to the arguments  $x$ ,  $t$  and  $u$ . This distinguishes this result from others which assume Hölder continuity of  $a$  with respect to  $x$ ,  $t$  and  $u$  and provide partial continuity for the spatial gradient of the solution. The talk will focus mostly on the subquadratic setting, where  $2n/(n+2)$ .

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### Some elliptic and parabolic problems with boundary conditions of diffusive type

**Ciprian Gal**

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We will mainly focus on parabolic and elliptic equations with boundary conditions of diffusive type, which are sometimes dubbed as dynamic(al) or Wentzell boundary-type conditions. We wish to discuss issues involving global well-posedness and regularity of solutions, the asymptotic behavior as time goes to infinity in terms of global/exponential attractors. Finally, some explicit dimension estimates are provided to show a different degree of structural complexity of these systems when compared to the same equations subject to the usual Dirichlet and Neumann-Robin type of boundary conditions.

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### The polarization in a ferroelectric thin film: local and nonlocal limit problems

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**Kamel Hamdache**

In this joint work with K. Hamdache (Ecole Polytechnique, Palaiseau, France), starting from classical non-convex and nonlocal 3D-variational model of the electric polarization in a ferroelectric material, via an asymptotic process we obtain a rigorous 2D-variational model for the polarization in a ferroelectric thin film. Depending on the initial boundary conditions, the limit problem can be either nonlocal or local. Thin films of ferroelectric material are used for the realization of "Ram" for computers and "RFID cards".

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### The wave equation with abstract nonlinear acoustic boundary conditions

**Jameson Graber**

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In this talk we introduce a wave equation with "abstract" nonlinear acoustic boundary conditions, which consists of a wave equation coupled with an abstract second-order PDE where the coupling occurs at the boundary interface. This system resembles the original model proposed by Morse and Ingard but is sufficiently general to cover a wide range of problems studied in the literature. We show that under quite general assumptions, the system generates a nonlinear semigroup, and, in the presence of appropriately selected nonlinear boundary damping, the semigroup is uniformly stable. These results serve to unify a number of the available studies in the literature on PDE models of acoustic/structure interactions. In addition, we study the effects of a "nonlinear coupling" which arises in the case of a porous structure.

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### Asymptotic behavior of solutions to nematic liquid crystal models

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**Wu Hao**

We present some recent results on the longtime behavior of solutions to some simplified versions of the Ericksen-Leslie model for the nematic liquid crystal flow. More precisely, we consider first the one

proposed by F.-H. Lin et al. In this case the evolution system consists of the Navier-Stokes equations coupled with a convective Ginzburg-Landau type equation for the (vector-valued) averaged molecular orientations. We discuss the convergence of given trajectories to single equilibria when the Dirichlet boundary conditions for the order parameter and the external force acting on the flow are time dependent. Then we introduce a more refined model proposed by C. Liu et al. and we show that the corresponding dynamical system in two spatial dimensions has a finite dimensional global attractor.

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### Global attractors for von Karman plate model with a localized damping

**Pelin Guven Geredeli**

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**Irena Lasiecka, Justin T. Webster**

In this study dynamic von Karman equations with localized interior damping supported in a boundary collar are considered. Hadamard well-posedness for von Karman plates with various types of nonlinear damping are well-known, and the long-time behavior of nonlinear plates has been a topic of recent interest. Since the von Karman plate system is of "hyperbolicity" with critical nonlinearity (noncompact with respect to the phase space), this latter topic is particularly challenging in the case of geometrically constrained and nonlinear damping. In this paper we first show the existence of a compact global attractor for finite-energy solutions, and we then prove that the attractor is both smooth and finite dimensional. Thus, the hyperbolic-like flow is stabilized asymptotically to a smooth and finite dimensional set.

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### Free liquid fibers and films

**Thomas Hagen**

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The mathematical description of free liquid fibers and films poses serious analytical challenges. In the simplest case of a highly viscous material, this description, based on a long-wave approximation of the Navier-Stokes equations with free surface, is essentially due to Matovich, Pearson and Yeow. It takes the form of a nonlinear transport equation coupled to an elliptic momentum equation or system of such equations. In this presentation I will address recent results on stability and global existence in the absence of surface tension for equations of this type. Some reduced models will also be addressed.

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### Non-linear plate coupled with Darcy flows for slightly compressible fluid

**Akif Ibragimov**

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**E.Aulisa, Y.Kaya-Cekin**

In this work, we consider the dynamical response of a non-linear plate with viscous damping, perturbed in both vertical and axial directions interacting with a Darcy flow. We first study the problem for the non-linear elastic body with damping coefficient. We prove existence and uniqueness of the solution for the steady state plate problem. We investigate the stability of the dynamical non-linear plate problem under some condition on the applied loads. Then we explore the fluid structure interaction problem with a Darcy flow in porous media. In an appropriate Sobolev norm, we build an energy functional for the displacement field of the plate and the gradient pressure of the fluid flow. We show that for a class of boundary conditions the energy functional is bounded by the flux of mass through the inlet boundary.

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### Nonlinear poroacoustic flow in rigid porous media

**Pedro Jordan**

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**J. K. Fulford**

An acoustic acceleration wave is defined as a propagating singular surface (i.e., wavefront) across which the first derivatives of the velocity, pressure, or density exhibit jumps. In this talk, the temporal evolution of the amplitude and speed of such waves are investigated in the context of nonlinear, fluid-acoustic propagation in rigid porous media, where the fluid-solid interaction is described by the resistance law of Darcy. It is shown that there exists a critical value, the constant  $\alpha^* (> 0)$ , of the initial jump amplitude. It is then established that the acceleration wave magnitude either goes to zero, as  $t \rightarrow \infty$ , or blows-up, in finite time, depending on whether the initial jump amplitude is less than or greater than  $\alpha^*$ .

Additionally, numerical solutions of an idealized initial-boundary value problem involving sinusoidal signaling in a fluid-saturated porous slab are used to illustrate the finite-time transition from acceleration to shock wave, which occurs when the initial jump amplitude is greater than  $\alpha^*$ , and comparisons with the linearized case (i.e., the damped wave equation) are presented whenever possible. Finally, the related phenomenon of weakly-nonlinear poroacoustic traveling waves, where an exact solution is possible in terms of the Lambert  $W$ -function, is briefly considered and connections to second-sound (i.e., thermal

wave) phenomena are noted. (Work supported by ONR funding.)

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### Remarks on the Fourier series method

**Vilmos Komornik**

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We review some recent applications of harmonic and nonharmonic Fourier series in control theory, including various results obtained in collaboration with C. Baiocchi, P. Loreti, M. Mehrenberger and A. Barhoumi. We discuss several possible generalizations of Parseval type inequalities due to Ingham and Beurling .

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### On the exterior problem for nonlinear wave equations in 2D

**Hideo Kubo**

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In this talk I wish to present a result on the exterior problem for the nonlinear wave equations in two space dimensions. Because the dispersive property for the solution to the wave equation is rather weak in 2D, it is not straightforward to get counterpart for the Cauchy problem. Nevertheless, under the geometric assumption on the obstacle, we are able to get an almost global solution for small initial data. Moreover, if we pose the null condition on the nonlinearity, then the solution actually exists globally.

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### Recovering sound speed and initial data for the wave equation by a single measurement

**Shitao Liu**

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We consider a problem of recovering the sound speed and an initial condition for wave equations which is motivated from the photo/thermo-acoustic imaging model. We will also show the connection between such a problem and the classical inverse hyperbolic problem with a single measurement.

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**Time optimal boundary controls for the heat equation****Sorin Micu**University of Craiova, Romania  
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The fact that the time optimal controls for parabolic equations have the bang-bang property has been recently proved for controls distributed inside the considered domain (interior control). The main result in this article asserts that the boundary controls for the heat equation have the same property, at least in rectangular domains. This result is proved by combining methods from traditionally distinct fields: the Lebeau-Robbiano strategy for null controllability and estimates of the controllability cost in small time for parabolic systems, on one side, and a Remez-type inequality for Müntz spaces and a generalization of Turán's inequality, on the other side.

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**Inverse problem for the heat equation and the Schrödinger equation on a tree****Ademir Pazoto**Federal University of Rio de Janeiro, Brazil  
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In this paper we establish global Carleman estimates for the heat and Schrödinger equations on a network. The heat equation is considered on a general tree and the Schrödinger equation on a star-shaped tree. The Carleman inequalities are used to prove the Lipschitz stability for an inverse problem consisting in retrieving a stationary potential in the heat (resp. Schrödinger) equation from boundary measurements.

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**Instability for nonlinear evolution equations****Petronela Radu**University of Nebraska-Lincoln, USA  
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We consider steady solutions of semilinear parabolic and hyperbolic equations; the hyperbolic equations exhibit a sign changing damping term. We prove that linear instability with a positive eigenfunction gives rise to nonlinear instability by either exponential growth or finite-time blow-up. We then discuss a few examples to which our main theorems are immediately applicable, including equations with supercritical and exponential nonlinearities.

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**Local and global well-posedness to a nonlinear model in viscoelasticity with  $m$ -Laplacian damping****Mohammad Rammaha**University of Nebraska-Lincoln, USA  
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We consider a general viscoelasticity model in a bounded domain  $\Omega \subset \mathbb{R}^3$  with a smooth boundary  $\Gamma$ :

$$u_{tt} - \Delta_p u - \Delta_m u_t = f(u) \quad \text{in } \Omega \times (0, T),$$

subject to Dirichlet boundary condition:  $u = 0$  on  $\Gamma \times (0, T)$ , or subject to  $p$ -generalized Robin boundary condition. We study the local and global solvability in the interesting case when  $p$  is between 2 and 3. In this case, if the interior source  $f(u)$  is of order  $r$  (i.e.,  $|f(u)| \leq c|u|^r$  for all  $|u| \geq 1$ ), then  $f(u)$  is locally Lipschitz from  $W^{1,p}(\Omega)$  into  $L^2(\Omega)$ , only for the values  $1 \leq r \leq \frac{3p}{2(3-p)}$ . Thus, our goal is to prove the local and global solvability of the problem when the interior source is of supercritical order, i.e.,  $\frac{3p}{2(3-p)} < r < \frac{3p}{3-p}$ .

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**Asymptotic behavior of the wave equation with dynamic boundary condition****Belkacem Said-Houari**KAUST university, Saudi Arabia  
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In this talk, we consider a semi-linear wave equation with a dynamic boundary conditions.

We prove the local existence and uniqueness of the solution of our problem. We use the Faedo-Galerkin approximation coupled with the fixed point theorem. Concerning the asymptotic behavior of the solution of this problems we prove the following :

When the initial data are large enough and the damping is nonlinear, then the energy solution is unbounded. In fact, we show that the  $Lp$ -norm of the solutions grows as an exponential function.

We prove that in the case of linear boundary damping, the solution blows up in finite time.

We showed also that if the initial data are in the stable set, the solution continues to live there forever.

In addition, we show that the presence of the strong damping forces the solution to go to zero uniformly and with an exponential decay rate. To obtain our results, we combine the potential well method with the energy method.

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### Solutions to the Euler equation on a domain with a moving boundary

**Amjad Tuffaha**

The Petroleum Institute, United Arab Emirates

**Igor Kukavica**

We present a new proof of local-in-time existence and regularity of solutions to the free boundary Euler Equation without surface tension under the Taylor-Rayleigh condition.

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### Simultaneous controllability and stabilization of some uncoupled wave and plate equations

**Louis Tebou**

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First, we discuss the simultaneous controllability of uncoupled wave equations with different speed of propagation in a bounded domain. The control is locally distributed, and the control region satisfies the geometric control condition of Bardos-Lebeau-Rauch. Thanks to the Hilbert uniqueness method of Lions, the controllability problem is reduced to an observability one. To solve the observability problem, we employ a combination of the observability result of Bardos-Lebeau-Rauch for a single wave equation, and a new uniqueness result for the uncoupled system. Then, we address the related stabilization problem with the help of a theorem of Haraux and the new uniqueness result. Afterwards, we investigate similar problems for uncoupled plate equations with the help of the Holmgren uniqueness theorem. Finally, we discuss some generalizations, and some open problems.

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### Carleman estimates and stabilization of hyperbolic systems in absence of geometric observability conditions

**Daniel Toundykov**

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**Matthias Eller**

The sharp sufficient conditions for observability in control systems are formulated via geometric optics and are linked to the structure of closed geodesics in the underlying physical domain. However, the necessary unique continuation property for PDEs is an intrinsically weaker requirement and does not impose such restrictions. It is, therefore, often possible to stabilize an evolution system by placing feedback controls on subsets of the domain that fail to satisfy

the geometric conditions. The price to pay is the necessity to work with smoother solutions and the stabilization rates obtained thereby are no longer exponential.

In this work we present specialized Carleman estimates and a generalization of a pioneering strategy due to G. Lebeau and L. Robbiano to prove uniform stability (for strong solutions) of 1st-order hyperbolic systems without reliance on the geometric observability assumptions.

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### An inverse problem for the ultrasound equation: uniqueness and stability

**Roberto Triggiani**

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We consider a third order (in time) ultrasound equation arising in acoustics and we recover a critical coefficient by means of boundary measurement. More specifically, we obtain uniqueness and stability of the recovery

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### Min-max game problem for coupled PDE system and an application to the fluid-structure interaction model

**Jing Zhang**

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**Irena Lasiecka, Roberto Triggiani**

We consider an established hyperbolic-parabolic fluid-structure interaction model with control and disturbance acting at the interface between the two media. Here, the structure (modeled by the system of dynamic elasticity) is immersed in a fluid (modeled by the linearized Navier-Stokes equations). A game theory problem between control and disturbance is studied, when both act at the interface. To this end, the main mathematical difficulty that one encounters is the fact that such model fails to satisfy the "singular estimate" from, say, control to state. This is a critical obstacle, as this is precisely the foundational property for a full theory to include solvability of the Differential Riccati equation in the study of the associated min-max game theory problem. Failure of the "singular estimate" property is due to a mismatch between the parabolic and hyperbolic component of the overall coupled dynamics. By introducing suitable observation or output operators, with incremental smoothness on the trajectory, it is shown that the resulting system satisfies a modified singular estimate, this time from the control to the observation space. This then allows one to adapt the complete min-max theory to the present

fluid-structure interaction model. More precisely, the approach followed is based on an abstract setting, of which the fluid-structure interaction model is a canonical illustration, which enjoys a desirable property not assumed for the abstract model.

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## Special Session 5: Hybrid Monte Carlo

**Elena Akhmatskaya**, Basque Center for Applied Mathematics, Spain  
**J. M. Sanz-Serna**, Universidad de Valladolid, Spain

The hybrid Monte Carlo (HMC) method was first introduced by Duane and his coworkers in 1987 in order to combine the best features of two well established simulation techniques, molecular dynamics and Monte Carlo. The HMC method proved itself to be both an efficient sampling device and an effective realization of a stochastic thermostat. Some of its weakness are its inability to reproduced dynamical properties and its lack of efficiency in the simulation of large systems.

An important development came in 1996, when Neal successfully applied HMC to neural network models, thus demonstrating its suitability for statistical computations. Recent developments have aimed at increasing the capability of simulating large molecules, retaining dynamical information and solving large dimensional statistical problems.

The purpose of this session is to bring together researchers with interests in different application fields in physics, chemistry and statistics, as well as in numerical and optimization techniques, to promote further development and dissemination of the method.

### Generalized shadow hybrid Monte Carlo: from theory to useful tools

**Elena Akhmatskaya**

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**S. Reich, B. Escribano, J.M. Asua, J.I. Mujika**

Recently introduced Generalized Shadow Hybrid Monte Carlo (GSHMC) method has found applications in areas such as atomistic simulation, particle simulation and Bayesian computation. This required the development of four variants of the original GSHMC technique: (1) the standard GSHMC (a thermodynamically consistent implementation of constant-temperature molecular dynamics); (2) MTS-GSHMC (a multi-time stepping GSHMC); (3) meso-GSHMC (a Metropolis corrected dissipative particle dynamics method) and (4) the GSHmMC (a Generalised Shadow Hamiltonian Monte Carlo approach employing modified Hamiltonians in the Metropolis test). We highlight the ideas behind the GSHMC methods, while focusing on their implementation and on the range of applications expected to benefit most from their introduction. We describe implementation of GSHMC and MTS-GSHMC in two popular molecular simulation software packages, GROMACS and PROTOMOL, aimed at high performance computers. Performance of GSHMC methods is analysed in some detail and illustrated by the examples of two on-going studies: dynamic modeling of multiphase polymer morphology development and large-scale simulations of Serum Transferrin- Aluminium complexes.

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### Constant-pH molecular dynamics using stochastic titration: theory and applications

**Antonio Baptista**

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**Miguel Machuqueiro, Sara R. R. Campos, Pedro R. Magalhaes**

Although the behavior of many biomolecules of interest depends markedly on the acid/base equilibrium of their protonatable groups, molecular dynamics (MD) simulations are traditionally performed using a fixed and somewhat arbitrary protonic state based on the  $pK_a$  values of those groups in solution. This talk discusses the stochastic titration method for constant-pH MD, where protonatable groups vary their state along the simulation according to the pH value of the solution, addressing its theoretical basis and some illustrative applications.

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### Hamiltonian Monte Carlo with endogenous splitting

**Martin Burda**

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**John Maheu**

Splitting methods for Hamiltonian Monte Carlo have been shown to exhibit improvements in sampling efficiency over standard Hamiltonian Monte Carlo approaches. Splitting methods separate the Hamiltonian function and apply varying effective stepsizes for the split terms. In this paper we explore Hamiltonian splitting determined endogenously based on the underlying structure of the economic model. Preliminary output suggests efficiency gains of the endogenous split method relative to the regular HMC.

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### Hybrid Monte Carlo for long timestep Langevin dynamics

**Jesus Izaguirre**

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**Haoyun Feng, Chris R. Sweet**

Langevin dynamics is an efficient thermostatted form of molecular dynamics. We have introduced the Normal Mode Langevin (NML) dynamics that accurately integrates low frequency motions and approximates fast frequency motions near equilibrium using Brownian dynamics. This allows for long timesteps (100 fs and above) and large speedups (10x to 50x) compared to plain Langevin dynamics. It is attractive to combine Langevin dynamics with Hybrid Monte Carlo to make sampling more rigorous. I will present Hybrid Monte Carlo extensions of NML that can be used for rigorous sampling of either the low or fast frequency motions or both. These HMC extensions will be derived using the Nonequilibrium Driven Dynamics approach recently introduced by Crooks and Chodera.

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### Monte Carlo sampling of RNA 3D graphs

**Namhee Kim**

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**Christian Laing, Shereef Elmetway, Segun Jung, Jeremy Curuku, Tamar Schlick**

RNA structure prediction is a mathematical and computational challenge with important biological applications in RNA structure and function. In this talk, I present a hierarchical Monte Carlo approach to predict RNA helical arrangements by a coarse-grained sampling of 3D graphs guided by knowledge-based potentials derived from geometrical measures based on solved structures. The coarse-grained model using newly developed 3D RNA graphs accelerate global sampling of candidate RNA topologies. Monte Carlo results are compared to reference graphs from both solved structures and predicted structures using current available programs. The comparison indicates promise for our graph-based sampling approach for characterizing 3D global helical arrangements in large RNA from a given secondary structure and offer reasonable candidate for further refinement.

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### Langevin dynamics with constraints

**Tony Lelievre**

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We consider Langevin processes with mechanical constraints. The latter are a fundamental tool in molecular dynamics simulation for sampling purposes and for the computation of free energy differences. We will present three main results: (i) We propose a simple discretization of the constrained Langevin process based on a splitting strategy. We show how to correct the scheme so that it samples exactly the canonical measure restricted on a submanifold, using a Metropolis-Hastings correction in the spirit of the Generalized Hybrid Monte Carlo (GHMC) algorithm. Moreover, we obtain, in some limiting regime, a consistent discretization of the overdamped Langevin dynamics on a submanifold, also sampling exactly the correct canonical measure with constraints. (ii) For free energy computation using thermodynamic integration, we rigorously prove that the long time average of the Lagrange multipliers of the constrained Langevin dynamics yields the gradient of a rigid version of the free energy associated with the constraints. A second order time discretization using the Lagrange multipliers is proposed. (iii) The Jarzynski-Crooks fluctuation relation is proved for Langevin processes with mechanical constraints evolving in time. An original numerical discretization without time discretization error is proposed, and its overdamped limit is studied. This talk is based on the following reference: T. Lelievre, M. Rousset and G. Stoltz, Langevin dynamics with constraints and computation of free energy differences, to appear in Mathematics of Computation.

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### A superconvergent method for configurational sampling using Langevin dynamics

**Charles Matthews**

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**B. Leimkuhler**

In Molecular Dynamics, trajectories are often treated from a statistical perspective: we use them to sample the overall phase space with respect to a desired statistical ensemble. In sampling the Canonical (NVT) ensemble, typically a stochastic perturbation to Newton's equations is used to generate such trajectories. However, once such a thermostatted system has been defined, the challenge remains to integrate the equations in an efficient and robust manner, particularly in a Molecular Dynamics context where rugged potentials and the high cost of force evaluations can stymie such efforts. In this talk, we will present some mathematical results on invariant measures for methods based on splittings of Langevin Dynamics,

and give some innovative numerical methods where the goal is exclusively configurational sampling. This talk describes joint work with Ben Leimkuhler.

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### Calculations of binding free energies with molecular simulations: progress and challenges

**David Mobley**

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**Gabriel Rocklin, Pavel Klimovich**

Calculation of binding strength, or affinity, between proteins, life's molecular machines, and small molecules is of substantial interest in pharmaceutical drug discovery. Accurate techniques for such calculations could yield substantial benefits in drug discovery pipelines. Here, we discuss progress calculating binding strengths, or free energies, from molecular simulations. We present the highlights of our calculations in several different binding sites, focusing on what we learned as well as on sampling challenges that can benefit from enhanced sampling techniques such as hybrid Monte Carlo. Overall, these calculations based on physical force fields are reaching the accuracy where they can, in favorable situations, yield sufficient accuracy to benefit pharmaceutical drug discovery, yet many challenges remain.

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### An application of a hybrid Monte Carlo method in path space

**Frank Pinski**

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**P.J. Malsom**

We are investigating collections of atoms as their positions evolve under Brownian (over-damped Langevin) dynamics. In the cases where the collection changes conformation, an energy barrier often exists. Such transitions are rare events when the thermal energy is small compared to the energy barrier. The understanding of such rare events is the goal of our studies. We sample the transition paths in a thermodynamically significant manner using a Hybrid Monte Carlo (HMC) Method in Path Space. The relative probability of paths is computed using expressions generated from Ito's equation and the Girsanov formula. In implementing the HMC, auxiliary variables (velocities) are introduced with the masses chosen to help equalize the time scales. Moreover the method preserves the quadratic variation along the paths. The immediate focus of this work is to study transitions in small Lennard-Jones clusters. We consider a low energy transformation

in 13-atom and 14-atom clusters. The 14-atom cluster consists of one atom sitting on the surface of a close-packed structure of the others. In this cluster, the single "surface" atom drives into the cluster and pushes an atom, initially on the opposite side of the cluster, onto the surface. In the 13-atom cluster, we investigated a similar mode.

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### Hybrid Monte Carlo on Hilbert spaces

**J Sanz-Serna**

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**A. Beskos, F.J. Pinski, A.M. Stuart**

We define a generalized Hybrid Monte Carlo algorithm which overcomes this problem for target measures arising as finite-dimensional approximations of measures  $\pi$  which have density with respect to a Gaussian measure on an infinite-dimensional Hilbert space. The performance of the algorithm does not deteriorate as the dimensionality of the problem increases.

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### Mass tensor molecular dynamics and hybrid Monte Carlo

**Robert D. Skeel**

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In hybrid Monte Carlo and for most application of molecular dynamics, the masses are relevant only to the extent that they enhance sampling. Back in 1975, it was proposed by C.H. Bennett that, in such cases, the physical masses be replaced by an arbitrary positive definite symmetric mass tensor. Much more recently, such an approach was developed independently for statistical applications by Girolami and Calderhead and applied with considerable success. Building on these insights, preliminary results will be presented concerning the possibility of applying this methodology in a rigorous fashion to molecular dynamics and hybrid Monte Carlo. This is joint work with Y. Fang, M. Calvo, and J.M. Sanz-Serna.

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### Riemann manifold hybrid Monte Carlo and alternative metrics

**Vassilios Stathopoulos**

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**Mark Girolami**

For the purposes of statistical inference taking a geometric perspective and defining Hybrid Monte Carlo on a differentiable manifold defined by a model

based Metric tensor and associated connection provides a powerful Markov chain Monte Carlo (MCMC) method. Although the Fisher Information (FI) is the natural metric for probability density functions there are cases where the metric can be degenerate or not analytically defined. In this talk an overview of the Riemann Manifold Hybrid Monte Carlo algorithm of Girolami and Calderhead (2010) will be given and example applications where the exact (FI) is degenerate or not analytic will be discussed. For such degenerate cases we can augment the FI with the Hessian of the prior while when the FI is not analytic we can consider local or finite estimate approximations of the FI. Examples will consist of a probit and logistic regression and finite Gaussian mixture models.

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## Special Session 6: Dispersal in Heterogeneous Landscape

Robert Stephen Cantrell, University of Miami, USA  
 Chris Cosner, University of Miami, USA  
 Yuan Lou, Ohio State University, USA  
 Juan Diego Davila, Universidad de Chile, Chile  
 Alexander Quaas, Universidad Tecnica Federico Santa Maria, Chile

This special session aims to address some recent developments in mathematical modeling and analysis of dispersal in heterogeneous landscape. Dispersal of organisms is a key component of ecological and evolutionary processes. Incorporating heterogeneity and dispersal into models for biological processes leads to new mathematical models and brings new mathematical challenges. More generally, understanding dispersal in spatially and/or temporally varying environments can significantly enhance our understanding of how diversity is maintained in complex foodwebs and how organisms can respond to global change.

### A new inequality on the spectral bound of resolvent positive operators that unifies results on the evolution of dispersal

Lee Altenberg

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The spectral bound,  $s(aA + bV)$ , of a combination of a resolvent positive linear operator  $A$  and an operator of multiplication  $V$ , was shown by Kato (1982) to be convex in  $b \in R$ . Kato's result is shown here to imply, through an elementary "dual convexity" lemma, that  $s(aA + bV)$  is also convex in  $a > 0$ , and notably,  $\partial s(aA + bV)/\partial a < s(A)$ . Diffusions typically have  $s(A) < 0$ , so that for diffusions with spatially heterogeneous growth or decay rates, greater mixing reduces growth. Models of the evolution of dispersal in particular have found this result when  $A$  is a Laplacian or second-order elliptic operator, or a nonlocal diffusion operator, implying selection for reduced dispersal. These cases are shown here to be part of a single, broadly general, "reduction" phenomenon. A key open problem is to characterize operators  $B$  for which  $s(aA + bB)$  decreases in  $a$ . By dual convexity, a sufficient conditions is that  $(aA + bB)$  be convex in  $b$ .

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### Regularity for solutions of non local parabolic equations

Hector Chang Lara

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Gonzalo Davila

We study the regularity of solutions of parabolic fully nonlinear nonlocal equations. We prove  $C^a$  regularity in space and time and, under different assumptions on the kernels,  $C^{1,a}$  in space for translation invariant equations. The proofs rely on a weak parabolic ABP in order to prove a Point Estimate which allows to decrease the oscillation of the solution geometrically in the interior. Our results remain uniform as  $s \rightarrow 2$  allowing us to recover most of

the classical regularity results for fully non linear parabolic equations.

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### Evolutionary stability of ideal free dispersal strategies: a nonlocal dispersal model

Chris Cosner

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R. S. Cantrell, Y. Lou, D. Ryan

An important problem in the study of the evolution of dispersal is determining what kinds of dispersal strategies are evolutionarily stable. This talk will present results on the evolutionary stability of ideal free dispersal strategies in the context of continuous time nonlocal dispersal models. This work extends recent results by Cosner et al. [1], where a class of ideal free dispersal kernels was introduced and results were obtained that suggested the dispersal strategies they define should be evolutionarily stable. This talk will describe a more general class of ideal free dispersal kernels and give results showing that the ideal free dispersal strategies they define are indeed evolutionarily stable. These results also partially extend some recent work on the evolutionary stability of ideal free dispersal for reaction-diffusion equations and patch models to nonlocal dispersal models.

[1] C. Cosner, J. Davila and S. Martinez, Evolutionary stability of ideal free nonlocal dispersal, J. Biological Dynamics, v.6 (2012), 395-405

This research was partially supported by the NSF grants DMS-0816068 (RSC, CC, DR) and DMS-1021179 (YL).

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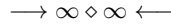
### Recent advance in heterogeneous nonlocal models for population dynamics

**Jerome Coville**

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to be announced



### A Lane-Emden equation with the fractional Laplacian

**Juan Davila**

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**Manuel del Pino, Yannick Sire**

We consider the Lane-Emden equation

$$(-\Delta)^s u = u^p, u > 0$$

either in entire space or a bounded domain and where  $p > 1$ . One of the contributions is the nondegeneracy of the bubble solution for the critical exponent, and its application for concentration phenomena for slightly subcritical equations. We also study the supercritical equation in entire space.



### Fish biomass production and dispersal across a seasonally flooded marsh

**Donald Deangelis**

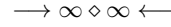
University of Miami, USA

don\_deangelis@usgs.gov

**Simeon Yurek, Joel C. Trexler, Fred Jopp, Douglas D. Donalson**

Small fishes in seasonally flooded environments such as the Everglades are capable of spreading into newly flooded areas and building up substantial biomass. We tested three hypothesized mechanisms for movement of fish into flooding areas; (1) spread through reaction-diffusion, (2) the refuge mechanism that remnant populations of small fishes survive the dry season in small permanent bodies of water, and (3) the dynamic ideal free distribution mechanism that a combination of expansion of habitat by flooding and consumption of prey by the fish creates a prey density gradient and that fish actively follow this. We concluded that although refugia may play an important role in recolonization by the fish population during reflooding, active movement up the prey gradient also seems capable of matching empirical observations. Further studies of the fish biomass dynamics on a seasonally varying, spatially heterogeneous wetland required simulations. We simulated population dynamics (growth, mortality, predation, movement patterns, and resource competition) of

three small fish functional groups across a grid-based spatially-explicit, heterogeneous, two-dimensional marsh slough landscape using hydrologic variability as the primary driver for macro-spatial scale movement. The model predicts the spatio-temporal pattern of fish biomass stranded (thus easily available for wading birds) as water levels decline.



### Linking individual movements and population patterns in dynamic landscapes

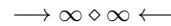
**William Fagan**

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**Thomas Mueller, Justin Calabrese, Chris Fleming, Chris Cosner, Steve Cantrell**

Real landscapes are dynamic in space and time, and the scale over which such variation occurs can determine the success of different strategies of population growth and movement. Real species rely on a variety of individual-level behaviors to move in and navigate through their landscapes. Such behaviors include (1) non-oriented movements based on diffusion and kinesis in response to proximate stimuli, (2) oriented movements utilizing perceptual cues, and (3) memory mechanisms that assume prior knowledge of movement targets. Species use of these mechanisms depends on life-history traits and resource dynamics, which together shape population-level patterns such as range residency, migration, and nomadism. This talk will draw upon empirical data, remote sensing imagery, and a variety of mathematical models to demonstrate the connections among individual movements, landscape dynamics, and population-level patterns. Preliminary work on our current project using integrodifferential equations to represent non-local information gathering will be discussed, focusing on results for bounded domains.



### Evolutionary ecology of habitat selection by predators and prey

**Samuel Flaxman**

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**Yuan Lou, Francois Meyer**

An essential key to explaining the mechanistic basis of ecological patterns lies in understanding the consequences of adaptive behavior for distributions and abundances of organisms. We developed a model that simultaneously incorporates (i) ecological dynamics across three trophic levels, (ii) evolution of behaviors via the processes of mutation, selection, and drift in populations of variable, unique individuals, and (iii) coevolution of traits mediating interactions between predators and prey. Results from the

model yield empirically testable predictions about dispersal strategies, spatial distributions, and ecological dynamics of predator-prey-resource systems.

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### Accelerating solutions in integro-differential equations

**Jimmy Garnier**

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I present the spreading properties of the solutions of an integro-differential equation of the form  $u_t = J * u - u + f(u)$ . I focus on equations with slowly decaying dispersal kernels  $J(x)$  which correspond to models of population dynamics with long-distance dispersal events. I show that for kernels  $J$  which decrease to 0 slower than any exponentially decaying function, the level sets of the solution  $u$  propagate with an infinite asymptotic speed. Moreover, I obtain lower and upper bounds for the position of any level set of  $u$ . These bounds allow me to estimate how the solution accelerates, depending on the kernel  $J$ : the slower the kernel decays, the faster the level sets propagate. My results are in sharp contrast with most results on this type of equation, where the dispersal kernels are generally assumed to decrease exponentially fast, leading to finite propagation speeds.

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### On pointwise estimates for non-local elliptic equations

**Nestor Guillen**

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**Russell Schwab**

Convexity has played an important role in fully non-linear equations, for instance in obtaining pointwise bounds for viscosity solutions, as in the celebrated Aleksandrov-Bakelman-Pucci theorem (ABP), or in understanding the regularity of the free boundary in the obstacle problem. In this talk I will describe the shortcomings of convexity when dealing with non-local problems and will discuss alternatives as well as their applications. In particular, I will describe a result obtained with Russell Schwab regarding pointwise bounds (analogous to the ABP) for weak solutions of non-local elliptic equations which is new even for non-local linear operators.

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### PDE vs ODE dynamics

**King-Yeung Lam**

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**Yuan Lou**

Dynamics, or behavior of solutions of a given nonlinear reaction-diffusion system is deeply related to that of its kinetic problem. In this lecture, we will use the classical Lotka-Volterra competition model as an example to illustrate some connections between the two. Also, the asymptotic behavior of the principal eigenvalue of linear cooperative elliptic systems, as the diffusion rates approach zero, will be motivated and studied.

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### Optimization of the persistence threshold in spatial environments with localized patches.

**Alan Lindsay**

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**M.J. Ward**

Determining whether a habitat with fragmented or concentrated resources best supports a contained population is a natural question to ask in Ecology. Such fragmentation may occur naturally or as a consequence of human activities related to development or conservation. In certain mathematical formulations of this problem, a critical value known as the persistence threshold indicates the boundary in parameter space for which the species either persists or becomes extinct. By assuming simple diffusive logistic dynamics for the population and accommodating the heterogeneous nature of the landscape with a spatially varying growth rate, a simple formulation for the persistence threshold is afforded in terms of an indefinite weight eigenvalue problem. In this talk I will show that for a growth rate with strongly localized patches of favorable habitat, the persistence threshold can be calculated implicitly and minimized with respect to the location and fragmentation of patches. This reveals an optimal strategy for minimizing the persistence threshold, thereby allowing the species to persist for the largest range of physical parameters. The techniques developed can be extended to study the effects of heterogeneity in a variety of ecological models.

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### Asymptotic behavior of a nonlocal inhomogeneous equation

**Salome Martinez**

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In this talk we will study the asymptotic behavior of the solutions of

$$u_t(x, t) = \int_{\mathbb{R}} J\left(\frac{x-y}{g(y)}\right) \frac{u(y, t)}{g(y)} dy - u(x, t),$$

with  $J : \mathbb{R} \rightarrow \mathbb{R}$  is a nonnegative even function with compact support such that

$$\int_{\mathbb{R}} J(y) dy = 1.$$

In this equation the dispersal is inhomogeneous in space and the step size,  $g(y)$ , of the dispersal depends on the position  $y$ . We assume that  $g(y) > 0$  in  $R$ . In particular, we will study the behavior of the steady state solutions of this equation.

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### Dynamics of a three species competition model

**Daniel Munther**

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**Yuan Lou**

We investigate the dynamics of a three species competition model, in which all species have the same population dynamics but distinct dispersal strategies. Gejji et al. (*Evolutionary convergence to ideal free dispersal strategies and coexistence*, Bull. Math. Biol., 74 (2012), 257-299) introduced a general dispersal strategy for two species, termed as an ideal free pair in this talk, which can result in the ideal free distributions of two competing species at equilibrium. We show that if one of the three species adopts a dispersal strategy which produces the ideal free distribution, then none of the other two species can persist if they do not form an ideal free pair. We also show that if two species form an ideal free pair, then the third species in general can not invade. When none of the three species is adopting a

dispersal strategy which can produce the ideal free distribution, we find some class of resource functions such that three species competing for the same resource can be ecologically permanent by using distinct dispersal strategies.

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### The dynamics of resource theft in a spatially continuous habitat

**Andrew Nevai**

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**Chris Cosner**

We describe the dynamics of a producer-scrounger system in a spatial habitat using a partial differential equation model. Both species are assumed to increase logistically and to move randomly in their environment. Although only producers can utilize the resource in its raw form, they are parasitized by nearby scroungers stealing resources from them. When possible, parameter combinations which allow producers and/or scroungers to persist either alone or together are distinguished from those in which they cannot.

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### A cross-diffusion model for avoidance behavior in an intraguild predation community

**Daniel Ryan**

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Intraguild predation (IGP) refers to a community module that blends competitive and predator-prey dynamics. Although IGP is widespread in nature, spatially homogeneous models for IGP communities predict that stable coexistence is only possible if restrictive conditions on resource productivity, competitive ability and predation susceptibility are satisfied. This talk will consider the population dynamics of an IGP module in a spatially heterogeneous landscape and examine how avoidance strategies deployed by the intraguild prey can lead to more robust coexistence states.

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## Special Session 7: Recent Progress in the Mathematical Theory of Compressible and Incompressible Fluid Flows

**Eduard Feireisl**, Mathematical Institute, Academy of Sciences, Prague, Czech Republic  
**Sarka Necasova**, Mathematical Institute, Academy of Sciences, Prague, Czech Republic

The session is devoted to the recent development of the mathematical theory of compressible and incompressible fluids in motion. The main topics include the qualitative properties of weak and strong solutions, long-time behavior, fluids confined to special geometries, rotating fluids, among others

### Decay estimates of the Oseen flow in the plane

**Toshiaki Hishida**

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We consider the initial value problem for the Oseen system in 2D exterior domains and study the local energy decay of solutions. For 3D case the theory was well developed by Kobayashi and Shibata, while 2D case has remained open. The result is applied to deduction of  $L^p$ - $L^q$  estimates of the Oseen semigroup. The dependence of estimates on the Oseen parameter is also discussed.

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### Fluid mechanical models of self-organized dynamics

**Trygve Karper**

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**Antoine Mellet, Konstantina Trivisa**

In nature one can find a wide range of species exhibiting self-organizing behavior. For instance, in a school of fish, or in a flock of birds, there are no external forces to coordinate the group, there is no leader to guide them. Starting from individual based models at the microscopic scale one can derive corresponding kinetic models for the mesoscopic scale. In this talk, we will rigorously study the hydrodynamic limit of a class of such kinetic models and obtain appropriate macroscopic models. The limiting system will be the compressible isentropic Euler equations with additional non-local terms modeling flock behavior such as attraction and alignment. We will also discuss several open problems related to the macroscopic model such as existence, regularity, and, in particular, large time behavior. The latter will be illustrated with numerical experiments.

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### Long time behavior of solutions to nonlinear flow-structure interactions

**Irena Lasiecka**

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**I. Chueshov, J. Webster**

We shall consider a model of flow-structure interaction which consists of perturbed wave equation coupled with a nonlinear plate. The interaction between two media takes place on the edge of the plate with the dissipation occurring in a small layer near the edge of the plate. We shall consider both subsonic and supersonic case. It is known that in the latter case the static problem loses ellipticity. Questions such as existence and uniqueness of finite energy solutions will be addressed first. The final goal is to determine geometric conditions for the configuration of the plate which would lead to existence of global attractors capturing solutions near the structure (wing of the airplane). Of particular interest are dissipation mechanisms which are geometrically constrained. It turns out that nonlinear effects in the model are critical in proving ultimate boundedness of solutions. The proofs rely on weighted energy methods with suitably constructed geometric multipliers combined with microlocal analysis and nonlinear elliptic theory which is degenerate.

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### Analysis and simulation of shear-dependent non-Newtonian fluids in moving domains

**Maria Lukacova**

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We present our recent results on mathematical modelling and numerical simulation of non-Newtonian flows in compliant two-dimensional domains having applications in hemodynamics. Two models of the shear-thinning non-Newtonian fluids, the power law Carreau model and the logarithmic Yeleswarapu model, will be considered. For the structural model the generalized string equation for radially symmetric tubes will be generalized to stenosed vessels and vessel bifurcations. The arbitrary Lagrangian-Eulerian approach is used in order to take into account moving

computational domains. The analytical result for the existence of a weak solution for the shear-thickening power-law fluid is based on the global iteration with respect to the domain deformation, energy estimates, compactness arguments using the semi-continuity in time and the theory of monotone operators. We will also present several numerical experiments for the Carreau and the Yeleswarapu model, comparisons of the non-Newtonian and Newtonian models and the results for hemodynamical wall parameters; the wall shear stress and the oscillatory shear index. Numerical experiments confirm high order accuracy and stability of new fluid-structure interaction methods. The results have been obtained in a cooperation with S. Necasova (Academy of Sciences, Prague) and A. Hundertmark, G. Rusnakova (University of Mainz).

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### Modeling effective pressure interface law between a free fluid and a porous medium

**Andro Mikelic**

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**Anna Marciniak-Czochra**

In this talk we present rigorous justification of the interface law describing contact between the flow in an unconfined fluid and a porous bed. The velocity of the free fluid dominates the filtration velocity, but the pressures are of the same order. Main results are the following:

1. We confirm Saffman's form of the Beavers and Joseph law in a new, more general, setting.
2. We show that a perturbation of the interface position, which is an artificial mathematical boundary, of the order  $O(\varepsilon)$  implies a perturbation in the solution of order  $O(\varepsilon^2)$ . Consequently, there is a freedom in fixing position of the interface. It influences the result only at the next order of the asymptotic expansion.
3. We obtain a uniform bound on the pressure approximation. Furthermore, we prove that there is a jump of the effective pressure on the interface and that it is proportional to the free fluid shear at the interface.

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### Some new regularity criteria for weak solutions of the Navier-Stokes equations

**Jiri Neustupa**

Czech Academy of Sciences, Institute of Mathematics, Czech Rep  
neustupa@math.cas.cz

We present two local criteria which imply that a chosen point  $(x,t)$  in the space-time cylinder is a regular point of solution  $v$ . One of the criteria imposes conditions on function  $v$  only in the exterior of a certain space-time paraboloid with the vertex at

point  $(x,t)$ . Furthermore, we present a global criterion which imposes conditions on a certain spectral projection of the associated vorticity or only on its one component.

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### Evolution of non-isothermal nematic liquid crystals flows

**Elisabetta Rocca**

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**Eduard Feireisl, Hana Petzeltova, Giulio Schimperna, Arghir Zarnescu**

We discuss two models describing the time evolution of nematic liquid crystals: one is the non-isothermal version of a variant of the celebrated Leslie-Ericksen model of liquid crystals, the other in the framework of DeGennes-Landau theory, where the natural physical constraints are enforced by a singular free energy potential proposed by J.M. Ball and A. Majumdar. Here the thermal effects are present through the component of the free energy that accounts for intermolecular interactions. The models are consistent with the general principle of thermodynamics and mathematically tractable. We identify the a priori estimates for the associated system of evolutionary partial differential equations and construct global-in-time weak solutions for arbitrary physically relevant initial data.

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### Fokker-Planck equations and neuroscience

**Maria Schonbek**

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**Jose Carrillo, Maria del Mar Gonzalez, Maria Gualdani.**

I will discuss the global existence of classical solutions to the initial boundary value problem for a nonlinear parabolic equation (Fokker-Planck type) describing the collective behavior of an ensemble of neurons. After some transformations the Fokker-Planck system will be handled by methods used for a classical Stefan problem.

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**Regularity problems related to Navier-Stokes equations with mass diffusion****Yongzhong Sun**Nanjing University, Peoples Rep of China  
sunyz@nju.edu.cn**Xiaoyun Cai, Liao Liangwen**

We discuss global in time well-posedness for the initial-boundary value problems of some coupled Navier-Stokes models with mass diffusion, for compressible or incompressible fluids. In the case of two dimensional space, we establish the global-in-time existence (and uniqueness) of strong solution with initial data of arbitrary size. In 3D case we give some regularity criteria.

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## Special Session 8: Propagation Phenomena Appearing in Reaction-Diffusion Systems

Hirokazu Ninomiya, Meiji University, Japan  
Masaharu Taniguchi, Tokyo Institute of Technology, Japan

This special session is concerned with mathematical analysis on propagation phenomena or pattern formation appearing in reaction-diffusion systems. Related topics are traveling waves, equilibrium states and asymptotic behavior of solutions.

### Existence and uniqueness of spiral waves of a wave front interaction model in a plane

**Yan-Yu Chen**

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**Jonq-Sheng Guo, Hirokazu Ninomiya**

To study the spiral wave in an unbounded excitable medium, we consider the wave front interaction model which derived by Zykov in 2007. This model consists of two systems of ordinary differential equations which describe the wave front and wave back, respectively. First, we derive some properties of the back by shooting argument and comparison principle. Next we show the global existence of the solution of the back. Then, we study its asymptotic behavior at infinity. Finally, we prove the uniqueness of the solution.

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### Monotone traveling waves of the nonlocal Fisher-KPP equation

**Jian Fang**

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**Xiaoqiang Zhao**

We consider traveling waves of the nonlocal Fisher-KPP equation. It is known that narrow nonlocal interactions do not change the monotonicity of traveling waves but some wide interactions do. We establish the critical value of the rate of nonlocal interactions for the existence of monotone traveling waves and further prove that such traveling wave is unique up to translation.

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### Dynamics of traveling fronts in some heterogeneous diffusive media

**Hideo Ikeda**

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We consider two component reaction-diffusion systems with a specific bistable and odd symmetric nonlinearity, which have the bifurcation structure of

pitchfork-type traveling front solutions with opposite velocities. We introduce a spatial heterogeneity, for example, a Heaviside-like abrupt change at the origin in the space, into diffusion coefficients. Numerically, the responses of traveling fronts via the heterogeneity can be classified into four types of behaviors depending on the strength of the heterogeneity: passage, stoppage, and two types of reflection. The goal is to reduce the PDE dynamics to finite-dimensional ODE systems on a center manifold and show the mathematical mechanism to produce the four types of responses in the PDE systems using finite-dimensional ODE systems. The reduced ODE systems include the terms (referred to as heterogeneous perturbations) originating from the interaction between traveling front solutions and the heterogeneity, which is very important in order to determine the dynamics of the ODE systems. Using these results, we discuss what play the role of separators of the four different behaviors.

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### Stability analysis for a planar traveling wave solution in an excitable system

**Kota Ikeda**

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Excitable systems appear in various phenomena in nature. The first example is the propagation of impulses along a nerve axon. The second one is the simplified chemical model, called Oregonator, of BZ reaction. The last one is the combustion in microgravity which has various spatial patterns depending on a typical parameter. These phenomena were modeled by reaction-diffusion equations which commonly exhibit excitability and generate a traveling wave solution with a pulse shape. On the other hand, a diffusive coefficient in the three equations changes widely in these three phenomena, which is crucial in the stability of planar traveling waves. In this talk, we show the relation between the diffusive coefficient and the stability of a traveling wave solution with a pulse shape.

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### Bifurcation structure of radially symmetric positive stationary solutions for a competition-diffusion system

**Yukio Kan-on**

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In this talk, we consider a reaction-diffusion system that describes the dynamics of population for two competing species community, and investigate the global bifurcation structure of radially symmetric positive stationary solutions for the system by assuming the habitat of the community to be a ball. To do this, we shall treat the dimension of the habitat and the diffusion rates of the system as bifurcation parameters, and employ the comparison principle and the bifurcation theory.

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### Semilinear solutions in a sector for a curvature flow equation

**Bendong Lou**

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We study a two-point free boundary problem in a sector for a curvature flow equation. The inhomogeneous boundary conditions are assumed to be spatially and temporally "similar" in a special way. We prove the existence and uniqueness of an expanding solution, as well as a shrinking solution, which is selfsimilar at discrete times.

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### Planar standing front waves of the FitzHugh-Nagumo system

**Yoshihisa Morita**

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**Chao-Nien Chen, Shih-Yin Kung**

We are dealing with the FitzHugh-Nagumo system,  $u_t = d\Delta u + f(u) - v$ ,  $v_t = \Delta v - \gamma v + u$ , where  $f(u) = u(u - \beta)(1 - u)$ ,  $\beta \in (0, 1/2)$ , in the whole space. We assume a balanced condition,  $\gamma = 9(2\beta^2 - 5\beta + 2)^{-1}$ . Then for  $d > \gamma^{-2}$  the existence of a planar standing front wave is shown by using variational method. We also show the uniqueness of the solution with a symmetry by the comparison method under an additional restriction for  $\beta$ . Moreover, we can discuss the dynamical stability of the solution in one-space dimensional setting if  $\beta$  is sufficiently close to  $1/2$ .

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### Existence of recurrent traveling waves in a two-dimensional undulating cylinder: the virtual pinning case

**Ken-ichi Nakamura**

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In this talk we study traveling wave solutions for a curvature-driven motion of plane curves in a two-dimensional infinite cylinder with undulating boundary. Here a traveling wave in non-periodic inhomogeneous media is defined as a time-global solution whose shape is "a continuous function of the current environment". Under suitable conditions on the boundary undulation we show the existence of traveling waves which propagates over the entire cylinder with zero lower average speed. Such a peculiar situation called "virtual pinning" never occurs if the boundary undulation is periodic.

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### Singular limit of a damped wave equation with bistable nonlinearity

**Mitsunori Nara**

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**Danielle Hilhorst**

We consider the interfacial phenomena and the singular limit of a damped wave equation with bistable type nonlinearity. Under the assumption that the damping effect is very strong, the solution will behave like that of parabolic equations. We establish the comparison principle for the damped wave equation and construct suitable supersolutions and subsolutions.

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### Stability and bifurcation of periodic traveling waves in a dispersive system

**Kunimochi Sakamoto**

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Stable periodic traveling waves bifurcate from a stable branch of trivial solutions. The trivial solutions do not change its stability (i.e., remain stable) before and after the periodic traveling waves bifurcate. Such bifurcation naturally seems contradictory in dissipative systems and stability is meant asymptotic stability. However, situation is different in dispersive systems and the stability is meant orbital stability. In this talk, an example of such bifurcations is presented in a derivative nonlinear Schrödinger equation with the periodic boundary condition.

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### Front speeds in reaction-diffusion systems: slow pushed and accelerated pulled fronts

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Matt Holzer

We show two examples where front speeds in coupled reaction-diffusion systems deviate from the typical distinction between pulled (slow linear) and pushed (fast nonlinear) predictions. In the first example, a Lotka-Volterra system, we show that the selected front speed is much slower than the linear prediction and give an asymptotic expansion using geometric desingularization of the traveling-wave problem. In the second example, we study multi-stage invasion problems, arising in pattern-forming systems (roll-hexagon competition), phase-separation problems (spinodal decomposition and coarsening fronts), and population models. We show that a primary front can accelerate the speed of the secondary invasion process, leading to locked modes of propagation and, more surprisingly, constant acceleration at arbitrary large distances. We give expansions for locked plateaus and determine the speed of the secondary front in the unlocked case.

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### Convergence and blow-up of solutions for a complex-valued heat equation with a quadratic nonlinearity

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Jong-Shenq Guo, Hirokazu Ninomiya, Eiji Yanagid

We study the Cauchy problem for a system of parabolic equations, which is derived from a complex-valued equation with a quadratic nonlinearity. Our equation has a strong relation with the viscous Constantin-Lax-Majda equation, which is a one dimensional model for the vorticity equation. We first study the asymptotic behavior for global solutions. We also construct solutions that both components of this system blow up simultaneously, although only the real part of the corresponding ODE blows up.

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### Precise asymptotic formulas of critical eigenfunctions for 1D bistable reaction diffusion equations

Tohru Wakasa

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In 1D bistable reaction diffusion equations with a small diffusion parameter, it is known that a transient dynamics of solutions are characterized by a super slow motion of thin interfaces connecting two stable states. In order to understand this pattern dynamics, the existence of critical eigenvalues of the corresponding linearized operators for nontrivial steady-states plays a crucial role. In this talk, we will introduce a precise asymptotic formulas for 1 dimensional linearized operators for n mode stationary solutions.

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### Nonplanar traveling wave solutions in Lotka-Volterra competition-diffusion system

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Wan-Tong Li, Shigui Ruan

We study the nonplanar traveling fronts of the Lotka-Volterra competition-diffusion system

$$\begin{cases} \frac{\partial}{\partial t} u_1(\mathbf{x}, t) &= \Delta u_1(\mathbf{x}, t) \\ &+ u_1(\mathbf{x}, t) [1 - u_1(\mathbf{x}, t) - k_1 u_2(\mathbf{x}, t)] \\ \frac{\partial}{\partial t} u_2(\mathbf{x}, t) &= d \Delta u_2(\mathbf{x}, t) \\ &+ r u_2(\mathbf{x}, t) [1 - u_2(\mathbf{x}, t) - k_2 u_1(\mathbf{x}, t)], \end{cases}$$

with  $\mathbf{x} \in \mathbb{R}^m, t > 0$ .

For the bistable case, namely  $k_1, k_2 > 1$ , it is known that the system admits an one-dimensional traveling front  $\Phi(x + ct) = (\Phi_1(x + ct), \Phi_2(x + ct))$  connecting two stable equilibria  $\mathbf{E}_u = (1, 0)$  and  $\mathbf{E}_v = (0, 1)$ , where  $c \in \mathbb{R}$  is the unique wave speed. Assume  $c > 0$ . For any  $s > c > 0$ , we establish the V-shaped fronts in  $\mathbb{R}^2$ , pyramidal traveling fronts and conical traveling fronts in  $\mathbb{R}^3$ . For the V-shaped fronts and pyramidal traveling fronts, we also prove their uniqueness and stability. For conical traveling fronts, we show that they locally uniformly converge to the planar traveling front when  $s$  tends to  $c$ . We also show the nonexistence of conical traveling fronts.

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### Asymptotic behavior in a 2-allele genetic model with population control

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The standard model for the evolution of the densities of the three genotypes  $aa$ ,  $aA$ , and  $AA$  is

$$\begin{aligned}\partial\rho_{aa}/\partial t - \Delta\rho_{aa} &= \frac{r[2\rho_{aa} + \rho_{aA}]^2}{2[\rho_{aa} + \rho_{aA} + \rho_{AA}]} - \tau_{aa}\rho_{aa} \\ \partial\rho_{aA}/\partial t - \Delta\rho_{aA} &= \frac{2r[2\rho_{aa} + \rho_{aA}][2\rho_{AA} + \rho_{aA}]}{2[\rho_{aa} + \rho_{aA} + \rho_{AA}]} \\ &\quad - \tau_{aa}\rho_{aa} \\ \partial\rho_{AA}/\partial t - \Delta\rho_{AA} &= \frac{r[2\rho_{AA} + \rho_{aA}]^2}{2[\rho_{aa} + \rho_{aA} + \rho_{AA}]} - \tau_{AA}\rho_{aa},\end{aligned}$$

where the parameters are constants. It has recently been shown by Souplet and Winkler that in the heterozygote intermediate case  $\tau_{aa} \geq \tau_{aA} \geq \tau_{AA}$ ,  $\tau_{aa} > \tau_{AA}$  the gene fraction  $u := [2\rho_{aa} + \rho_{aA}]/\{2[\rho_{aa} + \rho_{aA} + \rho_{AA}]\}$  converges to 0. However, the proof in the biologically interesting case depends on the fact that  $\rho_{AA}$  converges to infinity, which, as the authors pointed out, makes the model unrealistic. When  $[\tau_{aa} + \tau_{AA}]/2 \leq \tau_{aA} < \tau_{aa}$ , we produce a large class of models of the above form with the birth rate  $r$  replaced by a suitable density-dependent function. These models have the property that, for a reasonable set of conditions, the density  $\rho_{aa} + \rho_{aA}$  of those individuals which have at least one  $a$ -allele converges to zero uniformly in every bounded set as  $t$  approaches infinity, while the total population density remains bounded and uniformly positive.

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### Symmetric and asymmetric spikes for the two-dimensional Schnakenberg model

Matthias Winter

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Juncheng Wei

We consider the existence and stability of symmetric and asymmetric spikes for the two-dimensional Schnakenberg model in two space dimensions. For existence we will derive and solve an algebraic system

for the amplitudes using a nondegeneracy condition. The positions of the spikes are then given by the nondegenerate critical points of some Green's function. For stability we will investigate large eigenvalues of order  $O(1)$  and small eigenvalues of order  $o(1)$  separately, which are connected with the amplitudes and positions of the spikes, respectively.

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### The existence and stability of traveling front solutions for some autocatalytic systems

Yaping Wu

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Yi Li

In this talk we shall talk about the existence and stability of traveling fronts for the following typical autocatalytic chemical reaction system

$$\begin{cases} u_t = u_{xx} - u^q v^p, \\ v_t = dv_{xx} + u^q v^p. \end{cases}$$

For  $p \geq 1$ ,  $q \geq 1$  and  $d > 0$ , it is known that there exists a critical speed  $c^*(p, q, d)$  such that for any  $c \geq c^*(p, q)$  there exist travelling front solutions  $(u(x - ct), v(x - ct))$  connecting  $(0, 1)$  and  $(1, 0)$ . For the cases  $p > 1$  or  $q > 1$ , the travelling waves with noncritical speed decay algebraically in space at  $+\infty$  or  $-\infty$ .

In this talk we shall be more interested in the linear and nonlinear asymptotic stability of the waves with noncritical speeds and with algebraic spatial decay when  $d$  is near 1. We shall first talk about our recent work on the asymptotic stability of the waves with algebraic spatial decay in some polynomially weighted spaces for the system when  $d = 1$ . Further we shall introduce our recently obtained abstract results on the existence and analyticity of Evans function for the more general ODE systems with slow algebraic decaying coefficients, and our recent work on the linear exponential stability of waves with algebraic decay in some exponentially weighted spaces for the case  $p > 1$  and  $q \geq 1$  when  $d$  is near 1. Finally for the case  $p > 1$  and  $q = 1$  we shall prove the nonlinear asymptotic stability of the waves with noncritical speeds when  $d$  is near 1.

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## Special Session 9: Mathematics for Information Processing and Management

Jianhong Wu, York University, Canada  
Zongben Xu, Xian Jiaotong University, China

### ELECTRE ranking approach for benchmarking analysis in marketing sector

**Ai Ling Amy Poh**  
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The marketing mix is the lens through which the contemporary customer perceives value in retail stores on 4Ps is examined. From the model, we analyze the best practice among the four elements derived from a consensus ranking, a ranking method to identify the best in class. The analysis mainly depended on the outcome of what customer perceive towards the four marketing tactics. This paper discusses the introduction and use of a methodology for project ranking in retail store and, in particular, illustrates the use of a particular solution method called ELECTRE. In the effort of avoiding the shortcomings of the traditional methods based on the average aggregate monocriterion, outranking methods make it possible to deal with multicriteria benchmarking.

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### Discovering most collaborative teams of experts in social networks

**Aijun An**  
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**Mehdi Kargar**

We study the problem of discovering teams of experts from a social network. Given a project whose completion requires a set of skills, our goal is to find a set of experts that together have all of the required skills and also have the minimal communication cost among them. We present two communication cost functions designed for two types of communication structures. We show that the problem of finding the team of experts that minimizes one of the proposed cost functions is NP-hard. Thus, a 2-approximation algorithm is designed. We introduce the problem of finding a team of experts with a leader. The leader is responsible for monitoring and coordinating the project, and thus a different communication cost function is used in this problem. To solve this problem, an exact polynomial algorithm is proposed. We show that the total number of teams may be exponential with respect to the number of required skills. Thus, two procedures that produce top-k teams of experts with or without a leader in polynomial delay are proposed. Extensive experiments on real datasets

demonstrate the effectiveness and scalability of the proposed methods.

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### Time consistent multiperiod risk measure under generalized convex framework

**Zhiping Chen**  
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It is now accepted that time consistency should be a necessary requirement for multi-period risk measures. After investigating the relationship among different definitions of time consistency and discussing properties that a reasonable multi-period risk measure should possess, we propose a new class of time consistent multi-period risk measures under generalized convex framework. Basing on the new risk measure, we construct a corresponding multi-period portfolio selection model. Scenario technique is then used to efficiently compute the new multi-period risk measure and solve the multi-period portfolio selection problem. Preliminary numerical results are finally provided to illustrate the reasonability and practicality of our new multi-period risk measure and the corresponding multi-period portfolio selection model.

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### Firm clustering using standard-based financial statements

**Dazhi Chong**  
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**Dazhi Chong, Hongwei Zhu**

Classification and clustering of firms can produce useful insights about various aspects about the firms. Most existing approaches to classifying businesses only pay attention to the operation processes and outputs of firms. In this paper, we introduce a firm clustering method that uses firms' financial information. Publicly listed companies in the U.S. has been required to use a common data standards called the U.S. GAAP Taxonomy to submit their financial statements to the Securities and Exchange Commission. When constructing their financial statements, firms select data elements from the Taxonomy to tag their financial data. In this research, we construct the "social network" of firms using the shared data

elements in firms' financial statements. By implementing a spectral clustering method and applying it to the "social network", we identify clusters that differ from frequently used classification schemes. Firms within the same cluster share similar structures in their financial statements. Other firm characteristics are being explored to obtain additional insights about the clusters.

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### Network utility optimization and effect to response time

**Wenyang Feng**

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**Christopher Johns, Kevin Mak**

We study the optimization of network utilities under the constraints of limited resources. New models are developed and algorithms for theoretically solving the problems are presented. Particular cases are discussed. Applying simulation techniques, we also investigated the effects of different utility functions on network response time when the total utilization of the network is optimized.

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### The influence of media on social behaviour in an influenza pandemic

**Jane Heffernan**

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**Shannon Collinson**

Media reports affect social behaviour during epidemics and pandemics. Changes in social behaviour, in turn, affect key epidemic measurements such as peak magnitude, time to peak, the beginning and end of an epidemic. The extent of this effect has not been realized. We have developed mathematical models of influenza spread based on a Susceptible-Exposed-Infected-Recovered (SEIR) model including the effects of mass media. The models are used to evaluate different functions representing media impact and how these functions affect key epidemic measurements. We have also developed an agent based Monte Carlo (ABMC) simulation of influenza infection between hosts to determine variability in key epidemic measurements.

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### Beyond traditional search: probabilistic approaches and their applications

**Jimmy Huang**

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Most of the traditional Information Retrieval models are based on the assumption that query terms are independent of each other and a document is represented as a bag of words. Nevertheless this assumption may not hold in practice. In this talk, I will discuss how the query terms associate with each other and how to incorporate the term proximity information into the classical probabilistic IR models. Through extensive experiments on standard large-scale TREC Web collections, I will show that the extended models are able to markedly outperform the BM25 baseline and at least comparable to the state-of-the-art model. The talk will conclude with a discussion of novel challenges raised in extending probabilistic Information Retrieval and several applications such as promoting diversity in ranking for biomedical IR, sentiment analysis for predicting sales performance and EMR data analysis for effective health care.

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### Estimating a density in the presence of interval censoring

**Hanna Jankowski**

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In this talk I will compare several methods of density estimation, including parametric and nonparametric approaches. I will discuss the benefits and drawbacks of each approach. Finally, I will illustrate the various methods on data coming from the 2009 H1N1 pandemic in Ontario, Canada.

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### A divide-and-conquer approach to effective and efficient L1 norm matrix factorization

**Deyu Meng**

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**Zongben Xu**

The L1 norm low-rank matrix factorization has been recently attracting much attention due to its intrinsic robustness to outliers and missing data. However, most of the existing algorithms suffer the problems such as high complexity and/or low accuracy. In this paper, we propose a novel solution, which is essentially a divide-and-conquer approach, to robust L1 norm matrix factorization. The main idea is to break

the original difficult problem into many pieces of simple sub-problems, each involving only one unknown scalar parameter and having a closed-form solution. By recursively solving these small problems without time-consuming inner loop numerical optimization, an efficient algorithm can be readily constructed to tackle the original problem. The computational complexity of the proposed algorithm is approximately linear in both data size and dimensionality, making it be able to handle large-scale L1 norm matrix factorization problems. The extensive experimental results validate that our method outperforms state-of-the-arts in term of both computational time and accuracy, especially on large-scale applications such as face recognition and structure from motion.

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### Agent-based modelling frameworks for developing public health policies

**Seyed Moghadas**  
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Advances in Information and Communications Technologies (ICTs) during the past two decades have provided a major conduit for knowledge exchange, which conveys the notion of human connectivity within a complex network. These advances, combined with scientific discoveries, have had profound impacts on many aspects of real life, and led to dramatic changes in approaches to addressing societal issues. The underlying processes by which our knowledge influences public policy and decision-making involves retrieval and analysis of vast amounts of heterogeneous and complex data collected through the use of ICT tools. We show how an Agent-Based Modelling framework can integrate such data, and inform public health policies in the event of an emerging disease. We use the 2009 pandemic influenza H1N1 as a case study to illustrate how intervention strategies can be targeted to reduce the impact of disease on the population.

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### Tensor regression model for crime prediction

**Yang Mu**  
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**Yang Mu, Henry Lo, Wei Ding**

Crime incidents are affected by both spatial and temporal factors as well as other predictor variables. As such, crime prediction requires that we take spatiotemporal attributes into account when processing crime data. Yet, the structure and order inherent in space and time dimensions are not considered adequately using other mathematical techniques; it is

typical for prediction models to treat each spatial point as distinct and independent, even though in reality variables tend to be correlated with values in neighboring cells. In this work, we use a tensor regression model to predict a crime dataset that spans a northeastern city in last 5 years. This model aims to minimize the Ordinary Least Square (OLS) error and accepts tensor inputs directly. It greatly reduces the appearance of Small Sample Size (SSS) problem, which could easily occur in a vector based OLS case. Tensor features are extracted and used to train a tensor regression model in order to predict future crime. The tensor-based regression model is evaluated with comparison to a traditional point-prediction model, which considers only historical data but no spatial neighborhood data. Results show that the tensor model is far more effective in predicting hotspots of crime than the traditional model.

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### L1/2 regularization theory for sparse machine learning

**Zongben Xu (special session keynote speaker)**  
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**Baili Chen**

Machine Learning (ML) is a fundamental tool in data-driven information technologies, which aims at modeling a likely existing but unknown relationship between the input and output spaces based a given set of samples. In applications, ML leads to ill-posed problems and it is solved then through constraining the solutions to be found (via regularization). The constraints normally take the form of certain type of norms of the solution, say, the 2-norm, reflecting the expectation that the solution to be found is as smooth as possible, or the 1-norm, expecting the solution to be sparse and as simple as possible. I propose to apply the 1/2 quasi-norm in this talk in performing regularization. I develop an L(1/2) regularization based sparse ML theory, showing that the new regularization scheme preserves the classical Moreau's forward-backward splitting representation, can be very fast solved and is capable of yielding nearly sparsest solution with good generalization performance. I present also two applications to demonstrate the powerfulness of the developed theory. The first application is on image processing (the image representation, denoising, inpainting, restoration and identification in particular), which shows how the new theory can yield the state of the art techniques for those problems. The second application is on microwave imaging. In this later case I show that the new theory underlies a new microwave imaging principle, and furthermore, leads to a very useful, new SAR system.

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### Risk assessment of China's commercial banks: assessing data quality

**Feng Xu**

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**Tan Zhang, Yahui Li, Hongyun Zhang**

This research studies the function of analytical modeling in risk assessment of China's commercial banks. This mathematical model aims to solving the risk assessment problems, especially in data quality (Kaplan et al. 1998). We will use this analytical model as a systematic exploration from data quality perspective, which can be used to improve credit card businesses of China's commercial banks in the future. In addition, a decision support system is suggested so that financial regulators can use it for assessing data quality more effectively and efficiently. This research constructs a mathematical model and algorithms that support financial regulators to determine the target error classes as well as the minimum set of control procedures when assessing financial risks, which is the key to ensure high level of data quality, especially high level of reliability.

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### Delay differential equations and its application in developmental biology

**Xuguo Zhang**

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**Steven Bishop, Julian Lewis**

Delay Differential Equations (DDEs) play a fundamental role in the various fields such as Biology, Engineering and even Economics over recent years. This paper, with its focus on DDEs, investigate a specific model for cellular formation in Zebrafish developed by Lewis (2003), hereafter called Lewis model to illustrate the dynamic behaviour and the properties of Ordinary Differential Equations (ODEs) and DDEs extracted from Lewis work (Lewis, 2003). By using linearisation and some simple analytic skills

such as Bendixsons negative criterion and Liapunov function, it has been showed that system without delay cannot exhibit self-sustained oscillation. When model involves delays, we mainly analyse and explore the form of f function (Rate of synthesis for new mRNA molecules) extensively using Matlab and a new f function has been found to satisfy the DDE system. It has been showed that the amplitude of the oscillation against delay T or the power of protein n is monotonically changing when parameters are varied and meantime the appropriate bifurcation diagram is showed. This investigation have made a strong theoretical surport for Lewis experimental work and the significant analytic skills improved the results as well.

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### An improved model of anonymous entity resolution in the public sector

**Zhou Zhang**

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**Jing Chen, Hongyun Zhang**

Entity resolution (ER) is a body of knowledge and practice related to the activities supporting a process to decide whether two entity references are equivalent or not equivalent; entity reference refers to a collection of identity attribute values that describe a particular entity. A previous model was developed to deal with ER in the public sectors, where the information stored in different databases is prevented from data mining by a data warehouse because of policies and regulations (Talbur et al. 2005). In this paper, an improved model is proposed to deal with the problem of anonymous entity resolution in the Intranet. In addition, the model is more efficient with the use of Bloom Filter when accessing and comparing information between different sectors. Based on the improved model, this research is anticipated to improving data quality in the public sectors successfully.

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## Special Session 10: Computational and Nonautonomous Dynamics

**Michael Dellnitz**, University of Paderborn, Germany  
**Oliver Junge**, Technische Universitaet Muenchen, Germany  
**Stefan Siegmund**, Technische Universitaet Dresden, Germany

Computational techniques for dynamical and control systems have grown to powerful tools beyond mere simulation. In this session, state of the art computational methods, as well as theoretical foundations, for the analysis and synthesis of such (non-)autonomous systems are presented, with a focus on real world applications. One of our primary goals is to detect new links between dynamical systems resp. systems theory and such diverse fields as graph theory, numerics, mechanics, optimization, statistics, and topology.

### A computer assisted enclosure for invariant manifolds

**Gianni Arioli**

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**Davide Ambrosi, Hans Koch**

We describe a new technique to compute a very tight enclosure for the stable and unstable manifolds at a stationary point of an autonomous, finite dimensional dynamical system. We apply the technique to prove the existence of a traveling wave for a PDE modeling the propagation of electric signals in biological tissues, and provide a very accurate rigorous estimate of the wave speed.

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### Macro-state models for protein modeling

**Eric Darve**

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We present and discuss various numerical algorithms to calculate slow reaction rates in protein modeling. These rates typically correspond to conformational changes in protein such as folding or binding. These rates are associated with rare transition events from one metastable state to another metastable state. Such rare events, by definition, are difficult to observe through direct simulation and therefore getting accurate statistical information and kinetics is challenging. We will discuss methods that attempt to enhance the sampling and improve the accuracy using macro-states, which form a partition of the conformational space of the protein. By computing statistics of transition between these macro-states, in appropriate ways, one can reconstruct the kinetic information of interest.

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### Multiscaling and coarse-graining for coagulation processes in high dimension

**Lee Deville**

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**Matt West, Nicole Riemer**

We consider particle processes where the state-space has high dimension and pairwise events dominate the dynamics. (A concrete example of this is computing the time evolution of the multi-dimensional size distribution of interacting aerosol particles in a regime where coagulations dominate the cost of the computation.) We will discuss several methods of speeding up such particle methods when multiscaling is applicable and show that this applies to several concrete problems in atmospheric sciences.

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### Rotation number and QR based techniques for exponential dichotomy: numerical comparisons

**Cinzia Elia**

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**Roberta Fabbri**

We propose numerical techniques based on the continuous Iwasawa decomposition to compute the rotation number for Hamiltonian linear systems of dimension greater or equal than 2. The rotation number allows us to infer whether a given system has exponential dichotomy or not. We employ these techniques to study the Schroedinger operator and the Schroedinger equation in 2 dimensions. Comparisons with QR based techniques for exponential dichotomy allow us to recover new information on the spectrum of the Schroedinger operator. The talk focuses on the numerical aspects while theoretical results will be addressed in Roberta Fabbri's talk.

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### Rotation number and exponential dichotomy for linear Hamiltonian systems: theoretical and numerical aspects

**Roberta Fabbri**

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**Cinzia Elia**

The talk considers the properties of the rotation number for a family of linear nonautonomous Hamiltonian systems and its relation with the exponential dichotomy concept. In particular, we relate the rotation number to the presence of an exponential dichotomy for some Hamiltonian systems perturbed according to an Atkinson-type condition. Using the continuous Iwasawa decomposition of a symplectic matrix, we compute the rotation number of a linear Hamiltonian system and we employ these techniques to study the quasi-periodic Schroedinger operator in one dimension and the 2-dimensional Schroedinger equation. We refer to the talk of Cinzia Elia for the introduction and analysis of the numerical techniques considered.

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### Finite-time transport analysis for nonautonomous deterministic and stochastically perturbed systems

**Gary Froyland**

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We discuss finite-time transport for both deterministic dynamical systems and those subjected to stochastic perturbations. We will elucidate a new theory for characterising Lagrangian coherent sets in both deterministic and stochastic settings. The constructions are based around singular vectors of evolution operators (Perron-Frobenius operators). We also discuss how small random perturbations influence the maximal level of coherence.

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### Accelerating molecular dynamics: some ideas from robust and risk-sensitive control

**Carsten Hartmann**

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**Christof Schütte and Stefanie Winkelmann**

In the talk I will present some recent developments in robust control of bilinear systems that appear relevant in the context of nonequilibrium molecular dynamics and quantum control. A particular focus is on low-rank approximation techniques that make the calculation of an optimal control numerically feasible.

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### Computing invariant sets with Newton-type iterations: towards a covering approach

**Mirko Hessel-von molo**

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**Michael Dellnitz, Ioannis G. Kevrekidis**

The approximation of invariant sets is an important task in the numerical treatment of dynamical systems. Recent work approaches this problem by transferring it into a Banach space setting, using the formalism of directed sets, and employing Newton-type iterations for its solution. This approach is limited in the class of sets that can be approximated: only convex sets are feasible. We will present steps towards overcoming this limitation with a covering approach that represents invariant sets as (subsets of) unions of convex sets.

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### Characterization of pullback attractors

**José A. Langa**

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**A.N. de Carvalho, J.C. Robinson**

In this talk we present some recent results on the characterization of attractors for non-autonomous dynamical systems. We will pay attention in the upper and lower semicontinuity of attractors and on Morse decomposition of these families, which describes the internal dynamics on these sets. All our work is made in an infinite dimensional framework, so that specially well suited to apply to PDEs. Some open problems and future further research will be also presented.

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### Lazy global feedbacks for quantized nonlinear event systems

**Oliver Junge**

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**Stefan Jerg**

We consider nonlinear event systems with quantized state information and design a globally stabilizing controller from which only the minimal required number of control value changes along the feedback trajectory to a given initial condition is transmitted to the plant. In addition, we present a non-optimal heuristic approach which might reduce the number of control value changes and requires a lower computational effort. The constructions are illustrated by two numerical examples.

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**Invariant manifolds in finite-time dynamics****Daniel Karrasch**

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We present and discuss finite-time invariant manifolds from two perspectives. The first perspective is via finite-time hyperbolic trajectories and their (inherently non-unique) finite-time stable and unstable manifolds. We show an intrinsic (i.e. without infinite-time extensions) approach to the proof of a finite-time Local Stable Manifold Theorem and present a complete local description of the geometry close to hyperbolic trajectories. The second perspective is from ensemble dynamics, as often applied in the modelling of geophysical flows, for instance. We generalize a variational approach to hyperbolic Lagrangian Coherent Structures (LCS), recently proposed by Haller [Physica D, 2011], towards LCS of higher codimension and, second, to filtrations of embedded LCS.

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**Global stability design for non-linear dynamical systems****Péter Koltai**

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**Alexander Volf**

Given a stable non-linear system, one would like to compute its stability region. Often, the system depends on parameters, which should be tuned in order to shape this region according to one's needs. More generally, one would like to optimize the global stable behavior of the system; e.g. reducing transient motions.

In this talk I introduce a fairly general framework for doing this by approximating the dynamics by a finite dimensional stochastic process (closely related to the upwind scheme). Translating the desired objectives in the terms of this process enables the simple application of many optimization algorithms. In particular, no trajectory simulation is needed during the computation. The main advantage of the method lies in the resulting numerical efficiency and the general applicability for a wide range of different objectives.

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**Simple heteroclinic orbit examples in the plane****Stephen Lucas**

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**James S Sochacki**

There is much known about heteroclinic orbits in the plane and their properties. However, there are still open questions, and phase portraits highlighting unique features are still being sought. We present two classes of ODEs for the plane. The first one is a quadratic polynomial system that has a closed form solution, where every point is on a heteroclinic orbit with the same limit points. We demonstrate that this system has sensitive dependence on initial conditions and allows spiraling from one equilibrium to another. The second class of ODEs allows for an infinite number of heteroclinic limit points along arbitrary curves in the plane. We highlight an interesting quadratic polynomial that has its heteroclinic limit points defined by a single line in the plane.

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**Mesohyperbolicity and other ergodic theory concepts in nonautonomous dynamical systems****Igor Mezic**

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We discuss the concept of mesohyperbolicity for studying properties of non-autonomous dynamical systems. The idea was introduced in the context of two-dimensional, divergence free flows. We extend the analysis to higher-dimensional systems and provide connections with other concepts such as that of ergodic partition. We present some applications, including the study of oil and gas evolution on the subsurface and subsurface during the Deepwater Horizon oil spill in the Gulf of Mexico.

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**On the development and analysis of variational integrators for multirate dynamical systems****Sina Ober-Bloebaum**

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**Sigrid Leyendecker**

The use of discrete variational principles results in symplectic and momentum preserving variational integrators that exhibit excellent long-time behavior. In this talk variational integrators are developed for the integration of systems with dynamics on different time scales, for which the slow part of the system

is integrated with a relatively large step size while the fast part is integrated with a small time step to save function evaluations and decrease integration time. Based on a derivation in closed form via a discrete variational principle on a time grid consisting of macro and micro time nodes such a multirate variational integrator can be constructed. The structure preserving properties as well as the convergence behavior of the multirate integrator are analyzed and its performance is demonstrated by numerical examples.

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### Set-oriented numerical analysis of time-dependent transport

**Kathrin Padberg-Gehle**

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**Gary Froyland**

The numerical analysis of transport processes is central for understanding the macroscopic behavior of classical dynamical systems as well as time-dependent systems such as fluid flows. We review different theoretical concepts and their numerical implementation into a set-oriented framework. We demonstrate that the geometric approach based on invariant manifolds and Lagrangian coherent structures and the probabilistic concept which relies on transfer operators give consistent results. Finally, potential combinations of these techniques are discussed.

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### Rigorous connecting orbits from numerics

**Ken Palmer**

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**Brian A. Coomes, Huseyin Kocak**

A rigorous numerical method for establishing the existence of an orbit connecting two hyperbolic equilibria of a parametrized autonomous system of ordinary differential equations is presented. Given a suitable approximate connecting orbit and assuming that a certain associated linear operator is invertible, the existence of a true connecting orbit near the approximate orbit and for a nearby parameter value is proved. It turns out that inversion of the operator is equivalent to the solution of a boundary value problem for a nonautonomous inhomogeneous linear difference equation. A numerical procedure is given to verify the invertibility of the operator and obtain a rigorous upper bound for the norm of its inverse. Using this method, the existence of various kinds of homoclinic orbits including saddle-focus homoclinic orbits which imply the presence of chaos is demonstrated on examples including the Lorenz system.

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### The dynamics of non-autonomous Lotka-Volterra ODEs

**James Robinson**

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**Jose Langa, Antonio Suarez**

Lotka-Volterra equations, coupled systems of ODEs modelling two competing species, are a canonical example in the qualitative theory of low-dimensional dynamical systems. In this talk I will show how ideas from the theory of pullback attractors for non-autonomous dynamical systems can be used to analyse the same simple models when some of the parameters are allowed to depend on time. Although the problem and the analysis is relatively simple, it provides a good demonstration of many of the techniques that can be used in more complicated problems, including order-preserving properties, the existence of maximal elements of the attractor, and perturbation results for invariant manifolds.

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### Designing scalable algorithms for complex networks

**Tuhin Sahai**

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**Andrzej Banaszuk, Amit Surana, Alberto Speranzon**

Complex networks such as building systems, UAV swarms and communication networks are of paramount importance to modern day applications and particularly challenging from an analysis perspective.

For scalable analysis of large networks, our approach uses a novel decentralized clustering approach, based on propagating waves in the graph. The algorithm recovers the solution obtained from spectral clustering without the need for expensive eigenvalue/vector computations. We prove that, by propagating waves through the graph, a local fast Fourier transform yields the local component of every eigenvector of the Laplacian matrix, thus providing clustering information. For large graphs, the proposed algorithm is orders of magnitude faster than random walk based approaches.

We then use the above decentralized partitioning approach to develop polynomial chaos based methods for uncertainty quantification in large networks. Polynomial chaos is used extensively for propagating uncertainty through smooth dynamical systems. Though useful for systems of small to moderate dimension, the curse of dimensionality restricts the applicability of these methods to high dimensional dynamical systems. We construct iterative schemes (intrusive and non-intrusive), for scalable uncertainty quantification in complex networks

that utilize “weak” interactions between clusters to quickly compute the approximate output distributions in large networks with bounded error. We will demonstrate these techniques on models of energy efficient buildings.

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### **Towards automatic computation of the Conley index over a base**

**Jacek Szybowski**

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**Pawel Pilarczyk, Kinga Stolot**

We introduce an algorithmic method for the computation of the Conley index over the phase space for discrete-time semidynamical systems. This is a considerable step forward towards the computation of the Conley index over a base, and—to the best of our knowledge—the first algorithmized approach to this problem. In this talk, we recall the definition of the homological version of the index, we introduce an algorithm for the construction of appropriate index pairs using uniform cubical grids, and we show some examples in which we prove the lack of continuation between some dynamical systems, not distinguishable using the regular Conley index approach. This is joint work with P. Pilarczyk and K. Stolot.

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### **Sacker-Sell spectrum and Lyapunov spectrum for random dynamical systems**

**Guangwa Wang**

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**Yongluo Cao**

The significance and importance of Sacker-Sell spectrum and Lyapunov spectrum for dynamical systems is well-known. This talk is mainly concerned with the two kinds of spectra for random dynamical systems (RDS). More precisely, in the first part, we will establish the Sacker-Sell Spectral Decomposition Theorem in some different frameworks of RDS: finite dimensional case, infinite dimensional case with compactness and infinite dimensional case with some kind of weak compactness, respectively. In the second part, taking the ideas from Mane and Thieullen, we will obtain the Multiplicative Ergodic Theorem about Lyapunov exponents for a very general infinite dimensional RDS. Moreover, in the third part, the relations between Sacker-Sell spectrum and Lyapunov spectrum in three different frameworks of RDS will be discussed.

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### **Multiscale time evolution for Markov jump particle systems**

**Matthew West**

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We present a new time evolution scheme for efficiently generating realizations of Markov jump processes for particle systems, which can efficiently simulate highly multiscale particle distributions and event rates. Multiscale particle distributions are represented as weighted point samples, and time evolution occurs using a binned tau-leaping scheme with approximate rate sampling. A convergence proof as well as applications to aerosol particle simulation are given.

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## Special Session 11: Advances in Classical and Geophysical Fluid Dynamics

Madalina Petcu, Univ Poitiers, France  
 Roger Temam, Indiana University, USA  
 Shouhong Wang, Indiana University, USA

Geophysical Fluid Dynamics (GFD) came of age in the 1950s and 1960s, with the mathematically inspired work of Charney, Lorenz, Stommel, Veronis and von Neumann, among others. In the last twenty years or so, mathematical, statistical and numerical developments for the Navier-Stokes equations and basic geophysical fluid dynamics models are stimulating a deepening and broadening of its central concerns. The aim of this special session is to bring together a number of researchers who work on diverse frontiers of GFD and Navier-Stokes equations and their applications to climate dynamics and environmental problems to discuss how new ideas and methods will advance the field in the next decade.

### A model equation for water waves with dissipation

**Jerry Bona**

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**David Ambrose, David Nicholls**

The well-posedness of a model for water waves with dissipation put forward by Diaz, Dyachenko and Zhakharov is considered. Some associated ill-posedness results will also be outlined.

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### Invariant measures for dissipative dynamical systems: abstract results and applications

**Mickael Chekroun**

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**Nathan Glatt-Holtz**

In this talk we will study certain invariant measures that can be associated to the time averaged observation of a broad class of dissipative semigroups via the notion of a generalized Banach limit. Consider an arbitrary complete separable metric space  $X$  which is acted on by any continuous semigroup  $\{S(t)\}_{t \geq 0}$ . Suppose that  $\{S(t)\}_{t \geq 0}$  possesses a global attractor  $\mathcal{A}$ . We will show that, for any generalized Banach limit  $\text{LIM}_{T \rightarrow \infty}$  and any distribution of initial conditions  $m_0$ , that there exists an invariant probability measure  $m$ , whose support is contained in  $\mathcal{A}$ , such that

$$\int_X \phi(x) dm(x) = \text{LIM}_{T \rightarrow \infty} \frac{1}{T} \int_0^T \int_X \phi(S(t)x) dm_0(x) dt,$$

for all observables  $\phi$  living in a suitable function space of continuous mappings on  $X$ . This work is based on a functional analytic framework simplifying and generalizing previous works in this direction.

Two examples of concrete dynamical systems where the semigroup is known to be non-compact will be then examined in detail. We first consider the Navier-Stokes equations with memory in the diffusion terms. This is the so called Jeffery's model which describes certain classes of viscoelastic fluids. We then consider a family of neutral delay differential equations, that is equations with delays in the time derivative terms. These systems may arise in the study of wave propagation problems coming from certain first order hyperbolic partial differential equations; for example for the study of line transmission problems. For the second example the phase space is  $X = C([- \tau, 0], \mathbb{R}^n)$ , for some delay  $\tau > 0$ , so that  $X$  is not reflexive in this case.

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### Singular limits of geophysical fluid dynamics in spherical and bounded domains.

**Bin Cheng**

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We study 2D geophysical fluids that are inviscid and under strong Coriolis force. In particular, we address the challenges of nontrivial geometry of the physical domains such as on a sphere. We also consider general (a.k.a ill-prepared) initial data. The first model is incompressible Euler equations on a fast rotating sphere. Motivated by recent studies in geophysical and planetary sciences, we investigate the finite-time-average of the solution and prove that it stays close to a subspace of longitude-independent zonal flows. The second model is the rotating shallow water equations with solid-wall boundary conditions. We show that in the zero-Rossby-number limit, the solution comprises a quasi-geophysical flow and fast gravity waves, both subject to the same solid-wall boundary conditions.

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### Qualitative properties of eigenvectors related to multivoque operators

**Houssam Chrayteh**

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We introduce the notion of  $\vec{p}$ -multivoque Leray-Lions operators that are strongly monotonic on a Banach-Sobolev function space  $V$  and we study the generalized eigenvalue problem  $Au = \lambda \partial j(u)$ . Here  $\partial j$  denotes the subdifferential in the sense of convex analysis or more generally in the sense of H. Clarke. Connected with this problem, we also study a minimization problem with constraint then we give some qualitative properties of solutions by using relative rearrangement.

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**New results for the stochastic PDEs of fluid dynamics**

**Nathan Glatt-Holtz**  
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The addition of white noise driven terms to the fundamental equations of physics and engineering are used to model numerical and empirical uncertainties. In this talk we will discuss some recent results for the Stochastic Navier-Stokes and Euler Equations as well as for the Stochastic Primitive Equations, a basic model in geophysical scale fluid flows. For all of the above systems our results cover the case of a general nonlinear multiplicative stochastic forcing.

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**Viscous asymptotic models for water waves**

**Olivier Goubet**  
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In this talk, we discuss some issues related to various viscous asymptotic models for water waves. These models range from Kakutani-Maatsuchi models to models derived a decade ago independently by Dutykh and Dias, and Liu et Orfila. We focus on the decay rate of solutions to these models.

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**Time periodic solutions of the Primitive equations of the large-scale ocean**

**Chun-hsiung Hsia**  
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**Ming-Cheng Shiue**

This is a joint work with Ming-Cheng Shiue. For several decades, concerning the long time behavior of fluid motion, the time periodic flows have become an important type of flow patterns. In 1959, Serrin proposed a very heuristic method for proving the existence of asymptotic stable periodic solutions of the Navier-Stokes equations with small periodic forcing terms under suitable assumptions. Namely, in such case, one may prove that every small ( in a suitable sense ) solution would converge to a time periodic solution ( with the same period as the non-trivial forcing term ). In this article, we prove the existence of time periodic solution for the 3-D primitive equation with suitable time periodic forcing condition. Some related asymptotic behaviors of the solutions are also demonstrated.

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**The linearized 2D inviscid shallow water equations in a rectangle: boundary conditions and well-posedness**

**Aimin Huang**  
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**Roger Temam**

We consider the linearized 2D inviscid shallow water equations in a rectangle. A set of boundary conditions is proposed which make these equations well-posed. Several

different cases occur depending on the relative values of the reference velocities  $(u_0, v_0)$  and reference height  $\phi_0$  (sub- or super-critical flow at each part of the boundary).

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**Center manifold reduction for stochastic partial differential equations**

**Honghu Liu**  
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**Mickael D. Chekroun, Shouhong Wang**

We present – within the framework of random dynamical systems – explicit approximation formulas for center manifold reduction of a broad class of nonlinear stochastic evolution equations with linear multiplicative white noise. This reduction procedure allows us to derive explicit reduced equations to the corresponding random center manifold and conduct detailed analysis on the long term dynamics and bifurcation associated with both the reduced and the original equations. As an application, we analyze the bifurcation scenarios of the reduced equation of the Rayleigh-Benard convection with linear multiplicative noise. This is a joint work with Mickael D. Chekroun and Shouhong Wang.

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**Some equations with logarithmic nonlinear terms**

**Alain Miranville**  
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Our aim in this talk is to discuss the well-posedness and the asymptotic behavior, in terms of finite-dimensional attractors, of several equations with logarithmic nonlinear terms. Such equations arise, e.g., in phase transition.

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**The two layers shallow water equations**

**Madalina Petcu**  
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The aim of this talk is to model the flow of two superposed layers of fluid governed by the shallow water equations in space dimension one. Under some suitable hypotheses the governing equations are hyperbolic. We propose suitable boundary conditions and establish a result of existence and uniqueness of smooth solutions for a limited time for our model.

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**Lower bounds on blow-up solutions of the 3D Navier-Stokes equations**

**James Robinson**  
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**Ricardo Ferreira da Silva & Witold Sadowski**

If  $u(t)$  is a solution of the 3D Navier-Stokes equations with a regular initial condition  $u_0 \in H^s$  with  $s \geq 1/2$ , it is known that  $u(t)$  remains bounded in  $H^s$  for some short time interval. If one assumes that the solution "blows up" at some time  $t = T$ , then relatively simple arguments imply that  $\|u(T-t)\|_{H^1} \geq ct^{-1/4}$ , and such a blow up rate

is consistent with the natural scaling of the Navier-Stokes equations on the whole space. In this talk I will discuss what can be said about the blow up rate of solutions in other Sobolev spaces  $H^s$  for  $s \geq 1/2$ .

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### Quasi-hydrostatic modelling in geophysical fluid dynamics

**Antoine Rousseau**

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**James C. McWilliams**

This work deals with the derivation of the quasi-hydrostatic quasi-geostrophic (QHQG) equations of large scale ocean as the Rossby number goes to zero. We follow classical techniques for the derivation of the quasi-geostrophic (QG) equations but the primitive equations that we consider account for all the rotating terms. In particular, the traditional approximation on the Coriolis force (and consequently the hydrostatic approximation) are not made in the vertical momentum equation, which reduces to

$$-f^*u + \frac{\partial \varphi}{\partial z} = -\frac{g \varrho}{\varrho_0}.$$

We end up with a slightly different QG model:

$$D[q + \beta y] = 0, \quad (3)$$

$$\left( \Delta + \partial_z \left[ \frac{1}{N^2} \partial_z \right] \right) \phi = q, \quad (4)$$

where  $D = \partial_t + J[\phi, \cdot]$ ,  $\Delta = \partial_x^2 + \partial_y^2$ ,  $J[a, b] = a_x b_y - a_y b_x$ ,  $\partial_z = \partial_z + \lambda \partial_y$ , and  $\lambda = \frac{H}{L} \cot \theta_0$ .

The parameter  $\lambda$  above measures the non-traditional Coriolis effects ( $\lambda = 0$  in the traditional QG model). It is actually responsible for a (slight) tilt of the vertical direction ( $\partial_z = \partial_z + \lambda \partial_y$ ), which has been illustrated in previous works using the primitive equations. This work can be seen as a first mathematical illustration of these papers.

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### A numerical study of the boundary value problem for the shallow water equations

**Ming-Cheng Shiue**

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In geophysical fluid dynamics, Limited Area Models have been used to achieve high resolution over a region of interest. However, this forces us to face the choice of boundary conditions since there is no physical law which can provide natural boundary conditions at the lateral boundary. In this talk, one-layer and two-layer shallow water equations related to weather prediction and oceanography are considered. We will discuss the suitability of the proposed boundary conditions and the suitability of the proposed numerical schemes which are inspired by the semidiscrete central-upwind method.

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### Near-equilibrium behavior of 2-D stochastic Navier-Stokes equations

**Eric Simonnet**

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We present recent results on the near-equilibrium behavior of 2-D stochastic Navier-Stokes equations in doubly-periodic and channel flow domains and discuss its link with the statistical mechanics of 2-D Euler equation obtained by Robert, Sommeria and Miller in the 90s. We also discuss numerical algorithms for estimating extreme events and large-deviation rate functions in the context of PDEs.

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### Pullback attractors for the 2D Primitive equations of the ocean.

**Theodore Tachim Medjo**

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We derive a sufficient condition for the existence of a pullback attractor for closed cocycles acting on a Banach space, where the strong continuity is replaced by a much weaker requirement that the cocycle be a closed map. The proof of the existence of the attractor uses the concept of measure of non-compactness and a result from V. Pata and S. V. Zelik (2007). The result extends that obtained by H. Song and H. Q. Wu (2007). As application, we prove the existence of the pullback attractor of a cocycle associated with the z-strong solutions of a non-autonomous two-dimensional primitive equations of the ocean.

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### The Zakharov-Kuznetsov equation of plasma physics: the case of a bounded domain

**Roger Temam**

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**Jean-Claude Saut, Chuntian Wang**

In this lecture we will present some recent results on existence of solutions for the Zakharov-Kuznetsov equation of plasma physics (space dimension two and three). In space dimension two the solution constructed is unique, and more regular solutions can be also constructed. The case considered here is the case of a bounded domain which raises many new difficulties compared to the case of a flow in an infinite strip considered earlier.

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### Fluctuation-dissipation theory with time periodic forcing

**Xiaoming Wang**

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**Andrew Majda**

We present a new fluctuation-dissipation theory (linear response formulas) for chaotic (stochastic) dynamical systems with time periodic coefficients. Such time periodic systems arise naturally in climate change studies due to the seasonal cycle. These response formulas are developed through the mathematical interplay between

statistical solutions for the time-periodic dynamical systems and the related skew-product system. This interplay is utilized to develop new systematic quasi-Gaussian and adjoint algorithms for calculating the climate response in such time-periodic systems. New linear response formulas are also developed here for general time-dependent statistical ensembles arising in ensemble prediction including the effects of deterministic model errors, initial ensembles, and model noise perturbations simultaneously. An information theoretic perspective is developed in calculating those model perturbations which yield the largest information deficit for the unperturbed system both for climate response and finite ensemble predictions. This is a joint work with Andy Majda.

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### Dynamic transition theory for thermohaline circulation

**Shouhong Wang**  
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The main objective of this study is to derive a mathematical theory associated with the thermohaline circulations (THC). The results derived provides a general transition and stability theory for the Boussinesq system, governing the motion and states of the large-scale ocean circulation. First, it is shown that the first transition is either to multiple steady states or to oscillations (periodic solutions), determined by the sign of a nondimensional parameter  $K$ , depending on the geometry of the physical domain and the thermal and saline Rayleigh numbers. Second, for both the multiple equilibria and periodic solutions transitions, both Type-I (continuous) and Type-II (jump) transitions can occur, and precise criteria are derived in terms of two computable nondimensional parameters  $b_1$  and  $b_2$ . Third, a convection scale law is introduced, leading to an introduction of proper friction terms in the model in order to derive the correct circulation length scale. In particular, the dynamic transitions of the model with the derived friction terms suggest that the THC favors the continuous transitions to stable multiple equilibria. This is joint with Tian Ma

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### Navier-Stokes equations on the beta-plane and the sphere

**Djoko Wirosoetisno**  
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We prove that the solution of the Navier-Stokes equations on the beta-plane and a rotating sphere tends towards zonal flows as the rotation rate tends to infinity. The implications for the dimension of the global attractor is also discussed.

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## Special Session 12: Singular Perturbations and Boundary Layer Theory

**Makram Hamouda**, Indiana University, USA

**Chang-Yeol Jung**, UNIST, Korea

**Roger Temam**, Indiana University, USA

The main purpose of this workshop is to bring some new ideas and results of distinguished specialists in the field of singular perturbations and boundary layers for PDEs. The connection between applied mathematics with other fields constitutes a big challenge for the modern mathematics. In this direction, the studies focusing on the interface between fluid mechanics and PDEs are of great importance in our days. It is well-known that the convergence of the Navier-Stokes solutions to the ones of Euler on bounded domains is one of the challenging problems in fluid mechanics; this is an outstanding open problem because of the presence of boundary layers. The minisymposium is open both to young and established researchers and it constitutes a good platform to exchange the future of some relevant problems in fluid mechanics which are in connection with PDEs that develop boundary layers. It is also an opportunity to graduate students and post-docs to present and discuss their researches in this direction.

### On the boundary value problem for the parabolic equation with a small parameter

**Galina Bizhanova**

Institute of Mathematics, Kazakhstan

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We consider the boundary value problem for the parabolic equation with a small parameter at the time derivative in a boundary condition. This problem is a linearized one of a Stefan type problem with a small parameter at the velocity of a free boundary. There are studied the properties of the solution of the perturbed problem, the convergence of it as a small parameter tends to zero.

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### Boundary layer problem: Navier-Stokes equations and Euler equations

**Fernanda Cipriano**

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**N.V. Chemetov**

This work is concerned with the boundary layer turbulence. We consider an incompressible viscous fluid in domains with permeable walls. The permeability is described by the Navier slip boundary conditions. The goal is to study the fluid behavior at vanishing viscosity. We show that the vanishing viscous limit is a solution of the Euler equations with the Navier slip boundary condition on the inflow region of the boundary.

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### Boundary layers of the Navier-Stokes equations

**Gung-min Gie**

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**Makram Hamouda, Chang-Yeol Jung, James Kelliher, Roger Temam**

We study boundary layers of the Navier-Stokes equations at small viscosity, in a curved domain, under various boundary conditions. In this talk, using the curvilinear system adapted to the boundary, we will focus on the construction of an incompressible corrector.

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### Singular perturbations for the Primitive equations

**Makram Hamouda**

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We present some convergence results related to the Linearized Primitive Equations for which the solution develops a boundary layer as the viscosity goes to zero. More precisely, as one of the difficulties is related with the ill-posedness of the limit solution with any choice of *local* boundary conditions, it is necessary to choose *nonlocal* ones. The boundary layer analysis allows us to confirm rigorously this choice.

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**Boundary layer for a class of nonlinear pipe flow****Daozhi Han**Florida State University, USA  
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In this talk, we will consider a class of nonlinear pipe flow in which the Prandtl boundary layer theory can be verified in the Sobolev norm setting. In particular, we will derive the optimal  $L^\infty(L^\infty)$  convergence and the  $L^\infty(H^1)$  convergence under certain mild smoothness condition and compatibility condition on the data.

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**Nonpolynomial spline finite difference scheme for nonlinear singular boundary value problems with singular perturbation and its mechanization****Navnit Jha**Rajiv Gandhi Institute of Petroleum Technology,  
Rae Bareilly, (U.P.), India  
navnitjha@rgipt.ac.in**Roger Temam**

A general scheme for the numerical solution of nonlinear singular perturbation problems using nonpolynomial spline basis is proposed in the paper. The special nonequidistant formulation of mesh takes into account the boundary and interior layer structures. The proposed scheme is almost fourth order accurate and applicable to both singular and nonsingular cases. Convergence analysis of the scheme is briefly discussed. Maple program for the generation of difference scheme is presented. Computational illustrations characterized by boundary and interior layers show that the practical order of accuracy is close to the theoretical order of the method.

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**Singularly perturbed convection-diffusion equations on a circle domain****Chang-Yeol Jung**UNIST, Korea  
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We discuss the boundary layers generated by a convection-diffusion equation in a circle. In the model problem that we consider two characteristic points appear. I will present how to treat boundary layers in a systematic way with certain simplifying compatibility conditions. We also discuss noncompatible data issues and how to handle them.

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**Some recent results on an extended Navier-Stokes system****James Kelliher**UC Riverside, USA  
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We discuss recent results on the well-posedness and numerical stability of the extended Navier-Stokes equations. These results build on the earlier works of Liu, Liu, and Pego and Pego, Iyer, and Zarnescu.

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**Vanishing viscosity limit for a certain class of channel flows****Anna Mazzucato**Penn State University, USA  
alm24@psu.edu**Michael Taylor**

We study the vanishing viscosity limit for certain Taylor-Couette flows in channels. We establish convergence of the Navier-Stokes solution to the corresponding Euler solution as viscosity vanishes in various norms without compatibility conditions on the data. In the process we obtain a detailed analysis of the small-diffusion limit for a heat equation with drift, using a parametric construction.

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**Navier-Stokes equations in critical spaces: existence and stability of steady state solutions****Tuoc Phan**University of Tennessee, USA  
phan@math.utk.edu**Nguyen Cong Phuc**

In this talk, we discuss our recent results on the uniqueness existence of solutions to the stationary Navier-Stokes equations with small singular external forces belonging to a functional space introduced by Mazy'a and Verbitsky. The stability of the steady state solutions in such spaces is also obtained by a series of sharp estimates for resolvents of a singularly perturbed operator and the corresponding semigroup.

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**From vortex layers to vortex sheets****Marco Sammartino**University of Palermo, Italy  
marco@math.unipa.it**R.Caflisch, M.C.Lombardo**

In this talk we shall consider the case of a plane flow with initial datum of a vortex layer type, i.e. with vorticity concentrated around a curve. We shall prove that, under analyticity hypotheses, the solution keeps the structure of vorticity layer, for a time that does not depend on the thickness. Moreover, when the thickness goes to zero, the layer moves according to the Birkhoff-Rott equation.

 $\rightarrow \infty \diamond \infty \leftarrow$ **Convection-diffusion equation with small viscosity in a circle****Roger Temam**Indiana University, USA  
temam@indiana.edu**Chang-Yeol Jung**

In this talk we will discuss singular perturbation problems for convection-diffusion equations in a circle when the viscosity is small. Highly singular behaviors can occur at the characteristic points which render the analysis difficult. A detailed analysis of these singularities has been conducted, and the corresponding boundary layers have been made explicit. This simplified model shows how singular and involved the behaviors can be in incompressible fluid mechanics when the viscosity is small.

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## Special Session 13: Global Dynamics in Hamiltonian Systems

**Rafael de la Llave**, Georgia Institute of Technology, USA  
**Tere.M-Seara**, Univeritat Politecnica de Catalunya, Spain

This minisposium will concentrate on global properties of Hamiltonian systems. Including the invariant objects and the global dynamics that they produce. We plan to bring together mathematical theory about these questions, numerical calculations and applications to realistic systems.

### Zonal jets as transport barriers in the Earth's stratosphere

**Francisco Beron-Vera**

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**M. J. Olascoaga, M. G. Brown, H. Kocak**

Theoretical results relating to Kolmogorov–Arnold–Moser (KAM) theory have led to the expectation that associated with zonal (west–east) jet streams in the Earth's stratosphere should be barriers which inhibit meridional (south–north) transport. In this talk evidence will be provided for this expectation based on the analysis of stratospheric winds produced by a comprehensive general circulation model. This will follow a review of the relevant KAM theory results, namely, those dealing with the stability of a degenerate one-degree-of-freedom Hamiltonian system under a time-quasiperiodic perturbation.

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### Poincare-Birkhoff theorem in quantum mechanics

**Florentino Borondo**

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**F.J. Arranz, R.M. Benito, D. Wisniacki, M. Saraceno**

Quantum manifestations of the dynamics around resonant tori in perturbed Hamiltonian systems, dictated by the Poincare-Birkhoff theorem, are shown to exist. They are embedded in the interactions involving states which differ in a number of quanta equal to the order of the classical resonance. Moreover, the associated classical phase space structures are mimicked in the quasiprobability density functions and their zeros.

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### "Theory and computation of quasi-periodic solutions of conformally symplectic systems

**Renato Calleja**

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Quasi-Periodic motions appear in many different

physical systems (e.g. celestial mechanics, solid-state physics, non-equilibrium statistical mechanics, etc.) I will discuss an a-posteriori KAM theory for the existence and persistence of quasi-periodic solutions. The consequences of the a-posteriori format of KAM theory include local uniqueness, regularity results, and the rigorous justification of a criterion for the breakdown of analyticity. This format also suggests and validates very efficient numerical methods to approximate quasi-periodic solutions and their breakdown of analyticity and hyperbolicity.

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### Some topological aspects of Aubry-Mather theory

**Marian Gidea**

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We re-construct the Aubry-Mather theory in terms of a foliation of the annulus, by expressing the twist condition and the tilt condition in terms of such a foliation. Then the existence of Aubry-Mather sets and of orbits that shadow prescribed families of Aubry-Mather sets can be proved using the diagonal method of G. R. Hall.

A motivation for this work is the study of Hamiltonian flows on strictly convex energy levels in  $\mathbb{R}^4$ . Such flows possess a system of open book decompositions whose pages are disk-like global surfaces of sections to the flow. Each disk-like global surface of section is foliated by the pages of the complementary open book decomposition. The first return map to the disk determines a symplectic homeomorphism of an open annulus. Thus we can apply Aubry-Mather theory via foliations to this setting, assuming that the annulus map satisfies a twist condition or a tilt condition. Even if such a condition is not satisfied, some topological aspects of Aubry-Mather theory remain valid.

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### On stochastic sea of the standard map

**Anton Gorodetski**

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The standard map is one of the simplest and most famous conservative transformations that is still

far from being completely understood. We prove that stochastic sea (the set of orbits with non-zero Lyapunov exponents) of the standard map has full Hausdorff dimension for sufficiently large topologically generic parameters. In the proof we consider a generic one-parameter unfolding of a homoclinic tangency of an area preserving surface diffeomorphism, and show that it give birth to a hyperbolic set of large (almost full) Hausdorff dimension. The last statement has also applications in celestial mechanics (this is a joint work with V.Kaloshin), namely can be used to show that in some versions of the three body problem for many parameters the set of oscillatory motions has full Hausdorff dimension.

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### Generalized KAM cuves in time-aperiodic Hamiltonian systems

**George Haller**

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**Javier Beron-Vera, Mohammad Farazmand**

I show how the recent geodesic theory of transport barriers can be used to compute invariant regions bounded by generalized KAM curves (elliptic transport barriers) in temporally aperiodic, planar Hamiltonian systems. When advected by the flow map over a finite time interval, these curves preserve both their arclength and the area they enclose, much the same way as KAM curves do under the iteration of the associated Poincare map in time-periodic Hamiltonians. These properties render elliptic transport barriers as ideal boundaries of coherent vortices in fluid mechanics. I will illustrate this by applications to geophysical flows and 2D forced turbulence.

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### Singularity theory for non-twist KAM tori: a methodology

**Alex Haro**

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**Alejandra Gonzalez-Enriquez, Rafael de la Llave**

We present a novel method to find KAM tori in degenerate (nontwist) cases. We also require that the tori thus constructed have a singular Birkhoff normal form. The method provides a natural classification of KAM tori which is based on Singularity Theory. The method also leads to effective algorithms of computation, and we present some preliminary numerical results.

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### On the action-minimizing triple collision orbits in the planar three-body problem

**Hsin-yuan Huang**

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In this talk, we study the shape of the limiting configuration of the action-minimizing triple collision orbits in the planar Newtonian three-body problem with arbitrary masses. For a given non-collinear initial configuration, the minimizing triple collision orbit is collision-free until a simultaneous collision, and its limiting configuration is the Lagrangian configuration with the same orientation as the initial configuration. For the collinear initial configuration, under a certain technical assumption, there exist two minimizing orbits. The limiting configurations of these orbits are the two opposite Lagrangian configurations.

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### Extrapolation of frequencies of quasi-periodic orbits

**Alejandro Luque**

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**Jordi Villanueva**

Frequency analysis was introduced by Laskar to study secular motions of the planets in the Solar system. A significant refinement of Laskar's method, based in the simultaneous improvement of the frequencies and the amplitudes of the signal, was given later by G. Gómez, J.M. Mondelo and C. Simó. On another front, a methodology to compute rotation numbers of invariant curves (and more general objects) has been introduced recently in different works by A. Luque, T.M. Seara and J. Villanueva. The idea is to extrapolate the rotation number (and related quantities) from suitable averages of the iterates of an orbit. The goal of this talk is to present a methodology to compute the frequencies of a given quasi-periodic orbit with an arbitrary number of basic frequencies. The construction is a generalization of the mentioned averaging-extrapolation approach to study rotation numbers. We plan to describe informally this construction, that allows us to compute with high precision the components of the frequency vector. The main advantage over other high precision methods is that we do not require to compute nor to refine the amplitudes of the signal. As an illustration, we will consider quasi-periodic motions close to the point L5 in a restricted three-body problem and also include quasi-periodic perturbations from another bodies.

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### Hyperbolic cylinders and KAM tori in nearly integrable systems on $A^3$

**Jean Pierre Marco**  
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The aim of the talk is to present recent results on Arnold diffusion for three-degree-of-freedom systems. We will show how to construct a hyperbolic skeleton for diffusion, namely a generalized chain of hyperbolic cylinders connected by heteroclinic orbits, which is asymptotically dense on an energy level when the size of the perturbation tends to 0. All the conditions for constructing orbits shadowing this chain are fulfilled (torsion condition on the cylinders and tame homoclinic connections), so the existence of diffusion orbits follows from classical shadowing lemmas.

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### Generic super-exponential stability of invariant tori in Hamiltonian systems

**Laurent Niederman**  
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Morbidelli and Giorgilli have proved a result of stability over super-exponentially long times if one consider an analytic lagrangian torus, invariant for an analytic hamiltonian system, with diophantine translation vector which admit a sign definite torsion. More specifically, the solutions of the system moves very little over times which are super-exponentially long with respect to the inverse of the distance to the invariant torus. The proof is in two steps: first the construction of a Birkhoff normal form at a high order, then the application of Nekhoroshev theory. Bounemoura has shown that the second step of this construction remains valid if the Birkhoff normal linked to the invariant torus is in a generic set among the formal series. This is not sufficient to prove this kind of super-exponential stability results in a general setting. We should also establish that most strongly non resonant invariant torus in a Hamiltonian system admit a Birkhoff normal form in the set introduced by Bounemoura. We show here that this property is satisfied generically in the sense of the measure (prevalence) through methods similar to those developed by Kaloshin and Hunt in their work on the growth of the number of periodic points for prevalent diffeomorphisms.

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### Geometric control and dynamical systems

**Ludovic Rifford**  
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We show several applications of geometric control methods to Hamiltonian dynamics such as closing lemmas and Kupka-Smale like theorems.

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### A saddle in a corner - a model of collinear triatomic chemical reactions

**Vered Rom-kedar**  
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**L. Lerman**

A geometrical model which captures the main ingredients governing atom-diatom collinear chemical reactions is proposed. This model is neither near-integrable nor hyperbolic, yet it is amenable to analysis using a combination of the recently developed tools for studying systems with steep potentials and the study of the phase space structure near a center-saddle equilibrium. The nontrivial dependence of the reaction rates on parameters, initial conditions, and energy is thus qualitatively explained. Conditions under which the phase space transition state theory assumptions are satisfied and conditions under which they fail are derived.

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### Weak integrability of Hamiltonians in the two torus and rigidity

**Rafael Ruggiero Rodriguez**  
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**Jose Barbosa Gomes**

We show that a  $C^\infty$  k-basic Finsler metric in the two torus  $T^2$  whose geodesic flow preserves a codimension one  $C^{1,L}$  foliation is in fact flat. Although integrable high energy levels of Hamiltonians in the torus are not flat in general, the  $C^{1,L}$  integrability of k-basic Finsler geodesic flows in  $T^2$  implies flatness and in particular,  $C^\infty$  integrability. We also show that a codimension one  $C^1$  foliation invariant by the geodesic flow of the k-basic Finsler metric is  $C^\infty$ . A consequence of the above results is that the Hopf conjecture would be false for k-basic Finsler metrics in the two torus if and only if there exists a  $C^0$  integrable k-basic Finsler geodesic flow that is not  $C^1$  integrable.

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**Dimension reduction for Hamiltonian dynamics****Eli Shlizerman**

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**J. Nathan Kutz**

We study the dimension reduction of Hamiltonian PDEs using Proper Orthogonal Decomposition (POD), which allows one to project the full evolution equations onto an optimal set of modes that determine the dynamics. The optimal set of modes is constructed from observations of the dynamics of the governing equations. We describe the properties of such a reduction in the Hamiltonian framework and show how the reduced model can be used for analytical and numerical bifurcation study. Examples of the application of the approach to solitondynamics described by the nonlinear Schrodinger equation and computation of periodic surface waves will be given.

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## Special Session 14: Mathematical Models in Biology and Medicine

**Yang Kuang**, Arizona State University, USA  
**Bingtuan Li**, University of Louisville, USA  
**Jiaxu Li**, University of Louisville, USA  
**Andrew Nevai**, University of Central Florida, USA

This special session emphasizes the dual roles of both theory and applications of dynamical systems and differential equations to mathematical models in biology and medicine. Recent advances within mathematical ecology (including competition between species, predator-prey and cooperative systems, and evolutionary theory) will be presented alongside those for mathematical models in various fields of medicine (including immunology, epidemiology, oncology, endocrinology, and neurobiology).

### The dual oscillator model for pancreatic islets

**Richard Bertram**

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Insulin is a key hormone for glucose uptake and utilization in the body. It is known to be pulsatile, with a period of about 5 min, and this pulsatility is altered in type 2 diabetics and their nearest relatives. We describe a mathematical model that has been developed over the past decade for the electrical activity, calcium handling, and metabolism of insulin-secreting beta-cells within pancreatic islets. This Dual Oscillator Model consists of an oscillator involving ionic currents and calcium feedback, and a separate but coupled oscillator driven by oscillations in glycolysis. The model has been used successfully to make predictions that have been tested in the laboratory, as we discuss in this presentation.

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### Slow passage through a Hopf bifurcation in spatially extended excitable systems: some examples from neuroscience

**Lydia Bilinsky**

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**Steven M. Baer**

Hopf bifurcation is a common mechanism by which a dynamical system featuring a constant parameter  $p$  exhibits steady-state seeking behavior if  $p$  is less than some critical value  $p_{\text{Hopf}}$ , and sustained oscillations if  $p > p_{\text{Hopf}}$ . It is known that when  $p$  is not constant in time, but rather, slowly ramped up through  $p_{\text{Hopf}}$ , sustained oscillations do not ensue as soon as  $p$  exceeds  $p_{\text{Hopf}}$ . How delayed the onset of instability is depends on the exact time course of the ramp as well as its initial value  $p_0$ , and can be obtained from the WKB method of perturbation theory. In the current work, we have investigated the response of a spatially extended system to a slow parameter ramp. The particular system studied is one from neuroscience: a reaction-diffusion model of a dendrite studded with excitable spines, into one

end of which we inject current  $I$ ; here,  $I$  is the bifurcation parameter. It is found that the WKB method also provides the location along the cable at which instability first shows itself. By manipulating the current ramp, we can choose the location along the cable at which the approach to sustained oscillations is first apparent.

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### A mathematical model of Imatinib and Interferon-alpha combined treatment of chronic myeloid leukemia

**Svetlana Bunimovich**

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**L. Berezansky, A. Domoshnitsky**

We propose and analyze a mathematical model for the treatment of chronic myelogenous (or myeloid) leukemia (CML), a cancer of the blood. We introduce combined treatment of CML based on Imatinib therapy and Immunotherapy. Imatinib therapy is a molecular targeted therapy that inhibits the cell containing the oncogenic protein BCR-ABL, involved in the chronic CML pathogenesis. Immunotherapy based on interferon alfa-2a (IFN) effects on the cancer cells mortality and leads to improvement outcome of the combined therapy. We model the interaction between CML cancer cells in the body and effector cells of the immune system, using a system of differential equations. The proposed model belongs to a special class of nonlinear nonautonomous systems of ordinary differential equations (ODEs) with time-varying delays in the treatment. For this system the following results were obtained: existence of a unique global positive solution, existence of a unique nontrivial equilibrium, explicit local and global stability conditions for the nontrivial equilibrium.

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### Interpretation of the IVGTT by means of a distributed-controller model of the endocrine pancreas

**Andrea De Gaetano**

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**Claudio Gaz, Simona Panunzi, Pasquale Palumbo, Andrea De Gaetano**

Since the publication of the original model by Grodsky, a possible paradigm for the function of the endocrine pancreas has been that of a population of independent insulin-secreting controllers, coupled by the circulating levels of the controlled substrate, glucose. In 2009, an explicit non-reductionist stochastic-differential model of the 1 million independent human pancreatic islets was published, showing how the glucose threshold and recovery time distributions over the population of controllers could allow the model to simultaneously explain observed low- and high- frequency oscillations in the circulating levels of insulin. The present work shows how the model is able to replicate the observed morphology of glycemia and insulinemia time-courses after a standard perturbation experiment (Intra Venous Glucose Tolerance Test, IVGTT). The analysis of the model features necessary to faithfully reproduce observations links the present high-detail formulation with previous, much simpler delay-differential models for the same perturbation experiment, which were shown to achieve excellent predictive power in clinical research. Conclusions are drawn as to the likely essential components of the physiologic system under investigation.

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### A degenerate parabolic system

**Zhaosheng Feng**

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In this talk, we study a class of parabolic systems which can be used as a dispersion model of biological populations. Under certain parametric conditions, we apply the center manifold method and Bendixon's test to obtain the local behavior around a non-hyperbolic point of codimension one in the phase plane. Then we analyze the associated determining system through the linearized symmetry condition. Classical and non-classical symmetries are classified. Nontrivial infinitesimal generators are found, and exact solutions are established accordingly.

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### Mathematical models of the role of immune exhaustion in Hepatitis B and delta coinfection

**Jonathan Forde**

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**Yaixin Liu, Yixiao Sha**

Hepatitis Delta Virus (HDV) is a dependent satellite virus of the more common Hepatitis B Virus. HDV encodes only one protein of its own, relying on HBV to supply the additional proteins needed for its replication cycle. Although it HDV is noncytotoxic and present few targets for immune reaction, the symptoms of patients with HBV-HDV coinfection are much worse than those infected with HBV alone. The cause of this negative outcome is not clear. This work presents o.d.e. models for the interaction of HBV, HDV and the specific immune responses to each, and analyzes the implication of these models for understanding patient outcomes. In particular, the role of T cell exhaustion in chronic HBV is explored, and how superinfection with HDV may actually strengthen the HBV specific immune response, indirectly leading to the observed symptoms. Additionally, the role of nonspecific immune responses is explored.

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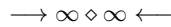
### Effects of surgery and chemotherapy on growth of metastases in prostate cancer: evidence from the natural history of the disease reconstructed through mathematical modeling

**Leonid Hanin**

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We present a comprehensive model of cancer progression accounting for primary tumor growth, shedding of metastases, their dormancy and growth at secondary sites. Model parameters were estimated from the volumes of the primary tumor and bone metastases collected from 12 prostate cancer patients. This enabled estimation of various unobservable characteristics of the individual natural history of cancer and the effects of treatment. We found that for all patients: (1) inception of the first metastasis occurred when the primary tumor was undetectable; (2) inception of all or most of the metastases occurred before the start of treatment; (3) the rate of metastasis shedding was essentially constant in time regardless of the size of the primary tumor and so it was only marginally affected by treatment; and, most importantly, (4) surgery and chemotherapy brought about a dramatic increase in the rate of growth of metastases. These conclusions agree with the findings of numerous clinical studies, animal experiments and epidemiological analyses conducted since the early

1900s. Our analysis supports the notion of metastasis dormancy and the existence of prostate cancer stem cells. This is a joint work with Dr. Marco Zaider from the Memorial Sloan-Kettering Cancer Center.



**Mathematical models of immunological tolerance and immune activation following prenatal infection with hepatitis B virus**

**Sarah Hews**

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**Stanca Ciupe**

We develop mathematical models for the role of hepatitis B e-antigen in creating immunological tolerance during hepatitis B virus infection and propose mechanisms for hepatitis B e-antigen clearance, subsequent emergence of a potent cellular immune response, and the effect of these on liver damage. We investigate the dynamics of virus-immune cells interactions, and derive parameter regimes that allow for viral persistence. We modify the model to account for mechanisms responsible for hepatitis B e-antigen loss, such as seroconversion and virus mutations that lead to emergence of potent cellular immune response to the mutant virus. Our models demonstrate that either seroconversion or mutations can induce immune activation and that instantaneous loss of e-antigen by either mechanism is associated with least liver damage and is therefore more beneficial for disease outcomes.



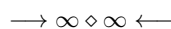
**A refuge-mediated apparent competition model**

**Sze-bi Hsu**

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**Lih-ing Wu Roeger**

We analyze a competition model of two plant species for a single-limited resource while the competition is apparent: an indirect interaction where the invading plants provide a refuge for a shared consumer, subsequently increasing the consumer pressure on the resident plant species. When there is no refuge effect, the resident species is a superior species. As the refuge effect increases, the coexistence state appears as a saddle point with a two-dimensional stable manifold while the two extinction equilibria are locally stable. Thus the refuge-mediated apparent competition creates an Allee effect for both the invading and the resident species. A Lyapunov function is found to show the global stability of the equilibrium in which only the resident species survives.



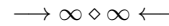
**Modeling the distribution of insulin in pancreas**

**Changbing Hu**

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**Jiaxu Li, James Johnson**

Maintenance of adequate pancreatic  $\beta$ -cell mass, via suppression of programmed cell death and/or sustained proliferation is critical for the prevention or delay of diabetes mellitus. In this talk we report a novel mathematical model to investigate the distribution and concentration of insulin within pancreas using existing physiological data and islet imaging data. Our studies reveal that insulin concentration along pancreas, related to glucose level, increases nearly linearly in the fashion of increasing quicker in tail area but slower in head area. We also showed that the factor of small diffusion with blood is negligible since the convection of blood flux dominates. To the best of our knowledge, our model is the first attempt to estimate the glucose and insulin distributions in pancreas. Our model is simple, robust and thus can be easily adopted to study more sophisticated cases.



**Multiple attractors in intraguild predation models with generalist/specialist predator**

**Yun Kang**

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**Lauren Wedekin**

Intraguild predation (IGP) is a combination of competition and predation which is the most basic system in food webs that contains three species where two species that are involved in a predator/prey relationship are also competing for a shared resource or prey. We formulate two intraguild predation (IGP: resource, IG prey and IG predator) models: one has a predator who is a generalist while the other one is a specialist. Both models have Holling-Type I functional response between resource-IG prey, resource-IG predator and Holling-Type III functional response between IG prey and IG predator. We prove sufficient conditions of the persistence and extinction of all possible scenarios for these two models, which give us a complete picture on their global dynamics. These analytical results indicate that IGP model with generalist predator has "top down" regulation while IGP model with specialist predator has "bottom up" regulation. In addition, we show that both IGP models can have multiple interior equilibria. Our analysis and numerical simulations suggest that 1. Both IGP models can have multiple attractors with complicated dynamical pattern; 2. Only IGP model with specialist predator can have both boundary attractor and interior attractor: i.e., whether the system has the extinction of one species or the

coexistence of three species dependent on initial conditions; 3. IGP model with generalist predator is prone to have stable dynamics.

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### Age structured population model with state dependent time delay

**Nemanja Kosovalic**

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**Felicia Magpantay, Jianhong Wu**

In this talk we consider an age structured population model in which the age to maturity at a given time depends on whether or not the food consumed by the immature population within that time span reaches a prescribed threshold value. This introduces a state dependent delay into the model. In contrast with other works on this problem, we consider it from the point of view of a hyperbolic partial differential equation with a state dependent time delay.

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### Designing optimal combined chemotherapy and immunotherapy protocols for a model of tumor immune interactions under drug pharmacokinetics

**Urszula Ledzewicz**

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**Mozhdeh Faraji Mosalman, Heinz Schüttler**

In the design of protocols for cancer treatment various questions need to be addressed related to dosage, frequency and in the case of more than one drug also their sequencing. In answering, from a more theoretical point of view tools of optimal control can be of use. In this talk we present such an analysis of a mathematical model for cancer combination therapy as an optimal control problem. Since the treatment involves combination of classical chemotherapy with an immune boost, the adequate model for consideration is the one that describes dynamics of the interactions between the tumor cells and the immune system. Within this framework we introduce into the dynamics the controls representing the action of each therapeutic agent. There exist various approaches for choosing the objective, but based on the dynamical behavior of the uncontrolled and fully controlled system, we choose the one to minimize the size of the tumor at the end of treatment while at the same time maximizing the immune-competent cell densities and keeping side effects of both treatments low. This leads to a multi-input optimal control problem with many challenging features. The complexity of the models becomes even higher if we also take into account the pharmacokinetics of both therapeutic agents. Analytical and numerical results about

the structures of optimal controls will be presented. Particularly the effect of the inclusion of pharmacokinetics of both drugs on the qualitative structure of solutions will be discussed. Overall the presented research is expected to provide insights into how to optimally design protocols for the combination of these two drugs in the sense of the scheduling the dosages and sequencing.

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### Global bifurcation branch of a spatially heterogeneous cooperative system with cross-diffusion

**Wan-tong Li**

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**Yu-Xia Wang**

In this paper, we will consider the strongly coupled cooperative system in a spatially heterogeneous environment with Neumann boundary condition. We show that the positive solution curve to form an unbounded fish-hook shaped continuum. Our results deduce that the spatial heterogeneity of environments can produce multiple coexistence states.

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### Modeling impulsive injections of insulin analogues: towards artificial pancreas

**Jiaxu Li**

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**Mingzhan Huang, Xinyu Song, Hongjian Guo**

We propose two novel mathematical models with impulsive injection of insulin or its analogues for type 1 and type 2 diabetes mellitus. One model incorporates with periodic impulsive injection of insulin. We analytically showed the existence and uniqueness of a positive globally asymptotically stable periodic solution for type 1 diabetes, which implies that the perturbation due to insulin injection will not disturb the homeostasis of plasma glucose concentration. We also showed that the system is uniformly permanent for type 2 diabetes, that is, the glucose concentration level is uniformly bounded above and below. The other model has the feature that determines the insulin injection by closely monitoring the glucose level when the glucose level reaches or passes a predefined but adjustable threshold value. We analytically proved the existence and stability of the order one periodic solution, which ensures that the perturbation by the injection in such an automated way can make the blood glucose concentration under control. Our numerical analyses confirm and further enhance the usefulness and robustness of our models. The first model has implications in clinic that the glucose level of a diabetic can be controlled within

desired level by adjusting the values of two model parameters, injection period and injection dose. The second model is probably the first attempt to conquer some critical issues in the design of artificial pancreas with closed-loop approach. For both cases, our numerical analysis reveal that smaller but shorter insulin delivery therapy is more efficient and effective. This can be significant in design and development of insulin pump and artificial pancreas.

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### Traveling wave solutions in delayed cooperative systems

**Bingtuan Li**

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**Liang Zhang**

We discuss the existence of traveling wave solutions for delayed cooperative recursions that are allowed to have more than two equilibria. We define an important extended real number that is used to determine the speeds of traveling wave solutions. The results can be applied to a large class of delayed cooperative reaction-diffusion models. We show that for a delayed Lotka-Volterra reaction-diffusion competition model, there exists a finite positive number that can be characterized as the slowest speed of traveling wave solutions connecting two mono-culture equilibria or connecting a mono-culture with the coexistence equilibrium.

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### The relative biologic effectiveness versus linear energy transfer curve as a phenotype

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**Paul DuChateau**

The magnitude of biological response varies with different radiation types. Using Linear Energy Transfer (LET) to differentiate types of incident radiation beam, the Relative Biologic Effectiveness (RBE) as a function of LET (RBE-LET) was found to have a peak around LET values 100-200 keV/nm. This general feature is believed to be a property of the incident beam. Our systems engineering model, however, suggests that the shape of the RBE-LET curve is a property of the cells. Here, we continue systems engineering analysis to show that the RBE-LET curve is a phenotype. Elementary state-space block diagram of the differential equation model suggests a genetic network and cellular context for this phenotype. Analogy to man-made analog computers suggests cell uses mathematical computations in its decision-making when faced with an external stress. The MRN protein complex, in particular, may perform addition to count the degree of DNA twisting

for the homeostatic regulation of DNA supercoiling. The ATM protein may act as a feedback amplifier.

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### Theoretical and numerical analysis of a class of dynamics models describing eutrophication of Lake Guishui

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**Zhanhui Wang, Wanbiao Ma, Hai Yan**

Firstly, two classes of dynamic models describing the growth of Microcystins (MCs) in lake are proposed. Secondly, locally asymptotic stability (LAS) of the equilibria and Hopf bifurcations of the models are analyzed by using theory of delay differential equations. Thirdly, the models are applied to the experimental data on eutrophication of Lake Guishui in Beijing, and the parameters in the models are determined. The numerical simulations suggest that the trajectories of the model fit the experimental data well.

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### Evolution of uncontrolled proliferation and the angiogenic switch in cancer

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**Dieter Armbruster**

The major goal of evolutionary oncology is to explain how malignant traits evolve to become cancer “hallmarks.” One such hallmark—the angiogenic switch—is difficult to explain for the same reason altruism is difficult to explain. An angiogenic clone is vulnerable to “cheater” lineages that shunt energy from angiogenesis to proliferation, allowing the cheater to outcompete cooperative phenotypes in the environment built by the cooperators. Here we show that cell- or clone-level selection explain the angiogenic switch, but not because of direct selection on angiogenesis factor secretion. We study a multiscale mathematical model that includes an energy management system in an evolving angiogenic tumor. The energy management model makes the counterintuitive prediction that ATP concentration in resting cells increases with increasing ATP hydrolysis, as seen in other theoretical and empirical studies. As a result, increasing ATP hydrolysis for angiogenesis can increase proliferative potential, which is the trait directly under selection. Intriguingly, this energy dynamic allows an evolutionary stable angiogenesis strategy, but this strategy is an evolutionary repeller, leading to runaway selection for extreme vascular hypo- or hyperplasia. The former case yields a tumor-on-a-tumor, or hypertumor,

as predicted in other studies, and the latter case may explain vascular hyperplasia in certain tumor types.

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### A PDE model for predator-prey dynamics with a resource subsidy

**Andrew Nevai**

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**Junping Shi**

The influence of a resource subsidy on predator-prey interactions is examined using a system of partial differential equations. The model arises from study of a biological system involving arctic foxes (predator), lemmings (prey), and seal carcasses (subsidy). The prey occurs in the interior of the habitat and the subsidy occurs near the edge. Criteria for feasibility and stability of the different equilibrium states are studied. At small subsidy input rates, there is a minimum prey carrying capacity needed to support both predator and prey. At intermediate subsidy input rates, the predator and prey can always coexist. At high subsidy input rates, prey persistence depends on a variety of factors. As the predator movement rate increases, the dynamic stability of the predator-prey-subsidy interactions also increases.

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### Dynamically consistent discrete-time SI, SIS, and SIR epidemic models

**Lih-ing Roeger**

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We construct discrete-time or difference equation *SI*, *SIS*, and *SIR* epidemic models such that they are dynamically consistent with their analog continuous-time models. The discrete-time models are constructed using the nonstandard finite difference discretization (NSFD) methods. All three basic standard incidence *SI*, *SIS*, and *SIR* models without births and deaths, with births and deaths, and with immigrations, are considered. The continuous models possess either the conservation law that the total population  $N$  is a constant or the total population  $N$  satisfies  $N'(t) = \lambda - \mu N$  and so that  $N$  approaches a constant  $\lambda/\mu$ . The difference equation systems via NSFD schemes preserve all properties including the positivity of solutions, the conservation law, and the local and some of the global stability of the equilibria. Hence they are said to be dynamically consistent with the continuous models. We show that the a simple criterion for choosing certain NSFD scheme such that the positivity solutions are preserved is usually an indication of an appropriate NSFD scheme for an epidemic model.

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### Periodic solutions of a predator-prey system with nonmonotonic response function and impulsive harvesting

**Xinyu Song**

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**Mingzhan Huang, Lansun Chen**

In this talk, we propose a predator-prey model with state dependent impulsive harvesting which use a nonmonotonic response function to model the phenomenon of group defense. We mainly discuss the existence, uniqueness and stability of the order  $k$  periodic solution by differential equation geometry theory and the method of successor functions. Moreover, we give the parametric conditions that the system exhibits the phenomenon of homoclinic bifurcation. We also explain the ecological applications of these results at last.

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### Optimization of *P. falciparum* gametocyte sex ratios via competitive and non-competitive strategies: the evolutionary implications

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**Thomas Yuster, Ha Nguyen, Ryan Warrior**

We analyze two distinct fitness optimization strategies for sex ratio determination for *P. falciparum* with varying fecundity. Initial results indicate that no polymorphic population containing both strategies is stable. The pure strategy that is the final evolutionary state depends heavily on the composition of the initial polymorphic population.

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### Epidemic model as controlled switched system

**Abdessamad Tridane**

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**Chakib Jerry**

The aim of this work is to study a SIR model controlled by impulsive vaccination and isolation control. The goal is to determine an optimal impulsive controls to minimize the total outbreak size over the course of the epidemic and using necessary condition of optimality, our goal is to draw conclusions about the effect of the shortage of the drug treatment on the management of strategies of control policy. In other words, we are going to answer two questions: What if the treatment runs out during the epidemic? If this happens, could we still control the spread of the disease?

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### Integro-differential age-structured system for the SAIQR influenza model

**Ana Vivas-barber**

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**Sunmi Lee**

In this work, we present an Integro-Differential System to study the dynamics of an age-structured SAIQR influenza model. The population in consideration is divided into five age-dependent epidemiological classes:  $S(a,t)$ , susceptible;  $A(a,t)$ , asymptomatic;  $I(a,t)$ , infectious;  $Q(a,t)$ , isolated (quarantined) and  $R(a,t)$ , recovered individuals. It is assumed that  $A(a,t)$ -individuals may be less infectious than those in the  $I(a,t)$ -class. We assume the “proportional mixing” property between age-groups. Existence of equilibria and the basic reproduction number are discussed. Furthermore, a multi age-group model and numerical simulations with influenza parameters are presented.

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### Coexistence of the unstirred chemostat model

**Jianhua Wu**

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**G.Wolkowicz, H.Nie, G.H.Guo**

In this talk, the coexistence of the unstirred chemostat model is given, and the related result is also surveyed if possible. The main ingredients include global bifurcation theory, fixed point index theory, perturbation method and simulations.

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### On latencies in malaria infections and their impact on the disease dynamics

**Yanyu Xiao**

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**Xingfu Zou**

We modify the classic Ross-Macdonald model for malaria disease dynamics by incorporating latencies for both human beings and female mosquitoes. Two general probability functions ( $P_1(t)$  and  $P_2(t)$ ) are introduced to reflect the fact that the latencies differ from individuals to individuals. We justify the well-posedness of the new model, identify the basic reproduction number  $\mathcal{R}_0$  for the model and analyze the dynamics of the model. When  $\mathcal{R}_0 < 1$ ,  $E_0$  becomes unstable. When  $\mathcal{R}_0 > 1$ , we consider two specific forms for  $P_1(t)$  and  $P_2(t)$ : (i)  $P_1(t)$  and  $P_2(t)$  are both exponential functions; (ii)  $P_1(t)$  and  $P_2(t)$  are both step functions. For (i), the model reduces to an ODE system, and for (ii), the long term disease dynamics are governed by a DDE system. In both cases, we are able to show that when  $\mathcal{R}_0 > 1$  then the disease will persist; moreover if there is no recovery ( $\gamma_1 = 0$ ), then all admissible positive solutions will converge to the unique endemic equilibrium. A significant impact of the latencies is that they reduce the basic reproduction number, regardless of the forms of the distributions.

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### Infectious diseases and demographic allee effect

**Abdul-Aziz Yakubu**

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**Avner Friedman**

If a healthy stable host population at the disease-free equilibrium is subject to the Allee effect, can a small number of infected individuals with a fatal disease cause the host population to go extinct? That is, does the Allee effect matter at high densities? In this talk, we use an SI epidemic model to answer these questions.

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## Special Session 15: Nonlinear Evolution Equations, Inclusions and Related Topics

Mitsuharu Otani, Waseda University, Japan  
 Tohru Ozawa, Waseda University, Japan  
 N. U. Ahmed, University of Ottawa, Canada  
 S. Migorski, Jagiellonian University, Poland  
 I. I. Vrabie, Al. I. Cuza University, Romania

This session will focus on the recent developments in the theory of Nonlinear Evolution Equations and Related Topics including the theory of abstract evolution equations in Banach spaces as well as the studies (the existence, regularity and asymptotic behaviour of solutions) of various types of Nonlinear Partial Differential Equations. Some new trends in both abstract linear and nonlinear evolution inclusions as well as concrete applications in Control Theory, Partial Differential Equations, Variational Inequalities, and mathematical models described by evolution inclusions will be discussed. Open problems and new results in the above mentioned areas based on an interplay between various disciplines as Multi-Valued Analysis, Measure Theory, Fourier Analysis and Spectral Theory, will find their place within this Special Session.

### Optimal feedback control for differential inclusions on Banach spaces

**Nasir Uddin Ahmed**

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In this paper we consider a class of partially observed semilinear dynamic systems on infinite dimensional Banach spaces governed by differential inclusion representing measurement uncertainty. In the paper we present a result on existence of an optimal operator valued function as the feedback control law minimizing the maximum risk. We also present necessary conditions of optimality for the feedback control law from a class of operator valued functions defined on finite intervals and taking values from the space of compact operators. This is then extended to cover systems with uncertainty in the dynamics itself. The necessary conditions remain valid also for a class of stochastic systems driven by Wiener martingale.

[1] N.U.Ahmed, Optimal Relaxed Controls for Systems Governed by Impulsive Differential Inclusions, *Nonlinear Funct. Anal. & Appl.*, 10(3), (2005), 427-460.

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### Doubly nonlinear parabolic equations with variable exponents

**Goro Akagi**

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This talk is concerned with doubly nonlinear parabolic equations involving variable exponents. Let  $\Omega$  be a bounded domain in  $\mathbb{R}^d$  with smooth boundary  $\partial\Omega$ . We discuss the following doubly non-

linear parabolic problem (P):

$$\begin{aligned} \partial_t \left( |u|^{m(x)-2} u \right) - \Delta_{p(x)} u &= 0 & \text{in } \Omega \times (0, T), \\ |u|^{m(\cdot)-2} u &= v_0 & \text{on } \Omega \times \{0\}, \end{aligned}$$

where  $\partial_t = \partial/\partial t$ ,  $T > 0$ ,  $v_0 = v_0(x)$  is a given initial data and  $\Delta_{p(x)}$  is the so-called  $p(x)$ -Laplacian, a typical example of nonlinearity with variable exponents, together with either the Dirichlet condition

$$u = 0 \quad \text{on } \partial\Omega \times (0, T),$$

or the Neumann condition

$$|\nabla u|^{p(x)-2} \partial_n u = 0 \quad \text{on } \partial\Omega \times (0, T),$$

where  $\partial_n u$  denotes the outward normal derivative of  $u$  on  $\partial\Omega$ . The existence of solutions is proved by developing an abstract theory on doubly nonlinear evolution equations governed by gradient operators. In contrast to constant exponent cases, two nonlinear terms have inhomogeneous growth and some difficulty may occur in establishing energy estimates. Our method of proof relies on an efficient use of Legendre-Fenchel transforms of convex functionals and a modified energy method.

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### Existence for a class of nonlinear delay reaction-diffusion systems

**Monica-Dana Burlica**

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**Daniela Rosu**

We consider an abstract nonlinear multi-valued reaction-diffusion system with delay and, using some compactness arguments coupled with metric fixed point techniques, we prove some sufficient conditions for the existence of at least one  $C^0$ -solution. A global

asymptotic stability result is also obtained and some specific examples are analyzed.

**Acknowledgements.** Supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-ID-PCE-2011-3-0052.

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### Dynamic analysis of a nonlinear Timoshenko equation

**Jorge Esquivel-Avila**

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An initial boundary value problem in a bounded domain for the Timoshenko equation with a nonlinear damping and a nonlinear source term is considered. Exploiting an idea of a potential well under suitable assumptions on the damping and the source terms, we prove blow-up as well as convergence to the zero and nonzero equilibria and we give rates of decay to the zero equilibrium.

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### Leadership in crowd dynamics: modelling via two-scale interactions

**Joep Evers**

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**Adrian Muntean**

We describe a two-scale model for the dynamics of a crowd by distinguishing between a number of leaders considered at the micro-scale and a large crowd viewed macroscopically. Leaders are of particular interest in view of crowd control. We encounter the following fundamental questions: - How to describe a leader in mathematical terms? - Does the presence of a leader essentially change the macro-scale behaviour? - Which specific macroscopic patterns can be caused by the leader(s)? - How can we validate our two-scale model with experimental data? The crowd in our model consists of two subpopulations in a corridor, assumed to be two-dimensional. To capture the evolution of the crowd, we use a time-dependent mass measure for each subpopulation. Each of the two mass measures satisfies the continuity equation. Hence, their evolution is governed by the partial velocity field, which we have to provide. The actual modelling consists therefore in finding the right velocity field, which we expect to depend functionally on both mass measures. In this talk, I explain the model we use, and the particular form we choose for the velocity field. Furthermore I give a flavour of the crowd-dynamical phenomena that arise from this model.

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### Nodal and multiple constant sign solutions for equations with the p-Laplacian

**Michael Filippakis**

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**Ravi P. Agarwal, Donal O'Regan, Nikolaos S. Papageorgiou**

We consider nonlinear elliptic equations driven by the p-Laplacian with a nonsmooth potential (hemivariational inequalities). We obtain the existence of multiple nontrivial solutions and we determine their sign (one positive, one negative and the third nodal). Our approach uses nonsmooth critical point theory coupled with the method of upper-lower solutions.

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### Asymptotic behavior of blow-up solutions for the heat equations with nonlinear boundary conditions

**Junichi Harada**

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We consider the heat equation equation on the half space with nonlinear boundary conditions ( $u_t = \Delta u$ ,  $\partial_\nu u = u^q$ ). Here we discuss the asymptotic behavior of blow-up solutions for the Sobolev subcritical case. In particular, we provide a sufficient condition on the initial data for the single point blow-up at the origin. Furthermore we study the spacial singularities of the blow-up profile.

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### Pohozaev-Ôtani type inequalities for weak solutions of some quasilinear elliptic equations in unbounded domains

**Takahiro Hashimoto**

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In this talk, we are concerned with the following quasilinear elliptic equations:

$$(E) \begin{cases} -\operatorname{div} \{a(x)|\nabla u|^{p-2}\nabla u\} = b(x)|u|^{q-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega$  is a domain in  $\mathbf{R}^N$  ( $N \geq 2$ ) with smooth boundary,  $1 < p, q < \infty$ ,  $a(x), b(x) \geq 0$ .

When  $a(x) = b(x) \equiv 1$ , a complementary existence result was founded for the problem of interior, exterior and whole space. The main purpose of this talk is to obtain the nonexistence results via Pohozaev-Ôtani type inequalities for some class of weak solutions of (E). We also discuss the existence result for exterior problem and the regularity of solutions of (E).

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### Fractional Sobolev spaces via Riemann - Liouville derivatives and some imbeddings

**Dariusz Idczak**  
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**Stanislaw Walczak**

Using the Riemann - Liouville derivatives of test functions we introduce fractional Sobolev spaces of functions of one variable, defined on a bounded interval. Next, we characterize elements of these spaces with the aid of the Riemann - Liouville derivatives as well as with the aid of some integral representations. We also study some imbeddings of the introduced spaces and prove their compactness.

**Acknowledgements.** The project was financed with funds of National Science Centre, granted on the basis of decision DEC-2011/01/B/ST7/03426.

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### Variational problems associated with Trudinger-Moser inequalities in unbounded domains

**Michinori Ishiwata**  
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In this talk, we are concerned with the existence and the nonexistence of maximizers for variational problems associated with the Trudinger-Moser inequality in unbounded domains. Particularly, we are interested in the variational problems for a version of TM-type inequality with singular weight functions.

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### Well-posedness for anisotropic degenerate parabolic equations with non-homogeneous boundary conditions

**Kazuo Kobayasi**  
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We study the well-posedness for anisotropic degenerate parabolic-hyperbolic equations with initial and non-homogeneous boundary conditions. We prove a comparison theorem for any entropy sub- and super-solution, which immediately deduces the  $L^1$  contractivity and therefore, uniqueness of entropy solutions. The method used here is based upon the kinetic formulation and the kinetic techniques developed by Lions, Perthame and Tadmor. By adapting and modifying those methods to the case of Dirichlet boundary problems for degenerate parabolic equations we can establish a comparison property. Moreover, in the quasi-isotropic case the existence of entropy solutions is proved.

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### A new class of nonlinear evolution equations

**Masahiro Kubo**  
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Codifying applications of subdifferential evolution equations to variational inequalities, we introduce a new class of evolution equations associated with subdifferentials depending on unknown functions. We consider three kinds of evolution equations: parabolic (I), parabolic (II) and hyperbolic evolution equations; and two types of dependence of the subdifferentials on the unknown functions: local and non-local types of dependence. We prove the existence of a solution to a hyperbolic problem with a non-local type dependence on unknown functions and study its singular limit to a solution to a parabolic (II) problem.

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### Existence and non existence of solutions to initial boundary value problems for nonlinear evolution equations with strong dissipation

**Akisato Kubo**  
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The main purpose is to investigate existence and non-existence of global solutions of the initial Dirichlet-boundary value problem for evolution equations with the strong dissipation. Many authors studied classes for which initial boundary value problems possess global solutions. We consider a related problem and seek global solutions and blow-up solutions of it depending on whether it belongs to such classes or not.

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### Existence of sliding motions for nonlinear evolution equations in Banach space

**Laura Levaggi**  
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In this work the existence problem for solutions of a class of controlled nonlinear evolution equations in a Banach space is presented, where the considered feedback laws can depend nonlinearly on the state variable. The proof is based on the usual Faedo-Galerkin approximation method and the use of techniques related to the monotonicity and coercivity assumptions on the evolution operator. The result is then applied to the existence of the so-called sliding motions for infinite dimensional systems, either under distributed or boundary control. Sliding mode methods are used to control finite-dimensional systems and are based on the idea of constraining the evolution on a manifold, designed in order to attain

the control aim. The idea here is to exploit these techniques to define a feedback control that is able to make each Galerkin approximation fulfill approximately the sliding constraint defining the prescribed manifold and show that the limit evolution is viable.

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### Fractional du Bois-Reymond lemma and its applications.

**Marek Majewski**

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**Rafal Kamocki, Stanislaw Walczak**

In the paper a fractional du Bois-Reymond lemma for functions of one variable with Riemann-Liouville derivatives of the order  $\alpha \in (n - \frac{1}{2}, n)$  with  $n \geq 1$  is presented. We use this lemma to show that any critical point of a fractional Lagrange functional is a solution to it's Euler-Lagrange equation. The above technique can be apply to the problem of the existence of solutions to the fractional counterpart of classical Dirichlet problem.

**Acknowledgements.** The project was financed with funds of National Science Centre, granted on the basis of decision DEC-2011/01/B/ST7/03426.

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### Boundary control and hidden trace regularity of a semigroup associated with a beam equation and non-dissipative boundary conditions

**Richard Marchand**

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**Irena Lasiecka, Timothy J. McDevitt**

New control-theoretic results will be presented for second order (in time) PDE scalar equations with non-monotone feedback boundary conditions. While the analysis of monotone structures is based on a suitable version of monotone semigroup theory, the non-monotone case seems to require detailed microlocal analysis on the boundary, showing that the underlying semigroup is of Gevrey's class. A particular type of "hidden regularity" exhibited by the boundary traces is instrumental for showing well-posedness of the associated control system within a standard finite energy space even when the controls are not necessarily collocated. Although the analysis is applicable to multidimensional problems, the talk will focus on a one-dimensional Euler-Bernoulli beam equation. Theoretical results will be complemented by numerical simulations that illustrate the spectral properties of the system operators.

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### Existence of time-periodic solutions for the micropolar fluid equations with the spin-vortex interaction boundary condition

**Kei Matsuura**

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**Mitsuharu Otani**

It will be presented that the time-periodic problem for the micropolar fluid equation in a regular bounded domain  $\Omega$  of  $\mathbb{R}^3$  is solvable if the norms of external forces are sufficiently small. As the boundary conditions on the velocity field  $u$  and the microrotation field  $\omega$  we impose the no-slip condition  $u|_{\partial\Omega} = 0$  on the velocity and assume that the spin and the vorticity are proportional on the boundary, i.e.,  $\omega = \frac{\theta}{2} \text{curl } u$  on  $\partial\Omega$  with some constant  $\theta \in [0, 1]$ . For the existence of time-periodic solutions the smallness of the parameter  $\theta$  is required as well. The stability and the uniqueness of time-periodic solutions will also be discussed.

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### Fuzzy stochastic differential equations-different approaches and recent results.

**Mariusz Michta**

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In the talk we present different approaches to the notion of fuzzy stochastic differential equations driven by semimartingales and also to the notion of their solutions. We also present existence and uniqueness theorems to such equations. Presented results extend to stochastic case possible approaches known from the theory of deterministic fuzzy differential equations.

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### Nonlinear subdifferential inclusions with applications to contact mechanics

**Stanislaw Migórski**

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We consider two classes of nonlinear subdifferential inclusions in a framework of evolution triple of spaces. The first class involves inclusions with a history-dependent term for which we provide an existence and uniqueness result. In the second class, we consider time-dependent possibly nonconvex nonsmooth functions and their Clarke subdifferentials operating on the unknown function. We prove the existence of a weak solution and study the asymptotic behavior of a sequence of solutions when a small parameter in the inertial term tends to zero. We prove that the limit function is a solution of a

first order inclusion. Finally, we give applications to quasi-static viscoelastic frictional contact problems.

**Acknowledgement.** Supported by the Marie Curie International Research Staff Exchange Scheme Fellowship within the 7th European Community Framework Programme under Grant Agreement No. 295118.

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### Stochastic delay inclusions with noncontinuous multifunctions

**Jerzy Motyl**

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The talk deals with the Ito's type delay stochastic differential inclusion

$$dX(t) \in F(X_t)dt + G(X_t)dW(t), \quad X_0 = \xi,$$

where  $W$  denotes an  $m$ -dimensional Wiener process,  $X_t(s) = X(t+s)$  and set-valued functions  $F, G$  defined on  $C([-r, 0], \mathbb{R}^d)$  take on closed and convex values. The existence of local and global solutions of the inclusion with so-called "upper separated" multifunctions will be discussed. Some examples of noncontinuous "upper separated" multifunctions will be also presented.

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### Regularity and singularities of optimal convex shapes in the plane

**Arian Novruzi**

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**Jimmy Lamboley, Michel Pierre**

We focus here on the analysis of the regularity or singularity of solutions  $\Omega_0$  to shape optimization problems among convex planar sets, namely:

$$J(\Omega_0) = \min\{J(\Omega), \Omega \text{ convex}, \Omega \in \mathcal{S}_{ad}\},$$

where  $\mathcal{S}_{ad}$  is a set of 2-dimensional admissible shapes and  $J : \mathcal{S}_{ad} \mapsto \mathbb{R}$  is a shape functional.

Our main goal is to obtain qualitative properties of these optimal shapes by using first and second order optimality conditions, including the infinite dimensional Lagrange multiplier due to the convexity constraint. We prove two types of results:

i) under a suitable convexity property of the functional  $J$ , we prove that  $\Omega_0$  is a  $W^{2,p}$ -set,  $p \in [1, \infty]$ . This result applies, for instance, with  $p = \infty$  when the shape functional can be written as  $J(\Omega) = R(\Omega) + P(\Omega)$ , where  $R(\Omega) = F(|\Omega|, E_f(\Omega), \lambda_1(\Omega))$  involves the area  $|\Omega|$ , the Dirichlet energy  $E_f(\Omega)$  or the first eigenvalue of the Laplace-Dirichlet operator  $\lambda_1(\Omega)$ , and  $P(\Omega)$  is the perimeter of  $\Omega$ ,

ii) under a suitable concavity assumption on the functional  $J$ , we prove that  $\Omega_0$  is a polygon. This result applies, for instance, when the functional is now written as  $J(\Omega) = R(\Omega) - P(\Omega)$ , with the same notations as above.

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### Splitting methods for semilinear evolution equations with applications to nonlinear Schrödinger equations

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We study splitting methods for semilinear evolution equation  $u' = Au + F(u)$ , where  $A$  is an  $m$ -dissipative operator in a Hilbert space  $X$ , and  $F : D(A) \rightarrow D(A)$  is a Lipschitz continuous function on bounded subsets of  $D(A)$ . Under appropriate assumptions on  $F$ , we prove the second order convergence of the Strang splitting method in  $X$  for initial data in  $D(A^2)$ . This abstract result unifies several known results for nonlinear Schrödinger equations.

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### Viability of a time dependent closed set with respect to a semilinear delay evolution inclusion

**Marius Popescu**

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**Mihai Necula**

We prove a sufficient condition for a time-dependent closed set to be viable with respect to a delay evolution inclusion governed by a strongly-weakly u.s.c. perturbation of an infinitesimal generator of a  $C_0$ -semigroup. This condition is expressed in terms of a natural concept involving tangent sets, generalizing tangent vectors in the sense of Bouligand and Severi.

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### Asymptotic behavior of solutions to a coupled system of Maxwell's equations and a controlled differential inclusion

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**Dina Kalinichenko, Sergey Skopinov**

We consider the parameter-dependent coupled system consisting of the one-dimensional Maxwell's

equations and the heat equation which is understood as differential inclusion. This system is called Maxwell's equations with thermal effect introduced by H.M. Yin. The use of a boundary control in the inclusion guarantees the boundedness and convergence of solutions and excludes the presence of a blow-up. The techniques which are used by us are Lyapunov functionals and multiscaling methods.

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### Global existence and exponential stability for a nonlinear delay evolution equation with non-local initial condition

**Daniela Rosu**

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**Monica-Dana Burlica**

We consider a delay evolution equation subjected to a nonlocal initial condition and governed by a nonlinear Lipschitz perturbation of an  $m$ -dissipative operator. We prove some existence and asymptotic stability results for global  $C^0$ -solutions and we exemplify the abstract results by a delayed porous media equation subjected either to periodic or to anti-periodic conditions.

**Acknowledgements.** Supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-ID-PCE-2011-3-0052.

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### Multiple positive solutions for periodic problems with concave terms

**Vasile Staicu**

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**S. Aizicovici, N. Papageorgiou**

We consider a nonlinear periodic problem  $(P_\lambda)$ , driven by the scalar  $p$ -Laplacian, with a parametric concave term  $\lambda|x|^{q-2}x$  and a Caratheodory perturbation  $f(t, x)$  such that  $x \rightarrow f(t, x)$  exhibits a  $(p-1)$ -superlinear growth near  $+\infty$  (the convex term). Using variational techniques, based on the critical point theory, and suitable truncation techniques we prove a bifurcation-type theorem describing the nonexistence, existence and multiplicity of positive solutions as the parameter varies. Namely, we show that a critical parameter value  $\lambda^* > 0$  exists such that: for all  $\lambda \in (0, \lambda^*)$ , the problem  $(P_\lambda)$  has at least two positive solutions; for  $\lambda = \lambda^*$ , the problem  $(P_\lambda)$  has at least one positive solution, and, for all  $\lambda > \lambda^*$ , the problem  $(P_\lambda)$  has no positive solutions.

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### On a nonstandard nonlinear parabolic problem for the coupling surface – deep ocean temperatures with latent heat and coalbedo terms.

**Lourdes Tello**

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**J.I. Diaz, A. Hidalgo**

We study a global climate model for the coupling of the mean surface temperature with the deep ocean temperature. The nonlinear model presents some nonstandard facts: the boundary condition, representing the mean surface temperature, is not only of dynamic type (involving the time derivative of the trace of the solution) but also a surface diffusive term. The model includes also some delicate nonlinear terms such as the coalbedo effect and the latent heat, which here are formulated in terms of suitable (multivalued) maximal monotone graphs of  $\mathbb{R}^2$ . We prove the existence of bounded weak solutions and show some numerical experiences. Other qualitative properties of the solutions will be also presented.

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### Robust feedback stabilization of solutions of stochastic evolution equations with delay

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**N.U. Ahmed**

In this talk, we consider the problem of state feedback robust stabilization against uncertainty in the class of relatively  $A$ - bounded operators of mild solutions of stochastic evolution equations with delay. An example is included to illustrate the theory.

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### Periodic solutions of some double-diffusive convection systems based on Brinkman-Forchheimer equation

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**Mitsuharu Ôtani**

We consider the following system:

$$\begin{cases} \partial_t \vec{u} = \nu \Delta \vec{u} - a\vec{u} - \nabla p + \vec{g}T + \vec{h}C + \vec{f}_1 & \text{in } \Omega \times [0, S], \\ \partial_t T + \vec{u} \cdot \nabla T = \Delta T + f_2 & \text{in } \Omega \times [0, S], \\ \partial_t C + \vec{u} \cdot \nabla C = \Delta C + \rho \Delta T + f_3 & \text{in } \Omega \times [0, S], \\ \nabla \cdot \vec{u} = 0 & \text{in } \Omega \times [0, S], \\ \vec{u}|_{\partial\Omega} = 0; T|_{\partial\Omega} = 0; C|_{\partial\Omega} = 0, \end{cases}$$

where  $N = 2, 3$  and  $\Omega$  is a bounded domain with smooth boundary. This system describes the double-diffusive convection between the temperature  $T$  and the concentration of solute  $C$ . The first equation of the system comes from the Brinkman-Forchheimer equation, which describes the behavior of the fluid velocity  $\vec{u}$  and the pressure  $p$  in some porous medium. In this talk, we discuss the existence of a solution for this system under the time periodic condition with period  $S$ .

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### Nonlinear delay evolution inclusions with non-local conditions on the initial history

**Ioan Vrabie**

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We consider a nonlinear delay evolution differential inclusion, governed by a multi-valued perturbation of an  $m$ -dissipative operator, subjected to a non-local condition on the initial history. Under some natural assumptions, allowing to handle periodic, anti-periodic and mean-type problems with delay, we prove the existence in the large of at least one  $C^0$ -solution. Two applications concerning nonlinear delay parabolic problems are presented.

**Acknowledgements.** Supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-ID-PCE-2011-3-0052.

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### Uniqueness and non-degeneracy of ground states of quasilinear Schrodinger equations

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We consider the following quasilinear elliptic problem:

$$-\Delta u - \kappa \Delta(|u|^\alpha)|u|^{\alpha-2}u = g(u) \quad \text{in } \mathbb{R}^N$$

where  $N \geq 3, \kappa > 0$  and  $\alpha > 1$ . This equation can be obtained as a stationary problem of modified Schrodinger equations which appear in the study of plasma physics.

In this talk, we discuss the existence and the variational characterization of the ground state and present our recent results on the uniqueness and non-degeneracy.

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### Life span of positive solutions for a semilinear heat equation with non-decaying initial data

**Yusuke Yamauchi**

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We consider the following semilinear heat equation:

$$u_t = \Delta u + u^p, \quad (x, t) \in \mathbb{R}^N \times (0, \infty),$$

where  $p > 1, N \geq 1$ . In this talk, we discuss the upper bound of the life span of positive solutions of the equation for initial data having positive limit inferior at space infinity. The proof is based on a slight modification of Kaplan's method.

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## Special Session 16: Reaction Diffusion Equations and Applications

Jerome Goddard, Auburn University, USA

Ratnasingham Shivaji, University of North Carolina at Greensboro, USA

Recent developments in reaction diffusion equations have greatly increased their importance and usefulness in modeling physical and biological phenomena in many disciplines. The application of reaction diffusion is seemingly endless with their use naturally arising in areas such as biology, ecology, chemistry, geology, physics, and engineering. Investigation of the structure of steady states for such models yields interesting nonlinear elliptic boundary value problems of varied types. Even with the study of elliptic boundary value problems having such a rich mathematical history dating back to the 1960s, much is still unknown about the structure of solutions to such problems. This session will facilitate the exploration of current applications of reaction diffusion, proof techniques, and open questions of nonlinear elliptic boundary value problems.

### Existence of solutions to boundary value problems at full resonance

Kristen Abernathy

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Jesus Rodriguez

The focus of this talk is the study of nonlinear differential equations of the form

$$\dot{x}_i(t) = a_i(t)x_i(t) + f_i(\epsilon, t, x_1(t), \dots, x_n(t)),$$

with  $i = 1, 2, \dots, n$ , subject to two-point boundary conditions

$$b_i x_i(0) + d_i x_i(1) = 0,$$

for  $i = 1, 2, \dots, n$ . We formulate sufficient conditions for the existence of solutions based on the dimension of the solution space of the corresponding linear, homogeneous equation and the properties of the nonlinear term when  $\epsilon = 0$ . We focus on the case when the solution space of the corresponding linear, homogeneous equation is  $n$ -dimensional; that is, when the system is at full resonance. The argument we use relies on the Lyapunov-Schmidt Procedure and the Schauder Fixed Point Theorem.

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### On the solvability of nonlinear Sturm-Liouville problems

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Jesus Rodriguez

In this talk, we establish sufficient conditions for the existence of solutions to the nonlinear differential equation

$$(p(t)x'(t))' + q(t)x(t) + \psi(x(t)) = G(x(t))$$

subject to general non-local boundary conditions of the form

$$\begin{cases} \alpha x(0) + \beta x'(0) + \eta_1(x) = \phi_1(x) \\ \gamma x(1) + \delta x'(1) + \eta_2(x) = \phi_2(x). \end{cases}$$

We will emphasize the relationship between the eigenvalues of a related linear Sturm-Liouville problem and the rate of growth of nonlinearities present in both the differential equation and boundary conditions. This relationship will then motivate explorations of singular Sturm-Liouville problems.

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### Existence of alternate steady states in a phosphorous cycling model

Dagny Butler

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Sarath Sasi, Ratnasingham Shivaji

We analyze the positive solutions to the steady state reaction diffusion equation with Dirichlet boundary conditions of the form:

$$\begin{cases} -\Delta u = \lambda[K - u + c \frac{u^4}{1+u^4}], & x \in \Omega \\ u = 0, & x \in \partial\Omega. \end{cases}$$

Here  $\Delta u = \text{div}(\nabla u)$  is the Laplacian of  $u$ ,  $\frac{1}{\lambda}$  is the diffusion coefficient,  $K$  and  $c$  are positive constants, and  $\Omega \subset \mathbb{R}^N$  is a smooth bounded region with  $\partial\Omega$  in  $C^2$ . This model describes the steady states of phosphorus cycling in stratified lakes. Also, it describes the colonization of barren soils in drylands by vegetation. In this paper, we discuss the existence of multiple positive solutions leading to the occurrence of an S-shaped bifurcation curve. We prove our results by the method of sub-super solutions.

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### Evolution of dispersal and the ideal free distribution

Robert Stephen Cantrell

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Chris Cosner, Yuan Lou

A general question in the study of the evolution of

dispersal is what kind of dispersal strategies can convey competitive advantages and thus will evolve. We consider a two species competition model in which the species are assumed to have the same population dynamics but different dispersal strategies. Both species disperse by random diffusion and advection along certain gradients, with the same random dispersal rates but different advection coefficients. We find a conditional dispersal strategy which results in the ideal free distribution of species, and show that it is a locally evolutionarily stable strategy. We further show that this strategy is also a globally convergent stable strategy under suitable assumptions, and our results illustrate how the evolution of dispersal can lead to an ideal free distribution. The underlying biological reason is that the species with this particular dispersal strategy can perfectly match the environmental resource, which leads to its fitness being equilibrated across the habitat.

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#### Existence and nonexistence of positive solutions for a special class of elliptic systems

**Maya Chhetri**  
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We consider an elliptic system of the form

$$\begin{cases} -\Delta u - \mu \Delta v = f(v) & \text{in } \Omega \\ -\Delta v - \lambda \Delta u = g(u) & \text{in } \Omega \\ u = 0 = v & \text{on } \partial\Omega, \end{cases}$$

where  $\lambda, \mu > 0$  are parameters,  $\Omega$  is a bounded domain in  $\mathbb{R}^N$  with smooth boundary  $\partial\Omega$ . The nonlinearities  $f, g : [0, \infty) \rightarrow \mathbb{R}$  are  $C^1$  functions that are sublinear at infinity. We discuss existence and nonexistence of positive solutions. We consider both positive and semipositone reaction terms in our analysis.

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#### Population models with diffusion, strong Allee effect, and nonlinear boundary conditions

**Jerome Goddard**  
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**E. Lee, R. Shivaji**

We discuss the steady state solutions of a diffusive population model with strong Allee effect. In particular, this study is focused on a population that satisfies a certain nonlinear boundary condition and on its persistence when constant yield harvesting is introduced. We prove the existence of at least two positive steady states of the model for certain parameter ranges. These existence results are established by the method of sub-super solutions.

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#### A reaction-diffusion problem with nonlocal reaction

**Georg Hetzer**  
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Incorporating a bio-feedback into an energy balance climate model leads to a nonautonomous functional reaction-diffusion problem with a set-valued reaction term which depends on a nonlocal Volterra operator. A global existence result for nonnegative solutions and the existence of a trajectory attractor are discussed.

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#### An existence result for an infinite semipositone problem

**Lakshmi Kalappattil**  
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**Jerome Goddard, Eun Kyoung Lee, R. Shivaji**

We consider the singular boundary value problem:

$$\begin{cases} -\Delta u = \frac{au - bu^2 - c}{u^\alpha}, & x \in \Omega \\ u = 0, & \text{on } \partial\Omega, \end{cases} \quad (5)$$

where  $\Delta$  is the Laplacian operator,  $\Omega$  is a smooth bounded domain in  $\mathbb{R}^n$  and  $a, b, c, \alpha$  are positive constants where  $\alpha$  is in  $(0, 1)$ . Let  $\lambda_1$  be the first eigenvalue of  $-\Delta$  with Dirichlet boundary conditions. When  $a > \lambda_1$ , we prove there exists a  $c^*(a, b, \alpha, \Omega)$  such that if  $c < c^*$ , the problem has a positive solution. We prove the existence result by the method of sub-super solutions. We also extend our results to the case when  $\Omega$  is an exterior domain.

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#### Analysis of class of elliptic equations with nonlinear boundary conditions arising in combustion theory

**Eunkyung Ko**  
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**Peter Gordon, R. Shivaji**

We study positive solutions to the boundary value problem

$$\begin{cases} -\Delta u = \lambda f(u), & x \in \Omega, \\ \mathbf{n} \cdot \nabla u + C(u)u = 0, & x \in \partial\Omega, \end{cases}$$

where  $C : [0, \infty) \rightarrow (0, \infty)$  is a  $C^1$  non decreasing function,  $\lambda > 0$ ,  $\Omega$  is a bounded domain in  $\mathbb{R}^N$ ,  $N \geq 1$  and  $f : [0, \infty) \rightarrow (0, \infty)$  is a  $C^1$  non decreasing function such that  $\lim_{u \rightarrow \infty} \frac{f(u)}{u} = 0$ . We establish the

existence of a positive solution for all  $\lambda > 0$ , and discuss the existence of multiple positive solutions and uniqueness results for certain ranges of  $\lambda$  when  $f$  satisfies certain additional assumptions. A simple model that satisfies all our hypotheses is  $f(u) = \exp[\frac{\alpha u}{\alpha + u}]$  for  $\alpha \gg 1$ . We prove our existence and multiplicity results by the method of sub-supersolutions, and our uniqueness result by establishing a priori estimates.

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### Existence of the second positive solution for a p-Laplacian problem

**Eun Kyoung Lee**

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We investigate the existence, nonexistence, and multiplicity of positive radial solutions for the p-Laplacian problem with boundary parameters. For proofs, we mainly use a combination of a fixed point theorem, the method of upper and lower solutions in the frame of the ordinary differential equations (ODE) technique. This is a joint work with Chan-Gyun Kim and Yong-Hoon Lee.

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### A balanced finite element method for singularly perturbed reaction-diffusion problems

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**Martin Stynes**

Consider the singularly perturbed linear reaction-diffusion problem  $-\varepsilon^2 \Delta u + bu = f$  in  $\Omega \subset \mathbb{R}^d$ ,  $u = 0$  on  $\partial\Omega$ , where  $d \geq 1$ , the domain  $\Omega$  is bounded with (when  $d \geq 2$ ) Lipschitz-continuous boundary  $\partial\Omega$ , and the parameter  $\varepsilon$  satisfies  $0 < \varepsilon \ll 1$ . It is argued that for this type of problem, the standard energy norm  $v \mapsto [\varepsilon^2 |v|_1^2 + \|v\|_0^2]^{1/2}$  is too weak a norm to measure adequately the errors in solutions computed by finite element methods: the multiplier  $\varepsilon^2$  gives an unbalanced norm whose different components have different orders of magnitude. A balanced and stronger norm is introduced, then for  $d \geq 2$  a mixed finite element method is constructed whose solution is quasi-optimal in this new norm. By a duality argument it is shown that this solution attains a higher order of convergence in the  $L_2$  norm. Error bounds derived from these analyses are presented for the cases  $d = 2, 3$ . For a problem posed on the unit square in  $\mathbb{R}^2$ , an error bound that is uniform in  $\varepsilon$  is proved when the new method is implemented on a Shishkin mesh. Numerical results are presented to show the superiority of the new method over the standard mixed finite element method on the same mesh for this singularly perturbed problem.

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### Strong bounded solutions for nonlinear parabolic equations

**Nsoki Mavinga**

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**M. Nkashama**

We are concerned with the existence of bounded solutions existing for all times for nonlinear parabolic equations with nonlinear boundary conditions on a domain that is bounded in space and unbounded in time (the entire real line). We establish a priori estimates for solutions to linear boundary value problems, and derive a weak maximum principle which is valid on the entire real line in time. We then use comparison techniques, a priori estimates, and nonlinear approximation methods to prove the existence and, in some instances, positivity and uniqueness of bounded solutions existing for all times.

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### A study of delayed cooperation diffusion system with Dirichlet boundary conditions

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**Dhirendra Bahuguna**

In this paper we study the existence and uniqueness of strong solution of a delayed cooperation diffusion system with Dirichlet boundary conditions using the method of semi discretization in time.

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### Asymptotic behavior of the solutions of the BVP governing Marangoni convection

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**J. Paultet**

The boundary value problem (BVP) governing the Marangoni convection over a flat surface is given by

$$f''' = \frac{2k+1}{3} f'^2 - \frac{k+2}{3} f f''$$

$$f(0) = 0, \quad f''(0) = -1, \quad f'(\infty) = 0,$$

where  $k > -1$  is the temperature gradient exponent. It has been proved by J. Paultet that for each  $k \in (-1, -1/2)$ , the BVP admits a continuum of solutions. In this talk, we will consider the asymptotics of these solutions. We will prove that for each  $k \in (-1, -1/2)$ , there exists a solution  $f_0$  of the BVP that satisfies

$$f'_0(\eta) \sim c_0 f_0(\eta)^{-\frac{3(k+1)}{k+2}} \exp\left(-\int_{\eta_0}^{\eta} f_0(s) ds\right)$$



as  $\eta \rightarrow \infty$  for some  $\eta_0 > 0$  sufficiently large, and a constant  $c_0 > 0$  that depends only on  $k$  and  $\eta_0$ . We conjecture that the BVP has exactly one solution that obeys the above asymptotics, i.e. its derivative decays to zero exponentially, while the derivatives of the other solutions decay to zero algebraically.

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#### Alternate stable states in ecological systems

**Sarath Sasi**

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We consider the existence of multiple positive solutions to the steady state reaction diffusion equation with Dirichlet boundary conditions of the form:

$$\begin{cases} -\Delta u = \lambda[u - \frac{u^2}{K} - c\frac{u^2}{1+u^2}], & x \in \Omega, \\ u = 0, & x \in \partial\Omega. \end{cases}$$

Here  $\Delta u = \text{div}(\nabla u)$  is the Laplacian of  $u$ ,  $\frac{1}{\lambda}$  is the diffusion coefficient,  $K$  and  $c$  are positive constants and  $\Omega \subset \mathbb{R}^N$  is a smooth bounded region with  $\partial\Omega$  in  $C^2$ . This model describes the steady states of a logistic growth model with grazing in a spatially homogeneous ecosystem. It also describes the dynamics of the fish population with natural predation. In this talk we discuss the existence of multiple positive solutions leading to the occurrence of an S-shaped bifurcation curve. We also introduce a constant yield harvesting term to this model and discuss the existence of positive solutions including the occurrence of a  $\Sigma$ -shaped bifurcation curve in the case of a one-dimensional model. We prove our results by the method of sub-super solutions and quadrature method.

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#### On radial solutions of polyharmonic equations with power nonlinearities

**Paul Schmidt**

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**Monica Lazzo**

A vast amount of literature on second-order semi-linear elliptic equations with power-like nonlinearities is concerned with the existence, uniqueness or multiplicity, and qualitative behavior of solutions to various types of boundary-value problems. Lacking a maximum principle, higher-order analogues of such problems require entirely new methods, even if expected results are similar to what is known in the second-order case. As a first step towards a better understanding of the higher-order case, we have been using dynamical-systems methods to study radially symmetric solutions of polyharmonic equations with

pure power nonlinearities. We give a survey of our results and their implications.

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#### Spatiotemporal mutualistic model of mistletoes and birds

**Junping Shi**

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**Chuncheng Wang, Rongsong Liu, Carlos Martinez del Rio**

A mathematical model which incorporates the spatial dispersal and interaction dynamics of mistletoes and birds is derived and studied to gain insights of the spatial heterogeneity in abundance of mistletoes. Fickian diffusion and chemotaxis are used to model the random movement of birds and the aggregation of birds due to the attraction of mistletoes respectively. The spread of mistletoes by birds is expressed by a convolution integral with a dispersal kernel. Two different types of kernel functions are used to study the model, one is Dirac delta function which reflects one extreme case that the spread behavior is local, and the other one is a general non-negative symmetric function which describes the nonlocal spread of mistletoes. When the kernel function is taken as the Dirac delta function, the threshold condition for the existence of mistletoes is given and explored in term of parameters. For the general non-negative symmetric kernel case, we prove the existence and stability of non-constant equilibrium solutions. Numerical simulations are conducted by taking specific forms of kernel functions. Our study shows that the spatial heterogeneous patterns of the mistletoes are related to the specific dispersal pattern of the birds which carry mistletoe seeds.

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#### Infinite semipositone problems with asymptotically linear growth forcing terms

**Ratnasingham Shivaji**

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**Hai Dang, Lakshmi Shankar**

We study positive solutions to the singular problem

$$\begin{cases} -\Delta u = \lambda f(u) - \frac{1}{u^\alpha} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $\lambda$  is a positive parameter,  $\Omega$  is a bounded domain in  $\mathbb{R}^n$ ,  $n \geq 1$  with smooth boundary  $\partial\Omega$ , 0

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**Existence of solutions for degenerate elliptic  $p(x)$ -Laplacian**

**Inbo Sim**

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**Yun-Ho Kim**

We study the following nonlinear problem

$$-\operatorname{div}(w(x)|\nabla u|^{p(x)-2}\nabla u) = \lambda f(x, u) \quad \text{in } \Omega$$

which is subject to Dirichlet boundary condition. Under suitable conditions on  $w$  and  $f$ , employing the variational method, we show the existence of solutions for the above problem in the weighted variable exponent Lebesgue-Sobolev spaces. Also we show the positivity of the infimum eigenvalue for the problem. This is a joint work with Yun-Ho Kim.

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**Dynamical behaviour of spatio-temporal plankton population model**

**Shilpee Srivastava**

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We investigate the dynamical behaviour of three dimensional plankton population model using analytical and numerical techniques, modified by the addition of diffusive terms to represent the effect of random motion. A comparative study of local stability in the presence and absence of diffusion has been performed.

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## Special Session 17: Singular Perturbations

**Freddy Dumortier**, Hasselt University, Belgium  
**Peter De Maesschalck**, Hasselt University, Belgium  
**Martin Wechselberger**, University of Sydney, Australia

The aim of this special session is to get informed about recent results on singular perturbations, both from a pure, applied and numerical point of view. Besides scheduling talks from established mathematicians, we will give opportunity to junior researchers to present their work. Topics include (non-exhaustive): (geometric) singular perturbation theory, mixed-mode oscillations, canards, singularly perturbed PDE, delay in bifurcations.

### Delayed Hopf bifurcation with a focus node transition

**Eric Benoît**

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The delayed Hopf bifurcation is studied by Neishtadt and Callot with the *relief* (real part of the integral of an eigenvalue in the complex domain). A new lemma, using fine majorations on complex paths allows to improve the classical result. This lemma solves the problem of delayed Hopf bifurcation when a focus-node transition is present. With this result, we can have a new point of view on some mixed mode oscillations.

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### Blow-up of vector fields and Painleve equations

**Hayato Chiba**

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Painleve property is one of the most fundamental concept in the study of ODEs in the complex plane. In this talk, it is shown that Painleve equations are obtained from blow-up of singularities in slow-fast systems. Many properties of the Painleve transcendents are given by means of the dynamical systems theory.

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### Slow-fast cycles with singular contact points

**Peter De Maesschalck**

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We consider slow-fast cycles on an orientable two-dimensional manifold, consisting of an arbitrary (finite) number of slow and fast branches, but where the contact points of the slow branches are either of jump type or of singular type. Depending on the order of the singularity of the contact points, and depending on a so-called slow divergence integral computed along the slow-fast cycle, we deal with

the unicity of nearby periodic orbits. This talk is a continuation of the talk of Freddy Dumortier, and is based on recent results obtained in collaboration with Freddy Dumortier and Robert Roussarie.

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### Numerical continuation techniques for planar slow-fast systems

**Mathieu Desroches**

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**Peter De Maesschalck**

Continuation techniques have been known to successfully describe bifurcation diagrams appearing in slow-fast systems with more than one slow variable. In this talk we will investigate the usefulness of numerical continuation techniques dealing with both open and solved problems in the study of planar singular perturbations. More precisely, we first verify known theoretical results (thereby showing the reliability of the numerical tools) on the appearance of multiple limit cycles of relaxation-oscillation type and on the existence of multiple critical periods in well-chosen annuli of slow-fast periodic orbits in the plane. We then apply the technique to study a notion of maximal canard, in the sense of maximal period.

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### Relaxation oscillations near common slow-fast cycles

**Freddy Dumortier**

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The talk deals with slow-fast cycles on orientable two-dimensional surfaces. The slow-fast cycles are common, in the sense that the slow curves are all attracting or all repelling. The contact points have a finite order, which can be arbitrary. Results are presented about the existence and unicity of relaxation oscillations near such slow-fast cycles. Attention is given to the proofs of the results. The talk is based on a recent paper by P.De Maesschalck, F.Dumortier and R. Roussarie.

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**Gasless combustion fronts with heat loss****Anna Ghazaryan**Miami University, USA  
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We consider a model of gasless combustion with heat loss, with the heat loss from the system to the environment modeled according to Newton's law of cooling. For the regime when the system contains two small parameters, a diffusion coefficient for the fuel and a heat loss parameter, we use geometric singular perturbation theory to show existence of traveling combustion fronts. We also study their spectral and nonlinear stability.

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**Using geometric singular perturbation techniques to analyse models of intracellular calcium dynamics****Emily Harvey**Montana State University, USA  
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Oscillations in free intracellular calcium concentration are known to act as signals in almost all cell types, controlling a huge range of cellular processes including muscle contraction, cellular secretion and neuronal firing. Due to the almost universal nature of calcium oscillations, understanding the mechanisms underlying them is of great physiological importance. A key feature of intracellular calcium dynamics is that some physiological processes occur much faster than others. This leads to models with variables evolving on very different time scales. This separation in time scales suggests that geometric singular perturbation techniques (GSPT) may be useful in explaining the observed dynamics, including mixed-mode oscillations. In this talk the results from analysing a range of representative models of intracellular calcium dynamics using GSPT will be presented. We will describe the important steps and parameters in the analysis and demonstrate the usefulness of these techniques and their limitations in this context.

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**Limit cycles in slow-fast codimension 3 saddle and elliptic bifurcations****Renato Huzak**Hasselt University, Belgium  
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In this talk we present singular perturbation prob-

lems occurring in planar slow-fast systems

$$\begin{cases} \dot{x} = y \\ \dot{y} = -xy + \epsilon \left( b_0 + b_1 x + b_2 x^2 \pm x^3 + x^4 + x^5 H(x, \lambda) \right) + y^2 G(x, y, \lambda), \end{cases}$$

where  $G$  and  $H$  are smooth,  $\epsilon > 0$  is the singular parameter that is kept small,  $(b_0, b_1, b_2)$  are regular perturbation parameters close to 0 and  $\lambda \in \Lambda$ , with  $\Lambda$  a compact subset of some euclidian space.

We investigate the number of limit cycles that can appear near the origin  $(x, y) = (0, 0)$ . When the sign in front of  $x^3$  is positive, we deal with slow-fast saddle bifurcation. If the sign in front of  $x^3$  is negative, the slow-fast systems under consideration are referred to as slow-fast elliptic bifurcations. In the saddle case we encounter canard-type relaxation oscillations of small amplitude. Hence the limit cycles in the saddle case are confined to small neighbourhood of the origin in the phase space and their size tends to 0 for  $(b_0, b_1, b_2) \rightarrow (0, 0, 0)$ . The slow-fast elliptic bifurcations allow detectable limit cycles to be present.

Using geometric singular perturbation theory, including blow-up, both saddle end elliptic case can be restricted to the well known jump case, slow-fast Hopf bifurcations and slow-fast Bogdanov-Takens bifurcations. The most difficult problem to deal with concerns an upper bound on the number of limit cycles that appears in slow-fast Hopf case.

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**Reinjected horseshoes****Vincent Naudot**Florida Atlantic University, USA  
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Horseshoes play a central role in dynamical system and are observed in many chaotic systems. These are invariant sets for both the forward and the backward iterations of the map that defines the dynamics. However most all the points outside this set escape from a neighborhood of the Horseshoe after finite iterations. In this work we construct a system that possesses a Horseshoe together with a nearby non-trivial attractor, i.e., this latter is transitive, stable and possesses a positive Lyapunov exponent. This system is obtained after reinjecting the Horseshoe thanks to the unfolding of a degenerate double homoclinic orbit.

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### Interval mappings for slow-fast models of neurons

**Andrey Shilnikov**

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**Jeremy Wojcik, Alex Neiman**

We present a thorough bifurcation analysis of transformations of bursting activity in slow-fast models of neurons through the computer-assisted reduction to one and two-dimensional Poincaré mappings of an voltage interval. We were able to examine in detail typical bifurcations that underlie the complex activity transitions between: tonic spiking and bursting, bursting and mixed-mode oscillations, including torus breakdown in generic slow-fast models.

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### Global attractors for damped semilinear wave equations with a Robin–acoustic boundary perturbation

**Joseph Shomberg**

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**Sergio Frigeri**

Under consideration is the damped semilinear wave equation

$$u_{tt} + u_t - \Delta u + u + f(u) = 0$$

on a bounded domain  $\Omega$  in  $\mathbb{R}^3$  with a perturbation parameter  $\varepsilon > 0$  occurring in an acoustic boundary condition, limiting ( $\varepsilon = 0$ ) to a Robin boundary condition. With minimal assumptions on the nonlinear term  $f$ , the existence and uniqueness of global weak solutions is shown. Also, the existence of a family of global attractors is shown to exist. After proving a general result concerning the robustness of a one-parameter family of sets, the result is applied to the family of global attractors. Because of the complicated boundary conditions for the perturbed problem, fractional powers of the Laplacian are not well-defined; moreover, because of the restrictive growth assumptions on  $f$ , the family of global attractors is obtained from the asymptotic compactness method developed by J. Ball for generalized semiflows.

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### Mixed-mode oscillations in a multiple time scale phantom bursting system

**Alexandre Vidal**

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**Martin Krupa, Mathieu Desroches,**

**Frédérique Clément**

During the past 20 years, studies have focused on the lowest dimension dynamics (two slow and one fast variables) that may display Mixed-Mode Oscillations (MMOs). However, the complex transition that adds a small oscillations to a periodic MMO orbit as a parameter varies has never been studied in detail since, in this context, one expects chaos to be the main underlying mechanism. I will present a new kind of MMOs in the case of a four dimensional system with three different time scales. I will show, using geometric singular perturbation theory, that the system admits a limit cycle generating these MMOs even during the canard-induced transition that adds a small oscillation.

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## Special Session 18: Qualitative Theory of Evolutionary Equation and its Application

**Xiaojie Hou**, UNCW, USA

**Yi Li**, Wright State University and Xi'an Jiaotong University, USA

**Wei-Ming Ni**, ECNU and University of Minnesota, China

**YuanWei Qi**, UCF, USA

**Yaping Wu**, Capital Normal University, Peoples Rep of China

The geometric theory of the semilinear parabolic equations has seen a rapid progress and new applications in recent years, in which the study of steady states and traveling wave solutions as well as pattern formation has achieved fruitful results. The purpose of this special section is to provide experts a platform to exchange ideas on new trends and developments as well as new applications of solutions (steady state and traveling wave solutions) of evolutionary equations; to enhance interactions/collaborations, and to provide graduate students and junior researchers opportunities to learn the frontier work in these directions and interact with experts. In particular, the following subareas are included: existence and multiplicity of solutions of semilinear elliptic problems; existence and stability of special solutions (steady-state, traveling solutions, etc) of reaction-diffusion (dispersion) problems; applications to geometry, physics, and biology, medical sciences.

### Limiting behavior dynamics of a two-predator one-prey population system with a Beddington-DeAngelis functional response

**Lorna Almcera**

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**Alexis Erich S. Almcera, Polly W. Sy**

We consider a population system of two competing predator species that exploit their consumption of a single renewable prey. In the system, Beddington-DeAngelis functional responses model each predator-prey interaction. We investigate on how the parameters can determine the limiting behavior of any solution in the given system. We solve differential inequalities to obtain parametric conditions that suffice for a given predator species to independently become extinct, hence being an “inadequate” competitor. We also show that competitive exclusion holds whenever the losing competitor is inadequate. Given that at least one competitor becomes extinct, we analyze subsystems and utilize the fluctuation lemma with Barbălat’s Lemma, to calculate the limiting values of the solution. In case neither competitor is inadequate, we establish conditions where all three species survive. Specifically, by solving differential inequalities, we find that a competitor can survive at a very low density. In addition, we use a Lyapunov function to establish coexistence through a global stability of the unique interior equilibrium point.

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### Flat stationary solutions of the Vlasov-Poisson system (flat galaxies)

**Jurgen Batt**

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The “flat” Vlasov-Poisson system of partial differential equations describes the evolution of an aggregation of mass particles under their mutual attraction in the plane under the preservation of the  $1/r$  singularity of the Newtonian potential of the three-dimensional case. The study of existence of flat stationary solutions leads to a nonlinear singular integral equation of convolution type on all over  $R_2$ . We present a new direct approach to the existence of its solutions beyond results of G. Rein, Comm.Math.Phys. 1999 (joint work with E.Jörn and Yi Li).

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### Traveling waves for nonlocal evolution systems

**Fengxin Chen**

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**P. W. Bates**

The existence, uniqueness and stability of traveling wave solutions for nonlocal evolution equations with bistable nonlinearity are discussed. The spectrum of the operator obtained by linearizing about a monotone traveling wave is also studied.

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### Traveling waves in high Lewis number combustion model

**Anna Ghazaryan**

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**Stephen Schecter, Yuri Latushkin, Jeffrey Humpherys, Joshua Lyttle**

High Lewis number combustion model is a model that is used to describe propagation of fronts in the process of burning of high density liquid fuels. Both existence and stability of such fronts are important issues in applications and, at the same time, are challenging mathematical problems. I will talk about the existence of fronts and show that, depending on the value of the exothermicity parameter, these fronts can be either convectively or absolutely unstable.

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### Traveling fronts for a nonlocal reaction diffusion system

**Xiaojie Hou**

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**Yi Li**

We study the long term effect of competitions between two species with nonlocal diffusion, and the competition is of a strong type. Using a new monotone iteration scheme we derive the existence of the front solutions. The uniqueness of the front solution corresponding to each propagation speed is proved by sliding domain method. We also derive the asymptotics of the fronts with critical and non-critical wave speeds. The asymptotic stability of the fronts are shown by a new method of spectral analysis in weighted Banach spaces. The results illustrate that under certain conditions the weaker competitor will die and the stronger one will survive in the long run, and a small change of the initial environment will not change the outcome of the competition.

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### Multiple solutions to an elliptic problem related to vortex pairs

**Yi Li**

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**Shuangjie Peng**

Let  $\Omega$  be a bounded domain in  $\mathbb{R}^N$  ( $N \geq 2$ ),  $\varphi$  is a harmonic function in  $\overline{\Omega}$ . In this paper we study the existence of solutions to the following problem arising in the study of vortex pairs

$$(P_\lambda) \quad \begin{cases} -\Delta u = \lambda(u - \varphi)_+^{p-1}, & x \in \Omega, \\ u = 0, & x \in \partial\Omega. \end{cases}$$

The set  $\Omega_p = \{x \in \Omega, u(x) > \varphi\}$  is called “vortex core”. Existence of solutions whose “vortex core” consisting of one component and asymptotic behavior of “vortex core” were studied by many authors for large  $\lambda$  recently. Under the condition that  $\varphi$  has  $k$  strictly local minimum points on the boundary  $\partial\Omega$ , we obtain in this paper that for  $\lambda$  large enough,  $(P_\lambda)$  has a solution with “vortex core” consisting of  $k$  components by a constructive way.

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### Traveling waves of thermal diffusivity system-existence and stability

**Yuanwei Qi**

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**Xinfu Chen, Guirong Liu**

In this talk, I shall discuss some of the recent results on existence and stability of Traveling Waves to Thermal Diffusivity System with non-KPP type of nonlinearity. In particular, we show how to get sharp estimates on the minimum speed by using a novel approach.

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### Spreading speed, traveling waves and linear determinacy for STDs models

**Joaquin Rivera**

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**Fernando Carreon, Carlos Castillo-Chavez**

In the talk, I will discuss a Susceptible-Infected-Susceptible (SIS) model for the spatial-spread of sexually transmitted diseases (STD), in spatially mobile heterosexually active populations. First, we will consider a single strain SIS reaction-diffusion model with density dependent recruitment rates. Then, we will reformulate the model to include multiple competing strains of the same pathogen. I will demonstrate the existence of a minimal speed at which the disease spreads to a non-infected region in the form of a traveling wave. Finally, I will discuss a single-strain model with multiple stages of infections.

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### A Graph-Theoretic Approach to Global Stability Problems in Some Discrete Diffusion Models

**Zhisheng Shuai**

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**Michael Y. Li**

Spatially discrete diffusion models can be regarded as coupled systems on networks. A new graph-theoretic approach is developed to guide the constructions of Lyapunov functions for coupled systems on networks, and can be applied to investigate global stability problems for some discrete diffusion models, such as a single-species diffusion model and a predator-prey model with prey movement.

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### Turing instability in a three species food chain model

**Zhifu Xie**

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In this paper, we study a strongly coupled reaction-diffusion system describing three interacting species in a food chain model, where the third species preys on the second one and simultaneously the second species preys on the first one. We first show that the unique positive equilibrium solution is globally asymptotically stable for the corresponding ODE system. The positive equilibrium solution remains linearly stable for the reaction diffusion system without cross diffusion, hence it does not belong to the classical Turing instability scheme. We further proved that the positive equilibrium solution is globally asymptotically stable for the reaction diffusion system without cross diffusion by constructing a Lyapunov function. But it becomes linearly unstable only when cross-diffusion also plays a role in the reaction-diffusion system, hence the instability is driven solely from the effect of cross diffusion. Our results also exhibit some interesting combining effects of cross-diffusion, intra-species competitions and inter-species interactions.

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### Some estimates of solutions to a quasilinear elliptic Dirichlet problem with large diffusion

**Chunshan Zhao**

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In this talk, some estimates of solutions to a quasilinear elliptic Dirichlet problem with large diffusion will be presented and it will be shown that the L-infinity norm has an algebraic growth as the diffusion coefficient increases, which is quite different from the corresponding Neumann case.

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### Non pattern formation in a chemo-repulsion problem

**Kun Zhao**

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In contrast to diffusion (random diffusion without orientation), chemotaxis is the biased movement of cells/particles toward the region that contains higher concentration of beneficial or lower concentration of unfavorable chemicals. The former often refers to the attractive chemotaxis and latter to the repulsive chemotaxis. Chemotaxis has been advocated as a leading mechanism to account for the morphogenesis and self-organization of a variety of biological coherent structures such as aggregates, fruiting bodies, clusters, spirals, spots, rings, labyrinthine patterns and stripes, which have been observed in experiments. In this talk, I will present some recent development on the rigorous analysis of a partial differential equation model arising from repulsive chemotaxis which is a system of conservation laws consisting of nonlinear and coupled parabolic and hyperbolic type PDEs. In particular, global well-posedness, large-time asymptotic behavior of classical solutions to such model are obtained which indicate that chemorepulsion problem of this type exhibits strong tendency against pattern formation. The results are consistent with general results for classical repulsive chemotaxis models.

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### Time periodic traveling wave solutions for periodic advection-reaction-diffusion systems

**Guangyu Zhao**

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**Shigui Ruan**

In this presentation, we will talk about time periodic traveling wave solutions to a class of periodic advection-reaction-diffusion systems. Among the basic questions, we shall concentrate on the existence of time periodic waves, the determination of critical wave speed, and the stability of traveling waves. Under certain conditions, we show that there exists a maximal wave speed  $c^*$  such that for each wave speed  $c \leq c^*$ , there is a time periodic traveling wave connecting two periodic solutions of the corresponding kinetic system. It is shown that such a traveling wave is unique modulo translation and is monotone with respect to its co-moving frame coordinate. We also show that the traveling wave solutions with wave speed  $c \leq c^*$  are asymptotically stable in certain sense.

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## Special Session 19: Waves and Convection

Sam Stechmann, University of Wisconsin-Madison, USA  
 Leslie Smith, University of Wisconsin-Madison, USA

This special session is focused on waves, convection, and their intertwined dynamics. Mathematical aspects include interesting new PDEs, multiscale asymptotics, numerical modeling, and data mining, among other topics. Applications include rotating and stratified fluid dynamics, atmospheric and oceanic science, turbulence, and biology, to name a few.

### Multi-modal dynamics in parallel and wave-induced stratified shear layers

**Roberto Camassa**  
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**Claudio Viotti**

In this talk we discuss the evolution of perturbations containing multiple non-normal eigenmodes in stratified shear flows. We perform detailed analysis of the Taylor–Goldstein equation, the stability operator in parallel stratified shear flows, and we present numerical observations of multi-modal dynamics arising in both parallel and non-parallel wave-induced shear layers. We show how the Taylor–Goldstein spectrum can be used to explain subtle aspects of perturbations evolution that affect the instability of internal waves beside other mechanisms, like Kelvin–Helmholtz instability, that so far have received most of the attention in such context.

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### Modulation of shallow water equatorial waves due to a varying equivalent height background

**Juliana Dias**  
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**P. L. Silva Dias, G. N. Kiladis**

The dynamics of equatorial convectively coupled Kelvin waves (CCKWs) is analyzed in an idealized model for the large-scale equatorial circulation. The model is composed of a linear rotating shallow water system with a variable equivalent depth, which is assumed to vary in space and time. This model is based on the hypothesis that moist convection acts to reduce the equivalent depth of a shallow water system. Here, asymptotic solutions are derived in the case of a small perturbation around a constant equivalent depth. The first order solutions correspond to the free normal modes of the shallow water system. The second order flow satisfies a forced shallow water system, where the forcing (representing the convection) is proportional to the divergence of the first order flow solution. We first demonstrate how solutions vary depending on the space-time behavior of the equivalent depth perturbation. In particular, propagating solutions exist in both limits where the equivalent depth oscillates fast

or slow in comparison to the first order wave scale. However, the second order wave amplitude decays as the period of the variable coefficient decreases. This result implies that the overall flow is less affected by high frequency equivalent depth oscillations. This analytical framework is applied to the study of a synoptic scale (1000 km) Kelvin wave propagating through a background where the equivalent depth oscillates at planetary scales (10000 km). The modeled flow share some remarkable similarities with observed CCKWs. First, as in observed CCKWs, the modeled wave develops a weak secondary meridional circulation. Second, not surprisingly, the phase speed and meridional trapping scale are modulated by changes in the equivalent depth. This modulation is consistent with the fact that CCKWs tend to propagate more slowly when they are embedded in a larger scale convective envelope such as the Madden Julian Oscillation. Based on space-time spectral analysis of tropical convection data, CCKWs propagate at speeds between 7-25 m/s. While highly idealized, the model used here provides a mechanism for both the variability in the observed phase speed of CCKWs, and for changes in the longitude-latitude structure of the observed waves in comparison to the dry modes.

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### Capturing intermittent and low-frequency variability in high-dimensional data through nonlinear Laplacian spectral analysis

**Dimitrios Giannakis**  
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**Andrew J. Majda**

Nonlinear Laplacian spectral analysis (NLSA) is a recently developed technique for spatiotemporal analysis of high-dimensional data, which represents temporal patterns via natural orthonormal basis functions on the nonlinear data manifold. Through the use of such basis functions, determined efficiently via graph-theoretic algorithms, NLSA captures intermittency, rare events, and other nonlinear dynamical features which are not accessible via linear approaches [e.g., singular spectrum analysis (SSA)]. Here, we apply NLSA in a comparative study of North Pacific SST data from extended control integrations of the CCSM3 and ECHAM5/MPI-OM models. Without performing spatial coarse graining (i.e., operating in ambient space dimensions up to  $1.6 \times 10^5$  after

lagged embedding), or seasonal cycle subtraction, the method reveals families of periodic, low-frequency, and intermittent spatiotemporal modes. The intermittent modes, which describe variability in the Western and Eastern boundary currents, as well as variability in the subtropical gyre with year-to-year reemergence, are not captured by SSA, yet are likely to have high significance in a predictive context and utility in cross-model comparisons.

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### A model of convective Taylor columns in rotating Rayleigh-Benard convection

**Ian Grooms**

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**Keith Julien, Jeffrey B. Weiss, Edgar Knobloch, Antonio M. Rubio**

Equations derived by asymptotic reduction of the rotating Navier-Stokes equations in the limit of strong rotation, weak stratification, and tall aspect ratio are presented, their relation to the hydrostatic equations and quasigeostrophic equations is explored, and a new multiscale interpretation of the Taylor-Proudman theorem is given. The discussion is then specialized to the problem of rotating Rayleigh-Bénard convection, and the results of numerical simulations in this setting are briefly summarized. In certain parameter regimes the simulations exhibit localized vortical structures termed ‘convective Taylor columns’; similar structures are also observed in laboratory experiments and in direct simulations of the unreduced Navier-Stokes equations. The ubiquity and dominance of the net heat transport by these structures motivates the derivation of a nonlinear, non-separable model which is an approximate solution of the reduced equations. The model specifies the horizontal structure in terms of the real part of a complex Hankel function, also known as a Bessel function of the third kind, while the vertical structure is specified by the numerical solution of a two-point boundary value problem.

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### Minimal models for precipitating organized convection

**Gerardo Hernandez-Duenas**

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**Andrew J. Majda, Leslie M. Smith, Samuel N. Stechmann**

Simulations of precipitating convection are usually carried out with cloud resolving models, which typically represent all the different phases of water: water vapor, cloud water, rain water and ice. Here

we investigate the question: what is the minimal possible representation of water processes that is sufficient for these models? The simplified models that we present assume fast auto conversion and neglect ice. To test the simplified models, we present simulations of squall lines and scattered convection and show that they qualitatively capture observations made in nature and also seen in more comprehensive cloud resolving models, such as propagation of squall lines with tilted profiles, cold pools, and scattered convection.

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### Low rossby number heat transport in rotating Rayleigh-Benard convection

**Keith Julien**

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**Antonio Rubio, Ian Grooms, Geoff Vasil, Edgar Knobloch**

Recent laboratory experiments of turbulent rotating Rayleigh-Benard convection, performed *entirely within* the regime of strong rotational constraint, have revealed a sharp transition in the scaling of the heat transport as a function of the thermal forcing. This is embodied by the nondimensional Nusselt-Rayleigh scaling law,  $Nu \propto Ra^\alpha$ , where a steep scaling regime ( $\alpha > 1$ ) gives way to a comparatively shallower regime ( $\alpha < 1/2$ ) typical of non-rotating turbulent convection. A crossover between the thermal and viscous boundary layers has been proposed as the root-cause of this remarkable result, yet a similar transition is found in the presence of stress-free boundary conditions where viscous layer boundary layers are absent. Unfortunately, the dynamics within the thermal boundary layer remain poorly understood due to resolution challenges at low Rossby number. Utilizing numerical simulations of the asymptotically exact nonhydrostatic balanced geostrophic equations we present an alternative explanation, not reliant on the form of the mechanical boundary conditions, but based on loss of geostrophic balance within the thermal boundary layers as a result of vigorous vortical motions. Furthermore, in contrast to nonrotating convection, we show prior to loss of balance that the bottleneck to heat transport is the turbulent interior not the boundary layers.

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### The vertical structure of baroclinic turbulence in the ocean

**Shane Keating**

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**K. Shafer Smith**

Mesoscale eddies in the ocean, which are primarily driven by baroclinic instability of the mean shear

and stratification profiles, dominate the ocean kinetic energy and play a crucial role in the transport of heat, salinity and biogeochemical tracers. However, no well-accepted scaling theory for baroclinic eddies exists, in part because of the complexity of the full nonlinear instability problem for realistic mean states, and this remains a critical limitation on efforts to develop mesoscale eddy parameterizations. In this study, we exploit the observation that the potential vorticity (PV) inversion problem for a general mean state can be decomposed into a part forced by surface boundary conditions and a part forced by the interior PV distribution. This allows for a convenient categorization of the mean state into Charney-type — in which surface-intensified modes interact with a background meridional PV gradient — or Phillips-type, which is unstable when there exists an inflection point in the mean PV profile. Within this framework, we examine idealized Charney, Phillips and mixed mean states in high-resolution quasigeostrophic simulations with the goal of elucidating the equilibration, vertical structure, and transport properties of the resulting baroclinic eddies.

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#### **A multiscale framework for analysis and simulation of the stratified wind-driven ocean surface boundary layer**

**Ziemowit Malecha**

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**Gregory Chini, Keith Julien**

A primary challenge in physical oceanography is to understand the interaction between small-scale turbulent convective flows in the upper ocean, particularly wind- and surface-wave-driven Langmuir circulation (LC), and submesoscale eddies, fronts, internal waves (IWs), and their associated instabilities. This problem is challenging because LC is strongly non-hydrostatic, is only indirectly affected by density stratification and the Earth's rotation, and has O(50) m length scales. In contrast, submesoscale flows are approximately hydrostatic, are strongly affected by density stratification and Coriolis accelerations, and have O(10) km lateral scales. In this investigation, we use multiscale asymptotic analysis to develop a physically consistent and computationally efficient model of the dynamics of the ocean surface boundary layer. Numerical experiments with this new model reveal novel dynamical phenomena induced by the two-way inter-scale coupling between submesoscale IWs and fine-scale LC: the IWs modulate the phase and intensity of the LC, while the rectified effect of the resulting non-uniformly distributed small-scale convective structures modifies the IW dynamics.

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#### **Internal solitary waves in two-layer flows with shear.**

**Paul Milewski**

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**Anakewit Boonkasame**

Internal solitary waves are ubiquitous in the ocean and atmosphere. They often coexist with a background shear and also generate shear at their crests or troughs. Most realistic models for these waves are ill posed, and this ill posedness is often misinterpreted as a consequence of the shear. Often ad-hoc filtering must then be used to stabilize the computations. We consider the weakly dispersive long wave limit of two layer flow in the presence of background (and also induced) shear and present a model that is stable as long as a depth dependent Richardson parameter is above a threshold. Comparisons of solitary waves of this model with those of other models will be made.

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#### **Rossby waves in rotating shallow water on the sphere**

**David Muraki**

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**Kevin Mitchell**

Linear waves in the tropics are well-understood by the equatorial beta-plane theory of Matsuno (1966). When linearized about a quiescent background flow, the Rossby wave dispersion relation allows for waves at all length scales near the equator. However, it is also well known that for climatologically-typical zonal shear, midlatitude Rossby waves are prevented from crossing the tropics due to the presence of critical latitudes where the phase speed matches that of the background wind. These seemingly contradictory behaviors are typically found in disjoint chapters in the textbooks on atmospheric dynamics and the consistency of their coexistence is not addressed. Based upon the Rotating Shallow Water (RSW) equations on the sphere, we present a unified understanding of how both phenomena fit into a consistent picture of the atmosphere in both the tropics and midlatitudes. This perspective also offers clarification on the RSW wave modes of Kasahara (1980), and observations of global Rossby waves in the recent review by Madden (2007).

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### Tropical cyclogenesis and vertical shear in a moist Boussinesq model

**Leslie Smith**

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**Leslie Smith, Qiang Deng, Andy Majda**

Tropical cyclogenesis is studied in the context of idealized three-dimensional Boussinesq dynamics with a simple model for bulk cloud physics. With low-altitude input of water vapor, numerical simulations capture the formation of vortical hot towers. From measurements of water vapor, vertical velocity, vertical vorticity and rain, it is demonstrated that the structure, strength and lifetime of the hot towers is similar to results from models including more detailed cloud microphysics. Furthermore, the idealized model captures merger of vortical hot towers into a larger-scale, cyclonic moist vortex. The effects of low-altitude vertical shear are investigated by varying the initial zonal velocity profile. In the presence of weak low-level vertical shear, the hot towers retain the low-altitude monopole vorticity structure characteristic of the zero-shear case (starting from zero velocity). For stronger vertical shear, the individual hot towers develop a vorticity dipole rather than a cyclonic monopole. Linear analysis helps to explain the transition from monopole to dipole vorticity structure as the shear increases. The dipoles are not as conducive to merger, and thus strong enough low-level shear prevents the vortical-hot-tower route to cyclogenesis.

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### Nonlinear dynamics and regional variations in the MJO skeleton

**Samuel Stechmann**

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**Andrew J. Majda**

The Madden-Julian Oscillation (MJO) is a propagating envelope of complex multi-scale convection/storms in the tropics. With characteristic scales of 30-60 days and 20,000 km, it significantly affects El Nino, monsoons, and midlatitude predictability. Despite its importance, no theory for the MJO has yet been generally accepted, and climate models typically have inadequate representations of it.

In this talk, a minimal, nonlinear oscillator model is analyzed for the MJO "skeleton," i.e., its fundamental features on intraseasonal/planetary scales: (i) slow eastward phase speed of roughly 5 m/s, (ii) peculiar dispersion relation with group velocity of roughly 0, and (iii) horizontal quadrupole vortex structure. Originally proposed in recent work by the authors, the fundamental mechanism involves neutrally stable interactions between (i) planetary-scale, lower-tropospheric moisture anomalies, and (ii)

the envelope of sub-planetary-scale, convection/wave activity. Here, the model's nonlinear dynamics are analyzed in a series of numerical experiments, using either a uniform sea surface temperature (SST) or a warm-pool SST. The results show both standing oscillations and eastward propagation, and there are significant variations in the number, strength, and/or locations of MJO events. Besides these numerical experiments, it is also shown that the nonlinear model conserves a total energy that includes a contribution from the convective activity.

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### The diurnal cycle and the meridional extent of the tropics

**Esteban Tabak**

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**Paul A. Milewski**

This talk proposes an explanation for the sharp transition between tropics and extra-tropics at a latitude of 30 degrees. This transition, at the outer edges of the Hadley cells, is marked by a steep jump in the height of the troposphere, from sixteen kilometers in the tropics to nine in the mid and high latitudes. The tropics, equatorwards of 30 degrees, are characterized by easterly surface winds -the Trades- and a strong diurnal signal in the wind, pressure and temperature. Polewards of 30 degrees, the winds are westerly, and the weather systems have longer spatio-temporal scales. This change of behavior can be explained in terms of diurnal baroclinic waves due to solar forcing and trapped equatorwards of 30 degrees by the Coriolis effect. Their effect can be illustrated in simple two-layer models for the meridional circulation, where both convection and the entrainment of stratospheric air into the troposphere are represented by energy-preserving shock waves.

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### Bioconvection revisited

**Jean-Luc Thiffeault**

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Bioconvection is convection driven by density differences due to concentration of swimming organisms. It is often driven by phototaxis or other biased motion. Here I will discuss a related phenomenon, where the swimming action of large swarms of microorganisms, such as some types of plankton, can actually drive large scale flows. I will also discuss the likelihood of observing such flows in the laboratory or the environment.

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## Special Session 20: Stochastic-Statistical Modeling of Climate

**Dimitris Giannakis**, New York University, USA  
**John Harlim**, North Carolina State University, USA  
**Andrew Majda**, New York University, USA

Fundamental barriers to advancing weather and climate prediction on time scales from several days to years are limited by the capability of contemporary operational and research prediction systems (GCMs) to represent coupled processes in the climate involving precipitating convection, atmospheric teleconnection patterns, low-frequency modes in the ocean, and variability in the cryosphere. A grand challenge of contemporary applied science is to understand these patterns and their changes in a globally warming world, as well as the impact of these changes on long range forecasting. These problems are beyond the regime of traditional weather forecast models, and instead both effects of the initial state and the change in mean forcing are important. In contrast to using comprehensive GCMs, it is very natural to develop stochastic-statistical models for these patterns, as well as their interaction, for use in long-range forecasting, sensitivity, and attribution studies. This special session aims to bring together researchers from across the spectrum of disciplines related to statistical-stochastic modeling of climate to discuss the development and application of emerging ideas and techniques for these important and difficult practical issues.

### Quantifying uncertainty for predictions with model error in non-Gaussian systems with intermittency

**Michal Branicki**  
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**Andrew J. Majda**

Synergy between empirical information theory and fluctuation-dissipation theorem provides a systematic framework for improving sensitivity and predictive skill for imperfect models of complex natural systems. We utilize a suite of increasingly complex nonlinear models with intermittent hidden instabilities and time-periodic features to illustrate the advantages of such an approach, as well as the role of model errors due to coarse-graining, moment closure approximations, and the memory of initial conditions in imperfect prediction.

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### Using stochastic models to diagnose the origins of leading atmospheric zonal modes

**Grant Branstator**  
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In terms of the meridional wind, the leading structures of interannual variability have a simple structure: to a first approximation each pattern is concentrated in a narrow band of latitudes and is dominated by a single zonal wavenumber. For example zonal wavenumber three variability is prominent at high latitudes while zonal wavenumber five variability is prominent in midlatitudes. Many general circulation models (GCMs) are not able to reproduce this behavior, which can affect their ability to respond properly to an external forcing, including increasing greenhouse gases. We have used mechanistic models generated by linearizing the governing equations

about climate mean states and driven by noise to determine aspects of the mean state that may be responsible for the prominent wavenumbers of variability in nature and GCMs. For example we find that whether the mean state has a strong projection onto zonal wavenumber three at high latitudes is a factor in determining whether a climate system will have prominent variability in that wavenumber in that region. We have also used response operators based on the fluctuation-dissipation theorem to determine regions from which interannual variability in tropical rainfall may force these prominent patterns, thus identifying another factor that may differentiate model behavior.

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### Stochastic subgrid-scale parameterization designed for a finite-difference model discretization

**Stamen Dolaptchiev**  
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**U. Achatz, I. Timofeyev**

We present a new approach for the construction of stochastic subgrid-scale parameterizations. Starting from a high-resolution finite-difference discretization of some model equations, the approach is based on splitting the model variables into fast, small-scale and slow, large-scale modes by averaging the model discretization over neighboring grid cells. After that, a closed form effective stochastic model for the slow modes is derived applying a stochastic mode reduction procedure. An advantage over heretofore applications of stochastic mode reduction to spectrally discretized models is that the resulting closure is local and thus remains applicable even if the number of slow variables is large. The new approach is implemented for the discretized Burgers equation and compared with other benchmark parameterizations.

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### Quantifying long-range predictability and model error through data clustering and information theory

**Dimitrios Giannakis**

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**Andrew J. Majda**

We present a framework blending data clustering and information theory to quantify long-range initial-value predictability and forecast error with imperfect models in complex dynamical systems. With reference to wind-driven ocean circulation, we demonstrate that the pertinent information for long-range forecasting can be represented via a coarse-grained partition of the set of initial data available to a model. A related formalism is applied to assess the forecast skill of Markov models of ocean circulation regimes.

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### Stochastic homogenization for an energy conserving multi-scale toy model of the atmosphere

**Georg Gottwald**

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**Jason Frank**

We study a simple Hamiltonian toy model for a Lagrangian fluid parcel in the semi-geostrophic limit which exhibits slow and fast dynamics. We first re-ject unresolved fast dynamics into the deterministic equation through stochastic parametrization which respects the conservation of the energy of the deterministic system. In a second step we use stochastic singular perturbation theory to derive an effective reduced stochastic differential equation for the slow dynamics. We verify the results in numerical simulations.

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### Optimal filtering of complex turbulent systems with memory depth through consistency constraints

**John Harlim**

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We present the AR(p)-filter for assimilating weakly chaotic dynamical systems with long memory depth. In particular, a data-driven autoregressive stochastic models with non-Markovian nature is used as surrogate filter prior models. We will show that the autoregressive filter is not as sensitive as standard ensemble filtering strategies to additional intrinsic model errors. Secondly, we will also discuss offline mathematical conditions for optimal autoregressive

filters. In particular, we will rigorously and numerically show that if the autoregressive model parameters are chosen to satisfy a certain subset of the consistency conditions and absolute stability of multistep numerical discretization scheme, then the optimal autoregressive filtering is guaranteed. We will also demonstrate how to apply this result to improve signals with long memory depth: the first Fourier coefficient of the truncated Burgers-Hopf model and the Lorenz-96 model in weakly chaotic regime.

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### A Bayesian approach to parameter estimation and model error quantification of stochastic models for turbulent signals

**Radu Herbei**

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**Dimitrios Giannakis, Andrew J. Majda**

We study the performance of Markov Chain Monte Carlo (MCMC) algorithms for parameter estimation and model error quantification in the setting of a model with time-dependent stochastic parameters, which has high skill in reproducing intermittency, transient instability, and other important features of turbulent signals. The test parameters of the model are chosen to simulate two distinct regimes: (1) frequent, short-lasting transient instabilities, and (2) large amplitude transient instabilities. We implement several state-of-art MCMC algorithms and study their ability to correctly recover these parameters from partial observations. Our main focus is to compare an adaptive MCMC approach to a particle MCMC approach under two scenarios: (a) correctly specified model and (b) with model error. We discuss the benefits and drawbacks of a Bayesian approach to this setting.

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### Stochastic data assimilation methods for estimating ocean eddy heat transport

**Shane Keating**

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**Andrew J. Majda, K. Shafer Smith**

The role of ocean eddies in redistributing heat from the tropics to the poles remains a poorly constrained feature of the global energy balance. Attempts to diagnose eddy transport are limited by the sparseness of available observations and the nonlinearity of the underlying dynamics. In this study, a suite of stochastic data assimilation methods are tested in idealized two-layer simulations of oceanic turbulence at high and low latitudes under a range of observation scenarios. A novel feature is the use of

inexpensive stochastic models to forecast the eddy dynamics. The stochastic model parameters can be estimated by regression fitting to climatological energy spectra and correlation times or by adaptively learning these parameters “on-the-fly” from the observations themselves. We show that, by extracting high-wavenumber information that has been aliased into the low wavenumber band, one can derive “stochastically superresolved” velocity fields with a nominal resolution increase of a factor of two or more. The filtered estimates of the upper and lower layer streamfunctions produce time-mean poleward eddy heat transports that are significantly closer to the true value when compared with standard estimates based upon optimal interpolation. Implications for estimating poleward eddy heat transport using current and next-generation altimeters are discussed.

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### Sparse adaptive polynomial chaos representations for ocean general circulation models

**Omar Knio**

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**P. Conrad, J. Winokur, I. Sraj, A. Alexandrian, M. Iskandarani, A. Srinivasan, Y. Marzouk, O. Knio**

A database of high-resolution HYCOM simulations of the oceanic circulation in the Gulf of Mexico is used to conduct an a priori analysis of the performance of adaptive refinement schemes for uncertainty quantification. The database includes realizations corresponding to isotropic sparse sampling of the uncertain model inputs, namely parameterizations of subgrid mixing and wind drag. The analysis is used to determine performance gains due to a sparse, adaptive, pseudospectral projection approach, and to study the impact of different refinement criteria. Predictions of adaptive refinement are validated against results obtained using a Latin hypercube sampling approach. Finally, the analysis is used to explore the potential of stochastic preconditioning in constructing sparse representations of model outputs as well as improving the performance of adaptive refinement schemes.

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### A digital filtering framework for the local ensemble transform Kalman filter

**Eric Kostelich**

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**Istvan Szunyogh**

Data assimilation refers to the process by which initial conditions for geophysical models are determined

from noisy observations, typically with maximum likelihood methods. In the classical Kalman filter, one seeks to minimize an appropriate sum of squares that is weighted according to one’s relative confidence in the observations and the model forecasts. This talk will explore alternative formulations to include an internal digital filter as a weak constraint within the Local Ensemble Transform Kalman Filter. Preliminary results of its application to the Global Forecast System atmospheric model will be described.

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### How do you determine whether the earth is warming up?

**Juan Restrepo**

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**Darin Comeau, Hermann Flaschka**

How does one determine whether the extreme summer temperatures in Moscow in 2010 was an extreme climatic fluctuation or the result of a systematic global warming trend? It is only under exceptional circumstances that one can determine whether a climate signal belongs to a particular statistical distribution. In fact, climate signals are rarely “statistical,” other than measurement errors, there is usually no way to obtain enough field data to produce a trend or a tendency, based upon data alone. We propose a trend or tendency methodology that does not make use of a parametric or statistical assumption. The most important feature of this trend strategy is that it is defined in very precise mathematical terms.

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### Blended reduced subspace algorithms for uncertainty quantification

**Themistoklis Sapsis**

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**Andrew J. Majda**

We study uncertainty-quantification (UQ) properties of the Quasi-linear Gaussian (QG) closure method and we compare it with the results from order-reduction based on dynamical orthogonality (DO). We find that each of these approaches suffer from disadvantages that can be overcome by combining them. Specifically, the QG method is incapable to capture strong energy transfers among linearized modes. On the other hand, due to the reduced order character the DO approach is incapable to capture the full-order effect of the linearized operator which in many cases (e.g. skew systems) can be critical for the correct evolution of the statistics. We formulate a blended approach based on these two methods which can be further improved by adding empirical information.

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### A stochastic model for tropical rainfall and extreme events

**Samuel Stechmann**

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**J. David Neelin**

Recently it has been discovered that tropical rainfall patterns, on scales of 20-200 km or larger, have statistics that resemble critical phenomena from statistical physics. Through, for instance, the power-law distributions and long-range correlations in these statistics, the characteristics of extreme rainfall events can be quantified. To gain further insight into these statistics and extreme events, a stochastic model is designed and analyzed to reproduce the statistics that are local in space (and evolving in time). The model includes the interaction of a stochastic jump process and Gaussian processes to represent different aspects of tropical convection, a highly complex system that, if fully resolved, involves nonlinear turbulent interactions of fluid dynamics and moist thermodynamics. The stochastic model can be thought of as a simplified subgrid-scale parameterization of moist convection for atmospheric models with grid spacings of 20-200 km.

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### Sub-sampling in parametric estimation of effective stochastic models

**Ilya Timofeyev**

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**R. Azencott, A Beri**

It is often desirable to derive an effective stochastic model for the physical process from observa-

tional and/or numerical data. Various techniques exist for performing estimation of drift and diffusion in stochastic differential equations from discrete datasets. In this talk we discuss the question of sub-sampling of the data when it is desirable to approximate statistical features of a smooth trajectory by a stochastic differential equation. In this case estimation of stochastic differential equations would yield incorrect results if the dataset is too dense in time. Therefore, the dataset has to be sub-sampled (i.e. rarefied) to ensure estimators' consistency. Favorable sub-sampling regime is identified from the asymptotic consistency of the estimators. Nevertheless, we show that estimators are biased for any finite sub-sampling time-step and construct new bias-corrected estimators.

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### Sampling in and out of equilibrium when the tails matter

**Jonathan Weare**

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Analyzing and simulation rare events in stochastic process is important in many areas. For example, in the context of weather and climate, prediction can be dramatically hampered by unlikely, but important transitions in the underlying system. The past decade or so has seen dramatic improvements in our ability to simulate and analyze these events. So far this progress has come mostly in the contexts of Chemistry and Computer Science but rare event ideas seem ripe for application in geophysical contexts. I will survey a bit of my work in rare event simulation generally as well as show some preliminary work toward geophysical applications.

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## Special Session 21: Dynamical Systems and Spectral Theory

David Damanik, Rice University, USA

Talks in this special session will address recent results and developments in dynamical systems and/or spectral theory.

### Derivation of NLS from an interacting Bose gas in $d = 3$ via Klainerman-Machedon type spaces

**Thomas Chen**

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**Natasa Pavlovic**

The Gross-Pitaevskii (GP) hierarchy is an infinite system of coupled linear non-homogeneous PDEs, which appear in the derivation of the nonlinear Schrödinger equation (NLS). In this talk we will discuss a new derivation of the defocusing cubic GP hierarchy in dimensions  $d = 2, 3$ , from an  $N$ -body Schrödinger equation describing a gas of interacting bosons in the GP scaling, in the limit  $N \rightarrow \infty$ . In particular, we prove convergence of the corresponding BBGKY hierarchy to a GP hierarchy in the spaces introduced in our previous work on the well-posedness of the Cauchy problem for GP hierarchies, which are inspired by solution spaces based on space-time norms introduced by Klainerman and Machedon. We note that in  $d = 3$ , this has been a well-known open problem in the field. While our results do not assume factorization of the solutions, consideration of factorized solutions yields a new derivation of the cubic, defocusing NLS in  $d = 2, 3$ .

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### Subshifts and low regularity potentials

**David Damanik**

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We discuss the spectral theory of Schrödinger operators with potentials defined by low regularity sampling functions along a suitable base transformation and approximations by subshifts over finite alphabets.

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### Inverse problems for Jacobi operators

**Rafael Del Rio**

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**M. Kudryavtsev**

We consider a linear finite spring mass system which is perturbed by modifying one mass and adding one spring. We study when masses and springs can be

recovered from the natural frequencies of the original and the perturbed systems. This is a problem about rank two or rank three perturbations of finite Jacobi matrices where we are able to describe quite explicitly the associated Green's functions. We give necessary and sufficient conditions for two given sets of points to be eigenvalues of the original and modified systems respectively.

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### Spectral properties for the quasi-periodic Schrödinger equation

**Roberta Fabbri**

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We consider the spectral properties of the quasi-periodic Schrödinger operator and the 2-dimensional Schrödinger equation using the relation between the rotation number and the uniform hyperbolicity of the corresponding differential systems. Using also some numerical computation (due to Cinzia Elia), we obtain information on the structure of the spectrum of the one-dimensional operator.

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### Properties of the IDS of the Fibonacci Hamiltonian

**Anton Gorodetski**

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**David Damanik**

The Trace Map techniques not only allow to describe the spectrum of the discrete Schrödinger operator with Fibonacci potential as a set, but also provide an insight into the properties of the integrated density of states (IDS) that turn out to be related to the measure of maximal entropy for the Trace Map. In particular, we show the exact dimensionality of the IDS, and provide sharp estimates on its Holder exponent.

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### Quasi-periodic Schrödinger operators beyond the almost Mathieu

**Alex Haro**

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**Joaquim Puig**

This talk is devoted to quasi-periodic Schrödinger operators beyond the Almost Mathieu, with more general potentials and interactions, considering the connections between the spectral properties of these operators and the dynamical properties of the associated quasi-periodic linear skew-products. In particular, we present a Thouless formula and some consequences of Aubry duality. We illustrate the results with numerical computations.

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### Recent developments for skew-shift Schroedinger operators

**Helge Krueger**

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The skew-shift is given by  $T : (x, y) \rightarrow (x + 2\omega, y + x)$  where  $\omega$  is an irrational number. Potentials given by evaluating a sampling function along the second coordinate of an orbit have many interesting properties. I will discuss properties related to the distribution of eigenvalues and the structure of the spectrum.

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### Jacobi matrices with decaying oscillatory coefficients

**Milivoje Lukic**

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We investigate decaying oscillatory perturbations of the free Jacobi matrix. The perturbation can, for instance, be a quasiperiodic sequence multiplied by  $\ell^p$  decay. Under mild conditions, we prove preservation of absolutely continuous spectrum and give bounds on the Hausdorff dimension of the singular part of spectral measures.

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### Analytic quasi-periodic cocycles with singularities and the Lyapunov Exponent of Extended Harper's model

**Christoph Marx**

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**S. Jitomirskaya**

We show how to extend (and with what limitations) Avila's global theory of analytic  $SL(2, \mathbb{C})$  cocycles to families of cocycles with singularities. This allows to develop a strategy to determine the Lyapunov exponent for extended Harper's model, for all values of parameters and all irrational frequencies. In particular, this includes the self-dual regime for which even heuristic results did not previously exist in physics literature. The extension of Avila's global theory is also shown to imply continuous behavior of the LE on the space of analytic  $M(2, \mathbb{C})$ -cocycles. This includes rational approximation of the frequency, which so far has not been available.

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### Orthogonal polynomials on the unit circle with almost periodic recursion coefficients

**Darren Ong**

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Given a probability measure on the unit circle, we perform a Gram-Schmidt orthogonalization process on  $\{1, z, z^2, \dots\}$  and obtain a sequence of orthogonal polynomials with respect to that measure. These polynomials obey a recurrence relation, and it is of natural interest to relate properties of the recurrence coefficients with properties of the probability measure. We present various results about the probability measure when the corresponding recurrence coefficients form an almost periodic sequence. We arrive at these conclusions by expressing the problem in terms of dynamically defined unitary operators, and by exploiting well-known connections between these unitary operators and the discrete Schrödinger Operator.

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### Fractals and dynamic

**Tatiana Rodrigues**

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Let  $A$  be an alphabet over 3 letters and  $A^*$  is the set of finite words written with the alphabet  $A$ . A *substitution*  $\sigma$  is a map from  $A$  to  $A^*$ . It is known that to any substitution we can associate a shift symbolic dynamical system. It is known that for a large class of substitutions  $\sigma$ , the associated dynamical system is measure theoretically isomorphic to an

exchange of pieces over a compact set  $\mathcal{K}_\sigma$  of  $\mathbb{R}^2$ . This set is called Rauzy Fractal and has many beautiful properties. In particular  $\mathcal{K}_\sigma$  induces a periodic tiling of the plane and moreover its boundary is fractal. In this work, we will present some geometrical and dynamical properties of the Rauzy fractal and its boundary.

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### Absolutely continuous spectrum and ballistic behavior for the Anderson model on the Bethe strip

**Christian Sadel**

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**Abel Klein**

The Bethe strip is the cross product graph of the Bethe lattice with a finite graph. We consider random Schroedinger operators on such graphs such as the Anderson model. For low disorder we find almost surely absolutely continuous spectrum in a certain interval. Moreover, the quantum dynamical wave spreading is ballistic.

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### Anderson localization for non-monotone Schroedinger operators

**Mira Shamis**

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**A. Elgart, S. Sodin**

We show how the fractional moment method of Aizenman and Molchanov can be applied to a class of Anderson-type models with non-monotone potentials, to prove (spectral and dynamical) localization. The main new feature of our argument is that it does not assume any a priori Wegner-type estimate: the (nearly optimal) regularity of the density of states is established as a byproduct of the proof. The argument is applicable to finite-range alloy-type models and a class of operators with matrix-valued potentials.

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### Spectral applications of McMullen's Hausdorff dimension algorithm

**Mihai Stoiciu**

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**K. Gittins, N. Peyerimhoff, D. Wirosoetisno**

We consider singular continuous measures on the unit circle obtained as limit measures of groups generated by reflections in the hyperbolic plane. We extend McMullen's Hausdorff dimension algorithm to approximate the moments of these measures. This allows us to study the corresponding orthogonal polynomials on the unit circle and to investigate various spectral properties of the associated CMV matrices.

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### Spectral analysis of tridiagonal Fibonacci Hamiltonians

**William Yessen**

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We consider a family of discrete Jacobi operators on the one-dimensional integer lattice, with the diagonal and the off-diagonal entries given by two sequences generated by the Fibonacci substitution on two letters. We show that the spectrum is a Cantor set of zero Lebesgue measure, and discuss its fractal structure and Hausdorff dimension. We also extend some known results on the diagonal and the off-diagonal Fibonacci Hamiltonians.

Our methods involve dynamical properties of the so-called Fibonacci trace map (a polynomial map of degree two on the three-dimensional Euclidean space).

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### Positive Lyapunov exponents for quasiperiodic Szego cocycles

**Zhenghe Zhang**

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I will talk about positivity of Lyapunov exponents for quasi-periodic Szégo cocycles. I will use different methods to consider potentials of different smooth categories: for  $C^0$  ones, I will use Avila and Damanik's technique for genericity of singular spectrum for Schrödinger operators; for  $C^r$  case,  $1 \leq r \leq \infty$ , I will use Lai-Sang Young's induction method, which in spirit is Benedicks-Carleson's method for Hénon map; for  $C^\omega$  ones, I will use subharmonicity and acceleration, which is recently introduced by Avila. In particular, new examples of analytic quasiperiodic Szégo cocycles with uniformly positive Lyapunov exponents (uniform in energy) will be constructed.

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## Special Session 22: Topological and Variational Methods for Boundary Value Problems

**John R. Graef**, University of Tennessee at Chattanooga, USA  
**Lingju Kong**, University of Tennessee at Chattanooga, USA  
**Bo Yang**, Kennesaw State University, USA

Topological methods have proved to be an important technique in the study of boundary value problems and related topics for ordinary and partial differential equations. Recently that has been a rapidly growing interest in applying variational methods and critical point theory to such problems. This session is devoted to the use of these methods in the study of boundary value problems including singular problems and those with multipoint conditions.

### Existence of multiple positive solutions for $p$ -Laplacian multipoint boundary value problem on time scales

**Abdulkadir Dogan**

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In this paper, we consider  $p$ -Laplacian multipoint boundary value problem on time scales. By using fixed point theorems, we prove the existence of at least three positive solutions to the boundary value problem. The interesting point is that the nonlinear term  $f$  depends on the first order derivative explicitly. As an application, an example is given to illustrate the result.

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### A note on a third-order multi-point boundary value problem at resonance

**Zengji Du**

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**Xiaojie Lin, Fanchao Meng**

Based on the coincidence degree theory of Mawhin, we prove some existence results for a third-order multi-point boundary value problem at resonance. In this talk, the dimension of the linear space  $\text{Ker } L$  is equal to 2. Since all the existence results for third-order differential equations obtained in previous papers are for the case  $\dim \text{Ker } L = 1$ , our work is new.

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### A class of decomposable nonlinear operators and its applications in BVP

**Wenying Feng**

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**Jianhong Wu**

We will introduce a class of nonlinear operators that can be decomposed into a linear operator and a nonlinear map. Some properties for the class are proved. Applications to existence of solutions for

some boundary value problems are given.

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### Generalized upper and lower solutions on fourth order Lidstone problems

**Joao Fialho**

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**Feliz Minhós**

In this work it is considered the nonlinear fully equation

$$u^{(iv)}(x) + f(x, u(x), u'(x), u''(x), u'''(x)) = sp(x) \quad (1)$$

for  $x \in [0, 1]$ , where  $f : [0, 1] \times \mathbb{R}^4 \rightarrow \mathbb{R}$  and  $p : [0, 1] \rightarrow \mathbb{R}^+$  are continuous functions and  $s$  a real parameter, coupled with the Lidstone boundary conditions,

$$u(0) = u(1) = u''(0) = u''(1) = 0, \quad (2)$$

These types of problems are known as Ambrosetti-Prodi problems, and they provide the discussion of existence, nonexistence and multiplicity results on the parameter  $s$ . More precisely, sufficient conditions, for the existence of  $s_0$  and  $s_1$ , are obtained, such that:

- if  $s < s_0$ , the problem has no solution.
- if  $s = s_0$ , the problem has a solution.
- if  $s \in ]s_0, s_1]$ , the problem has at least two solutions.

In this work it is discussed how conditions in the lower and upper definitions influence the main results and vice-versa. This "power shift" between the Definition and Theorem makes it possible to extend some results to a functional version of (1)-(2). In addition we replace the usual bilateral Nagumo condition by a one-sided condition, allowing the nonlinearity to be unbounded.

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### On a notion of category depending on a functional and an application to Hamiltonian systems

**Marlene Frigon**

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A notion of category depending on a functional is introduced. This notion permits to obtain a better lower bound on the number of critical points of a functional than the classical Lusternik-Schnirelman category. A relation between this notion and the linking of type splitting spheres is discussed. An application to Hamiltonian systems is also presented.

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### Existence of nontrivial solutions to systems of multi-point boundary value problems

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**Lingju Kong, Shapour Heidarkhani**

The authors consider the system of  $n$  multi-point boundary value problems

$$\begin{cases} -(\phi_{p_i}(u'_i))' = \lambda F_{u_i}(x, u_1, \dots, u_n), & x \in (0, 1), \\ u_i(0) = \sum_{j=1}^m a_j u_i(x_j), & u_i(1) = \sum_{j=1}^m b_j u_i(x_j), \end{cases}$$

for  $i = 1, \dots, n$ . The existence of at least one non-trivial solution is proved using variational methods and critical point theory.

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### Existence and multiplicity for positive solutions of a system of higher-order multi-point boundary value problems

**Johnny Henderson**

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**Rodica Luca**

We investigate the existence and multiplicity of positive solutions of multi-point boundary value problems for systems of nonlinear higher-order ordinary differential equations.

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### Solvability of second order three-point boundary value problem at resonance

**James S. W. Wong**

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**C. H. Ou**

We are interested in existence theorems for second order three-point boundary value problems at resonance. The usual method of proof is based upon coincidence degree theory which differs from the fixed point theorem approach for problems subject to non-resonant boundary conditions. We introduce an alternate method by reducing the original problem to a two-point problem with non-homogeneous boundary condition containing a parameter  $\mu$ . We then apply fixed point theorem and shooting method to determine the proper  $\mu$  which gives rise to a solution of the original three-point problem.

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### On a discrete fourth order periodic boundary value problem

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**John R. Graef, Min Wang**

By using the variational method and critical point theory, we obtain criteria for the existence of multiple solutions of the discrete fourth order periodic boundary value problem

$$\begin{aligned} \Delta^4 u(t-2) - \alpha \Delta^2 u(t-1) + \beta u(t) &= f(t, u(t)), \quad t \in [1, T]_{\mathbb{Z}}, \\ \Delta^i u(-1) &= \Delta^i u(T-1), \quad i = 0, 1, 2, 3, \end{aligned}$$

where  $T \geq 2$  is an integer,  $[1, T]_{\mathbb{Z}} = \{1, 2, \dots, T\}$ ,  $\alpha, \beta \geq 0$  are parameters, and  $f : [1, T]_{\mathbb{Z}} \times \mathbb{R} \rightarrow \mathbb{R}$  is a continuous function. Examples are included to illustrate the results.

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**A fourth-order functional problem at resonance****Nickolai Kosmatov**University Of Arkansas at little Rock, USA  
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We study the differential equation

$$u^{(4)}(t) - \omega^4 u(t) = f(t, u(t), u'(t), u''(t), u'''(t)),$$

satisfying linear functional conditions

$$B_i u = 0, \quad i = 1, \dots, 4.$$

$$\longrightarrow \infty \diamond \infty \longleftarrow$$

**Boundary data smoothness for solutions of nth order nonlocal boundary value problems****Jeffrey Lyons**Texas A&M University - Corpus Christi, USA  
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In this talk, we investigate boundary data smoothness for solutions of the nonlocal boundary value problem,  $y^{(n)} = f(x, y, y', \dots, y^{(n-1)})$ ,  $y^{(i)}(x_j) = y_{ij}$  and  $y^{(i)}(x_k) - \sum_{p=1}^m r_{ip} y(\eta_{ip}) = y_{ik}$ . Essentially, we show under certain conditions that partial derivatives of the solution to the problem above exist with respect to boundary conditions and solve the associated variational equation. Lastly, there will be a corollary and nontrivial example.

$$\longrightarrow \infty \diamond \infty \longleftarrow$$

**Two-point boundary value problems with impulses.****Daniel Maroncelli**North Carolina State University, USA  
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The authors look at nonlinear boundary value problems of the form

$$x'(t) = A(t)x(t) + f(t, x(t)), \quad t \in [0, 1] \setminus \{t_1, \dots, t_k\}$$

$$x(t_i^+) - x(t_i) = J_i(x(t_i)), \quad i = 1, \dots, k$$

subject to

$$Bx(0) + Dx(1) = 0.$$

We focus on the resonant case, that is, the case in which the solution space of the linear homogeneous problem is nontrivial. In particular, we use degree theory to prove the existence of solutions when the solution space has dimension greater than 1.

$$\longrightarrow \infty \diamond \infty \longleftarrow$$

**Existence and multiplicity of solutions in fourth order BVPs with unbounded nonlinearities****Feliz Minhos**University of Evora, Portugal  
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This work studies the Ambrosetti-Prodi fourth order nonlinear fully equation

$$u^{(4)}(x) + f(x, u(x), u'(x), u''(x), u'''(x)) = s p(x)$$

for  $x \in [a, b]$ ,  $f : [a, b] \times \mathbb{R}^4 \rightarrow \mathbb{R}$ ,  $p : [a, b] \rightarrow \mathbb{R}^+$  continuous functions and  $s \in \mathbb{R}$ , with the boundary conditions

$$u(a) = A, \quad u'(b) = B, \quad u'''(a) = C, \quad u'''(b) = D.$$

In this work it will be presented an Ambrosetti-Prodi type discussion on  $s$ , with some new features: the existence part is obtained in presence of nonlinearities not necessarily bounded, and in the multiplicity result it is not assumed a speed growth condition or an asymptotic condition, as it is usual in the literature for these type of higher order problems.

The arguments used apply lower and upper solutions technique and topological degree theory.

An application to a continuous model of the human spine, used in aircraft ejections, vehicle crash situations and some forms of scoliosis, will be referred.

$$\longrightarrow \infty \diamond \infty \longleftarrow$$

**Extremal points for an nth order three point boundary value problem****Jeffrey Neugebauer**Eastern Kentucky University, USA  
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We characterize first extremal points of an  $n$ th order three point boundary value by using a substitution method and working with the 4th order problem. These results are then used to find positive solutions of the  $n$ th order nonlinear problem.

$$\longrightarrow \infty \diamond \infty \longleftarrow$$

### Oscillation results for fourth order nonlinear mixed neutral differential equations

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**A. K. Tripathy, R. Basu**

Oscillatory and asymptotic behaviour of a class of nonlinear fourth order neutral differential equations with positive and negative coefficients of the form

$$(r(t)(y(t) + p(t)y(t - \tau)))'' + q(t)G(y(t - \alpha)) - h(t)H(y(t - \beta)) = 0,$$

and

$$(E) (r(t)(y(t) + p(t)y(t - \tau)))'' + q(t)G(y(t - \alpha)) - h(t)H(y(t - \beta)) = f(t)$$

are investigated under the assumption

$$\int_0^\infty \frac{t}{r(t)} dt = \infty$$

for various ranges of  $p(t)$ . Using Scauder's fixed point theorem, sufficient conditions are obtained for the existence of positive bounded solutions of (E).

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### Boundary value problems governing fluid flow and heat transfer over an unsteady stretching sheet

**Joseph Pualet**

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This talk will consider two situations involving unsteady laminar boundary layer flow due to a stretching surface in a quiescent viscous incompressible fluid. In one configuration, the surface is impermeable with prescribed heat flux, and in the other, the surface is permeable with prescribed temperature. The boundary value problems governing a similarity reduction for each of these situations are investigated and existence of a solution is proved for all relevant values of the physical parameters. Uniqueness of the solution is also proved for some (but not all) values of the parameters. Finally, a priori bounds are obtained for the skin friction coefficient and local Nusselt number.

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### Existence analysis for nonlocal Sturm-Liouville boundary value problems

**Jesus Rodriguez**

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**Zach Abernathy**

We establish conditions for the existence of solutions to nonlinear differential equations subject to nonlocal boundary conditions. The problems considered are of the form

$$(p(t)x'(t))' + q(t)x(t) + \psi(x(t)) = G(x(t))$$

subject to the global boundary condition

$$\begin{aligned} \alpha x(0) + \beta x'(0) + \eta_1(x) &= \phi_1(x), \\ \gamma x(1) + \delta x'(1) + \eta_2(x) &= \phi_2(x). \end{aligned} \quad (1)$$

We assume  $\psi : \mathbb{R} \rightarrow \mathbb{R}$  is continuously differentiable,  $p(t) > 0$  and  $q(t)$  is real valued on  $[0, 1]$ ,  $p, p', q$  are continuous on  $(0, 1)$ , and the boundary conditions (1) are such that  $\alpha^2 + \beta^2 \neq 0$ ,  $\gamma^2 + \delta^2 \neq 0$ . Further,  $G, \eta_1, \eta_2, \phi_1$ , and  $\phi_2$  shall be nonlinear operators defined on a function space.

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### Applications of variational methods to anti-periodic boundary value problem for second-order differential equations

**Yu Tian**

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**Johnny Henderson**

We discuss the existence of multiple solutions to a second-order anti-periodic boundary value problem by using variational methods and critical point theory. Furthermore, we get the existence of periodic solutions for corresponding second-order differential equations. In constructing variational structure, we prove a fundamental lemma, which plays an important role in prove the critical point of functional is just the solution of original problem.

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### Fractional boundary value problems with integral boundary conditions

**Min Wang**

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**John R. Graef, Lingju Kong, Qingkai Kong**

The authors study a type of nonlinear fractional boundary value problem with integral boundary conditions. By constructing an associated Green's function, applying spectral theory, and using fixed point theory on cones, they obtain criteria for the existence, multiplicity, and nonexistence of positive solutions.

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### Existence, location and approximation results for some nonlinear boundary value problems

**Guangwa Wang**

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**Mingru Zhou, Li Sun**

The significance and importance of boundary value problems (BVPs) is well-known. The aim of this talk is to present some results of existence, location and approximation for some nonlinear BVPs. In our study, the theory of differential inequalities plays a very important role. More precisely, in the first part, we will present the existence and location criteria of solutions for the general nonlinear system with the general nonlinear boundary conditions. Here, we introduce a new concept of bounding function pair and a method, which may be called simultaneous modification. In the second part, using the generalized quasilinearization method, we will study two classes of second-order nonlinear BVPs with nonlocal boundary conditions. We will establish some sufficient conditions under which corresponding monotone sequences converge uniformly and quadratically to the unique solution of the problem. The interesting point is that our boundary condition is nonlinear and nonlocal.

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### New periodic solutions for N-body-type problems with prescribed energies

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**Shiqing Zhang**

In this paper, we use a variant of the Benci-Rabinowitz Theorem to study the solutions of N-body-type problems with prescribed energies.

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## Special Session 23: Topological and Combinatorial Dynamics

**Lluís Alseda**, Universitat Autònoma de Barcelona, Spain  
**Francisco Balibrea Gallego**, Universidad de Murcia, Spain  
**Piotr Oprocha**, AGH University, Poland

The session will be focused on the topological and combinatorial aspects of low dimensional discrete dynamical systems including entropy, chaos, limit sets, sets of periods, rotation theory and the like. Applications of this theory to Economics, Physics, Engineering, Biology and others are highly welcome. Related areas of dynamical systems, combinatorial aspects of dynamical systems and ergodic theory are not excluded.

### Simple permutations with order $4n + 2$

**Primitivo Acosta-Humanez**  
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**Eduardo Martínez-Castiblanco**

The problem of genealogy of permutations has been solved partially by Stefan (odd order) and Acosta-Humanez & Bernhardt (power of two). It is well known that Sharkovskii's theorem shows the relationship between the cardinal of the set of periodic points of a continuous map, but simple permutations will show the behavior of those periodic points. This paper studies the structure of permutations of mixed order  $4n + 2$ , its properties and a way to describe its genealogy by using Pasting and Reversing as in the case of simple permutations with order a power of two.

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### Topological and algebraic reducibility for patterns on trees

**Lluís Alseda**  
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**David Juher, Francesc Mañosas**

We extend the classical notion of block structure for periodic patterns of interval maps to the setting of tree maps and study the algebraic properties of the Markov matrix of a periodic tree pattern having a block structure. We also prove a formula which relates the topological entropy of a pattern having a block structure with that of the underlying periodic pattern obtained by collapsing each block to a point, and characterize the structure of the zero entropy patterns in terms of block structures. Finally, we prove that an  $n$ -periodic pattern has zero (positive) entropy if and only if all  $n$ -periodic patterns obtained by considering the  $k$ -th iterate of the map on the invariant set have zero (respectively, positive) entropy, for each  $k$  relatively prime to  $n$ .

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### Hofbauer towers and inverse limit spaces

**Lori Alvin**  
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We use Hofbauer towers for unimodal maps to study the collection of endpoints of the associated inverse limit spaces. We show that if  $f$  is a unimodal map for which the kneading map  $Q_f(k)$  tends to infinity and  $f|_{\omega(c)}$  is one-to-one, then the collection of endpoints of  $(I, f)$  is precisely the set  $\mathcal{E}_f = \{(x_0, x_1, \dots) \in (I, f) \mid x_i \in \omega(c) \text{ for all } i \in \mathbb{N}\}$ .

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### Li-Yorke chaos in rational difference equations

**Francisco Balibrea**  
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**Antonio Cascales**

Given the difference equation

$$x_{k+1} = f(x_k)$$

where  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is differentiable, a result of Marotto (1978) stated that if  $f$  has a *snap-back repeller*, then the equation is chaotic in Li-Yorke sense. When every component of  $f$  is a quotient of  $n$ -variables polynomials, we have a *rational difference equation*. In such case  $f$  can have discontinuity points. The union of such points is the *forbidden set of the equation*. In the case of rational difference equations with non-empty forbidden set, the existence of snap-back repellers does not guarantee Li-Yorke chaos. We will present examples on it and talk on the general validity of Marotto's result. In particular we will use the additional condition on  $f$  of having the property of *compact preimage* as a tool to obtain Li-Yorke chaos. Additionally, we will explain the obtained results using as a model the discontinuous rational equation  $x_{k+1} = \frac{1}{x_k^2 - 1}$

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### Omega-limit sets of quadratic maps on their Julia sets

**Andrew Barwell**

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**Brian Raines**

We provide a characterization of *omega*-limit sets for dendritic Julia sets of quadratic maps, using Baldwin's symbolic representation of these spaces as a non-Hausdorff itinerary space. The property of shadowing tells us that a map is stable under small perturbations of orbits, and internal chain transitivity tells us that pairs of points can always be linked using arbitrarily small perturbations of orbits. We show that the quadratic maps with dendritic Julia sets have shadowing, and that for all such maps, a closed set is an *omega*-limit set if, and only if, it has internal chain transitivity.

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### Periods of periodic orbits for vertex maps on graphs

**Christopher Bernhardt**

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Given a graph  $G$  with  $n$  vertices, a *vertex map*  $f$  from  $G$  to itself is a continuous map that permutes the vertices of  $G$ . We only consider examples for which the  $n$  vertices form one periodic orbit. The basic question is to classify the set of the periods of the periodic orbits that  $f$  must have. In this setting, Sharkovsky's Theorem gives the answer when the underlying graph is simply a path. The set of periods that must occur in this case is the set of positive integers forced in the Sharkovsky order by  $n$ .

This talk will look at Sharkovsky-type orderings for vertex maps of general graphs.

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### Aperiodic Cantor dynamics

**Sergey Bezuglyi**

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A homeomorphism  $T$  of a Cantor set  $X$  is called aperiodic if for every point  $x \in X$  the orbit  $(T^n(x))$  is infinite. The pair  $(X, T)$  represents a Cantor aperiodic dynamical system. In my talk I will first discuss the topological properties of the set of all aperiodic homeomorphisms considered as a subset of  $\text{Homeo}(X)$ . Every aperiodic homeomorphism admits a realization as a Vershik map acting on a path space of a Bratteli diagram. The second part of my talk will be focused on aperiodic homeomorphisms

whose Bratteli-Vershik models are represented by Bratteli diagrams of the simplest form: stationary and finite rank diagrams. For such diagrams we can explicitly describe the set of ergodic invariant measures.

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### Patterns, topological transitivity, and entropy

**Louis Block**

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**Dennis Ledis**

Let  $f$  denote a continuous map of the compact interval  $I$  to itself. We use the term pattern to denote an ordered pair  $(P, \pi)$  where  $P = \{x_1 < x_2 < \dots < x_n\}$  is an ordered finite subset of the real line, and  $\pi : P \rightarrow P$ . We say that  $f$  exhibits the pattern  $(P, \pi)$  if and only if there exists a finite subset  $Q = \{y_1 < y_2 < \dots < y_n\}$  of  $I$  such that  $f(y_i) = y_j$  if and only if  $\pi(x_i) = x_j$ . Given a pattern  $(P, \pi)$  we have an associated piecewise linear  $L_{(P, \pi)}$  sometimes called the linearization or connect the dots map. Suppose that  $f$  exhibits a pattern  $(P, \pi)$ . Suppose that  $f$  is topologically transitive. We prove that the topological entropy of  $f$  is equal to the topological entropy of  $L_{(P, \pi)}$  if and only if  $f$  is topologically conjugate to  $L_{(P, \pi)}$ . It is known that the topological entropies of the maps  $L_{(P, \pi)}$ , where the supremum is taken over all patterns exhibited by  $f$ . In fact, it is sufficient to take only those patterns  $(P, \pi)$  such that  $\pi$  is a cyclic permutation. Our theorem implies that for topologically transitive maps the supremum is actually a maximum if and only if  $f$  is topologically conjugate to  $L_{(P, \pi)}$ , for some pattern  $(P, \pi)$  exhibited by  $f$ .

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### On dynamics of surface homeomorphisms with invariant continua

**Jan Boronski**

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Examples of homeomorphisms of  $\mathbb{S}^2$  with circle-like continua as minimal sets, or attractors, are well known. For example, there is the  $C^\infty$  area-preserving diffeomorphism of Handel that is minimal on the pseudo-circle. On the other end of the spectrum a hereditarily decomposable example was given by Walker, by an extension of a minimal homeomorphism similar to the one constructed by Gottschalk and Hedlund on a union of infinitely many copies of the Warsaw circle. We shall discuss the dynamics of self-homeomorphisms of hereditarily decomposable circle-like continua, including the existence of homeomorphisms of type  $2^\infty$ . As a tool, a characterization of covering spaces of these continua will be given. The proofs are built upon related results

obtained by Barge&Gilette, Bellamy&Lewis, Heath, Ingram, Minc&Transue, Mouron, and Ye. Time permitting we shall also discuss some new fixed point results for torus homeomorphisms.

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### Analysis of an infinite dynamical system using substitution systems

**Maria Correia**

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**Carlos Ramos, Sandra Vinagre**

We consider an infinite dynamical system obtained by iteration of functions of a class of differentiable functions,  $\mathcal{A}$ , under  $m$ -modal maps  $f$ . If we consider functions in the class  $\mathcal{A}$  whose critical values are periodic points for  $f$  then we define and characterize a substitution system associated with the dynamical system. With these substitution systems, we analyze the behavior of the whole system. Moreover, we consider a new subclass of functions where the substitution rules are independent of the initial conditions.

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### Dynamics on tiling spaces, invariant measures and generalized Thurston semi-norm

**Francois Gautero**

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**Jean-Marc Gambaudo**

The problem of deciding whether a given finite set of tiles can tile the euclidean plane is known to be an undecidable problem. An aim of this work is to translate this undecidability in a purely topological and geometrical way. When non-empty the set of tilings of the euclidean plane constructed from a given finite set of tiles  $\mathcal{T}$  inherits a natural structure of compact metric space: this is a compact laminated space with transverse structure a Cantor set, equipped with an action of  $R^2$  on the leaves of the lamination. There is then a non-empty set of invariant measures: each one defines a certain homology-class in the second homology group of a branched surface constructed from  $\mathcal{T}$ . The aim of the current work is to characterize, among all the homology classes, those coming from invariant measures on the laminated space. This is done by the introduction of a kind of Thurston semi-norm: the homology-classes one is looking for are exactly those on which this semi-norm vanishes.

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### Ergodic theory and topological intersections as a tool to solve geometrical problems

**Roland Gunesch**

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This talk presents a method to reduce certain (very difficult) geometrical problems to (less difficult) topological intersection problems, and reduce these again to (even less difficult) problems in ergodic theory. While the resulting problems are still not easy, they can actually be solved. The talk illustrates these methods to solve some old problems concerning dynamical systems arising from Riemannian geometry.

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### Maximally transitive semigroups of matrices

**Mohammad Javaheri**

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We discuss the problem of finding a pair of  $n \times n$  matrices that generates a topologically  $n$ -transitive semigroup action on  $\mathbb{K}^n$ , where  $\mathbb{K} = \mathbb{R}$  or  $\mathbb{C}$ . Equivalently, we construct dense 2-generator subsemigroups of  $GL(n, \mathbb{K})$  as well as  $SL(n, \mathbb{K})$  for all  $n \geq 1$ .

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### Devaney chaos and singularities of invertible piecewise isometric dynamics

**Byungik Kahng**

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It is known that piecewise isometric dynamical systems can exhibit Devaney-chaos in an appropriate invariant set. Clearly, the chaotic behavior comes from singularity, because the piecewise isometric dynamics is innocuous besides the singularity. However, the clear connection between the afore-mentioned Devaney-chaos and the properties of singularity has never been established. This talk aims to discuss and investigate such connection.

In general, the singularities of bounded invertible piecewise isometric dynamical systems in Euclidean plane can be classified as, removable, sliding and shuffling singularities according to their geometrical aspects. Furthermore, it is proved that the removable singularities and the shuffling singularities do not generate the Devaney-chaos. Thus, the presence of the sliding singularities is necessary in order to have the chaotic dynamics. Through this talk, the speaker will survey these results and discuss some new results on the unsolved half, the sufficiency. If time permits, the speaker will talk about a possible connection between the shuffling singularity and the ergodicity as well.

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### Infinite-dimensional topology and the Hilbert-Smith conjecture

**James Keesling**

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**James Maissen, David Wilson**

The Hilbert-Smith Conjecture states that if  $G$  is a compact topological group acting effectively on a connected manifold  $M^n$ , then  $G$  is a Lie group. The conjecture is false if and only if it is not possible for any  $p$ -adic group  $\Delta_p$  to act effectively on some manifold. There are classic results showing that the conjecture is true for  $n = 1$  and  $n = 2$ . It has also been recently shown for  $n = 3$  by John Pardon.

We present a new approach to this classic problem using the theory of compactifications. We prove several theorems using this approach and give several examples from infinite-dimensional topology illustrating their application.

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### Bizarre topology is natural in complex dynamical systems

**Judy Kennedy**

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A continuum is a closed bounded connected set. When a dynamical system is complex the presence of complicated continua can be expected. A much-studied example is the Poincaré map on the cylinder determined by the forced damped pendulum for some parameters. In this example there are at least three disjoint basins of attraction with the boundary of the basins being a Lakes of Wada continuum - a thin fractal set that has the property that each point of the continuum is in the boundary of all three basins. We discuss this example and a number of others. Some of the work is joint with Miguel Sanjuan and James Yorke and some is due to other authors.

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### A lower bound for the maximum topological entropy of $4k + 2$ -cycles

**Deborah King**

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**Lluis Alsedà, David Juher**

In recent work, we have started an investigation of the outstanding cases for the problem of classifying cycles with maximum topological entropy. We will present an overview of the problem, discuss the state of known results, and introduce our recent work. For continuous interval maps we formulate a conjecture

on the shape of the cycles of maximum topological entropy of period  $4k + 2$ . We also present numerical support for the conjecture.

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### On almost specification and average shadowing properties

**Dominik Kwietniak**

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**Marcin Kulczycki, Piotr Oprocha**

I am going to present our results describing the properties of systems exhibiting average shadowing and/or almost specification property. We have explored recurrence properties of dynamical systems with the average shadowing property and proved that every dynamical system with the average shadowing property and full invariant measure is topologically weakly mixing. Moreover, I will present examples showing that without the assumption on the invariant measure, there is no recurrence property that is implied by the average shadowing property. Another theorem states that the almost specification implies average shadowing property and that  $f$  has the average shadowing property if  $f$  restricted to the closure of the union of all supports of invariant measures has the average shadowing property.

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### Shadowable chain transitive sets of $C^1$ -vector fields

**Manseob Lee**

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**Keonhee Lee**

Let  $X$  be a  $C^1$ -vector field on a closed smooth manifold  $M$ . We show that for  $C^1$ -generic  $X$ , if a locally maximal chain transitive set is shadowable for  $X$ , then the chain transitive set is hyperbolic.

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### Robust dynamics of $C^1$ -generic diffeomorphisms

**Keonhee Lee**

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**Jumi Oh**

A basic problem in differentiable dynamical system is to understand how a robust dynamical property (meaning a property that holds for a system as well as all  $C^1$  nearby systems) on the underlying manifold would influence the behavior of the tangent map on

the tangent bundle. In this talk we discuss some recent and some ongoing works on the robust dynamics of  $C^1$  generic diffeomorphisms.

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### Central strips of sibling leaves in laminations of the unit disk

**John Mayer**

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**Jeffrey Houghton, Luka Mernik, Joseph Olson**

Quadratic laminations of the unit disk were introduced by W. Thurston as a vehicle for understanding the (connected) Julia sets of quadratic polynomials and the parameter space of quadratic polynomials. The “Central Strip Lemma” plays a key role in Thurston’s classification of gaps in quadratic laminations, and in describing the corresponding parameter space. We generalize the notion of *Central Strip* to laminations of degree  $d \geq 2$  and prove a Central Strip Lemma for degree  $d \geq 2$  that may play a similar role for higher degree laminations. We provide an example of an application to cubic laminations.

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### Tiles in convex dynamics; error diffusion on simplices: invariant regions, tessalations and acuteness

**Tomasz Nowicki**

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**R. Adler, T. Nowicki, G. Świrszcz, C. Tresser, S. Winograd**

The error diffusion algorithm can be represented as a time dependent dynamical system, generated by piecewise isometries, in fact by translations acting on a partition of the phase space. Long term behaviour of the algorithm can be deduced from the asymptotic properties of invariant sets, especially from the absorbing ones. We study a special case of the translations generated by vectors from the vertices of a simplex to its internal point. Each translation acts on an element of partition, which in a special case is a Voronoi partition relative to the vertices of this simplex. Such systems arise from the error diffusion algorithm with constant input.

We study the properties of the minimal absorbing invariant set and prove that typically those sets are bounded fundamental sets for a discrete lattice generated by the simplex and that the partition parts of those sets are fundamental sets for specific derived lattices.

Theorem A [Ergodic Inputs] For acute simplices the minimal absorbing set for the error diffusion with an ergodic constant input is a fundamental set for the

lattice generated by the simplex. Theorem B [Sub-Tiles] If a bounded forward invariant set of a generalized (arbitrary partition) error diffusion on a simplex is fundamental for the simplex lattice, then each part of this invariant set (intersection with the partition) is a fundamental for a derived lattice.

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### Weak product recurrence and related properties

**Piotr Oprocha**

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A point  $x$  is F-product recurrent, if it is recurrent in pair with any point whose recurrent times belong to family F. In this talk we will survey recent advances on classification of product recurrence and remaining open problems.

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### Memory loss for time-dependent dynamical systems

**William Ott**

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**Chinmaya Gupta, Andrew Török**

We discuss recent results on memory loss for time-dependent dynamical systems.

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### Distributional chaos – recent progress and open problems

**Jaroslav Smital**

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In 1994, the notion of distributional chaos was introduced for continuous maps of the interval. This notion, now denoted as DC1, is applicable for continuous maps of a compact metric space. Later two weaker notions, DC2 and DC3, were introduced. Currently, there are many versions of this type of chaos, and hundreds paper has been written on this subject. It is known that positive topological entropy does not imply the strongest version of distributional chaos. The most recent, and very nice result by Tomasz Downarowicz, that for a continuous map of a compact metric space, positive topological entropy implies DC2, has several consequences and yields interesting open problems. We point out some of them.

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### Strange chaotic triangular maps

**Marta Stefankova**

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We show that in the class  $\mathcal{T}$  of the triangular maps  $(x, y) \mapsto (f(x), g_x(y))$  of the square there is a family of maps of type  $2^\infty$  which are Li-Yorke chaotic on a minimal set, but not distributionally chaotic in the weaker sense, *DC2*. This result makes possible to answer an open question concerning classification of maps in  $\mathcal{T}$  with zero topological entropy, and contributes to an old problem formulated by A. N. Sharkovsky.

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**Disjunctive cuts, lattice-free sets and mixed-integer programming****Grzegorz Swirszcz**

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**Sanjeeb Dash, Neil Dobbs, Oktay Gunluk,  
Tomasz Nowicki**

In the talk we describe the topic of mixed-integer programming. The talk is addressed to a wide audience, no specific knowledge of MIP algorithms is assumed. The results are an outcome of combining forces of domain expert computer scientists and mathematicians not working on MIP problems before. We discuss how this subject is related to geometry of convex sets by means of disjunctive cuts introduced by Li and Richard in 2008. We present new results describing the complexity of such algorithms which lead to interesting geometric questions about lattice-free convex bodies. By analyzing  $n$ -dimensional lattice-free sets, we prove that every facet-defining inequality of the convex hull of a mixed-integer polyhedral set with  $n$  integer variables is a  $t$ -branch split cut for some

positive integer  $t$ . Moreover, this number  $t$  does not depend on the data defining the polyhedral set and is bounded by a function of the dimension  $n$  only. We use this result to give a finitely convergent cutting-plane algorithm to solve mixed-integer programs. We also show that the minimum value  $t$ , for which all facets of polyhedral mixed-integer sets with  $n$  integer variables can be expressed as  $t$ -branch split cuts, grows exponentially with  $n$ . In particular, when  $n = 3$ , we observe that not all facet-defining inequalities are 6-branch split cuts. We analyze the cases when  $n = 2$  and  $n = 3$  in detail, and show that an explicit classification of maximal lattice-free sets is not necessary to express facet-defining inequalities as branching disjunctions with a small number of atoms.

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**Nonlinearly perturbed heat equation: a symbolic approach****Sandra Vinagre**

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**M. F. Correia, J. Machado, C. C. Ramos**

We consider the linear heat equation with appropriate boundary conditions in order to model the temperature on a wire with adiabatic endpoints. We assume there is some perturbation which provokes a change in the temperature of the wire. This perturbation, which is modelled by an iterated nonlinear map of the interval  $f$ , occurs periodically and in this case we observe a stabilization on the number of new critical points of heat function. We study the parameters in order to characterize these behaviours and its dependence on the topological invariants of  $f$ .

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## Special Session 24: Geometric Mechanics

**Tom Mestdag**, Ghent University, Belgium  
**Manuel de Leon**, Instituto de Ciencias Matematicas (CSIC-UAM-UC3M-UCM), Spain  
**Frans Cantrijn**, Ghent University, Belgium  
**Aziz Hamdouni**, University of La Rochelle, France  
**Dina Razafindralandy**, LEPTIAB, France

Mathematical modelling of dynamical systems plays an important role in many branches of science. Since the second half of last century, differential geometry has developed into a mathematical discipline with an ever growing impact on the construction of such models. In particular, "geometric mechanics" has become the common name that is given to those research activities that are devoted to the application of differential geometry in various fields of theoretical physics such as classical mechanics (Newtonian, Lagrangian and Hamiltonian mechanics), continuum mechanics, dynamical systems theory, control theory and quantum mechanics.

Using the powerful tools and techniques of Riemann geometry, contact geometry, symplectic and Poisson geometry, and exploiting the properties of Lie groups, fibre bundles, jet bundles, connections, distributions, etc., geometric mechanics has contributed a lot to the description and analysis of the structure and properties of mechanical systems. In addition, many of these geometrical ideas have found an extension to field theories, classical and quantum, such as general relativity, classical and quantum gauge theories.

This session has a double goal: to promote the AIMS Journal of Geometric Mechanics by bringing together excellent researchers in the field and to provide a scientific platform for the international partners within the 'Geometric Mechanics' network (a network within Marie Curie's International Research Staff Exchange Scheme (IRSES) in the 7th European Framework Program).

### Generalized Navier-Stokes flows

**Marc Arnaudon**  
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**Ana Bela Cruzeiro**

We introduce a notion of generalized Navier-Stokes flows on manifolds, that extends to the viscous case the one defined by Brenier. Their kinetic energy extends the kinetic energy for classical Brownian flows, defined as the  $L^2$  norm of their drift. We prove that there exists a generalized flow which realizes the infimum of kinetic energies among all generalized flows with prescribed initial and final configuration. Following a method of Ocone and Pardoux we construct generalized flows from solutions to finite variation transport equations. They are flows with prescribed drift, and their kinetic energy is smaller than the  $L^2$  norm of the solutions to the transport equation.

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### On the geometry of nonholonomic systems

**Paula Balseiro**  
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As it is known, nonholonomic systems are characterized by the failure of the Jacobi identity of the bracket describing the dynamics. In this talk I will present different (geometric) technics to deal with the failure of the Jacobi identity and we will see how twisted Poisson structures might appear once we reduce the system by a group of symmetries.

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### The geometry of integrable and gradient flows and dissipation

**Anthony Bloch**  
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In this talk I will discuss the dynamics and geometry of various integrable systems that exhibit asymptotic stability and dissipative behavior, as well as dissipative perturbations of integrable systems. Examples include the finite Toda lattice, the dispersionless Toda partial differential equation and certain nonholonomic systems I will describe the geometric structures, including metric and complex structures, that give rise to some of these flows and determine their behavior. This includes work with P. Morrison and T. Ratiu.

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### Some applications of some geometric integrators

**Marx Chhay**  
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PDE's geometric structure contains the physical informations traduced by the mechanical model. One can expect from numerical methods to preserve this structure. We present different constructions of some popular geometric methods. Comparisons and performances are performed on illustrative examples.

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### On the geometry of mechanical control systems on Lie groups

**Leonardo Colombo**

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**David Martín de Diego**

In this talk we will describe a geometric setting for higher-order lagrangian problems on Lie groups. Using left-trivialization of the higher-order tangent bundle of a Lie group and an adaptation of the classical Skinner-Rusk formalism, we will deduce an intrinsic framework for this type of dynamical systems. Interesting applications as, for instance, a geometric derivation of the higher-order Euler-Poincaré equations, optimal control of underactuated control systems with symmetries, etc, will be considered.

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### Stochastic Euler-Poincaré reduction on Lie groups

**Ana Bela Cruzeiro**

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**Marc Arnaudon, Xin Chen**

A Euler-Poincaré reduction theorem for stochastic processes taking values in a Lie group is presented, as well as some examples of its application to  $SO(3)$  and to the group of diffeomorphisms.

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### Hamilton-Jacobi theory for classical field theories

**Manuel de Leon**

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We will present a Hamilton-Jacobi theory for classical field theories based on the multisymplectic formalism. In addition, we will develop the corresponding theory on the space of Cauchy data.

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### Variational integrators for hamiltonizable nonholonomic systems

**Oscar Fernandez**

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**Anthony M. Bloch, Peter J. Olver**

I will discuss some new applications of the Poincaré and Sundman time-transformations to the simulation of nonholonomic mechanical systems. We will

see how the application of these transformations permits the usage of variational integrators for these non-variational mechanical systems. Two new geometric integrators for nonholonomic systems known to be Hamiltonizable (briefly, nonholonomic systems whose constrained mechanics are Hamiltonian after a suitable reparameterization of time) will be discussed, along with examples and numerical results comparing the results of the new integrators to those obtained by applying a standard nonholonomic integrator.

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### Invariant higher-order variational problems and computational anatomy

**Francois Gay Balmaz**

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**D. Holm, D. Meier, T. Ratiu, F.-X. Vialard**

Motivated by applications in computational anatomy, we consider a second-order problem in the calculus of variations on object manifolds (or shapes) that are acted upon by Lie groups of smooth invertible transformations. This problem leads to solution curves known as Riemannian cubics on object manifolds that are endowed with normal metrics. The prime examples of such object manifolds are the symmetric spaces. We characterize the class of cubics on object manifolds that can be lifted horizontally to cubics on the group of transformations. Conversely, we show that certain types of non-horizontal geodesics on the group of transformations project to cubics. Finally, we apply second order Lagrange-Poincaré reduction to the problem of Riemannian cubics on the group of transformations. This leads to a reduced form of the equations that reveals the obstruction for the projection of a cubic on a transformation group to again be a cubicon its object manifold.

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### Lagrangian submanifolds and classical field theories of first order on Lie algebroids

**Elisa Lavinia Guzman Alonso**

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**Juan Carlos Marrero, Joris Vankerschaver**

A description of classical field theories in the context of Lie Algebroids in terms of Lagrangian submanifolds of premultisymplectic manifolds is presented. For this purpose, a Tulczyjew's triple associated with a fibration is discussed. The triple is adapted to the extended Hamiltonian formalism. Using this triple, we prove that Euler-Lagrange and Hamilton equations are the local equations defining Lagrangian submanifolds of a premultisymplectic manifold.

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**Geometric control of electromagnetic docking****Marin Kobilarov**California Institute of Technology, USA  
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The talk considers geometric control method for stabilization and trajectory tracking of electromagnetic propulsion systems. The work is motivated by the development of robust rendezvous and docking systems for small satellites. The main difficulty lies in dealing with complex pose-dependent control vector fields arising from the magnetic field interaction. We will address the issue of underactuation and discuss stability results.

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**Schroedinger problem and Ricci curvature of graphs****Christian Leonard**Universite Paris Ouest, France  
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The Schroedinger problem is a statistical physics analogue of the Monge-Kantorovich optimal transport problem. Its solution is a stochastic process with prescribed initial and final marginal probability measures. The time-marginal flow of this optimal process is an entropic interpolation between the endpoint marginals which is a stochastic deformation of McCann's interpolation. It allows both recovering the basic results of the Bakry-Emery theory on a Riemannian manifold and extending it to a graph structure, suggesting a natural definition of Ricci curvature on a graph. We also recover modified logarithmic inequalities and derive transport inequalities under the assumption that the Ricci curvature is bounded below. This approach can be viewed as entering a stochastic deformation of the Lott-Sturm-Villani theory of curvature of metric measure length spaces.

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**Hamilton-Jacobi theory and hamiltonian systems with respect to fiber-wise linear Poisson structures****Juan Carlos Marrero**University of La Laguna, Spain  
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It is well-known that the existence of a fiber-wise linear Poisson structure on a vector bundle is equivalent to the existence of a Lie algebroid structure on the dual bundle. Assume that this Lie algebroid is integrable, that is, it is the Lie algebroid  $AG$  of a Lie groupoid  $G$  and that  $h : A^*G \rightarrow R$  is a hamiltonian function on the dual bundle  $A^*G$ . Then, in

this talk, we will see that it is possible to reconstruct the flow of the corresponding hamiltonian vector field from a complete solution of the time-dependent Hamilton-Jacobi equation associated with a suitable hamiltonian function on the cotangent bundle  $T^*G$  of the Lie groupoid  $G$ .

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**On discrete mechanics for optimal control theory****David Martin de Diego**ICMAT, Spain  
david.martin@icmat.es**F. Jimenez, M. Kobilarov**

During this talk we will discuss numerical methods for optimal control of mechanical systems in the Lagrangian setting. It extends the theory of discrete mechanics to enable the solutions of optimal control problems through the discretization of variational principles. The key point is to solve the optimal control problem as a variational integrator of a specially constructed higher-dimensional system. The developed framework applies to systems on general manifolds, Lie groups, underactuated and nonholonomic systems, and can approximate either smooth or discontinuous control inputs. The resulting methods inherit the preservation properties of variational integrators and result in numerically robust and easily implementable algorithms. The control of an underwater vehicle, will illustrate the application of the proposed approach.

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**Involutive distributions and dynamical systems of second-order type****Tom Mestdag**Ghent University, Belgium  
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We investigate the existence of coordinate transformations which bring a given vector field on a manifold equipped with an involutive distribution into the form of a second-order differential equation field with parameters. We define associated connections and we give a coordinate-independent criterion for determining whether the vector field is of quadratic type. Further, we investigate the underlying global bundle structure of the manifold under consideration, induced by the vector field and the involutive distribution. We illustrate the results in the context of so-called mechanical control systems, and we apply them to Routh reduction of mechanical systems with an Abelian symmetry group.

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### An extension of the Marsden-Weinstein reduction process to the symplectic algebroid setting

**Edith Padron**

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**J.C. Marrero, M. Rodríguez-Olmos**

In this talk we will show a reduction theorem for Lie algebroids with respect to a Lie group action by complete lifts. This result allows to obtain a Lie algebroid version of the classic Marsden-Weinstein reduction theorem for symplectic manifolds. Additionally, we will apply it to the particular case of the canonical cover of a fiberwise Poisson structure.

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### Projective symmetry in Randers spaces

**Mehdi Rafie-Rad**

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A significant extent of researches in the contexts is devoted to study symmetries in Mathematical Physics to be applied in dynamical systems. In each case, characterizing the maximum symmetry is of interests and includes considerations on the dimension of the transformation groups and the Lie algebra of vector fields. A class of important symmetries in Physics are the projective symmetries. On the other hand, the Randers metrics are the most original Finsler metrics in Physics. This work is to study the projective symmetry in Randers spaces. In particular, we prove that the Randers spaces with maximum projective symmetry are locally projectively flat. This extends an analogue result in Riemannian geometry. Some non-Riemannian invariants and specifically important projective symmetry are also studied.

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### Lie group theory in turbulence

**Dina Razafindralandy**

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**Aziz Hamdouni**

Lie group theory is used to analyse turbulent non-isothermal fluid flows. Using the symmetry properties of the correlation equations, new scaling laws for velocity and for temperature are computed. Next, a symmetry-preserving turbulence model for the sub-grid stress tensor and the sub-grid heat flux is developed and numerically tested.

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### Newtonoids vector fields and conservation laws on the Lagrangian k-symplectic formalism

**Modesto Salgado**

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In this talk we discuss symmetries, Newtonoid vector fields, conservation laws, Noether Theorem and its converse, in the framework of the k-symplectic formalism. For the k=1 case it is well known that Cartan symmetries induce and are induced by conservation laws, and these results are known as Noether Theorem and its converse. For  $k \geq 1$ , we provide a new proof that Noether Theorem is true, and hence each Cartan symmetry induces a conservation law. We show that under some assumptions, the converse of Noether Theorem is true and provide examples when this is not. We also study the relation between dynamical symmetries, Newtonoid vector fields, Cartan symmetries and conservation laws, showing when one will imply the others. We use several examples of partial differential equations to illustrate when these concepts are related and when they are not.

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### Invariant metrics on Lie groups

**Gerard Thompson**

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We will investigate integrability properties of left-invariant metrics on low-dimensional Lie groups. We will consider Killing's equations of degree one and higher and also Hamiltonian-Jacobi separable systems.

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### Stochastic methods for Navier-Stokes equations

**Gazanfer Unal**

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Using a stochastic transformation we obtain stochastic Navier-Stokes equations in the sense of Ito-Skorokhod. We seek for exact solutions to the latter by utilizing both anticipative calculus and symmetry methods.

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### The geometry of multi-Dirac structures

**Joris Vankerschaver**

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**Hiroaki Yoshimura, Melvin Leok**

In this talk, I will introduce the concept of a multi-Dirac structure, which is a graded analogue of the concept of a usual Dirac structure. After discussing some aspects of the geometry of multi-Dirac structures, I will point out how these structures can be used for the description of classical field theories, and I will show how a multi-Dirac structure on a jet bundle gives rise to an infinite-dimensional Dirac structure on the space of fields when a 3+1 decomposition has been selected.

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### Hamilton-Jacobi theory for Singular Lagrangians

**Miguel Vaquero**

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**Manuel de León, Juan Carlos Marrero, David Martín de Diego**

In classical mechanics regular lagrangians leads to canonical hamiltonian formalism. For singular lagrangians Dirac developed a theory of constraints that describes the dynamics for such lagrangians. It is well known the important role played by the Hamilton-Jacobi theory in integrating Hamilton's equation when the lagrangian is regular. In this talk, we introduce a Hamilton-Jacobi theory for almost regular lagrangians in the Skinner-Rusk setting. Some examples and future developments will be also discussed.

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### A very general Hamilton-Jacobi theorem

**Luca Vitagliano**

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The classical Hamilton-Jacobi theorem for an Hamiltonian system with Hamiltonian  $H = H(x, p)$  states that  $S = S(x)$  is a solution of the Hamilton-Jacobi equation iff for any solution  $x(t)$  of the ODE  $\frac{dx}{dt} = \frac{\partial H}{\partial p}(x, dS(x))$ ,  $(x(t), dS(x(t)))$  is a solution of the Hamilton equations. Thus it may be understood as a way of finding (some) solutions of a certain ODE

by lifting solutions of a simpler ODE. I will present a wide, field theoretic generalization of the Hamilton-Jacobi theorem within the geometric theory of PDEs on jet spaces.

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### Twisted angles for central configurations formed by two twisted regular polygons

**Xiang Yu**

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**Zhang Shiqing**

In this paper, we study the necessary conditions and sufficient conditions for the twisted angles of the central configurations formed by two twisted regular polygons, in particular, we prove that for the 2N-body problem, the twisted angles must be  $\theta = 0$  or  $\theta = \pi/N$ . And we study also the necessary conditions and sufficient conditions for the existence of the central configurations formed by two twisted regular polygons.

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### Stochastic geometric mechanics

**Jean-Claude Zambrini**

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We shall describe a way to deform stochastically the main tools of Geometric Mechanics. And, in particular, consider various aspects of integrability in this context.

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### Variational principles for Hamel's equations

**Dmitry Zenkov**

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**Kenneth Ball, Anthony Bloch**

Hamel's equations are a generalization of the Euler-Lagrange equations of Lagrangian mechanics obtained by measuring the velocity components relative to a frame that is not associated with system's configuration coordinates. These equations often simplify the representation of system's dynamics. This talk will elucidate the variational nature of Hamel's equations.

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## Special Session 25: Dynamics in Complex Biological Systems

**Bijoy K. Ghosh**, Texas Tech University, USA  
**Akif Ibraguimov**, Texas Tech University, USA  
**Qishao Lu**, Beihang University, China  
**Jianzhong Su**, University of Texas at Arlington, USA

Many biological systems such as neuronal systems, immune systems and gene regulatory systems have highly nonlinear elements and form complex networks that may feature nontrivial coupling (feedback) and architectures. Experimentally, they have been observed for generating complex spatio-temporal activity patterns. Understanding the mechanisms underlying such dynamic patterns, as well as their transient behavior that may emerge in these networks, represents a mathematical challenge of current interest. The aim of this special session is to survey some recent results in which dynamical systems theory or computational and statistical methods have been developed and applied to these dynamical patterns, examples of which include oscillation, bursting and other neuronal activities in complex neuron circuits, dynamical patterns in complex inflammatory systems that are related to wound healing, immune reactions, and cell differentiations in gene regulations. The common themes of these dynamics include many excitatory/inhibitory coupling or in general positive and negative feedbacks, and their effects on the overall dynamics; the influence of stochastic noise may be another significant contributor that are not only as act as perturbations to these dynamics patterns, but also give rise to new dynamics altogether.

This special session offers a forum for new viewpoints and results related to this rapidly developing field. The aim is to foster and encourage communication and interaction between researchers in these interesting directions. The topics include mathematical models and theoretical analysis, computational and statistical methods of dynamical systems and differential equations, as well as applications, in neurodynamics, biological circuitry, biophysics and biochemistry, in several scales.

The topics may include but not restrict to: 1. Neuronal dynamics: Mathematical modeling and dynamical analysis of biological neurons, synapses, neuronal networks and brain, especially with effects of time delay and noise; Nervous signal generation, encoding and transduction; Cognitive information processing, learning and memory functions in brain; Mathematical and computational analyses in artificial intelligence and other biological and medical applications; 2. Immune dynamics: Modeling biomedical processes including tumor growth, cardio-vascular disease, infection, and healing are mediated by immunologic mechanisms; Nonlinear differential equations arises in immunology and medical applications; Analysis of mathematical models for features of solutions such as instabilities, bifurcations, symmetries, and blow-up that provide insight into the nature of the underlying bio-physical mechanisms; 3. Systems Biology: Dynamical system models in Gene regulation, expression, identification and network; Computational evolutionary biology; Algorithms, models, software, and tools in Bioinformatics.

### A mathematical model for the origin of left-handedness

**Daniel Abrams**  
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**Mark J. Panaggio**

An overwhelming majority of humans are right-handed. Numerous explanations for individual handedness have been proposed, but this population-level handedness remains puzzling. I will present a novel mathematical model and use it to test the idea that population-level hand preference represents a balance between selective costs and benefits arising from cooperation and competition in human evolutionary history. I will also present evidence of atypical handedness distributions among elite athletes, and show how our model can quantitatively account for these distributions within and across many professional sports. The model predicts strong lateralization for social species with limited combative interaction, and elucidates the absence of consistent population-level "pawedness" in many animal species.

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### Investigating bacteria-immune dynamics in premature infants

**Julia Arciero**  
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**G. Bard Ermentrout, Richard Siggers, Amin Afrazi, David Hackam, Yoram Vodovotz, Jonathan Rubin**

Necrotizing enterocolitis (NEC) is a severe disease of the gastrointestinal tract in premature infants characterized by an impaired epithelium and exaggerated inflammatory response. Toll-like receptors-4 (TLR4) are located on epithelial cells and function in sensing bacteria and protecting against bacterial translocation. However, TLR4 activation can also lead to an uncontrolled inflammatory response and has been identified as a key contributor to the development of NEC. Toll-like receptors-9 (TLR9) can suppress TLR4 activation and potentially reduce inflammation, but they may also compromise the beneficial antibacterial effects of TLR4. A mathematical model of bacteria-immune interactions within the intestine

is developed here to analyze conditions in which inhibiting TLR4 activation may regulate inflammation and prevent tissue invasion by bacteria. The model offers a means for exploring the competing effects of TLR4 and TLR9 and the influence of therapeutic intervention on system dynamics. In particular, model predictions are used to suggest three interventions for NEC: reducing bacteria in the mucus layer, administering probiotic treatment, and blocking TLR4 activation. Situations in which the proposed treatments would be most helpful or potentially harmful are identified using the model.

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### Determining the conductance for a neuronal cable model defined on a tree graph via a boundary control method

**Jonathan Bell**

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Dendrites of nerve cells can be expected to have non-uniform conductances that one would like to determine. Can this be done by boundary measurements? Motivated by this question, we solve the inverse problem of recovering a single spatially distributed conductance parameter in a cable equation model defined on a finite tree graph that represents a dendritic tree of a neuron. We employ a boundary control method that gives a unique reconstruction and an algorithmic approach.

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### Dynamics of sleep-wake states: a stochastic process, random graph model

**Janet Best**

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We pass our lives alternating behavioral states of sleep and wakefulness. Underlying this alternation are two interacting neuronal networks: wake-active cells and sleep active cells. Moreover, for many mammals the dynamics of the state oscillation are those of a renewal process such that the distribution of wake episodes is power-law distributed while the distribution of durations of sleep episodes is exponentially distributed. In order to study how these dynamics may arise from interacting networks, we consider a directed random graph model consisting of excitatory wake- and sleep-active clusters that compete via inhibitory inter-cluster connections. Each node (neuron) can be in any of three states: basal, excited or inhibited. Activity spreads as a stochastic process on the full network, and the clusters alternate periods of high activity. We apply both standard and novel probabilistic approximations to analyze the resulting dynamics.

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### Complex immune responses: modeling & control

**Judy Day**

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Host-pathogen interactions consist of a complex cascade of events involving a multitude of immune cells and molecules concentrated on the goal of eliminating the offending agent and restoring equilibrium. The immune response cannot always restore homeostasis on its own, and appropriate interventions are needed. However, the administration of therapeutics helping to guide desirable outcomes is not as precise as one might think. Tools such as Model Predictive Control (MPC) have been suggested as a way to systematically determine the correct timing and amount of therapies to achieve a specified health goal. In this talk I will give an overview of modeling immune event cascades to various pathogens and introduce the MPC methodology applied to a four dimensional ordinary differential equations model of systemic inflammation.

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### Bursting oscillation in the pre-Bötzinger complex

**Lixia Duan**

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**Zhaide Hong**

In this talk, we study and classify the bursting in a two-cell network of excitatory neuron within the pre-Bötzinger complex of the mammalian brain stem. We investigated the effects of different parameters on the bursting generation and pattern transitions in the two-cell model network with synaptic coupling by the fast-slow decomposition and bifurcation analysis approach. Comparing the firing patterns of the uncoupled and coupled cells, we found that the bursting patterns are the same both for a single and two-cell model network with the parameter  $g_{Na}$  changed, while they are different with the parameter  $g_K$  changed. Our results show the ions  $Na^+$  and  $K^+$  have different effects on the complex firing activities in the network when the single cells coupled by excitatory synapse.

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### Bifurcations and chaos in a three dimensional discrete time Lotka-Volterra model

**Yinghui Gao**

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**Bing Liu, Wei Feng**

Three dimensional discrete time competitive Lotka-Volterra map was studied by Gardini et al. Bischi et al. extended their results to discrete time prey predator model. Bischi et al. investigated some local and global behaviors of this three dimensional map by numerical explorations. In this paper, this three dimensional map will be further studied from another point of view. Specifically speaking, the conditions of existence for flip bifurcation and Neimark-Sacker bifurcation are derived using the center manifold theorem and the bifurcation theory. In addition, we prove that there exists chaotic behavior in the sense of Marotto's definition of chaos. And numerical simulations show the consistence with the theoretical analysis.

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### Standing and travelling patterns in a neural field model

**Yixin Guo**

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**Dennis Guang Yang**

We study both standing and traveling patterns in a firing rate model with 'Mexican hat' type of synaptic couplings and non-saturated firing rate functions. We present an intrinsic relationship between the underlying integral equation and a system of ODEs. Then we establish the existence of standing and travelling solutions through the system of ODEs. We further analyze the linear stability of the traveling patterns. We also analyze the coexistence and bifurcations of the standing and traveling solutions.

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### Equilibria and linearization of the one-dimensional Forchheimer equation for incompressible two-phase flows

**Thinh Kieu**

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**Luan Hoang, Akif Ibragimov**

We derive the 1-D Forchheimer equation for incompressible two-phase fluid flows in porous media in the presence of capillary forces. This type of equation is used to model dynamics of different biological system. The original problem is reduced to an initial value problem for a non-linear system of

parabolic equations. We find six classes of equilibria and present their qualitative analyses. We show that for appropriate capillary forces and relative permeability these equilibrium states are linearly stable.

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### Neutrophil dynamics in response to chemotherapy and G-CSF

**Jinzhi Lei**

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We have used a mathematical model of the combined dynamics of the hematopoietic stem cells and the differentiated neutrophil progeny to examine the effects of periodic chemotherapy in generating neutropenia, and the corresponding response of this system to granulocyte colony stimulating factor given to counteract the neutropenia. We find that there is a significant period of chemotherapy delivery that induces resonance in the system (at a period twice the average neutrophil lifespan from commitment to death) and a corresponding neutropenia suggesting that myelosuppressive protocols should avoid this period to minimize hematopoietic damage. The response to G-CSF is highly variable.

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### Bursting dynamics of pancreatic beta-cells with electrical and chemical couplings

**Qishao Lu**

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**Pan Meng, Zhuoqin Yang**

Pancreatic beta-cells located in islets of Langerhans in the pancreas are responsible for the synthesis and secretion of insulin in response to a glucose challenge. Bursting electrical activity is also important for pancreatic beta-cells as it leads to oscillations in the intracellular free calcium concentration, which in turn leads to oscillations in insulin secretion. The minimal model for a single pancreatic beta-cell was extended by introducing one additional ionic current was considered. The synchronization behaviors of two identical pancreatic beta-cells connected by electrical (gap-junction) and chemical (synaptic) couplings, respectively, are investigated based on bifurcation analysis by extending the fast-slow dynamics from single cell to coupled cells. Various firing patterns are produced in coupled cells under proper coupling strength when a single cell exhibits tonic spiking or square-wave bursting individually, no matter the cells are connected by electrical or chemical coupling. The above analysis of bursting types and the transition may provide us with better insight into understanding the role of coupling in the dynamic behaviors of pancreatic beta-cells.

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**Foam cell formation in atherogenesis****Lake Ritter**Southern Polytechnic State University, USA  
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The onset of atherosclerosis is characterized by the uptake of lipids by immune cells within the arterial wall. Once lipid laden, these immune cells are dysfunctional and runaway inflammation may occur. We consider a model of the formation of foam cells (lipid laden macrophages) with the view that aggregation corresponds to a mathematical instability. The disease mitigating effects of anti-oxidant and high density lipoproteins are investigated.

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**Using a mathematical model to analyze the treatment of a wound infection with oxygen therapy****Richard Schugart**Western Kentucky University, USA  
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A mathematical model was developed to treat a wound with a bacterial infection using oxygen therapy. The model describes the relationship among neutrophils, bacteria, oxygen, cytokines, and reactive oxygen species. A quasi-steady-state assumption was introduced to reduce the model down systems of two and three equations. A mathematical analysis on the reduced model and simulation results will be presented in this talk.

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**1. Control of attractors in nonlinear dynamical systems using external noise/ 2. Effects of noise on synchronization phenomena****Masatoshi Shiino**Showa Pharmaceutical University, Japan  
masatshiino@yahoo.co.jp**Keiji Okumura**

Synchronization phenomena as a result of the occurrence of cooperative ones are ubiquitous in nonequilibrium physical and biological systems and are of vital importance in information processing in the brain. Those systems are, in general, subjected to various kinds of noise. While in equilibrium thermodynamic systems external Langevin noise is considered to play the role of heat bath in terms of temperature, few systematic study has been conducted to explore effects of noise on large degrees of nonlinear dynamical systems exhibiting limit cycle oscillations and chaotic motions, due to their complexity. Considering nonlinear models as simple as

possible that allow rigorous analyses based on use of nonlinear -Fokker-Planck equations, we have conducted systematic studies to observe effects of noise and changes in types of attractors with changes in several kinds of parameters characterising mean-field coupled oscillator and (or) excitable element ensembles. The first title deals with general aspects shown by our models and the second one specialises understanding of results of noise effects in terms of phase diagrams to show the appearance and disappearance of synchronization of limit cycle oscillations.

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**Complexity in a Leslie-Gower delayed prey-predator model****Anuraj Singh**Graphic Era University, Dehradun, India  
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The complex dynamics is explored in a prey predator system with multiple delays. The predator dynamics is governed by Leslie-Gower scheme. The existence of periodic solutions via Hopf-bifurcation with respect to delay parameters are established. To substantiate analytical findings, numerical simulations are performed. The system shows rich dynamic behavior including chaos and limit cycles.

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**A model for foreign body and its stability analysis****Jianzhong Su**University of Texas at Arlington, USA  
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Implanted medical devices often trigger immunological and inflammatory reactions from surrounding tissues. The foreign body-mediated tissue responses may result in varying degrees of fibrotic tissue formation. There is an intensive research interest in the area of wound healing modeling, and quantitative methods are proposed to systematically study the behavior of this complex system of multiple cells, proteins and enzymes. This paper introduces a kinetics-based model for analyzing reactions of various cells/proteins and biochemical processes as well as their transient behavior during the implant healing in 2 dimensional space. In particular, we provide a detailed modeling study of different roles of Macrophages (MΦ) and their effects on fibrotic reactions. The main mathematical result indicates that the stability of the chronically inflamed state depends primarily on the reaction dynamics of the system, spatial diffusion and chemotaxis can not de-stabilize an equilibrium which is stable in the reaction-only system. However if the equilibrium

is unstable by its reaction-only system, the spatial diffusion and chemotactic effects can help to stabilize when the model is dominated by classical and regulatory macrophages over the inflammatory macrophages. The mathematical proof and counter examples are given for these conclusions.

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### Mathematical modeling of immune response to influenza infection

**David Swigon**

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Influenza is a contagious acute respiratory disease caused by a highly cytopathic virus attacking the epithelium cells of the respiratory tract. Human body responds to the infection by initiating a spectrum of immune responses, ranging from innate to adaptive cellular and antibody, which are regulated by an intricate network of signaling interactions that have not yet been completely characterized. Although in most cases the disease is mild and results in a full recovery, in some cases complications may lead to severe trauma or death. In the talk I will outline a series of simplified models that provide qualitative and quantitative prediction of the time course of the disease, aid in understanding of the mechanisms of the immune response, and have been utilized in the study the effects of an antiviral drug treatment. Our latest effort has been focused on ensemble models that reflects the uncertainty about parameter values, data sparseness, and the likely variation of the disease outcome across a population exposed to IAV. The technique is useful when the model contains many unknown parameters, such as reaction rate constants of biochemical processes, which are poorly constrained and their direct measurement in vivo is not feasible.

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### Eye/head movement dynamics satisfying the Donders' law

**Indika Wijayasinghe**

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**Eugenio Aulisa, Bijoy K. Ghosh, Akif Ibragimov**

The problem of head movement has been studied in conjunction with the eye movement problem for well over hundred years and the names of Helmholtz, Donders and Listing are associated with it. The basic problem is that the eye and the head movement can be looked at as a rotational dynamics on the space  $SO(3)$  with constraints that has to do with the axis of rotation. In general, Donders' Law specifies that the axis of rotation has a small torsional

component that can be expressed as a function of the horizontal and vertical components. In this research, head movement dynamics is constructed by writing down a suitable Riemannian Metric on the constrained space of rotation matrices, together with a suitable form of "coordinate free" potential energy and a damping term.

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### Effect of the channel block on the spatiotemporal dynamics in stochastic Hodgkin-Huxley neuronal networks

**Ying Wu**

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The influence of sodium and/or potassium ion channel blocking on the evolution of spiral wave is investigated in a stochastic two-dimensional network of Hodgkin-Huxley neurons. Some interesting results are approached. (1) With the decreasing of factor for sodium ion channel block, the mean interspike interval of action potential and the wave length increase, and the conductive velocity keeps stable for a larger membrane patch size, however, the evolution trends of spiral wave are almost unchanged, for a smaller membrane patch size. The result confirms that the evolution of the spiral waves is dependent on the sodium ion channel block in the neural network with weak strength noise being considered. As a result, the noise weakens the influence of the sodium ion channel poisoning on the evolution of spatiotemporal pattern in neural network. (2) In the case of potassium ion channel is blocked, no matter how large the patch sizes are, the spiral waves become denser and denser with the factor of poisoning decreasing, and mean interspike interval of action potential of a single neuron decrease. As the factor of poisoning decreases, the trends of the mean interspike interval of action potential, the wave length and the conductive velocity in neural network with a larger membrane patch size are similar with the trends in the system with a smaller membrane patch size. In other words, the influence of potassium ion poisoning on the evolution of spatiotemporal pattern is independent of the strength of noise. Furthermore, the effect of noise on the evolution of spatiotemporal patterns in the neural network is analyzed by calculating the firing possibility and the factor of synchronization.

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### Oscillatory dynamics induced by time delay in an internet congestion control model with a ring topology

**Jian Xu**

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**Shu Zhang, Yu Huang**

Internet is one of the most complex artificial systems and congestion control is among the major tasks of designing this system. In this paper, an n-dimensional congestion control model with ring topology is considered to study the oscillation induced by time delay via Hopf bifurcation. Firstly, time delay is difficult to design or estimate, therefore its influence is of major concern. Secondly, ring topology can be used to model the rapidly developing Ethernet ring as well as some local and campus area network systems, therefore it is worth attention. Then, given the assumption that the variance of hops of all the transmissions is zero, the expression of the possible critical delay for Hopf bifurcation is obtained by analyzing the linear part of the system around the equilibrium and verified by the numerical simulation. Based on that, the method of multiple scales is employed to obtain the quantitative relation between the delay and the oscillation induced by it. The analytical results agree well with those obtained by numerical method. Furthermore, the effects of other parameters are studied either. The discussions reveal that excessive hops may cause the oscillation and increasing the link capacity can control this unexpected dynamics.

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### Multiscale analysis of bacterial chemotaxis

**Chuan Xue**

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Chemotaxis is the active movement of cells or organisms in response to external chemical signals. The chemotactic movement of bacteria such as *E. coli* has been described using both continuum models and cell-based models. In continuum models, the evolution of cell density is described by partial differential equations (PDEs), such as the classical Patlak-Keller-Segel (PKS) chemotaxis equations. However, these models are phenomenological and their relation to mechanistic cellular processes such as signal transduction and movement is not well understood. In contrast, cell-based models can integrate great details of fundamental cellular mechanisms and address population behavior. However, for problems that involve large numbers of cells, they are computationally intensive and time-consuming.

In this talk, I will elucidate the connections between continuum/phenomenological models and cell-based/mechanistic models of bacterial chemotaxis in shallow and steep signal gradient. The derived macroscopic PDEs show good agreement with the stochastic simulations of the cell-based models.

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### Different bursting patterns in two-parameter bifurcation plane of fast subsystem

**Zhuoqin Yang**

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Firing activities during insulin secretion in pancreatic islets are principally characterized by bursts of action potentials. In view of diversities of electrical bursting patterns in pancreatic  $\beta$ -cell, a simple but representative Vries-Sherman model with fast and slow time scales is considered to study different existing regions of different bursting patterns and transition mechanism between them by means of two-parameter bifurcation analysis of fast subsystem with respect to a kinetic parameter and a slow variable in a varying range of the slow variable. Since the number and types of bifurcation curves are different in different regions in two-parameter bifurcation plane, topological types and dynamic behaviors of different bursting patterns in the different regions are revealed by fast/slow analysis with respect to the slow variable. Summarily, codimension-1 and -2 bifurcation analysis is very important to explore dynamic behavior of different electrical bursting and transition mechanism between them, moreover, it can also provide a theoretical basis for physiological experiment design.

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### Robustness of global dynamics for reversible Schnackenberg equations

**Yuncheng You**

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Schnackenberg equations originally introduced as a simplified model for some tri-molecular autocatalytic biochemical reactions, including pattern formations in embryogenesis. This talk will focus on the global dynamics of the reversible Schnackenberg equations on a bounded 3D domain. The robustness of the global attractors for the family of solution semiflows with respect to the reverse reaction rate as it converges to zero will be shown.

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## Special Session 26: Qualitative Aspects of Nonlinear Boundary Value Problems

**Marta Garcia-Huidobro**, Catholic University of Chile, Chile  
**Raul Manasevich**, University of Chile, Chile  
**James Ward**, University of Alabama at Birmingham, U.S.A.

In this session we will deal with qualitative aspects of nonlinear boundary value problems such as existence and multiplicity of solutions, dependence upon data and parameters, bifurcation, and topological properties of solutions and global structures. In the case of the periodic boundary value problem, qualitative aspects include Lyapunov stability or instability of solutions. We will consider scalar as well as systems of boundary value problems

### Variational methods for nonlinear perturbations of the mean curvature operator in Minkowski space

**Cristian Bereanu**

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**P. Jebelean, J. Mawhin**

In this talk we present various existence and multiplicity result for Neumann boundary value problems associated to nonlinear perturbations of the mean curvature operator in Minkowski space. The main tool is Szulkin's critical point theory for smooth perturbations of convex and lower semi-continuous functionals.

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### On bound state solutions with a prescribed number of sign changes

**Carmen Cortazar**

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**Marta Garcia Huidobro, Cecilia Yarur**

We consider radial solutions of the problem

$$\begin{aligned} \Delta u + f(u) &= 0 \\ \lim_{r \rightarrow \infty} u(r) &= 0 \end{aligned}$$

with a prescribed number of sign changes. Conditions on  $f$  are established that guarantee the existence and the uniqueness of such solutions.

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### On positive solutions for a class of Caffarelli-Kohn-Nirenberg type equations

**David Costa**

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**J. Chabrowski**

We investigate the solvability of singular equations of Caffarelli-Kohn-Nirenberg type having a critical-like nonlinearity with a sign-changing weight function.

We examine how the properties of the Nehari manifold and the fibering maps affect the question of existence of positive solutions.

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### Nonlocal maximum principles and applications

**Ricardo Enguiça**

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**Luis Sanchez**

We will make some remarks about the equation

$$-u''(t) - \frac{k}{t}u'(t) + \lambda^2 u(t) = h(t), \quad t \in (0, 1],$$

considering some weighted Sobolev spaces and analyse special features of the associated Green's function. This will be used to derive some nonlocal maximum principles relevant in the search for solutions between lower and upper solutions of the nonlocal boundary value problem

$$\begin{cases} -u''(t) - \frac{n-1}{t}u'(t) = f\left(u(t), \omega_n \int_0^1 s^{n-1}g(u(s)) ds\right) \\ u'(0) = 0 = u(1). \end{cases}$$

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### Boundary blow up of nonnegative solutions of some elliptic systems

**Marta Garcia Huidobro**

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We study the nonnegative solutions of the elliptic system

$$\Delta u = |x|^a v^\delta, \quad \Delta v = |x|^b u^\mu,$$

where  $a, b$  are real numbers such that  $a, b > \max\{-2, -N\}$ , in the superlinear case  $\mu\delta > 1$ , which blow up near the boundary of adomain of  $\mathbb{R}^N$ , or at one isolated point. In the radial case we give the precise behavior of the large solutions near the boundary in anydimension  $N$ . We also show the existence of infinitely many solutions blowingup at 0. Furthermore,

we show that there exists a global positive solution in  $\mathbb{R}^N \setminus \{0\}$ , large at 0, and we describe its behavior. We apply the results to the sign changing solutions of the biharmonic equation

$$\Delta^2 u = |x|^b |u|^\mu.$$

Our results are based on a new dynamical approach of the radial system by means of a quadratic system of order 4. This talk is part of a joint work with Prof. M. Véron and Prof. C. Yarur.

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### Solutions for a semilinear elliptic equation in dimension two with supercritical growth.

**Ignacio Guerra**

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**Manuel del Pino, Monica Musso**

We consider the problem

$$\begin{aligned} -\Delta u &= \lambda u e^{u^p}, \quad u > 0, \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega, \end{aligned}$$

where  $\Omega \subset \mathbb{R}^2$  and  $p > 2$ . Let  $\lambda_1$  be the first eigenvalue of the Laplacian. For each  $\lambda \in (0, \lambda_1)$ , we prove the existence of solutions for  $p$  sufficiently close to 2. In the case of  $\Omega$  a ball, we also describe numerically the bifurcation diagram  $(\lambda, u)$  for  $p > 2$ .

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### Convergence versus periodicity in a dynamical system arising in the study of a higher-order elliptic PDE

**Monica Lazzo**

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**Paul G. Schmidt**

Trying to understand the blow-up behavior of large radial solutions of a polyharmonic PDE with power nonlinearity, we are led to analyze the dynamics of a parameter-dependent single-loop positive-feedback system; the dimension of the system corresponds to the order of the PDE. In low dimensions, we observe convergence to equilibrium; in high dimensions, multiple periodic orbits arise via successive Hopf bifurcations. We discuss the dynamics of the system and consequences for the underlying PDE.

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### A mean curvature type of geometric parabolic equation

**Junfang Li**

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**Pengfei Guan**

In this talk, we will present a parabolic PDE defined on hypersurfaces and its fully non-linear analogue. For any closed star-shaped smooth hypersurface, this flow exists for all time  $t \geq 0$  and exponentially converges to a round sphere. Moreover, we will show that all the quermass integrals evolve monotonically along this flow. Consequently, we prove a class of isoperimetric type of inequalities including the classical isoperimetric inequality on star-shaped domains. We will also present a fully non-linear analogue of this flow. More specifically, we study a fully non-linear parabolic equation of a function on the standard sphere and discuss its long-time existence and exponential convergence. As applications, we recover the well-known Alexandrov-Fenchel inequalities on bounded convex domains in Euclidean space.

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### Sign changing solutions with compact support for a nonlinear equation with a $p$ -Laplace operator

**Raul Manasevich**

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**Jean Dolbeault**

In this talk we will consider radial solutions for a nonlinear equation with a  $p$ -Laplace operator. By a shooting method we prove the existence of solutions with compact support that have any prescribed number of zeros.

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### On the energy of the current vector of a complex valued function in $\mathbb{R}^3$

**Alberto Montero**

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For a complex valued function  $u : \Omega \subset \mathbb{R}^3 \rightarrow \mathbb{C}$ , the current vector is usually defined as  $j(u) = \text{Im}(\bar{u} \nabla u)$ . In this talk I will present an estimate for  $\int_{\Omega} |j(u)|^2$  in terms of quantities determined by a given curve  $\Gamma$  in  $\Omega$ . These estimates are valid when  $\nabla \times j(u)$  and  $\Gamma$  are sufficiently close as functionals on vector fields.

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### Eigenvalue-curves and nonlinear second order elliptic equations with nonlinear boundary conditions

**M. Nkashama**

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**N. Mavinga**

We are concerned with the existence of eigenvalue-curves connecting the Steklov and Neumann-Robin spectra of linear second order elliptic equations. We then discuss nonlinear elliptic problems with nonlinear boundary conditions when the reaction and boundary nonlinearities stay in some sense between two consecutive eigenvalue-curves.

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### Nonlinear periodic boundary value problems via initial value problems: generalized quasilinear techniques

**Sudhakar Pandit**

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**J. O. Adeyeye, D. H. Dezern**

It is well known that the Monotone Iterative (MIT) and the Generalized Quasilinear Technique (GQT) are two important techniques for obtaining solutions, in a constructive way, of a variety of nonlinear Initial value Problems (IVPs) and Periodic Boundary Value Problems (PBVPs). Whereas the rate of convergence of the iterates in the former technique is linear, that in the latter is quadratic, and hence more rapid. In this talk, we present a new approach to solve nonlinear PBVPs by employing the GQT. The fact that the iterates in our approach are solutions of (linear) IVPs (as opposed to PBVPs, in the conventional approach), and converge to the multiple solutions (or the unique solution, under additional conditions) of the given nonlinear PBVP, allows us to dispense with many a hypothesis required in the conventional approach. We provide graphical and numerical examples to support the results in our new approach.

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### Hyperbolic fractional Laplacian

**Mariel Saez**

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**V. Banica, M.d.M. González, M. Sáez**

In this talk I will discuss an appropriate definition for the fractional Laplacian on Hyperbolic Space. In the spirit of the work of L. Caffarelli and L. Silvestre, we show that we can associate an extension problem to our definition.

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### Oscillatory entire solutions of polyharmonic equations with subcritical growth

**Paul Schmidt**

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**Monica Lazzo**

Much of the literature on entire solutions for semilinear elliptic equations with power nonlinearities focuses on positive solutions, whose existence requires critical or supercritical growth. Even assuming radial symmetry, the subcritical case is completely understood only for second-order problems. We recently established the existence and uniqueness (up to scaling) of oscillatory entire radial solutions for subcritical fourth-order problems. We discuss the asymptotic behavior of these solutions and possible generalizations for higher-order problems.

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### On the nonuniqueness of positive solutions for a class of superlinear problems

**Satoshi Tanaka**

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The following two-point boundary value problem is considered:  $u'' + h(x)f(u) = 0$  on  $(-1, 1)$ ;  $u(-1) = u(1) = 0$ . Here,  $h \in C[-1, 1] \cup C^1([-1, 1] \setminus \{0\})$ ,  $h(x) > 0$  on  $[-1, 1] \setminus \{0\}$ ,  $f \in C^1[0, \infty)$ ,  $f(s) > 0$  for  $s > 0$  and  $f(0) = 0$ . In addition, the superlinear condition  $sf'(s) > f(s)$  on  $(0, \infty)$  is satisfied. In many cases, the positive solution of this problem is unique. Indeed, if  $h(x) = 1$ , then the problem has at most one positive solution. However, for example, if  $f(s) = s^p$ ,  $p > 1$ ,  $h(x) = |x|^l$  and  $l > 0$  is sufficiently large, then there exist three positive solutions. In this talk, a sufficient condition is derived for the existence of three positive solutions.

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### On some connecting orbits for a class of singular second order Hamiltonian systems

**Hossein Tehrani**

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We present some existence results for connecting orbits emanating from 0 of a class of singular second order Hamiltonian equations of the form

$$u'' + V_u(t, u) = 0, \quad t \in \mathbb{R}$$

where the singular potential  $V(t, u)$  is  $T$  periodic and satisfies Strong-Force condition at the singularity.

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## Special Session 27: Transport Barriers in Dynamical Systems

**George Haller**, McGill University, Canada  
**Wenbo Tang**, Arizona State University, USA

Transport barriers are ubiquitous generators of coherent patterns in complex dynamical systems. In recent years, significant advances have been made on the mathematics and computations of such barriers in temporally aperiodic flows. This special session surveys recent analytic and computational techniques for the identification of generalized transport barriers in dynamical systems, and reviews applications to geophysical and engineering flows.

### Transport barrier detection via braid theory

**Michael Allshouse**

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**Jean-Luc Thiffeault and Thomas Peacock**

The definition of transport barriers as minimally stretching material lines has recently been established. A detection method was developed for regions where the full velocity field is known; however this is not always the case. Our work attempts to detect similar boundaries in situation where only sparse trajectory information is known. We do this through the use of tools from topology, in particular braid theory. We will briefly review our detection method and a proof of concept in a mixing example. Additionally, we will analyze trajectories in the double-gyre system to try to establish the relationship between transport barriers detected using the geodesic method and our braid based method.

### Explicit stable and unstable manifolds in a class of unsteady 2D and 3D flows

**Sanjeeva Balasuriya**

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Stable and unstable manifolds strongly govern fluid transport in unsteady flows, yet are often difficult to determine even numerically. This talk introduces a class of unsteady Eulerian velocity fields in both two- and three-dimensions, in which the time-varying invariant manifolds can be expressed explicitly. It is shown that exponential dichotomy conditions – which ensure that fluid particles on these manifolds decay exponentially to hyperbolic trajectories – are satisfied for these flows. Methods for generating finite-time analogues of these manifolds are also discussed; in particular, it is shown how the violation of “infinite-timeness” can be quantified and tuned using a parameter. While there is in reality no chaotic motion associated with this class of examples, Lagrangian trajectories which are seemingly arbitrarily complicated can be generated. As such, these will serve as useful testbeds for researchers developing and improving diagnostic methods for extracting flow barriers in genuinely time-dependent two- and three-dimensional flows.

### Uncovering the Lagrangian skeletons of oceanic and atmospheric flows

**Francisco Beron-Vera**

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**George Haller**

Haller and Beron-Vera (2012) have recently developed a geodesic theory for the objective (i.e., frame-independent) identification of key material curves (transport barriers) which shape global transport patterns in unsteady 2-d flows defined over a finite-time interval. In the geodesic theory transport barriers emerge as material curves closely shadowed by least-stretching geodesics for the Cauchy–Green tensor. In the incompressible case, the geodesic theory identifies hyperbolic (generalized stable and unstable manifolds), elliptic (generalized eddy boundaries), and parabolic (generalized shear jets) transport barriers. Such transport barriers, which can be regarded as generalized Lagrangian Coherent Structures or LCSs, compose the Lagrangian skeleton of an incompressible 2-d flow. Here we apply the geodesic theory to oceanic and atmospheric velocity fields in an attempt to uncover their Lagrangian skeletons.

### Lagrangian coherent structures, biological invasions, and limits of forecasting

**Amir Bozorgmagham**

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**S. Ross, D. Schmale, P. Tallapragada**

The language of Lagrangian coherent structures (LCSs) provides a new means for discussion of transport and mixing of pathogens that are transported through the atmosphere, paving the way for new modeling and management strategies for the spread of infectious diseases affecting plants, domestic animals, and humans. Our research group is investigating the influence of atmospheric dynamical structures on the aerocology of pathogen populations collected with autonomous unmanned aerial vehicles (UAVs). Recent work analyzing collections of pathogens across multiple UAV sampling missions has demonstrated a statistical correlation between punctuated changes in the population structure and

passages of LCSs over the sampling location, based on data from archived meteorological models which constitute our best estimate of the atmospheric flow. If LCSs prove to be important for understanding the atmospheric transport of invasive microbial species into previously unexposed regions, accurate forecasting of LCSs could be incorporated into early warning systems for plant pathogens. We will consider the consequences of wind forecast errors on the finite-time Lyapunov exponent (FTLE) field and the results from proper orthogonal decomposition for comparing forecast and pastcast LCS features. Finally, we describe stochastic FTLE fields which take into account the uncertainty of forecasts.

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### KAM-like Lagrangian coherent structures in geophysical flows

**Michael Brown**

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**H. Kocak, F. J. Beron-Vera, M. J. Olascoaga**

Transport barriers in the vicinity of zonal (east-west) jets in planetary atmospheres are investigated. Recent results relating to KAM (Kolmogorov-Arnold-Moser) theory predict that when such a flow is perturbed, invariant tori in the vicinity of the jet core should be present and will serve as impenetrable barriers to meridional (north-south) transport. Numerical estimates of invariant tori of this type are referred to as KAM-like Lagrangian Coherent Structures (LCSs). Relevant theory and numerical methods will be reviewed, and applications, including the annually recurring Antarctic ozone hole and Jupiter's belts and zones, will be discussed.

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### Escape and diffusion through small holes

**Carl Dettmann**

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**O. Georgiou, R. Klages, G. Knight**

A dynamical system may be "opened" by allowing trajectories to leak out through one or more holes, subsets of phase space. Given a distribution of initial conditions, we can then pose questions about the probability of surviving within the system, as a function of time, the size and position of the hole(s). Open billiard dynamics can be related to a number of physical experiments and applications involving escape of particles from a cavity. In several geometries the leading coefficient of the survival probability can be determined, including connections with the Riemann Hypothesis and phenomena such as asymmetric transport. A chain of systems linked

by their holes can also model deterministic diffusion. Very recent results for escape and diffusion in one-dimensional maps will be discussed, including an expansion for the escape rate beyond linear order in hole size, and an exact additivity formula for diffusion coefficients.

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### Transport in time-dependent dynamical systems: finite-time coherent sets

**Gary Froyland**

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**Naratip Santitissadeekorn, Adam Monahan**

We study the transport properties of nonautonomous chaotic dynamical systems over a finite time interval. We are particularly interested in those regions that remain coherent and relatively nondispersive, despite the chaotic nature of the system. We detect maximally coherent transport pathways using singular vectors of a matrix of transitions induced by the dynamics. The methodology is illustrated on an idealized stratospheric flow and two and three-dimensional analyses of European Centre for Medium Range Weather Forecasting reanalysis data.

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### Using LCS to study the transition vortex shedding on a cylinder in cross-flow

**Melissa Green**

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The transition from steady separation to unsteady vortex shedding downstream of a circular cylinder in cross-flow is examined using a Lagrangian coherent structure analysis. Velocity data is gathered from both 2D and 3D simulations at Reynolds numbers close to transition ( $Re = 47$ ). The LCS is compared against common Eulerian criteria, such as vorticity, the  $Q$  criterion, and the acceleration minima. At transition, when flow begins to entrain into and de-train from the cylinder wake, the wake as described using LCS undergoes a distinct qualitative change. This event in the evolution of the LCS will offer new information about possible timing and location at which to implement effective flow control to mitigate the shedding and unsteady forces on the cylinder body.

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### Geodesic theory of transport barriers

**George Haller**

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**Francisco Beron-Vera**

I describe a unified approach to locating key material transport barriers in unsteady flows induced by two-dimensional, non-autonomous dynamical systems. Seeking transport barriers as minimally stretching material lines, one obtains that such barriers must be shadowed by minimal geodesics under the metric induced by the Cauchy-Green strain tensor field associated with the flow map. As a result, snapshots of transport barriers can be explicitly computed as trajectories of ordinary differential equations. Using this approach, hyperbolic barriers (generalized stable and unstable manifolds), elliptic barriers (generalized KAM curves) and parabolic barriers (generalized shear jets) can be found with high precision in temporally aperiodic flows defined over a finite time interval. I illustrate these results on unsteady flows arising in mechanics and fluid dynamics. Further applications to geophysical flows will be discussed by Francisco Beron-Vera in another talk within this special session.

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### Out of flatland: 3D aspects of Lagrangian coherent structures in oceanography

**A. Kirwan**

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**Mohamed H. Mahmoud Sulman, Helga S. Huntley, B. L. Lipphardt, Jr.**

Since the seminal work of Haller (Chaos, 99-108, 2000) and Haller and Yuan (Physica D, 352-370, 2000) finite time Lyapunov exponents (FTLE) have been used to identify Lagrangian coherent structures (LCS). Until very recently applications of this methodology in oceanography have been restricted to analyses along a few selected surfaces even though the theory is readily extended to fully three-dimensional flows. In rotating stratified geophysical scale fluids LCS are 2D surfaces embedded within a finite fluid volume so the intersections of these surfaces with a few level surfaces clearly are inadequate descriptors of true LCS. The only study that reports on the 2D structure of LCS in oceanography that we are aware of is Branicki and Kirwan (Int. J. Engr. Sci., 1027-1042, 2010). In their investigation of a large anticyclonic ring in the Gulf of Mexico they reported that the 2D LCS were nearly vertical and that there was entrainment into the ring near its base and detrainment near the surface. However, for technical reasons peculiar to most data assimilating ocean and atmospheric general circulation models they elected not to use the vertical velocity

in their analysis. Thus the question addressed in this talk: how representative of true 2D LCS are reduced FTLE representations? Specifically we discuss two strategies for computing 2D LCS from reduced representations of the Cauchy Green tensors. Our analysis indicates that under typical oceanographic conditions it is important to account for the vertical shear of the horizontal currents in constructing a reduced representation of this tensor. We apply this approach to a high-resolution data assimilating general circulation model to report on the vertical structure of 2D LCS in the Gulf of Mexico.

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### Transport in transitory dynamical systems

**James Meiss**

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**B.A. Mosovsky**

A transitory dynamical system is nonautonomous only on a compact interval of time. With this particularly simple form, coherent structures can be identified with those of the “past” or “future” autonomous vector fields, and transport is quantified by a transition map. We study Liouville dynamics in which the incompressible vector field has a Lagrangian generating form. In this case the volume of regions bounded by past-unstable and future-stable surfaces can be efficiently computed using an action-like formula on heteroclinic orbits. Three-dimensional examples using the ABC flow and a model of droplet flow in a twisted pipe will be presented.

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### Predicting instabilities in environmental pollution patterns using LCS-core analysis

**Maria Olascoaga**

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**G. Haller**

I present a methodology to predict major short-term changes in environmental contamination patterns, such as oil spills in the ocean and ash clouds in the atmosphere. The approach is based on new mathematical results on the objective (frame-independent) identification of key material surfaces that drive tracer mixing in unsteady, finite-time flow data. Some of these material surfaces, broadly known as Lagrangian Coherent Structures (LCSs), turn out to admit highly attracting cores that lead to inevitable material instabilities even under future uncertainties or unexpected perturbations to the observed flow. These LCS cores have the potential to forecast imminent shape changes in the contamination pattern, even before the instability builds up and brings large

masses of water or air into motion. Exploiting this potential, the resulting LCS-core analysis provides a model-independent forecasting scheme that relies only on already observed or validated flow velocities at the time the prediction is made. The methodology is used to obtain high-precision forecasts of two major instabilities that occurred in the shape of the Deepwater Horizon oil spill in the Gulf of Mexico.

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### Finite-time entropy: a probabilistic approach for measuring nonlinear stretching in dynamical systems

**Kathrin Padberg-Gehle**

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**Gary Froyland**

We propose an entropy-based methodology for estimating nonlinear stretching in dynamical systems based on the evolution of probability densities. We present a very efficient computational approach that makes direct use of a discretized transfer operator. The novel methodology is illustrated by several example systems, highlighting the similarity to the frequently used concept of finite-time Lyapunov exponents. Thus the finite-time entropy approach is a first step towards formally linking geometric and probabilistic methods for the numerical analysis of transport.

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### Inertial manifold dimensionality and finite-time instabilities in transient turbulent flows

**Themistoklis Sapsis**

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We examine the geometry of the inertial manifold associated with fluid flows described by Navier-Stokes equations and we relate its nonlinear dimensionality to energy exchanges between the mean flow and stochastic modes of the flow. Specifically, we employ a stochastic framework based on the dynamically orthogonal field equations to perform efficient order-reduction in terms of time-dependent modes and describe the inertial manifold in the reduced-order phase space in terms of the associated probability measure. We introduce the notion of local fractal dimensionality and we establish a connection with the finite-time Lyapunov exponents of the reduced-order dynamics. Based on this tool we illustrate in 2D Navier-Stokes equations that the underlying mechanism responsible for the finite dimensionality of the inertial manifold is, apart from the viscous dissipation, the reverse flow of energy from the stochastic fluctuations (containing in general smaller length-scales) back to the mean flow (which is characterized by larger spatial scales).

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### Investigating fluid flows via individual trajectory complexity methods

**Sherry Scott**

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**I.I. Rypina, L.J. Pratt, M.G. Brown**

We consider the analysis of flows in terms of the complexity of the fluid particle trajectories. Two complexity measures - the correlation dimension and the ergodicity defect - are explored in the context of several examples. Both measures yield structures resembling Lagrangian coherent structures in the examples. The possible advantage of these individual trajectory complexity methods over more traditional approaches in the analysis of typical ocean float and drifter data sets is discussed.

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### Maximal stretching surfaces as potential platelet activation pathways

**Shawn Shadden**

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As platelets are transported they are continuously stretched, compressed and sheared by local gradients in the flow. Exposure to elevated gradients can cause platelets to actively react with conformational, chemical and enzymatic responses, i.e. becoming activated. Once switched to the activated state, platelets perform multifaceted roles to orchestrate clotting. When one measures the activation potential for platelets in blood flow, the measure tends to be maximized along distinguished material surfaces that are otherwise referred to as "LCS." Furthermore, comparison of these structures with data on platelet deposition near arterial stenoses suggest that in fact, clot formation may be strongly influenced by these structures. Moreover, these results shed insight into the possible synergies between force-mediated and transport-mediated biomechanical processes, which have previously been considered independently.

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### Mapping unstable manifolds using floats in a Southern Ocean field campaign

**Emily Shuckburgh**

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**J. B. Sallee, F. d'Ovidio**

Ideas from dynamical systems theory have been used in an observational field campaign in the Southern Ocean to provide information on the transport and mixing structure of the flow. Satellites can provide



information concerning surface currents at a scale of 10 km or so. This information was used in near-real time to provide an estimate of the location of stable and unstable manifolds in the vicinity of the Antarctic Circumpolar Current. As part of a large US/UK observational field campaign (DIMES: Diapycnal and Isopycnal Mixing Experiment in the Southern Ocean) a number of floats were then released (at the surface and at a depth of approximately 1km) close to the estimated intersection of the two manifolds in several locations with apparently different dynamical characteristics. The subsequent trajectories of the floats has allowed the unstable manifolds to be tracked, and the relative separation of pairs of floats has allowed an estimation of Lyapunov exponents. The results of these deployments have given insight into the strengths and limitations of the satellite data which does not resolve small scales in the velocity field, and have elucidated the transport and mixing structure of the Southern Ocean at the surface and at depth.

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### Topological chaos in systems ‘stirred’ by almost-cyclic sets

**Mark Stremler**

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**Mohsen Gheisarieha, Piyush Grover, Pankaj Kumar, Shane D. Ross**

In many dynamical systems, there are regions of phase space that remain coherent for an extended period of time. Identification of these coherent regions can be accomplished by, for example, computing the eigenvectors of the discretized Perron-Frobenius transfer operator via a set-oriented approach. The eigenvector(s) give the locations(s) of these regions, known as Almost-Invariant Sets (AIS), and the corresponding eigenvalue(s) quantify their ‘leakiness’. In some cases, there exist disconnected components of a single AIS that form Almost-Cyclic Sets (ACS). We extend this set-oriented approach by considering the relative interaction of trajectories from within the ACS. In certain (2+1)-dimensional systems, the braiding of these trajectories provides a framework for analyzing chaos in the system through application of the Thurston-Nielsen classification theorem. Sufficiently complex braiding patterns lead to what is known as ‘topological chaos’, enabling predictions of system behavior based solely on the spatio-temporal ACS structure. This approach makes it possible to investigate and quantify chaos in complex dynamical using limited information about the relative motion of a few coherent regions. We demonstrate this approach by considering stirring and mixing in a two-dimensional, time-dependent fluid system.

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### Lagrangian coherent structures and eddy diffusion

**Wenbo Tang**

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Lagrangian coherent structures (LCS) are known as the templates for chaotic mixing in nonlinear aperiodic dynamical systems, such as geophysical flows. In recent years, theoretical developments on the deterministic LCS have allowed the objective identification of mixing barriers and enhancers in geophysical flows. Stochastic transport associated with LCS, on the other hand, is less studied, partly due to the inherent scale separation between coherent structures and molecular diffusion. However, sub-grid scale uncertainty of geophysical flows cannot be neglected when one tries to quantify diffusive transport of substances. In this talk we will discuss some recent efforts on quantifying diffusive mixing associated with the LCS. In particular, eddy diffusivity tensors associated with advection-diffusion are constructed based on Lagrangian measures from LCS. Some archetypal examples will be discussed.

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### Moving walls accelerate mixing

**Jean-Luc Thiffeault**

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**Emmanuelle Guillard, Olivier Dauchot**

Mixing in viscous fluids is challenging, but chaotic advection in principle allows efficient mixing. In the best possible scenario, the decay rate of the concentration profile of a passive scalar should be exponential in time. In practice, several authors have found that the no-slip boundary condition at the walls of a vessel can slow down mixing considerably, turning an exponential decay into a power law. This slow-down affects the whole mixing region, and not just the vicinity of the wall. The reason is that when the chaotic mixing region extends to the wall, a separatrix connects to it. The approach to the wall along that separatrix is polynomial in time and dominates the long-time decay. However, if the walls are moved or rotated, closed orbits appear, separated from the central mixing region by a hyperbolic fixed point with a homoclinic orbit. The long-time approach to the fixed point is exponential, so an overall exponential decay is recovered, albeit with a thin unmixed region near the wall.

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## Special Session 28: Analysis and Numerics of Differential Equations and Dynamical Systems in Mathematical Fluid Mechanics

Changbing Hu, University of Louisville, USA

Ning Ju, Oklahoma State University, USA

Theodore Tachim-Medj, Florida International University, USA

In the last decades great progresses have been made in the field of mathematical fluid mechanics both theoretically and numerically by using theory of differential equations and dynamical systems. The aim of this special session is to bring together researchers in this area to present most recent results, ideas and discuss on future research directions. The topics in this special session will range from mathematical modeling, theoretical analysis including well-posedness and asymptotic behavior with respect to small parameters and large time, and computational methods.

### Dynamics of particle settling and resuspension in viscous liquids

**Andrea Bertozzi**

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We derive and study a dynamic model for suspensions of negatively buoyant particles on an incline. Our theoretical model includes the settling/sedimentation due to gravity as well as the resuspension of particles induced by shear-induced migration, leading to different regimes observed in the experiments. Using an approach relying on asymptotics, we systematically connect our dynamic model with the previously developed equilibrium theory for particle-laden flows. We show that the resulting transport equations for the liquid and the particles are of hyperbolic type, and study the dilute limit, for which we compute exact solutions. We also carry out systematic experimental study of the settled regime, focusing on the motion of the liquid and the particle fronts. Finally, we carry out numerical simulations of our transport equations. We show that the model predictions agree well with the experimental data.

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### Gevrey regularity for dissipative equations with applications to decay.

**Animikh Biswas**

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**Hantaek Bae**

The regular solutions of a large class of dissipative equations are well-known to be analytic in both space and time variables. Moreover, the space analyticity radius has an important physical interpretation. It demarcates the length scale below which the viscous effect dominates the (nonlinear) inertial effect. Foias and Temam introduced an effective approach to estimate space analyticity radius via the use of Gevrey norms which, since then, has become a standard tool for studying analyticity properties for dissipative equations. We extend this approach to a class

of (possibly nonlocal) equations with analytic nonlinearity in critical invariant spaces. As application, we obtain large time decay of higher derivatives. Applications include the Navier-Stokes, surface quasi-geostrophic, Burger's and Cahn-Hilliard equations among others. This is a joint work with Hantaek Bae.

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### On a 1D alpha-patch model

**Hongjie Dong**

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**Dong Li**

We consider a one-dimensional  $\alpha$ -patch model. A dichotomy result between the finite time blowup and the global in time regularity is obtained. The result is sharp in terms of the range of  $\alpha$ .

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### Parameter estimation for nonlinear stochastic partial differential equations

**Nathan Glatt-Holtz**

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**Igor Cialenco**

While the general form of a model is commonly derived from the fundamental properties of a physical process under study, frequently parameters arise in the formulation which need to be specified or determined on the basis of empirical observation. Given in particular the growing significance of nonlinear stochastic partial differential equations (SPDE) in applications there is a clear need to develop the theory of parameter estimation for such systems. Under the assumption that a phenomenon of interest follows the dynamics of such an SPDE, and given that some realizations of this process are measured, we wish to find these unknown parameters appearing in the model, such that the equations fit or predict as much as possible this observed data. In this work we discuss some recent results concerning the estimation of

the “drift” parameter for a general class of nonlinear SPDE, based on the first  $N$  Fourier modes of a single sample path observed on a finite time interval. In particular, we exhibit specific estimators for the viscosity coefficient for the 2D stochastic Navier-Stokes equations, and study asymptotic properties of these estimators.

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### Alternating direction second order method for the Navier-Stokes equations

**Daniel Guo**

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A fully discretized projection method is studied. It contains a parameter operator. Depending on this operator, we can obtain a first-order scheme, which is appropriate for theoretical analysis, and a second-order scheme, which is more suitable for actual computations. In this method, the boundary conditions of the intermediate velocity field and pressure are not needed. For the comparison, we apply the alternating direction method to the discretized projection method for the Navier-Stokes equations.

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### Some mathematical theory of viscous Camassa-Holm equations

**Changbing Hu**

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Viscous Camassa-Holm equations, or Navier-Stokes-alpha (NS- $\alpha$ ) equations were introduced as a natural mathematical generalization of the integrable inviscid 1D Camassa-Holm equation through a variation formulation. Mathematical theory, including well-posedness, and the existence of global attractors and their Hausdorff and fractal dimension, of viscous Camassa-Holm equations has been established by Foias, Holm and Titi. In this talk we address some further topics regarding the viscous Camassa-Holm equations, including the Gevrey regularity of the solutions, some bifurcation results and lower bounds of the global attractor.

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### Bounded vorticity, bounded velocity (Serfati) solutions to 2D Euler equations

**James Kelliher**

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**David Ambrose, Helena Nussenzveig Lopes, Milton Lopes Filho**

In a short note in 1995, Philippe Serfati established the existence and uniqueness of solutions to the 2D Euler equations in the whole plane when the initial vorticity and initial velocity are bounded. We describe an extension of this result to an external domain in the plane, and discuss related issues of stability.

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### On the arrow of time

**Y. Charles Li**

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**Hong Yang**

We propose a Theory to resolve the problem of the Arrow of Time. The Theory is composed of three ingredients: 1. The equations of the dynamics of gas molecules, 2. Chaotic instability of the equations of the dynamics, 3. unavoidable perturbations to the gas molecules. Numerical simulations on the Theory are conducted.

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### Determination of viscosity in an incompressible fluid

**Xiaosheng Li**

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In this talk we study the inverse boundary value problems in fluid mechanics. We present the unique determination of the viscosity function in an incompressible fluid by boundary measurements of the velocity and the force.

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### Invariant manifolds of Euler equations

**Zhiwu Lin**

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We consider a linearly unstable steady flow  $v_0$  of the Euler equation in a fixed bounded domain in  $\mathbf{R}^n$ . By rewriting the Euler equation as an ODE on an infinite dimensional manifold of volume preserving mapping in  $H^k$  ( $k > 1 + \frac{n}{2}$ ), the unstable and stable

manifolds of  $v_0$  is constructed under certain spectral gap condition which is verified for both 2D and 3D examples. This implies the nonlinear instability of  $v_0$  in the sense that arbitrarily small  $H^k$  perturbations can lead to  $L^2$  derivation of the solutions.

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### Effective viscosity in dilute suspensions

**Anna Mazzucato**

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**Brian Haines**

We give a rigorous derivation of Einstein's formula for the effective viscosity of a dilute suspension of spheres in a finite-size container. The spheres are arranged in a cubic lattice. We employ boundary integral equations for the Stokes operator and include boundary effects. Our proof admits a generalization to other particle shapes and the inclusion of point forces to model self-propelled particles.

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### On the exponential decay of the power spectrum and the finite dimensionality for the solutions of the three dimensional primitive equations

**Madalina Petcu**

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In this talk we study the three dimensional primitive equations, describing the motion of the oceans and of the atmosphere. More exactly, we prove the exponential decay of the spatial power spectrum for the solutions, as well as the finite dimensionality of the flow described by our model.

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### On the long-time stability of the implicit Euler scheme for the 2d thermohydraulics equations

**Florentina Tone**

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In this article we consider the two-dimensional thermohydraulics equations, we discretize these equations in time using the implicit Euler scheme and with the aid of the classical and uniform discrete Gronwall lemmas, we prove that the scheme is H1-uniformly stable in time.

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### A high order WENO scheme for detonation waves

**Wei Wang**

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**Chi-Wang Shu, Helen Yee, Bjorn Sjogreen**

We propose a high order finite difference WENO method with Harten's ENO subcell resolution idea for the chemical reactive flows. In the reaction problems, when the reaction time scale is very small, e.g., orders of magnitude smaller than the fluid dynamics time scales, the problems will become very stiff. Wrong propagation of discontinuity occurs due to the underresolved numerical solutions in both the space and time. The proposed method is a modified fractional step method which solves the convection step and reaction step separately. A fifth-order WENO is used in convection step. In the reaction step, a modified ODE solver is applied but with the flow variables in the discontinuity region modified by the subcell resolution idea.

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### A level set approach for dilute non-collisional fluid-particle flows

**Zhongming Wang**

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**Hailiang Liu, Rodney Fox**

Gas-particle and other dispersed-phase flows can be described by a kinetic equation containing terms for spatial transport, acceleration, and particle processes (such as evaporation or collisions). However, computing the dispersed velocity is a challenging task due to the large number of independent variables. A level set approach for computing dilute non-collisional fluid-particle flows is presented. We will consider the sprays governed by the Williams kinetic equation subject to initial distributions away from equilibrium of the form  $\sum_{i=1}^N \rho_i(x) \delta(\xi - u_i(x))$ . The dispersed velocity is described as the zero level set of a smooth function, which satisfies a transport equation. This together with the density weight recovers the particle distribution at any time. Moments of any desired order can be evaluated by a quadrature formula involving the level set function and the density weight. It is shown that the method can successfully handle highly non-equilibrium flows (e.g. impinging particle jets, jet crossing, particle rebound off walls, finite Stokes number flows).

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**Global regularity results for 2D generalized MHD equations****Xinwei Yu**

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**Chuong V. Tran, Zhichun Zhai**

In this talk I will present several recent results on the global regularity for the 2D generalized MHD equations, where the dissipation terms in the usual MHD system are replaced by fractional powers of the Laplacian. I will also discuss some new criteria for global regularity of 2D and 3D generalized MHD system. This is joint work with Prof. Chuong V. Tran of University of St. Andrews, and Dr. Zhichun Zhai.

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## Special Session 29: Self-Organized Behavior of Nonlinear Elliptic Equations and Pattern Formation of Strongly Interacting Systems

Susanna Terracini, University of Milano-Bicocca, Italy  
 Jun-cheng Wei, Chinese University of Hong Kong, Hong Kong

Several phenomena can be described by a number of densities (of mass, population, probability, ...) distributed in a domain and subject to diffusion, reaction, and either cooperative or competitive interaction. Whenever the interaction is the prevailing mechanism, one can reasonably expect pattern formation and, in the competitive case, that the several densities can not coexist and tend to segregate hence determining a partition of the domain.

We focus on the analysis of the qualitative properties of solutions of systems of semilinear elliptic equations, whenever the interaction is not negligible with respect to diffusion (e.g. when the parameter describing the competition diverges to infinity). Prototype of variational systems are those of Gross-Pitaevski equations, describing the motion of a Bose-Einstein condensated quantum gas with different spin states. Strong competition appears in association with diverging interspecific scattering lengths. The limiting behaviour is known for the ground state stationary solutions and solitary waves: the wave amplitudes segregate, that is, their supports tend to be disjoint (phase separation). Of course, the partition becomes the main object of investigation both from the analytical point of view than from the geometric, with emphasis on the multiple points of multiple intersection. When the system possesses a variational structure, one can associate an optimal partition problem with the ground states. Conversely, one can regard at optimal partitions related to linear or nonlinear eigenvalues as limits of competing systems as the competition parameter diverges.

A key role is played by the entire solutions to systems, with polynomial interactions. In the case of an eigenvalue, the variational properties of the nodal partition associated with an eigenfunction is deeply connected to the number of nodal domains it is possible to link the minimizing property of the nodal partition to that of being sharp with respect to the Courant's nodal Theorem.

### On a prescribed mean curvature equation in modeling MEMS

Nicholas Brubaker

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Alan E. Lindsay, John A. Pelesko

Mathematical models of the form  $\mathfrak{M}u = f(u; \lambda)$ , where  $\mathfrak{M}$  is the nonparametric mean curvature operator and  $\lambda$  is a nonlinear eigenvalue, have been studied since the early 1800s, beginning with the work of Thomas Young and Pierre-Simon Laplace. Recently, it has been shown that when  $f(u; \lambda) = \lambda(1 + u)^{-2}$  the aforementioned model describes the shape of an electrostatically deflected thin, elastic membrane. In this talk, we look at the contrast between this particular prescribed mean curvature equation and its standard approximation  $\Delta u = f(u)$ .

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### Solutions with point singularities for a MEMS equation with fringing field

Juan Davila

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We construct solutions of the equation

$$-\Delta u = \frac{\lambda(1 + |\nabla u|^2)}{(1 - u)^2}, \quad 0$$

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### Singularity of eigenfunctions at the junction of shrinking tubes

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Susanna Terracini

Consider two domains connected by a thin tube: it can be shown that, generically, the mass of a given eigenfunction of the Dirichlet Laplacian concentrates in only one of them. The restriction to the other domain, when suitably normalized, develops a singularity at the junction of the tube, as the channel section tends to zero. Our main result states that, under a nondegeneracy condition, the normalized limiting profile has a singularity of order  $N - 1$ , where  $N$  is the space dimension. We give a precise description of the asymptotic behavior of eigenfunctions at the singular junction, which provides us with some important information about its sign near the tunnel entrance. More precisely, the solution is shown to be one-sign in a neighborhood of the singular junction. In other words, we prove that the nodal set does not enter inside the channel.

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### Solutions for a semilinear elliptic equations involving critical exponents.

**Ignacio Guerra**

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**Juncheng Wei**

Let  $\Omega$  be a bounded domain in  $\mathbb{R}^N$ . We consider the problem

$$\begin{aligned} -\Delta u &= u^p + \lambda u^q, & u > 0 & \text{ in } \Omega \\ u &= 0 & \text{ in } \partial\Omega, \end{aligned}$$

where  $p = (N + 2)/(N - 2)$ . We describe large solutions for this problem, and we obtain precise estimates for  $\lambda = \lambda(c)$  where  $c := \max_{\Omega} u$  and tends to infinity. This depends strongly in the dimension  $N$  and the value of  $q$ . The case  $q = 1$  is the well know Brezis-Nirenberg problem. We also study a characterization of the point in  $\Omega$  where solution concentrates as  $c \rightarrow \infty$ .

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### Hot-spot solutions in a model of crime

**Theodore Kolokolnikov**

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**Michael Ward, Juncheng Wei**

We study a model of crime developed recently by Short and collaborators at UCLA. This model exhibits hot-spots of crime – localized areas of high criminal activity. In a certain asymptotic limit, we use singular perturbation theory to construct the profile of these hot-spots and then study their stability.

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### Concentration behaviour in biharmonic equations of MEMS.

**Alan Lindsay**

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**J. Lega, M.J. Ward, K.B. Glasner**

A Micro electro-mechanical systems (MEMS) capacitor is a microscopic device consisting of two plates held opposite one other. The lower plate is immobile while the upper plate is fixed along its edges but free to deflect in the presence of an electric potential towards the lower plate. The deflecting upper plate may reach a stable equilibrium, however, if the applied potential exceeds a threshold, known as the pull-in voltage, the upper plate will touchdown on the lower plate. When certain physical assumptions are applied, the deflection of the upper surface can be modeled as a fourth order PDE with a singular

non-linearity. It is shown that the model captures the pull-in instability of the device and provides a prediction of the pull-in voltage. When the pull-in voltage is exceeded, the equations develop a finite time singularity (FTS) and it is demonstrated that this FTS forms on multiple isolated points or on a continuous set of points. The touchdown set is predicted for certain domains by means of asymptotic expansions. We also illustrate a new model which regularizes the infinite electric field and allows the dynamics to be continued beyond the initial FTS.

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### The effects of alliances in competing species models

**Julian Lopez-Gomez**

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**M. Molina-Meyer**

We analyze the effects of spatial refuges in heterogeneous competing species systems as well as the effects of strategic alliances in competitive environments. The spatial refuges under strong competition enhance segregation and, hence, diversity. The facilitative effects increase dramatically the stability of the ecosystems.

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### Triple-junctions in a strong interaction limit of a three-component system

**Hideki Murakawa**

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**Hirokazu Ninomiya**

We consider a three-component reaction-diffusion system with a reaction rate parameter, and investigate its singular limit as the reaction rate tends to infinity. The limit problem is described by a nonlinear cross-diffusion system. The system is regarded as a weak form of a free boundary problem which possesses three types of free boundaries. Triple junction points appear at the intersection of the three interfaces. Furthermore, the dynamics is governed by a system of equations in each region separated by the free boundaries.

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### Convergence of minimax and continuation of critical points for singularly perturbed systems

**Benedetta Noris**

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**Hugo Tavares, Susanna Terracini, Gianmaria Verzini**

In the recent literature, the phenomenon of phase separation for binary mixtures of Bose–Einstein condensates can be understood, from a mathematical point of view, as governed by the asymptotic limit of the stationary Gross–Pitaevskii system

$$\begin{cases} -\Delta u + u^3 + \beta uv^2 = \lambda u \\ -\Delta v + v^3 + \beta u^2 v = \mu v \\ u, v \in H_0^1(\Omega), \quad u, v > 0, \end{cases}$$

as the interspecies scattering length  $\beta$  goes to  $+\infty$ . For this system we consider the associated energy functionals  $J_\beta$ ,  $\beta \in (0, +\infty)$ , with  $L^2$ -mass constraints, which limit  $J_\infty$  (as  $\beta \rightarrow +\infty$ ) is strongly irregular. For such functionals, we construct multiple critical points via a common minimax structure, and prove convergence of critical levels and optimal sets. Moreover we study the asymptotics of the critical points.

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### Nonnegative solutions of elliptic equations and their nodal structure

**Peter Polacik**

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We consider the Dirichlet problem for a of class nonlinear elliptic equations on reflectionally symmetric bounded domains. We are mainly interested in nonnegative, nonzero solution which are not strictly positive. We will discuss the existence of such solutions and their symmetry properties, including the symmetry of their nodal set.

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### A double bubble solution in a ternary system with long range interaction

**Xiaofeng Ren**

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**Juncheng Wei**

We consider a ternary system of three constituents, a model motivated by the triblock copolymer theory. The free energy of the system consists of two parts: an interfacial energy coming from the boundaries separating the three constituents, and a longer range

interaction energy that functions as an inhibitor to limit micro domain growth. We show that a perturbed double bubble exists as a stable solution of the system. Each bubble is occupied by one constituent. The third constituent fills the complement of the double bubble. The location of the double bubble is determined by the Green's function of the Laplace operator on the sample domain.

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### Optimal partition problems involving Laplacian eigenvalues

**Hugo Tavares**

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**Susanna Terracini**

Given a bounded domain  $\Omega \subseteq \mathbb{R}^N$ ,  $N \geq 2$ , and a positive integer  $m \in \mathbb{N}$ , consider the following optimal partition problem:

$$\inf \left\{ \sum_{i=1}^m \lambda_k(\omega_i) : \omega_i \subset \Omega \text{ open } \forall i, \right. \\ \left. \omega_i \cap \omega_j = \emptyset \text{ whenever } i \neq j \right\},$$

where  $\lambda_k(\omega)$  denotes the  $k$ -th eigenvalue of  $-\Delta$  in  $H_0^1(\omega)$ . Approximating this problem by a system of elliptic equations with competition terms, we show the existence of regular optimal partitions. Moreover, multiplicity of sign-changing solutions for the approximating system is obtained.

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### Extremality conditions for optimal partitions

**Susanna Terracini**

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We deal with the free boundary problem associated with optimal partitions related with linear and nonlinear eigenvalues. We are concerned with extremality conditions and the regularity of the interfaces. These properties are then linked with extremality conditions of the nodal set of eigenfunctions and the number of their nodal components.

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### Natural constraints in variational methods and superlinear Schroedinger systems

**Gianmaria Verzini**

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**B. Noris**

For a  $C^2$ -functional  $J$  defined on a Hilbert space  $X$ , we consider the set

$$\mathcal{N} = \{x \in A : \text{proj}_{V_x} \nabla J(x) = 0\},$$

where  $A \subset X$  is open and  $V_x \subset X$  is a closed linear subspace, possibly depending on  $x \in A$ . We study sufficient conditions for a constrained critical point of  $J$  restricted to  $\mathcal{N}$  to be a free critical point of  $J$ , providing a unified approach to different natural constraints known in the literature, such as the Birkhoff-Hestenes natural isoperimetric conditions and the Nehari manifold. As an application, we prove multiplicity of solutions to a class of superlinear Schroedinger systems on singularly perturbed domains.

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### Qualitative properties and existence results for an nonlinear elliptic system

**Juncheng Wei**

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**H. Berestycki, TC Lin, S. Terracini, Kelei Wang, CY Zhao**

We consider the following nonlinear elliptic system

$$\Delta U = UV^2, \Delta V = VU^2, U, V > 0 \text{ in } R^n$$

We first prove the existence, asymmetry, stability and local uniqueness of the one-dimensional solution. Then we prove the two-dimensional De Giorgi Conjecture and Stability Conjecture, under some growth condition. Finally we construct solutions with any polynomial growth at  $+\infty$ .

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### Spiky patterns in a consumer chain model

**Matthias Winter**

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**Juncheng Wei**

We study a consumer chain model which is based on Schnakenberg type kinetics. This model is realistic in a predator-prey context if cooperation of predators is prevalent in the system. The system also serves as a model for a sequence of irreversible autocatalytic reactions in a container which is in

contact with a well-stirred reservoir. In this model there is a middle predator feeding on the prey and a final predator feeding on the middle predator. This means that the middle predator plays a hybrid role: it acts as both predator and prey. We will consider two cases: (i) Final predator has smaller diffusivity than the rest. (ii) Middle predator has smaller diffusivity than the rest.

We will rigorously prove the existence of spiky patterns for this system. We will also study their stability properties.

The analytical results are confirmed by numerical computations. Biological applications are discussed.

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### Monge-Ampere equations on exterior domains

**Lei Zhang**

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**Jiguang Bao, Haigang Li**

We consider the Monge-Ampere equation  $\det(D^2u) = f$  where  $f$  is a positive continuous function in  $R^n$  and is a perturbation of a positive constant at infinity. First we prove that every globally defined solution is close to a parabola plus a logarithmic term in two dimensional spaces and is close to a parabola in higher dimensional spaces. Then we show that given any prescribed asymptotic behavior mentioned above, there exists a unique global solution corresponding to it. Finally we solve the exterior Dirichlet problem with given function on the boundary of a bounded convex set and a prescribed asymptotic behavior at infinity.

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### Regularity results for boundary partition problems

**Alessandro Zilio**

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**Susanna Terracini, Gianmaria Verzini**

We analyze the regularity of solutions to boundary partition problems. In particular, we consider a system of elliptic equations arising as constrained minima of a certain energy functional, under the condition that the solutions have traces with disjoint support on some portion of the boundary. Thanks to the link between boundary dynamics and fractional diffusion processes, our results extend also to segregation problems involving fractional powers of the laplace operator.

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## Special Session 30: Recent Developments on Turbulence

**Eleftherios Gkioulekas**, University of Texas-Pan American, USA  
**Michael Jolly**, Indiana University, USA

Turbulence is often referred to as the last open problem of classical mechanics. It is a problem of both fundamental importance, posing unique mathematical challenges, and with a wide range of applications, in earth and atmospheric science, plasma physics, aerospace engineering, and many other areas. Despite nearly a century of efforts by the best minds, many questions remain open, and considerable progress has only been achieved for some idealized situations, such as homogeneous and isotropic turbulence. The goal of the session is to bring together specialists from various areas of turbulence research.

### Non-linear cascades in rotating stratified Boussinesq flows

**Hussein Aluie**

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**Susan Kurien**

We use high-resolution simulations of Boussinesq flows, forced in the large-scales, with fixed rotation and stable stratification along the vertical axis, to study the downscale cascades of energy and potential enstrophy in three different regimes of stratification and rotation. In addition, we analyze the spatial distribution of the cascades using a coarse-graining approach which allows simultaneous resolution of the dynamics in scale and in space.(1) For strongly stratified flows with moderate rotation, we observe anisotropic fluxes of energy and potential enstrophy into Fourier modes with large vertical component  $k_z$ , predominantly due to a highly non-local transfer from the large-scales directly to the smallest scales. The energy cascade is predominantly due to three vortical-mode interactions.(2) For strongly rotating flow with moderate stratification, there are anisotropic fluxes to modes with large  $k_h$ , due to a “diffusely” local transfer much like in isotropic Navier-Stokes turbulence. The energy cascade is primarily due to three vortical-mode interactions, as in the strongly stratified case, although wave-vortical-wave and vortical-wave-vortical interactions also make a noticeable contribution.(3) In the third case of equally strong rotation and stratification, there are only slightly anisotropic fluxes, mostly to modes with large  $k_h$ , due to an ultra-local transfer in which the energy gained by an inertial scale comes almost exclusively from the adjacent larger scales. We confirm that the cascades in the third regime are primarily due to wave-vortical-wave interactions, in agreement with previous work.

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### Maximal spatial analyticity radius for the Navier-Stokes equations

**Animikh Biswas**

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**C. Foias, M. Jolly**

The regular solutions of the Navier-Stokes equations (NSE) are well-known to be analytic in both space and time variables. Moreover, the space analyticity radius has an important physical interpretation. It demarcates the length scale below which the viscous effect dominates the (nonlinear) inertial effect. Foias and Temam introduced an effective approach to estimate space analyticity radius via the use of Gevrey norms which, since then, has become a standard tool for studying analyticity properties for dissipative equations. Using this approach, we study the maximal space analyticity radius of solutions of the 2D and 3D NSE as a function of time. Our main tool is an ODE associated to the NSE which can be solved on a maximal domain. The upper boundary of this domain is the maximal analyticity radius of the solution to the NSE. The main objective is to study the connection between (topological) features of this domain and physical phenomena such as intermittency, energy and enstrophy cascades. This is a jointwork with Professors C. Foias and M. Jolly.

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### Analytical approach to intermittency in turbulence

**Alexey Cheskidov**

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**Roman Svydkoy**

We introduce in precise mathematical terms some of the empirical concepts used to describe intermittency in fully developed turbulence. We give definitions of the active turbulent region, volume, eddies, energy dissipation set, and derive rigorously some power laws of turbulence. In particular, the formula for the Hausdorff dimension of the energy dissipation set will be justified, and upper/lower bounds on the energy spectrum will be obtained.

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### Geodesic detection of coherent vortices in two-dimensional turbulence

**Mohammad Farazmand**  
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**George Haller**

We study the evolution of vortices in two-dimensional turbulence through the recently developed geodesic theory of transport barriers [Haller & Beron-Vera (2012)]. First, we review the main results of the theory and its numerical implementation. We then use the theory to locate hyperbolic and elliptic Lagrangian Coherent Structures (LCSs) in two-dimensional turbulence. The hyperbolic LCSs correspond to generalized stable and unstable manifolds while the elliptic LCSs correspond to the cores of Lagrangian vortices. In addition, we use these structures to explore the Lagrangian signature of vortex merger.

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### Energy and potential enstrophy flux constraints in the two-layer quasi-geostrophic model

**Eleftherios Gkioulekas**  
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We investigate an inequality constraining the energy and potential enstrophy flux in the two-layer quasi-geostrophic model. This flux inequality is unconditionally satisfied for the case of two-dimensional Navier-stokes turbulence. However, it is not obvious that it remains valid under the multi-layer quasi-geostrophic model. We derive the general form of the energy and potential enstrophy dissipation rate spectra for a generalized multi-layer model. We then specialize these results for the case of the two-layer quasi-geostrophic model under dissipation configurations in which the dissipation terms for each layer are dependent only on the streamfunction or potential vorticity of that layer. We derive sufficient conditions for satisfying the flux inequality and discuss the possibility of violating it under different conditions.

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### Bounds on energy, enstrophy for the 2D NSE with single mode forcing

**Michael Jolly**  
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**C. Foias, Ming Yang**

We examine how the global attractor of the 2-D periodic Navier-Stokes equations projects in the energy-enstrophy-plane when the force is an eigenvector of the Stokes operator. We prove the existence

semi-integral curves, which form the upper boundary. Along the way we find regions of the plane in which the energy and enstrophy must decrease, and calculate the curvature of the projected solution at certain initial data. Finally, we obtain restrictions on solutions that project onto a single point in the plane.

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### Recent numerical results for the 3D MHD-Voigt model and related models

**Adam Larios**  
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**Evelyn Lunasin, Edriss S. Titi**

Recently, the Voigt-regularization, which is related to the alpha-models of turbulent flows, has been investigated as a regularization of various fluid models. It overcomes many of the problems present in other alpha-models; for example, it is well-posed in bounded domains, and its global bounds in the relevant spaces are independent of viscosity. Moreover, in studying the limit as the regularization parameter tends to zero, a new criterion for the finite-time blow-up of the original equations arises. I will discuss recent analytical and numerical work on the Voigt-regularization in the context of the 3D MHD equations and, given time, the 2D Boussinesq equations.

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### Optimal stirring strategies with fixed energy, fixed power or fixed palenstrophy constraint

**Evelyn Lunasin**  
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**Zhi Lin, Alexei Novikov, Anna Mazzucato, Charles Doering**

We consider passive scalar mixing by a prescribed divergence-free velocity vector field in a periodic box and address the following question: Starting from a given initial inhomogeneous distribution of passive tracers, and given a certain energy budget, power budget or finite palenstrophy budget, what incompressible flow field best mixes the scalar quantity? We adopt the optimal stirring strategy recently proposed by Lin, Thiffeault and Doering which subsequently determine the flow field that instantaneously optimizes the depletion of the  $H^{-1}$  mix-norm. In this work we bridge some of the gap in the best available *a priori* analysis and the simulation results. We recall some previously derived rigorous analysis and then present a new explicit analytical example establishing finite-time perfect mixing with finite energy constraint. On the other hand, using techniques pioneered by Yudovich in proving uniqueness of solutions to the 2-d Euler equations, we establish that

if the flow is constrained to have constant palenstrophy  $\mathcal{P} := \|\Delta u\|_{L^2}^2$ , then the  $H^{-1}$  mix-norm decays at most  $\sim e^{-c\mathcal{P}t^2}$ . That is, finite-time perfect mixing is certainly ruled out when too much cost is incurred by small scale structures in the stirring. As observed from direct numerical simulations, we discuss the impossibility of finite-time perfect mixing for flows with fixed power constraint and conjecture an exponential lower bound on the  $H^{-1}$  mix-norm. Interestingly, we discuss results about some related problems from other areas of analysis which are quantitatively equivalent or similarly suggestive of an exponential lower bound for the  $H^{-1}$  mix-norm.

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### Dissipative length scales of the Navier-Stokes equations

**Vincent Martinez**

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The radius of analyticity of the Navier-Stokes equations indicates a length scale below which viscous effects dominate the inertial ones, and in the context of turbulence can be couched in terms of the so-called Kolmogorov length-scale, the unique length scale determined by the viscosity and energy dissipation rate alone. This talk will address a semigroup method initiated by [Biswas-Swanson '07] for obtaining a lower bound on this radius in terms of the Gevrey norm of the initial data. While this approach does not improve the best known estimate obtained by [Kukavica '98] it does recover and generalize the estimate made by [Doering-Titi '95], as well as identify a Kolmogorov-type length scale based on a quantity formally analogous to the energy dissipation rate.

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### A new instability that breaks the spatial homogeneity symmetry in wave turbulence

**Benno Rumpf**

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**Alan C. Newell, Vladimir E. Zakharov**

Spatial homogeneity, the symmetry property that all statistical moments are functions only of the relative geometry of any configuration of points, can be spontaneously broken by the instability of the finite flux Kolmogorov-Zakharov spectrum in certain (usually one dimensional) situations. As a result, the nature of the statistical attractor changes dramatically, from a sea of resonantly interacting dispersive waves to an ensemble of coherent radiating pulses.

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### Characterizing the layer thickness in unit- and small-aspect-ratio rotating Boussinesq turbulence

**Leslie Smith**

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**Susan Kurien**

High-resolution simulations are used to quantify how small domain aspect ratio conspires with strong stratification to set the dynamical vertical length scale of 'pancake' structures in strongly stratified Boussinesq flow. All simulations are in an asymptotic parameter regime defined by quadratic potential enstrophy. The relevant parameters of the problem are the buoyancy frequency  $N$ , the Coriolis parameter  $f$ , the domain height  $H$ , the domain length and width  $L$ , and the Burger number  $NH/(fL)$ . There are two sets of calculations with (i) Burger number fixed at unity and decreasing domain aspect ratio  $H/L = 1, 1/4, 1/8$  and  $1/16$ , and (ii) increasing  $N/f = 4, 8, 16$  and aspect ratio  $H/L$  fixed at unity. The first set of calculations is relevant for mid-latitude atmosphere/ocean dynamics with Burger number close to unity.

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### Phase transitions in optical turbulence

**Natalia Vladimirova**

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**Gregory Falkovich**

We consider turbulence in the Gross-Pitaevsky model and study the creation of a coherent condensate via an inverse cascade originated at small scales. The growth of the condensate leads to a spontaneous breakdown of symmetries of small-scale over-condensate fluctuations: first, statistical isotropy is broken, then series of phase transitions mark the change of symmetry from the two-fold to three-fold to four-fold. At the highest condensate level reached, we observe a short-range positional and long-range orientational order (similar to a hexatic phase in the condensed matter physics). In other words, the longer one pumps the system the more ordered it becomes. We show that these phase transitions happen when the driving term corresponds to an instability but not when the system is pumped by a random force. Thus we demonstrate non-universality of the inverse-cascade turbulence.

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**Fractal contours of scalar in smooth flows****Marija Vucelja**Courant Institute of Mathematical Sciences, USA  
vucelja@cims.nyu.edu**Gregory Falkovich, Konstantin Turitsyn**

A passive scalar field was studied under the action of pumping, diffusion and advection by a smooth flow with a Lagrangian chaos. We present theoretical arguments showing that scalar statistics is not conformal invariant and formulate a new effective semi-analytic algorithm to model scalar turbulence. We then carry out massive numerics of scalar turbulence focusing on nodal lines. The distribution of contours over sizes and perimeters is shown to depend neither on the flow realization nor on the resolution (diffusion) scale for scales exceeding this scale. The scalar isolines are found fractal/smooth at the scales larger/smaller than the pumping scale. We characterize the statistics of bending of a long isoline by the driving function of the Loewner map, show that it behaves like diffusion with diffusivity independent of resolution yet, most surprisingly, dependent on the velocity realization and time (beyond the time on which the statistics of the scalar is stabilized).

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## Special Session 31: Mathematical Models of Cancer and Cancer Therapy

Yangjin Kim, KonKuk University, Korea

Cancer is a complex, multiscale process, in which genetic mutations occurring at a sub-cellular level manifest themselves as functional changes at the cellular and tissue scale. The main aim of this session is to discuss current development and challenges in modeling tumor growth and cancer therapy. Specific goals of the session are to bring together (both computational and analytical) solutions to challenges in mathematical modeling, laboratory experimentation, and clinical diagnosis, and to improve our understanding of fundamental mechanism of tumor development leading to better clinical outcomes. Both the immediate microenvironment (cell-cell or cell-matrix interactions) and the extended microenvironment (e.g. vascular bed) are considered to play crucial roles in tumour progression as well as suppression. Stroma is known to control tumor growth and invasion to surrounding tissue. However, it also prohibits therapeutics from accessing the tumor cells, thus causing drug resistance. Therefore, a thorough understanding of the microenvironment would provide a foundation to generate new strategies in therapeutic drug development. At the cellular level, cell migration is a key step for metastasis and further development of cancer in a given microenvironment. Thus, understanding of cell motility under the control of signal transduction pathways would improve technical advances in cancer therapy by targeting the specific pathways that are associated with the diseases. Analysis of mathematical models would identify fundamental (abstract) structure of the model system and shed a light on our understanding of tumor growth in the specific host tissue environment and interactions between players in cancer progression. More comprehensive multi-scale (hybrid) models can be used to meet the needs of developing patient-specific drugs. The focus of this session is threefold: (a) to present mathematical models of tumor growth and analysis of the models, (b) to discuss many aspects of (patient specific) drug development, (c) to showcase mathematical models incorporating mechanical aspects of movement and growth of cancerous cells and tissues.

### An agent based evolutionary model of prostate cancer

David Basanta

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Heterogeneity in prostate cancer is the driving force in somatic evolution which explains the emergence of resistance to therapies and relapse. We will show a computational agent based model of prostate cancer where tumour cells can adapt to the microenvironment and show how this evolutionary process is responsible and exploits the heterogeneity that makes prostate tumours so difficult to treat.

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### Cell migration features in glioma growth and invasion: mathematical modeling and analysis

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Gliomas are very aggressive brain tumors, in which tumor cells gain the ability to penetrate the surrounding normal tissue. The invasion mechanisms of this type of tumors are not yet fully understood. In a first part, we will present mathematical approaches to model and investigate one particular mechanism, the migration/proliferation dichotomy, i.e. the antagonistic migratory and proliferating cellular behaviors in a glioma cell population, which has been hypothesized as playing a central role in

the development of these tumors. By using a combination of numerical and analytical techniques, we will show how these models provide insights into the dynamics of avascular glioma growth and capture in vitro observations. In a second part, and based on recent in vivo data, we will present preliminary analysis of the effect of vascularization on glioma growth, which is known to be a major component of in vivo development.

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### Hypoxia inducible factors mediate the inhibition of cancer by GM-CSF: a mathematical model

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Julie M. Roda, Clay B. Marsh, Timothy D. Eubank, Avner Friedman

Under hypoxia, tumor cells and tumor-associated macrophages produce VEGF (vascular endothelial growth factor), a signaling molecule that induces angiogenesis. The same macrophages, when treated with GM-CSF (granulocyte/macrophage colony-stimulating factor), produce sVEGFR-1 (soluble VEGF receptor-1), a soluble protein that binds with VEGF and inactivates it. The production of VEGF by macrophages is regulated by HIF-1 (hypoxia inducible factor-1), and the production of sVEGFR-1 is mediated by HIF-2. Recent experiments were conducted to measure the effect of inhibiting tumor growth by GM-CSF treatment in mice with HIF-1-deficient macrophages or HIF-2-deficient macrophages. In the present work we rep-

resent these experiments by a mathematical model based on a system of partial differential equations. We show that the model simulations agree with the above experiments. The model can then be used to suggest strategies for inhibiting tumor growth. For example, the model quantifies the extent to which GM-CSF treatment in combination with a small molecule inhibitor that stabilizes HIF-2 will reduce tumor volume and angiogenesis.

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### Stochastic differential models of tumor spheroid growth

**Andrea De Gaetano**

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**Simona Panunzi, Andrea De Gaetano**

The mathematical modeling of tumor growth has become an important avenue for understanding cancer biology and eventually controlling neoplastic diseases. Tumor spheroids represent a simplified biological situation, in which a cluster of tumor cells is allowed to replicate and grow in vitro, subject to varying substrate and oxygen concentrations, allowing the experimenter to obtain serial measurements of the attained size. Models for these data have ranged from simple Gompertz growth to complex spatial PDE descriptions taking into account nutrient diffusion and local cell replication. The goal of the present work is to propose a simple Stochastic Differential model of tumor spheroid growth and to estimate its parameters from series of historical tumor spheroid growth data. The model is able to account for both the overall trend in tumor expansion (dependent on predictors such as oxygen and nutrient availability) and for its random fluctuations. Implications of the assessment of tumor growth volatility on the clinical management of cancer patients are discussed.

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### Mechanistic modeling of myoferlin effects on cancer cell invasion

**Marisa Eisenberg**

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**Yangjin Kim, Ruth Li, William E. Ackerman, Douglas A. Kniss, Avner Friedman**

Myoferlin is a member of the evolutionarily conserved ferlin family of proteins, noted for their roles in a variety of membrane processes, including membrane fusion and repair, vesicle transport, receptor recycling and stability, and cell motility. Thus, one might expect the ferlin family to be strong candidates for cancer proteins, although they have not previously been investigated in this capacity. In this

talk, I will discuss our recent work showing that myoferlin plays a previously unrecognized role in cancer cell invasion, using a combination of mathematical modeling and in vitro experiments. Using a real-time impedance-based invasion assay, we have shown that lentiviral-based knockdown of myoferlin significantly reduced invasion of MDA-MB-231 breast cancer cells. Based on these experimental data, we developed a partial differential equation model of myoferlin effects on cancer cell invasion which we used to generate mechanistic hypotheses. Our model predictions revealed that matrix metalloproteinases (MMPs) may play a key role in modulating this invasive property, which was supported by experimental data using qRT-PCR screens. These results suggest that MYOF may be a promising new target for biomarkers or drug target for metastatic cancer diagnosis and therapy, perhaps mediated through MMPs.

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### Non-stem cancer cell kinetics modulate solid tumor progression

**Heiko Enderling**

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**Charles I Morton, Lynn Hlatky, Philip Hahnfeldt**

Solid tumors are heterogeneous in composition. Cancer stem cells (CSCs) are believed to drive tumor progression, but the relative frequencies of CSCs versus non-stem cancer cells span wide ranges even within tumors arising from the same tissue type. Tumor growth kinetics and composition can be studied through an agent-based cellular automaton model using minimal sets of biological assumptions and parameters. Herein we describe a pivotal role for the generational life span of non-stem cancer cells in modulating solid tumor progression in silico. We demonstrate that although CSCs are necessary for progression, their expansion and consequently tumor growth kinetics are surprisingly modulated by the dynamics of the non-stem cancer cells. Simulations reveal that slight variations in non-stem cancer cell proliferative capacity can result in tumors with distinctly different growth kinetics. Longer generational life spans yield self-inhibited tumors, as the emerging population of non-stem cancer cells spatially impedes expansion of the CSC compartment. Conversely, shorter generational life spans yield persistence-limited tumors, with symmetric division frequency of CSCs determining tumor growth rate. We show that the CSC fraction of a tumor population can vary by multiple orders of magnitude as a function of the generational life span of the non-stem cancer cells. Our study suggests that variability in the growth rate and CSC content of solid tumors may be, in part, attributable to the proliferative capacity of the non-stem cancer cell population that arises during asymmetric division of CSCs. In our model,

intermediate proliferative capacities give rise to the fastest-growing tumors, resulting in self-metastatic expansion driven by a balance between symmetric CSC division and expansion of the non-stem cancer population. Our results highlight the importance of non-stem cancer cell dynamics in the CSC hypothesis, and may offer a novel explanation for the large variations in CSC fractions reported in vivo.

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### Are more complicated tumor control probability models better?

**Thomas Hillen**

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**J. Gong, M.M. dos Santos, C. Finlay**

Mathematical models for the tumor control probability (TCP) are used to estimate the expected success of radiation treatment protocols of cancer. There are several mathematical models in the literature and we made the experience that simple and complex models often make the same predictions. Here we compare six of these TCP models: the Poisson TCP, the Zaider-Minerbo TCP, a Monte Carlo TCP, and their corresponding cell cycle (two-compartment) models. Several clinical non-uniform treatment protocols for prostate cancer are employed to evaluate these models. These include fractionated external beam radiotherapies, and high and low dose rate brachytherapies. We find that in realistic treatment scenarios, all one-compartment models and all two-compartment models give basically the same results. A difference occurs between one compartment and two compartment models due to reduced radiosensitivity of quiescent cells. Based on our results, we can recommend the use of the Poissonian TCP for every day treatment planning. More complicated models should only be used when absolutely necessary.

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### Modeling the effects of drug binding on the dynamic instability of microtubules

**Peter Hinow**

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**Vahid Rezaia , Jack Tuszynski, Manu Lopus, Mary Ann Jordan)**

We propose a stochastic model that accounts for the growth, catastrophe and rescue processes of steady state microtubules assembled from MAP-free tubulin in the possible presence of a microtubule associated drug. As an example for the latter, we both experimentally and theoretically study the perturbation of microtubule dynamic instability by S-methyl-D-DM1, a synthetic derivative of the microtubule-targeted agent maytansine and a potential anticancer

agent. We find that among drugs that act locally at the microtubule tip, primary inhibition of the loss of GDP tubulin results in stronger damping of microtubule dynamics than inhibition of GTP tubulin addition. On the other hand, drugs whose action occurs in the interior of the microtubule need to be present in much higher concentrations to have visible effects.

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### PDE tumor models - mathematical analysis and numerical method

**Bei Hu**

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**Avner Friedman, Andrew Sommese, Wenrui Hao, Jon Hauenstein**

We shall discuss the recent progress (joint work with many others) on the PDE tumor models, the stability of the tumor, the bifurcation diagram near the bifurcation point, the numerical methods and simulations, the bifurcation diagram extensions, and the method for finding other possible stationary solutions.

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### A model of prostate cancer progression under androgen ablation therapy

**Harsh Jain**

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**Arvinder Bhinder, Steven Clinton, Avner Friedman**

Due to its dependence on androgens, metastatic prostate cancer is treated with the elimination of testosterone. However, such interventions are not curative because cancer cells evolve via multiple mechanisms to a castrate-resistant state, allowing progression to a lethal outcome. It is hypothesized that administration of anti-androgen therapy in an intermittent, as opposed to continuous, manner may bestow improved disease control. In this talk, I will present a biochemically motivated mathematical model of anti-androgen therapy that can be tested prospectively as a predictive tool. The model includes 'personalized' parameters, which address the heterogeneity in the predicted course of the disease under various androgen-deprivation schedules. Model simulations are able to capture a variety of clinically observed outcomes for 'average' patient data under different intermittent schedules. The model predicts that in the absence of a competitive advantage of androgen-dependent cancer cells over castration-resistant cancer cells, intermittent scheduling can lead to more rapid treatment failure as compared to continuous treatment. However, increasing a competitive advantage for hormone-



sensitive cells swings the balance in favor of intermittent scheduling. Given the near universal prevalence of anti-androgen treatment failure in the absence of competing mortality, such modeling has the potential of developing into a useful tool for incorporation into clinical research trials and ultimately as a prognostic tool for individual patients.

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### Multiscale study of angiogenesis from molecule to tissue

**Yi Jiang**

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Angiogenesis, growth of new blood vessels from existing ones, is an important process in cancer development. Anti-angiogenesis approach is still being intensively studied as a potential cancer therapy. To understand angiogenesis in cancer, we need to first understand normal angiogenesis, e.g. in development. In close connection with experiments, we have developed two cell-based multiscale models of angiogenesis, in retina and tumor, respectively. Our models consider intracellular signaling pathways (VEGF and Notch/Delta), cell dynamics, cell-cell and cell-environment interactions. The models reproduced sprouting morphology and dynamics. The simulations showed that 1) diffusible and matrix-bound VEGF isoforms result in distinctively different morphology, 2) VEGF and Notch/Delta pathways determine the vascular network patterns through dynamic regulation of endothelial phenotypes, and 3) extracellular matrix not only guides the collective migration of endothelial cells, but also participates in cell phenotype regulation. These results highlight the important role of extracellular environment, both biochemical and biomechanical, in sprouting morphogenesis, and suggest new hypotheses to be tested in experiments.

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### Signal transduction pathways in the growth and invasion of glioblastoma: a mathematical model

**Yangjin Kim**

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**Soyeon Roh, Avner Friedman, Sean Lawler**

Glioblastoma is a highly invasive brain tumor. This invasive behavior of tumor cells is responsible for low survival rates. Microenvironment plays an important role in the active migration and steady growth of glioma cells. A thorough understanding of the microenvironment would provide a foundation to generate new strategies in therapeutic drug development.

Recently, miR451 and its counterpart AMPK complex were recognized as key regulators of a balance between the migratory phase and proliferation mode. We developed a mathematical model of this signaling pathway to investigate the role of changing environmental factors such as fluctuating glucose levels in creating different invasion and growth patterns. We then analyze the growth behaviors of glioma cells in response to various challenges in the media and show that the various patterns observed in experiments can be obtained by simulating the model with tight regulation of this miR451-AMPK pathway. Using a hybrid model, we also develop several strategies to kill cancerous cells hidden in the microenvironment in addition to surgical resection of the main tumor core, leading to better clinical outcomes.

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### A clinical data validated mathematical model of prostate cancer growth with hormone therapy

**Yang Kuang**

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**Travis Portz, John D. Nagy**

Prostate cancer is commonly treated by a form of hormone therapy called androgen suppression. This form of treatment, while successful at reducing the cancer cell population, affects quality of life and accelerates a recurrence of the cancer in an androgen-independent form. Intermittent androgen suppression (IAS) aims to alleviate some of these problems by cycling the patient on and off-treatment. Clinical studies have suggested that intermittent therapy is capable of maintaining androgen dependence over more treatment cycles while increasing quality of life during off-treatment periods. We presents a mathematical model of prostate cancer to study the dynamics of androgen suppression therapy and the production of prostate specific antigen (PSA), a clinical marker for prostate cancer. Preliminary models were based on the assumption of an androgen-independent (AI) cell population with constant net growth rate. These models gave poor accuracy when fitting clinical data during simulation. The final model presented hypothesizes an AI population with increased sensitivity to low levels of androgen. It also hypothesizes that PSA production is heavily dependent on androgen. The high level of accuracy in fitting clinical data with this final model strongly support these hypotheses, which are also consistent with biological evidences.

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### Robustness and sensitivity of optimal protocols for mathematical models for multi-drug cancer treatments

**Urszula Ledzewicz**

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**Heinz Schüttler**

In this talk we review some results and outline challenges one encounters in analyzing various models for cancer treatments as optimal control problems. The resulting optimal controlled trajectories, representing cancer treatments along with the corresponding response of the system, exhibit various levels of robustness and sensitivity. These are not only related to what is commonly understood and routinely tested when constructing the models which is the sensitivity to the values of the parameters. We also want to discuss how variations in the model assumptions, like the choice of the growth function for the cells, or moving from simpler to more complicated models, like adding more precise aspects such as pharmacokinetics of the drugs, drug resistance or cell cycle specificity, effect the solutions to the problem. We will illustrate it on various models for cancer treatment including models for cell cycle specific chemotherapy, antiangiogenic treatments alone and in combination with chemotherapy and radiotherapy as well as combinations of chemo- and immunotherapy. Examples of both very robust and very sensitive behaviors will be given and the effect of it on the qualitative and quantitative structures of solutions will be discussed.

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### A mathematical model of lung cancer progression

**Seongwon Lee**

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**Yangjin Kim, Hyung Ju Hwang, Yoon-Keun Kim**

Lung cancer originates in the epithelial lining of the airway, but it can evolve into invasive carcinoma. Understanding the progression and how to effectively intervene in it presents a major scientific challenge. The extracellular matrix (ECM) in stroma contains several types of cells and several types of growth factors that are known to individually affect tumor growth, but at present the complex biochemical and mechanical interactions of these stromal cells with tumor cells is poorly understood. Here we develop a mathematical model that incorporates the cross-

talk between stromal and tumor cells, and which can predict how perturbations of the local biochemical and mechanical state influence tumor evolution. We study a hybrid model for the interaction of cells with the tumor microenvironment (TME), in which epithelial cells (ECs) and other immune cells are modeled individually while the ECM is treated as a continuum, and show how these interactions affect the early development of tumors. Finally, we incorporate breakdown of the epithelium into the model and predict the early stages of tumor invasion into the stroma. Our results shed light on the interactions between growth factors, mechanical properties of the ECM, and feedback signaling loops between stromal and tumor cells. We suggest how immune response changes in lung tissue affect tumor progression.

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### The interplay between microenvironmental heterogeneity and anticancer drug dynamics: a computational study

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Poor penetration of tumor tissue by drug particles contributes to low efficacy of therapeutic compounds, and, may result in the failure of the Phase II clinical trials. This may be attributed, at least partially, to the fact that experimental models do not recreate the process of drug penetration into the tumor tissue in a way it takes place in the patient body. We developed a computational model of drug penetration that operates on the microscopic tissue scale and recreates various physico-chemical conditions of the tumors. This model includes explicitly defined tissue morphology that is comprised of individual cells surrounded by the interstitial space filled with the fluid that impacts drug transport. We investigated the dynamics of a class of drugs activated in regions of low oxygen, and showed that they may lead to shifting of the tissue metabolic profile. Our computational results showed a non-linear relation between tissue permeability, its cellular density and penetration of drug molecules due to the convective interstitial transport. Moreover, we demonstrated that heterogeneity in tissue composition, such as irregular cell configurations, might solely be responsible for the emergence of tissue zones that are not exposed to drugs in concentrations sufficient to provide therapeutic action.

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## Special Session 32: Existence and Multiplicity Results in Elliptic Variational Problems

**G. Bonanno**, University of Messina, Italy  
**S. Carl**, University of Halle, Germany  
**S. A. Marano**, University of Catania, Italy  
**D. Motreanu**, University of Perpignan, France

This session is considered as a platform for the presentation of very recent topics and results in the qualitative study of nonlinear elliptic problems such as, e.g., existence, multiplicity, and comparison principles. Emphasis is put on variational techniques, combined with topological arguments and sub-super-solution methods, in both the smooth and non-smooth framework. Thus, a wide range of elliptic problems in bounded or unbounded domains, with or without constraints will be covered by the speakers of this session.

### Multiple solutions for a class of superlinear Neumann problems

**Sergiu Aizicovici**

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**N. S. Papageorgiou, V. Staicu**

We prove the existence of multiple nontrivial smooth solutions, with precise sign information, for a Neumann boundary value problem governed by the p-Laplacian, with a superlinear perturbation.

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### Multiplicity results to elliptic problems in $\mathcal{R}$

**Giuseppina Barletta**

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We present some existence and multiplicity results for the following elliptic problem:

Find  $u \in W^{1,p}(\mathcal{R})$  satisfying

$$(P_\lambda) \quad -(|u'(x)|^{p-2}u'(x))' + B|u(x)|^{p-2}u(x) = \lambda\alpha(x)g(u(x)) \text{ a.e. in } \mathcal{R},$$

where  $\lambda$  is a real positive parameter,  $B$  is a real positive number  $\alpha$  and  $g$  are nonsmooth functions.

We give an existence result for  $(P_\lambda)$ , when the parameter  $\lambda$  varies in a suitable interval. We also show that under additional assumptions on the behavior of  $g$  at infinity or at zero the interval is unbounded from above or its lower bound is zero. Finally, we present some multiplicity results for the problem.

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### A characterization of the mountain pass geometry and applications to nonlinear differential problems

**Gabriele Bonanno**

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A characterization of the mountain pass geometry is presented. Relations between the mountain pass theorem and local minima are pointed out. As consequences, multiplicity results for nonlinear elliptic Dirichlet problems are obtained.

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### Some existence results for a perturbed asymptotically linear problem

**Anna Maria Candela**

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Let us consider the following semilinear elliptic problem

$$(P_\varepsilon) \quad \begin{cases} -\Delta u - \lambda u = f(x, u) + \varepsilon g(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega$  is an open bounded domain of  $\mathbb{R}^N$  ( $N \geq 3$ ) with smooth boundary  $\partial\Omega$ ,  $\varepsilon \in \mathbb{R}$  and  $p, g$  are given real functions on  $\Omega \times \mathbb{R}$ . If the problem  $(P_\varepsilon)$  perturbs an asymptotically linear problem, i.e.

$$\lim_{|t| \rightarrow +\infty} \frac{f(x, t)}{t} = 0$$

uniformly with respect to a.e.  $x \in \Omega$ , we prove that the number of distinct critical levels of the functional associated to the unperturbed problem is “stable” under small perturbations also in lack of symmetry, both in the non-resonant and in the resonant case, even when the perturbation term  $g$  is continuous but satisfies no growth assumption.

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### Three solutions for a quasilinear elliptic problem via critical points in open level sets and truncation principle

**Pasquale Candito**

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The aim of this talk is to present a novel approach, jointly developed with S. Carl and R. Livrea, to investigate the existence of at least three solutions for the following quasilinear elliptic problem depending on a parameter  $\lambda$ , with homogeneous Dirichlet boundary conditions in a smooth bounded domain  $\Omega$ ,

$$-\Delta_p u = \lambda f(u) \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega.$$

More precisely, the existence of at least two constant sign solutions is established owing to an abstract localization principle of critical points for functionals of the form  $\mathbb{E} = \Phi - \lambda\Psi$  on open sublevels  $\Phi^{-1}(]-\infty, r[)$  recently obtained in collaboration with G. Bonanno. The existence of a sign-changing solution is pointed out adapting the sub-supersolution method recently developed by S. Carl and D. Motreanu, where, variational and topological arguments, such as the Mountain Pass Theorem, in conjunction with comparison principles and truncation techniques are the main tools.

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### Elliptic variational inequalities with discontinuous multifunctions

**Siegfried Carl**

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We consider multi-valued elliptic variational inequalities for operators of the form

$$u \mapsto Au + \Theta(u),$$

where  $A$  is a second order elliptic operator of Leray-Lions type, and  $u \mapsto \Theta(u)$  is a multi-valued lower order term that may neither be lower nor upper semi-continuous. Our main goal is to provide an analytical framework for this new class of multi-valued variational inequalities that includes variational-hemivariational inequalities as special case.

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### Multiple solutions for Dirichlet problems involving the $p(x)$ -Laplace operator

**Antonia Chinni**

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We investigate the existence and multiplicity of weak solutions for Dirichlet problems involving the  $p(x)$ -Laplace operator where  $p$  is a continuous function defined on an open bounded domain with smooth boundary  $\Omega \subset R^N$ . In particular, in the case  $N < p^-$ , under an appropriate oscillating behaviour of the nonlinearity, the existence of infinitely many weak solutions is established while, in the more general case  $1 < p^-$  and under a suitable growth condition of the nonlinear term, we obtain the existence of at least three weak solutions. The approach is based on variational methods. Bonanno G. and Chinni A., Existence results of infinitely many solutions for  $p(x)$ -Laplacian elliptic Dirichlet problems, Complex Variables and Elliptic Equations,(2012)

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### Multiplicity results for elliptic Neumann problems

**Giuseppina D'Agui**

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In this talk we present some multiplicity results for an elliptic Neumann problem. Precisely, recent critical point results for differentiable functionals are exploited in order to prove the existence of a determined open interval of positive eigenvalues for which the problem admits multiple weak solutions.

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### Variational methods for differential equation with small impulsive effects

**Beatrice Di Bella**

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In this talk we will deal with the following nonlinear Dirichlet value problem with small perturbations of impulse:

$$(1) \begin{cases} -u''(t) + q(t)u(t) = \lambda f(t, u(t)) & t \in [0, T], \quad t \neq t_j \\ u(0) = u(T) = 0 \\ \Delta u'(t_j) = u'(t_j^+) - u'(t_j^-) = \mu I_j(u(t_j)), \quad 1 \leq j \leq p \end{cases}$$

By using the critical points theorems obtained in [1] and [2] we will prove the existence and multiplicity of solutions for problem (1) when the parameters  $\lambda$  and  $\mu$  lie in precise intervals (see [3]).

[1] Bonanno G. and Candito P., Non-differentiable functionals and applications to elliptic problems with discontinuous nonlinearities, *J. Differential Equations*, 244 (2008), 3031–3059.

[2] Bonanno G. and Marano S.A., On the structure of the critical set of non-differentiable functionals with a weak compactness condition, *Appl. Anal.*, 89 (2010), 1–10.

[3] Bonanno G. and Di Bella B., A Dirichlet boundary value problem with small perturbations of impulse, preprint.

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### Multiple critical orbits for a class of lower semicontinuous functionals

**Petru Jebelean**

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We deal with a class of functionals  $I$  on a Banach space  $X$ , having the structure  $I = \Psi + \mathcal{G}$ , with  $\Psi : X \rightarrow (-\infty, +\infty]$  proper, convex, lower semicontinuous and  $\mathcal{G} : X \rightarrow \mathbb{R}$  of class  $C^1$ . Also,  $I$  is  $G$ -invariant with respect to a discrete subgroup  $G$  of  $X$ , with  $\dim(\text{span } G) = N$ . Under some appropriate additional assumptions we prove that  $I$  has at least  $N + 1$  critical orbits. As a consequence, we obtain that the periodically perturbed  $N$ -dimensional relativistic pendulum equation has at least  $N + 1$  geometrically distinct periodic solutions. Based on joint work with Cristian Bereanu.

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### Standing waves of nonlinear Schrodinger equation

**Shibo Liu**

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We consider the standing wave solutions for a wide class of superlinear Schrodinger equations with periodic potential and asymptotically linear Schrodinger equations whose nonlinearity is not sublinear. In both cases the linear part of the equations may be indefinite.

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### Some recent existence and multiplicity results for second order Hamiltonian systems

**Roberto Livrea**

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In this talk we will show an overview on some existence and multiplicity results of periodic solutions for wide classes of second order Hamiltonian systems that are included in the following general case

$$(P)_\lambda \quad \begin{cases} -\ddot{u}(t) = \nabla_u F(t, u(t), \lambda) \text{ a.e. in } [0, T] \\ u(T) - u(0) = \dot{u}(T) - \dot{u}(0) = 0, \end{cases}$$

where  $T > 0$ ,  $F : [0, T] \times \mathbf{R}^N \times ]0, +\infty[ \rightarrow \mathbf{R}$  is a smooth function. In particular, exploiting some abstract critical points theorems, there will be analyzed several situations when  $F$  satisfies different conditions at zero and/or at infinity with respect to  $u$ .

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### Multiple solutions to Dirichlet eigenvalue problems with p-Laplacian

**Salvatore Marano**

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The existence of both constant -sign and nodal solutions to homogeneous Dirichlet problems with p-Laplacian and reaction term depending on a positive parameter is investigated via variational as well as topological methods, besides truncation techniques.

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### Elliptic variational problems with nonlocal operators

**Michael Melgaard**

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We study the nonlocal and nonlinear problem

$$\begin{aligned} L\phi + V\phi - |\phi|^2 * W\phi &= -\lambda\phi, \\ \|\phi\|_{L^2} &= 1, \end{aligned}$$

for a large class of potentials  $V$  and  $W$  on  $R^3$ . The operator  $L = \sqrt{-\alpha^{-2}\Delta + \alpha^{-4}} - \alpha^{-2}$  (the quasirelativistic Laplacian), with  $\alpha$  being Sommerfeld's fine structure constant, is a nonlocal, pseudo differential operator of order one. We prove the existence of multiple solutions for two separate cases: (1) unconstrained problem; (2) constrained problem.

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### Existence and non-existence of solutions for $p$ -laplacian equations with decaying cylindrical

**Olimpio Miyagaki**

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**P. Carriao, R. Demarque, O. Miyagaki**

In this paper we deal with a class of quasilinear elliptic problem in whole space, involving  $p$ -laplacian operator and the Hardy-Sobolev critical exponent. The potential has a unbounded singular set. Combining a version of the concentration compactness result by Solimini, Hardy-Sobolev type inequality with the Mountain Pass Theorem, existence of non-trivial solutions are obtained. Decay properties of these solutions are showed by applying Vassilev results. Pohozaev type identities are established in order to get non-existence results.

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### Variational problems in geometrical analysis

**Giovanni Molica Bisci**

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We present some existence results, obtained in collaboration with G. Bonanno and V. Rădulescu, for a parametric elliptic problem on a Riemannian manifold without boundary. In particular, for a precise location of the parameter, is established the existence of a nontrivial solution, without requiring any asymptotic condition at zero or at infinity on the nonlinearity. In the case of sublinear terms at the origin, we deduce the existence of solutions for small positive values of the parameter and we obtain that the corresponding solutions have smaller and smaller energies as the parameter goes to zero. It is worth noticing that these results also hold in presence of nonlinearities with critical growth. Finally, a multiplicity result is obtained and concrete examples of applications are provided. A basic ingredient in our arguments is a recent local minimum theorem for differentiable functionals due to G. Bonanno.

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### Count and symmetry of global and local minimizers of the Cahn-Hilliard energy functional over cylindrical domains.

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**João Biesdorf, Janete Crema**

We address the problem of minimization of the Cahn-Hilliard energy functional under a mass constraint

over two and three-dimensional cylindrical domains. Although existence is presented for a general case the focus is mainly on rectangles, parallelepipeds and circular cylinders. According to the symmetry of the domain the exact number of global and local minimizers are given as well as their geometric profile and interface location; all are one-dimensional increasing/decreasing and odd functions for domains with lateral symmetry in all axes and also for circular cylinders. The selection of global minimizers by the energy functional is made via the smallest interface area criterion. The approach utilizes  $\Gamma$ -convergence techniques to prove existence of an one-parameter family of local minimizers of the energy functional for any cylindrical domain. The exact number of global and local minimizers as well as their geometric profiles are accomplished via suitable applications of the unique continuation principle while exploring the domain geometry in each case and also the preservation of global minimizers through the process of  $\Gamma$ -convergence.

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### Bernstein-Nagumo conditions and solutions to nonlinear differential inequalities

**Loc Nguyen**

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**Klaus Schmitt**

For  $\Omega$ , an open bounded subset of  $\mathbb{R}^N$ , with smooth boundary, and  $1 < p < \infty$ , we establish  $W^{1,p}(\Omega)$  *a priori* bounds and prove the compactness of solution sets to differential inequalities of the form

$$|\operatorname{div} A(x, \nabla u)| \leq F(x, u, \nabla u),$$

which are bounded in  $L^\infty(\Omega)$ . The main point in this work is that the nonlinear term  $F$  may depend on  $\nabla u$  and may grow as fast as a power of order  $p$  in this variable. Such growth conditions have been used extensively in the study of boundary value problems for nonlinear ordinary differential equations and are known as Bernstein-Nagumo growth conditions. In addition, we use these results to establish a subsolution theorem.

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### Multiplicity theorems for $(p,2)$ equations

**Nikolaos Papageorgiou**

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We consider a parametric nonlinear elliptic equation driven by the  $(p,2)$ -differential operator. We prove multiplicity theorems when the parameter is bigger than the second eigenvalue of the  $p$ -Laplacian. We establish three or four nontrivial smooth solutions

with sign information. Our approach combines variational methods with Morse theory.

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### Some results for impulsive problems

**Kanishka Perera**

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**T. Gnana Bhaskar**

Impulsive problems arise naturally in processes that involve abrupt changes in the state of the system. In this talk we will present some existence and multiplicity results for asymptotically piecewise linear elliptic problems with superlinear impulses using variational methods.

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### Symmetric problems in unbounded domains

**Addolorata Salvatore**

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We present some existence and multiplicity results concerning a special class of radially symmetric elliptic systems in unbounded domains. Furthermore, in unbounded cylinders we study also partially radially symmetric problems.

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### Mountain pass and linking solutions for fractional Laplacian equations

**Raffaella Servadei**

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**Enrico Valdinoci**

Motivated by the interest shown in the literature for non-local operators of elliptic type, in some recent papers, joint with Enrico Valdinoci, we have studied problems modeled by

$$\begin{cases} (-\Delta)^s u - \lambda u = |u|^{q-2}u & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega, \end{cases} \quad (1)$$

where  $s \in (0, 1)$  is fixed and  $(-\Delta)^s$  is the fractional Laplace operator,  $\Omega \subset \mathbb{R}^n$ ,  $n > 2s$ , is open, bounded and with Lipschitz boundary,  $\lambda > 0$ ,  $2^* = 2n/(n - 2s)$  is the fractional critical Sobolev exponent and  $2 < q \leq 2^*$ , that is problems with subcritical ( $q < 2^*$ ) and critical growth ( $q = 2^*$ ). In problem (1) the Dirichlet datum is given in  $\mathbb{R}^n \setminus \Omega$  and not simply on  $\partial\Omega$ , consistently with the non-local character of the operator  $(-\Delta)^s$ .

Problem (1) represents the non-local counterpart of the following nonlinear elliptic equation

$$\begin{cases} -\Delta u - \lambda u = |u|^{q-2}u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with  $2 < q \leq 2^*$ , where  $2^* = 2n/(n - 2)$  and  $n > 2$ .

Aim of this talk will be to present some results which extend the validity of some existence theorems known in the classical case of the Laplacian to the non-local framework. In particular, in the critical setting our theorems may be seen as the extension of the classical Brezis-Nirenberg result to the case of non-local fractional operators.

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### A concentration phenomenon for a semilinear elliptic equation

**Andrzej Szulkin**

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We consider the problem

$$-\Delta u + V(x)u = Q(x)|u|^{p-2}u, \quad x \in \Omega, \quad u \in H_0^1(\Omega),$$

where  $\Omega \subset \mathbb{R}^N$  is a domain containing the origin,  $2 < p < 2^*$ ,  $V$  is bounded,  $V \geq 0$  and  $\sigma(-\Delta + V) \subset (0, \infty)$ . Further, we assume that  $Q$  is bounded, positive on a small ball centered at the origin and negative outside a slightly larger ball. We show that the solutions of this problem concentrate at the origin as the size of the ball tends to 0. We also consider the same problem with  $Q$  positive on two spots of small size and show that ground state solutions concentrate at one of these spots.

This is joint work with Nils Ackermann.

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### Three weak solutions for elliptic Dirichlet system

**Elisabetta Tornatore**

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Consider the following elliptic Dirichlet system

$$\begin{cases} -\Delta u = \lambda \nabla_u F(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where  $\Omega \subset \mathbb{R}^N$  (with  $N \geq 3$ ) is a non-empty bounded open set with a smooth boundary  $\partial\Omega$ ,  $\lambda$  is a positive real parameter and  $m \geq 1$ ,  $F : \Omega \times \mathbb{R}^m \rightarrow \mathbb{R}$  is a  $C^1$ -Caratheodory function,  $F(x, 0) = 0$  for every  $x \in \Omega$  and  $\nabla_u F = (F_{u_i})_{i=1, \dots, m}$  where  $F_{u_i}$  denotes the partial derivative of  $F$  respect on  $u_i$  ( $i = 1, \dots, m$ ). Under suitable assumptions on  $F$ , the existence of three non zero weak solutions is obtained. The approach is based on critical points theorems.

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## Special Session 33: Nonlinear Elliptic and Parabolic Problems in Mathematical Sciences

Yoshihisa Morita, Ryukoku University, Japan  
Junping Shi, College of William and Mary, USA

Recent developments of mathematical study for nonlinear PDEs (partial differential equations) provide new ideas and various techniques based on calculus of variations, dynamical systems, asymptotic analysis, qualitative theory etc. In this session we bring together researchers in this research area to present new results for nonlinear parabolic and elliptic equations arising from mathematical science and related problems. Various lectures will be delivered by both senior and junior experts in the field.

### Uniqueness of solutions in a gravitational gauge field theory with coexistence of vortices and antivortices

**Jann-Long Chern**  
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**Sze-Guang Yang**

We consider an elliptic equation arising from the study of static solutions with prescribed zeros and poles of the Einstein equations coupled with the classical sigma model and an Abelian gauge field. In the radially symmetric cases, the solutions can be completely described. We also establish the uniqueness of solutions with the presence of vortices and antivortices.

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### Reduction of parabolic PDEs

**Hayato Chiba**  
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It is known that the Swift-Hohenberg equation can be reduced to the Ginzburg-Landau equation (amplitude equation) by means of the singular perturbation method. This means that a solution of the latter equation provides an approximate solution of the former one on a certain time interval. In this talk, a reduction of a certain class of (a system of ) nonlinear parabolic equations is proposed. An amplitude equation of the system is defined and an error estimate of solutions is given. Further, it is proved under certain assumptions that if the amplitude equation has a stable steady state, then a given equation has a stable periodic solution . In particular, near the periodic solution, the error estimate of solutions holds uniformly in  $t > 0$ .

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### Evolution and long-time behavior of the free boundary in nonlinear Stefan problems

**Yihong Du**  
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We consider the following free boundary problem

$$\begin{cases} u_t - d\Delta u = f(u) & \text{for } x \in \Omega(t), t > 0, \\ u = 0 \text{ and } u_t = \mu |\nabla_x u|^2 & \text{for } x \in \Gamma(t), t > 0, \\ u(0, x) = u_0(x) & \text{for } x \in \Omega_0, \end{cases}$$

where  $\Omega(t) \subset \mathbb{R}^n$  ( $n \geq 2$ ) is bounded by the free boundary  $\Gamma(t)$ , with  $\Omega(0) = \Omega_0$ ,  $\mu$  and  $d$  are given positive constants. Our assumptions on  $f(u)$  include monostable, bistable and combustion type nonlinearities.

We show that the free boundary  $\Gamma(t)$  is  $C^1$  outside the convex hull of  $\Omega_0$ , and as  $t \rightarrow \infty$ , either  $\Gamma(t)$  remains bounded and  $u(t, \cdot) \rightarrow 0$  in the  $L^\infty$  norm, or  $\Gamma(t)$  goes to infinity in the sense that it is contained in an annulus of the form  $\{R(t) - C_0 \leq |x| \leq R(t)\}$ , with  $R(t) \rightarrow \infty$  as  $t \rightarrow \infty$ . Moreover,  $R(t)/t \rightarrow k_0 > 0$  as  $t \rightarrow \infty$ .

This is joint work with Hiroshi Matano (Tokyo) and Kelei Wang (Sydney).

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### Multiple positive solutions for p-Laplacian equation with allee effect growth rate

**Chan-gyun Kim**  
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**Junping Shi**

A  $p$ -Laplacian equation with Allee effect growth rate and Dirichlet boundary condition is considered. The existence, multiplicity and bifurcation of positive solutions are proved with comparison and variational techniques. The existence of multiple positive solutions implies that the related ecological system may exhibit bistable dynamics.

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### Bifurcation from a degenerate simple eigenvalue

Ping Liu

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Junping Shi, Yuwen Wang

It is proved that a symmetry-breaking bifurcation occurs at a simple eigenvalue despite the usual transversality condition fails, and this bifurcation from a degenerate simple eigenvalue result complements the classical one with the transversality condition. The new result is applied to an imperfect pitchfork bifurcation, in which a forward transcritical bifurcation changes to a backward one. Several applications in ecological and genetics models are shown.

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### Bifurcation analysis for the Lugiato-Lefever equation in two space dimensions

Tomoyuki Miyaji

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I. Ohnishi, Y. Tsutsumi

We study the steady-state bifurcation of a spatially homogeneous stationary solution for cubic nonlinear Schrödinger equation (CNLS) with damping, detuning, and driving force. It is a model equation derived by L.A. Lugiato and R. Lefever for describing the evolution of the envelope of electric field in an optical cavity with a Kerr medium. It is known by numerical simulations that the Lugiato-Lefever equation (LL equation) in one or two dimensional space has spatially localized solutions in a certain range of parameters. In contrast to CNLS, LL equation does not satisfy any conservation law of CNLS. A localized structure of LL equation appears as a stationary solution, and lots of localized patterns coexist at the same parameter value. It is also known by simulation that, for 2D LL equation, it undergoes a Hopf bifurcation, and a spatially localized and temporary periodic pattern occurs. We analyze the steady-state mode interactions near the pattern-forming instability. Especially, we focus on the solutions which are periodic with respect to a planar square or hexagonal lattice.

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### Stable patterns and Morse index one solutions

Yasuhito Miyamoto

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In this talk we study the shapes of the stable patterns of two problems. One is the stationary problem of a shadow reaction-diffusion system and the other is

the minimization problem of a functional with constraint. We will show the following: If a steady state of a shadow system is stable, then the Morse index of the solution is zero or one, and if a critical point of the minimization problem with constraint is a local minimizer, then the Morse index of the solution of the associated Euler-Lagrange equation is zero or one. It is well-known that the solution with Morse index zero is constant provided that the domain is convex. Therefore we are interested in the shape of the Morse index one solutions.

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### Gradient-like property of a reaction-diffusion system with mass conservation

Yoshihisa Morita

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We consider the following two-component system of reaction-diffusion equations

$$u_t = d\Delta u - g(u + v) + v, \quad v_t = \Delta v + g(u + v) - v,$$

in a bounded domain  $\Omega$  with the homogeneous Neumann boundary conditions. We first prove that the system possesses a Lyapunov function if  $d$  is less than 1. Then we establish a comparison theorem for the spectrum of the linearized operators around an equilibrium solution of the system and a related scalar elliptic equation with a nonlocal term. This talk is based on the recent joint work with Shuichi Jimbo (Hokkaido University).

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### Diffusion-induced blowup and bifurcation from infinity of reaction-diffusion systems

Hirokazu Ninomiya

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Chihiro Aida, Chao-Nien Chen

The diffusion process is usually thought as a trivializing one. However, for some reaction-diffusion systems, the blowup of solutions may occur, though the corresponding ODE possesses a globally attractor. This is called diffusion-induced blowup. To study this phenomenon, in this talk, we consider the bifurcation from infinity. In some class of reaction-diffusion systems, this bifurcation takes place by adding the diffusion.

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### Long time existence of shortening-straightening flow for non-closed planar curves with infinite length

**Shinya Okabe**

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We consider a steepest descent flow for the modified total squared curvature defined on curves. We call the flow the shortening-straightening flow. First it has been proved by A. Polden (1996) that the flow admits smooth solutions globally defined in time, when the initial curve is smooth, closed, and has finite length. In 2002, G. Dziuk, E. Kuwert, and R. Schätzle extended Polden's result to closed curves with finite length in  $n$ -dimensional Euclidean space. We are interested in the following problem: "What is a dynamics of non-closed planar curves with infinite length governed by shortening-straightening flow?" In this talk, we will talk about a long time existence of solution of shortening-straightening flow starting from non-closed planar curve with infinite length. This work is a joint research with M. Novaga (Padova Univ.).

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### Dynamics for an evolution equation describing micro phase separation

**Yoshihito Oshita**

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We study the free boundary problem describing the micro phase separation of diblock copolymer melts in the regime that one component has small volume fraction such that micro phase separation results in an ensemble of small spheres of one component. On some time scale, the evolution is dominated by coarsening and subsequent stabilization of the radii of the spheres. Starting from the free boundary problem restricted to spheres (particles) we derive the effective equations describing the dynamics in this time regime called mean-field models. We first consider the spatially uniform mean-field models in the dilute case, where the self-interaction of particles is dominated. We identify all the steady states and their stabilities, and show the convergence of solutions. Furthermore we see that the steady states are of the form of the sum of at most two Dirac deltas except for disappeared particles. We then consider the models for the joint distribution of particle radii and centers in the screened case, which is an inhomogeneous extension of the dilute case, where a small migration term is remained. We obtain the form of steady states for this model.

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### Connection graphs for Sturm attractors of $S^1$ -equivariant parabolic equations

**Carlos Rocha**

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**Bernold Fiedler, Mathias Wolfrum**

We consider a semilinear parabolic equation of the form  $u_t = u_{xx} + f(u, u_x)$  defined on the circle  $x \in S^1 = \mathbb{R}/2\pi\mathbb{Z}$ . For a dissipative nonlinearity  $f$  this equation generates a dissipative semiflow in the appropriate function space, and the corresponding global attractor  $\mathcal{A}_f$  is called a Sturm attractor. We use the Sturm permutation  $\sigma_f$  introduced for the characterization of Neumann flows to obtain a purely combinatorial characterization of the Sturm attractors  $\mathcal{A}_f$  on the circle. With this Sturm permutation  $\sigma_f$  we show how to construct a connection graph  $\mathcal{G}_f$  representing the Sturm attractor  $\mathcal{A}_f$ .

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### Turing type instabilities in diffusion systems

**Kunimochi Sakamoto**

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The purpose of this talk is to display various situations in which Turing type instabilities occur. The essential ingredients are: (1) diffusion and interaction; (2) existence of unstable subsystems in a stable full system. The diffusion and interaction may take place in the interior or on the boundary of domains for interaction-diffusion systems. In particular, we will show that linear diffusion in the interior and nonlinear interaction on the boundary tend to produce temporally oscillating patterns with nontrivial spatial modes.

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### Wavenumber selection in closed reaction-diffusion systems

**Arnd Scheel**

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Motivated by the plethora of patterns observed in precipitation experiments starting with Liesegang's 1896 study, we investigate pattern formation in the wake of fronts in closed reaction-diffusion systems. We will briefly describe some models and the relation to phase separation models such as the Cahn-Hilliard equation and the Phase-Field System. We will then present results that characterize patterns formed in the wake of fronts.

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### Time delay induced instabilities and Hopf bifurcations in general reaction-diffusion systems

**Junping Shi**

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**Shanshan Chen, Junping Shi, Junjie Wei**

The distribution of the roots of a second order transcendental polynomial is analyzed, and it is used for solving the purely imaginary eigenvalue of a transcendental characteristic equation with two transcendental terms. The results are applied to the stability and associated Hopf bifurcation of a constant equilibrium of a general reaction-diffusion system or ordinary differential equation with delay effects. Examples from chemical reaction and predator-prey models are analyzed using the new techniques. In particular the stability and associate Hopf bifurcation of a Gierer-Meinhardt system with the gene expression time delays is analyzed by using our method.

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### Spatially inhomogeneous time-periodic solutions in delayed Nicholson's blowflies model

**Ying Su**

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**Xingfu Zou**

In this talk, we focus on the spatially inhomogeneous and time periodic solutions of a delayed diffusive Nicholson's blowflies model subject to the no flux boundary condition. In particular, we investigate the existence and stability of the spatially inhomogeneous periodic solutions as well as their dependence on some model parameters, especially the diffusive rate of the mature population.

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### Stability of patterns in some reaction-diffusion systems with the diffusion-driven instability

**Kanako Suzuki**

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We consider the following system of a single reaction-diffusion equation coupled with an ordinary differential equation:

$$u_t = f(u, v), \quad v_t = D\Delta v + g(u, v).$$

Here,  $u = u(x, t)$ ,  $v = v(x, t)$  for  $x \in \Omega$ ,  $t > 0$ , and  $\Omega$  is a bounded domain with smooth boundary. We impose the Neumann boundary condition on  $v$ . This

type of models exhibits the diffusion-driven instability. However, we show that, under some assumptions on non-linearities, all regular stationary solutions are unstable. This talk is devoted to understanding its mechanism. This work is a joint work with Anna Marciniak-Czochra (University of Heidelberg) and Grzegorz Karch (University of Wrocław).

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### Isolated singularities of nonlinear polyharmonic inequalities

**Steven Taliaferro**

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We discuss how the blow-up behavior at an isolated singularity of solutions of nonlinear polyharmonic inequalities depends on an exponent in the inequalities. In particular, we show that for certain exponents there exists an a priori bound on the rate of blow-up and for other exponents the solutions can blow up arbitrarily fast. Remarkably, the optimal bound for solutions may itself not be a solution.

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### Diffusion driven instabilities on evolving surfaces

**Necibe Tuncer**

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**Anotida Madzwamuse, A.J. Meir**

Reaction diffusion systems defined on evolving surfaces has many application in mathematical biology. Examples of such applications include tumor growth, pattern formation on seashells, butterfly wing pigmentation patterns and animal coat markings. We develop and analyze a finite element method to approximate solutions of nonlinear reaction diffusion systems defined on evolving surfaces. The method we propose is based on radially projected finite elements.

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### On global bifurcation of bifurcation curves of some multiparameter problems

**Shin-Hwa Wang**

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**Kuo-Chih Hung**

We study the global bifurcation of bifurcation curves of positive solutions for some nonlinear multiparameter problems of the form

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**Qualitative analysis of a diffusive predator-prey model with modified Leslie-Gower and Holling-type II schemes**

**Jun Zhou**

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**Junping Shi**

We investigate the existence, multiplicity and stability of positive solutions to a prey-predator model with modified Leslie-Gower and Holling-Type II schemes

$$\begin{cases} -\Delta u = u \left( a_1 - bu - \frac{c_1 v}{u + k_1} \right) & \text{in } \Omega, \\ -\Delta v = v \left( a_2 - \frac{c_2 v}{u + k_2} \right) & \text{in } \Omega, \\ u \geq 0, v \geq 0 & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega \subset \mathbb{R}^N$  ( $N \geq 1$ ) is a bounded domain with a smooth boundary  $\partial\Omega$ , the parameters  $a_i, b, c_i, k_i$  ( $i = 1, 2$ ) are positive numbers,  $u$  and  $v$  are the respective populations of prey and predator. Here, we say  $(u, v)$  with  $u|_{\partial\Omega} = v|_{\partial\Omega} = 0$  is a positive solution if  $(u, v)$  is a solution and  $u, v > 0$  in  $\Omega$ .

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## Special Session 34: Multi-phase Flows in Porous Media and Related Systems

**David Ambrose**, Drexel University, USA  
**Xiaoming Wang**, Florida State University, USA  
**Steven Wise**, University of Tennessee, USA

Multiphase fluid flow in porous media is of great importance in many areas of science and engineering applications. Two well-known examples are the flow of groundwater and oil recovery in petroleum engineering. There are also classical applications in materials science (such as Hele-Shaw cells), as well as emerging applications in fuel cell technology (water management in PEM fuel cells), and biomedical science (tumor growth modeled as flow in porous media). The associated mathematical problems are intriguing and challenging, involving free boundaries as well as singular perturbations and stiff problems.

Much effort has been devoted to the investigation of multi-phase flows in porous media, with much recent progress on the mathematical theories for the physical models. Nevertheless there are still many unresolved mathematical and physical issues. Hence it seems a good time to have a special session devoted to the problem of multi-phase flows in porous media and related problems. There are two principal approaches to multi-phase flow. The first approach treats the interface as a sharp one with zero width. The second approach recognizes the micro-scale mixing and hence treats the interface as a transition layer with finite (small) width (the so-called diffuse interface model or phase field model). Our plan is to have some of the leading experts from both camps, including people working on modeling, analysis and simulation, to report on recent progress and future challenges associated with multi-phase flow in porous media and closely related systems.

### Removing the stiffness from 3D interfacial flow with surface tension

**David Ambrose**  
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**Michael Siegel, Svetlana Tlupova**

We present a boundary integral method for the efficient computation of three-dimensional irrotational free surface flows in the presence of surface tension. Specifically, we will consider a model problem for porous media flow. By considering a model problem, we are able to focus on the removal of stiffness, rather than other issues such as computing the Birkhoff-Rott integral.

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### Numerical methods for some phase-field models

**Franck Boyer**  
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**C. Lapuerta, S. Minjeaud, B. Piar**

In this talk, I will describe some numerical methods that we propose for the computation of multi-phase flows through a phase-field model consisting in the coupling of a Cahn-Hilliard system and the Navier-Stokes equation.

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### Solvability of a generalized Buckley-Leverett model

**Nikolai Chemetov**  
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**W. Neves**

We propose a new mathematical modelling of the Buckley-Leverett system, which describes the two-phase flows in porous media.

We prove the solvability of the initial-boundary value problem for a deduced model, being a coupled system of hyperbolic/elliptic type equations. In order to show the solvability result, we consider an approximated parabolic-elliptic system. Since the approximated solutions do not have ANY type compactness property, the limit transition is justified by the kinetic method.

The main issue is to study a linear (kinetic) transport equation, instead of the nonlinear original system.

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### Functionalized Cahn-Hilliard equation: competitive evolution of bilayers and pores

**Shibin Dai**  
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**Keith Promislow**

The functionalized Cahn-Hilliard equation is introduced as a phase field model to describe the evolution of complex nanoscale structures similar to those observed in Polymer Electrolyte Membrane (PEM) fuel cells. Such complex structures include single lay-

ers, bi-layers, pore networks and micelles, etc. We concentrate on the motion of closed bi-layers and pores. Using asymptotic analysis, we analyze their inner structures and derive the leading order normal velocity in different time scales. Also we will show the mechanism under which they compete with each other.

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### Cahn-Hilliard-Navier-Stokes systems with nonlocal interactions

**Maurizio Grasselli**

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**Sergio Frigeri**

A well-known diffuse interface model consists of the Navier-Stokes equations nonlinearly coupled with a convective Cahn-Hilliard equation. This system describes the evolution of an incompressible isothermal mixture of fluids and it has been investigated by many authors. Here we discuss a variant of this model where the standard Cahn-Hilliard equation is replaced by its nonlocal version. More precisely, the gradient term in the free energy functional is replaced by a spatial convolution operator acting on the order parameter. We intend to present some recent results on the global longtime behavior of the (weak) solutions.

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### A rescaling scheme and its applications to free boundary problems

**Shuwang Li**

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In this talk, I present a time and space rescaling scheme for the computation of moving interface problems. The idea is to map time-space such that the interfaces can evolve exponentially fast in the new time scale while the area/volume enclosed by the interface remains unchanged. The rescaling scheme significantly reduces the computation time (especially for slow growth), and enables one to accurately simulate the very long-time dynamics of moving interfaces. We then implement this scheme in a Hele-Shaw problem, examine the dynamics for a number of different injection fluxes, and present the largest and most pronounced viscous fingering simulations to date. I then generalize the idea to be able to compute multiple interfaces. I will also give examples of the curvature weakening model of the Hele-Shaw problem.

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### Stability of 2D incompressible flows under 3D perturbations

**Milton Lopes Filho**

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**C. Bardos, D. Niu, H. Nussenzveig Lopes, E. Titi**

We discuss results on preservation of 2D symmetry under 3D viscous flows with low regularity and corresponding results for inviscid flow.

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### Liquid drops sliding down an inclined plane

**Antoine Mellet**

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**Inwon Kim**

We investigate the properties of a model describing the motion of liquid drops sliding down an inclined plane (the so-called quasi-static approximation model). We prove existence and uniqueness of a solution and investigate its long time behavior for both homogeneous and inhomogeneous medium (i.e. constant and non-constant contact angle).

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### A Cahn-Hilliard model with dynamic boundary conditions

**Alain Miranville**

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Our aim in this talk is to discuss the dynamical system associated with the Cahn-Hilliard equation with dynamic boundary conditions. Such boundary conditions take into account the interactions with the walls for confined systems. We are in particular interested in a model which accounts for the conservation of mass, both in the bulk and on the walls.

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### Capillary driven viscous fingering in Buckley-Leverett models

**Iain Moyses**

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The modelling of two fluid displacement in porous media is of great industrial interest in soil remediation and environmental engineering. One of the phenomena of interest is the formation of fingering patterns that emerge at the fluid interface. There

has been a large focus from the academic community to accurately model the formation of fingers under various physical scenarios. In this talk, we look at an oil-water in situ flushing model based on Buckley-Leverett formulations. We start with an overview of the Taylor-Saffman instability applied to lateral flow when we neglect surface tension between fluids. We then consider the effect of introducing a small local capillary effect at the interface between fluids, motivated by a physical scaling based on industrially relevant parameters. With the introduction of the capillary effect we present some singular perturbation analysis as well as numerical computations to analyze and compare the finger growth to that of the case with no surface interaction. Depending on the capillary-saturation dependence, the steady-state solutions that form can either be  $C^\infty$  smooth or  $C^0$  with compact support; this is due to the appearance of degenerate terms in the differential equations. We present results under degenerate and non-degenerate conditions.

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### Thermal compositional model based on the diffuse interface assumption

**Paul Papatzacos**

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A thermal compositional model for three phase flow must face a number of challenges. (1) The internal degrees of freedom of the atoms must be taken into account in the thermodynamic description of the fluid. (2) The transport coefficients must be known functions of the temperature and the densities, even for unstable states. (3) Numerical instabilities due to small thermal conductivities must be overcome. These challenges are discussed and solutions presented. A numerical example involving injection of cold fluid in a warmer reservoir is presented.

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### A nonstiff boundary integral method for 3D interfacial flow with surface tension

**Michael Siegel**

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**David Ambrose, Svetlana Tlupova**

A nonstiff boundary integral method for moving boundaries in 3D flow is presented, with an application to porous media flow. The velocity of the interface is given in terms of the Birkhoff-Rott (B-R) integral. Numerical stiffness due to surface tension is removed by using a generalized isothermal parameterization of the surface and approximating the B-R integral at high wavenumbers by a small scale decomposition. Results are presented for a doubly-periodic

interface and a closed interface, the latter of which is discretized using moving coordinate patches and partitions of unity.

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### 3-D pore-scale resolved model for coupled species/charge/fluid transport in a vanadium redox flow battery

**Ying Sun**

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**Gang Qiu, Abhijit S. Joshi, C.R. Dennison, K.W. Knehr, E.C. Kumbur**

The vanadium redox flow battery (VRFB) has emerged as a viable grid-scale energy storage technology that offers cost-effective energy storage solutions. In this paper, a novel methodology is introduced for modeling of the transport mechanisms of electrolyte flow, species and charge in the VRFB at the pore scale of the electrodes; that is, at the level where individual carbon fiber geometry and electrolyte flow are directly resolved. The detailed geometry of the electrode is obtained using X-ray computed tomography (XCT) and calibrated against experimentally determined pore-scale characteristics. The processed XCT data is then used as geometry input for modeling of the electrochemical processes in the VRFB. The flow of electrolyte through the pore space is modeled using the lattice Boltzmann method (LBM) while the finite volume method (FVM) is used to solve the coupled species and charge transport and predict the performance of the VRFB under various conditions. An electrochemical model using the Butler-Volmer equations is used to provide species and charge coupling at the surfaces of the carbon fibers. Results are obtained for the cell potential distribution, as well as local concentration, overpotential and current density profiles undergalvanostatic discharge conditions. The cell performance is investigated as a function of the electrolyte flow rate and external drawing current. The model developed here provides a useful tool for building the structure-property-performance relationship of VRFB electrodes.

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### A coupled level set-moment of fluid method for incompressible two-phase flows

**Mark Sussman**

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**M. Jemison, E. Loch, M. Shashkov, M. Arienti, M. Ohta, Y. Wang**

A coupled level set and moment of fluid method (CLSMOF) is described for computing solutions to incompressible two-phase flows. The local piecewise linear interface reconstruction (the CLSMOF

reconstruction)uses information from the level set function, volume of fluid function, and reference centroid, in order to produce a slope and an intercept for the local reconstruction. The level set function is coupled to the volume-of-fluid function and reference centroid by being maintained as the signed distance to the CLSMOF piecewise linear reconstructed interface. The nonlinear terms in the momentum equations are solved using the sharp interface approach recently developed by Raessi and Pitsch (2009). We have modified the algorithm of Raessi and Pitsch from a staggered grid method to a collocated grid method and we combine their treatment for the nonlinear terms with the variable density, collocated, pressure projection algorithm developed by Kwatra et al (2009). A collocated grid method makes it convenient for using block structured adaptive mesh refinement (AMR) grids. Many 2D and 3D numerical simulations of bubbles, jets, drops, and waves on a block structured adaptive grid are presented in order to demonstrate the capabilities of our new method.

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### **Two phase fluid flows with biological microstructures**

**Xiaoqiang Wang**

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Phase field method has become a widely used tool to characterize the evolution of surfaces in material science, biology, imaging processing, etc. In this talk the speaker presents how to apply phase field method to the evolution of biological microstructures in fluid. The content includes the study of the shape of lipid vesicles and its transformation and oscillations in fluid systems.

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## Special Session 35: Qualitative Theory of Nonlinear ODEs and Applications

Fabio Zanolin, University of Udine, Italy

The study of qualitative aspects of nonlinear ordinary differential equations is a classical and important topic which is widely studied for its interconnections with nonlinear analysis and its applications to mathematical sciences. The aim of this session is to put together different experts in this area in order to show the central role of the qualitative theory of ODEs both from the point of view of the applications and also as a source of new and interesting problems which may lead to the introduction and development of new abstract tools. Some of the main topics to be included in this section are: Autonomous and non-autonomous systems, Limit cycles, Oscillation theory and comparison theory, Boundedness of solutions, Nonlinear oscillations, Coupled oscillators, Transformation and reduction of equations and systems, Bifurcation, Periodic solutions, Complex behavior and chaotic systems, Dynamical systems, ODEs methods for particular solutions of PDEs (radially symmetric solutions, traveling front solutions), Homoclinic and heteroclinic solutions, Applications of variational and topological methods to nonlinear boundary value problems for ODEs, Equations and systems on manifolds, Asymptotic behavior of the solutions, Applications to ODE models in mathematical sciences.

### A local minimum theorem and applications to nonlinear ordinary differential problems

**Gabriele Bonanno**

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A local minimum theorem for a continuously Gâteaux differentiable function, possibly unbounded from below and without any weak continuity assumption, is presented. As a consequence, some classes of nonlinear ordinary differential problems are investigated.

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### Positive solutions to second order ODEs with indefinite weight: multiplicity and complex dynamics

**Alberto Boscaggin**

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**F. Zanolin**

We consider scalar second order differential equations of the type  $u'' + a(t)g(u) = 0$ , being  $a(t)$  a function which changes its sign ("indefinite weight"). In the past few decades, this class of equations has been widely investigated, as a simple model exhibiting high multiplicity of sign-changing solutions (with different boundary conditions) and, eventually, chaotic dynamics. Here, we re-consider the problem from the point of view of positive solutions only, presenting some recent results which show that, again, multiplicity and complex dynamics can occur.

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### Chaotic dynamics in some pendulum type equations

**Lakshmi Burra**

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We investigate the presence of chaotic-like dynamics for a class of second order ODEs (including some pendulum-type equations) using the concept of "stretching along paths" and the theory of "linked twist maps". The chaotic dynamics considered is of the coin-tossing type as in the Smale's Horseshoe. The proof relies on some recent results about chaotic planar maps combined with the study of geometric features which are typical of linked twist maps.

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### Existence and multiplicity of solutions for a Dirichlet boundary value problem

**Pasquale Candito**

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We present some recent results concerning existence, multiplicity and localization of solutions for a Dirichlet boundary value problem involving the  $p$ -Laplacian. The approach adopted is chiefly based on critical point theory.

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### Existence results of solutions for $p(x)$ -Laplacian elliptic Dirichlet problems

**Antonia Chinni**

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We investigate the existence and multiplicity of weak solutions for Dirichlet problems that involve the  $p(x)$ -Laplace operator. The results are based on two multiple critical points theorems. The first theorem, established by G. Bonanno and P. Candito [Non-differentiable functionals and applications to elliptic problems with discontinuous nonlinearities, *J. Differential Equations*, 244, 3031-3059, (2008)], is applied to obtain three weak solutions for a Dirichlet problem in case of small perturbations of the nonlinear term and when  $p^- > 1$ . The second theorem, established by G. Bonanno and S.A. Marano [On the structure of the critical set of nondifferentiable functions with a weak compactness condition, *Appl. Anal.*, 89,1-10 (2010)], is applied to obtain three weak solutions for a Dirichlet problem in absence of small perturbations and whenever  $p^- > N$ .

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### Mixed boundary value problems with Sturm-Liouville equations

**Giuseppina D'Agui**

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We present some existence results of solutions for a mixed boundary value problem with Sturm-Liouville equation where no asymptotic condition on the nonlinear term either at zero or at infinity is requested. The approach is based on variational methods

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### Radial solutions of Dirichlet problems with concave-convex nonlinearities

**Francesca Dalbono**

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**Walter Dambrosio**

We prove the existence of a double infinite sequence of radial solutions for a Dirichlet concave-convex problem associated with an elliptic equation in a ball. We are interested in relaxing the classical positivity condition on the weights, by allowing the weights to vanish. The idea is to develop a topological method and to use the concept of rotation number. The solutions are characterized by their nodal properties.

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### Existence of solutions to second order boundary value problems.

**Nicholas Fewster**

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**C.C. Tisdell**

This talk examines the existence of solutions to nonlinear singular boundary value problems. This area of differential equations arises constantly in the modeling of dynamical phenomenon, for example; thermal explosions, electrohydrodynamics and radially symmetric nonlinear diffusion in the  $n$ -dimensional sphere. This naturally motivates the study for further understanding of the solutions of the qualitative nature. The main focus is generating novel a priori bounds on all possible solutions via differential inequalities. Then by using these new a priori bounds and existence theorems of D. O'Regan to produce existence of solution(s).

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### Star flows and singular hyperbolicity

**Shaobo Gan**

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**Yi Shi, Lan Wen**

A vector field is called star vector field if every singularity and periodic orbit of its small perturbations are hyperbolic. We will prove that every Lyapunov stable chain recurrent class of a star flow is singular hyperbolic. Especially, if dimension of phase space is less than 5, then every chain recurrent class is singular hyperbolic.

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### Nonlinear first order systems in the plane with positively homogeneous principal term

**Maurizio Garrione**

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**A. Boscaggin, A. Fonda**

In the first part of the lecture, we will overview some basic features of positively homogeneous Hamiltonian systems in the plane, i.e., systems of the type

$$Ju' = \nabla H(u), \quad u \in \mathbb{R}^2,$$

where

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

is the standard symplectic matrix and  $H$  is a  $C^1$ -function, with locally Lipschitz continuous gradient,

satisfying

$$0 < H(\lambda u) = \lambda^2 H(u), \quad u \in \mathbb{R}^2, \lambda > 0.$$

We will then focus on the  $T$ -periodic boundary value problem ( $T > 0$ ) for a nonlinear planar system like

$$Ju' = \nabla H(u) + R(t, u),$$

with  $H(u)$  as above and  $R(t, u)$  having a sublinear growth in the variable  $u$ . Our attention will be devoted to the possible occurrence of resonance phenomena, depending on some minimal period associated with the autonomous system  $Ju' = \nabla H(u)$ . We will explore some recent results of existence and multiplicity of solutions for both the nonresonant and the resonant case (these last ones mainly relying on a suitable planar version of the Landesman-Lazer condition). With a similar approach, we can also deal with other kinds of boundary value problems, as it will be briefly shown at the end of the seminar.

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#### Existence and multiplicity results for second order differential problems

**Roberto Livrea**

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Some results concerning the existence and the multiplicity of solutions for second order boundary value problems will be pointed out. The approach adopted is fully variational and it relies on recent general abstract critical points theorems.

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#### Existence results for parameterized Emden-Fowler equations

**Giovanni Molica Bisci**

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We present some recent results, obtained in collaboration with G. Bonanno and V. Rădulescu on the existence of multiple solutions for a class of Emden-Fowler equations. Some comparisons with several results present in literature are pointed out.

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#### Periodic solutions of the prescribed curvature equation

**Pierpaolo Omari**

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We discuss existence, multiplicity and regularity of periodic solutions of the one-dimensional curvature equation

$$-\left(u'/\sqrt{1+u'^2}\right)' = f(t, u)$$

in the space of bounded variation functions. We study the case where  $f$  interacts with the first and the second eigenvalue (or more generally with the first non-trivial branch of the Fucik spectrum) of the one-dimensional 1-Laplace operator with periodic boundary conditions. The problem of the existence of subharmonic solutions is considered as well.

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#### Liapunov-type integral inequalities for higher order dynamic equations on time scales

**Saroj Panigrahi**

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In this paper, we obtain Liapunov-type integral inequalities for certain nonlinear, nonhomogeneous dynamic equations of higher order without any restriction on the generalized zeros of their higher-order delta derivatives of the solutions by using elementary time scale analysis. As an applications of our results, we show that oscillatory solutions of the equation converge to zero as  $t \rightarrow \infty$ . Using these inequalities, it is also shown that  $(t_{m+k} - t_m) \rightarrow \infty$  as  $m \rightarrow \infty$ , where  $1 \leq k \leq n - 1$  and  $\langle t_m \rangle$  is an increasing sequence of generalized zeros of an oscillatory solution of  $D^n y + y(\sigma(t))f(t, y(\sigma(t)))|y(\sigma(t))|^{p-2} = 0$ ,  $t \geq 0$ , provided that  $W(\cdot, \lambda) \in L^\mu([0, \infty)_{\mathbb{T}}, \mathbb{R}^+)$ ,  $1 \leq \mu \leq \infty$  and for all  $\lambda > 0$ . A criterion for disconjugacy of nonlinear homogeneous equation is obtained in an interval  $[a, b]_{\mathbb{T}}$ .

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#### Periodic solutions and chaotic dynamics in 3D equations with applications to Lotka Volterra systems

**Alfonso Ruiz-Herrera**

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**F. Zanolin**

We discuss a geometric configuration for a class of homeomorphisms in  $\mathbb{R}^3$  producing the existence

of infinitely many periodic points as well as a complex dynamics due to the presence of a topological horseshoe. We also show that such a class of homeomorphisms appear in the classical Lotka- Volterra system.

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### Symbolic dynamics for the $N$ -centre problem at negative energies

**Nicola Soave**

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**Susanna Terracini**

We consider the planar  $N$ -centre problem, namely the study of the ODE

$$\ddot{x}(t) = - \sum_{k=1}^N \frac{m_k}{|x(t) - c_k|^3} (x(t) - c_k),$$

where  $x : I \subset \mathbb{R} \rightarrow \mathbb{R}^2$  and the positions of the centres  $c_k$  are fixed in  $\mathbb{R}^2$ . We prove the existence of infinitely many collisions-free periodic solutions with small (in absolute value) and negative energy; these solutions satisfy particular topological constraints, which are expressed in terms of the partitions of the centres in two different non-empty sets. The proof is based upon perturbative, variational and geometric techniques. As a consequence, for small and negative values of the energy, the dynamical system associated to the motion equation has a symbolic dynamics, where the symbols are the partitions of the previous type.

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### An even solution to a fourth-order ODE

**Greg Spradlin**

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We study a fourth-order ordinary differential equation of the form  $u^{(4)} - cu'' + a(x) = f(u)$ , where  $f$  is similar to a power function  $|q|^{p-2}q$  ( $p > 1$ ) and  $a$  is even with  $a(x) \rightarrow l > 0$  as  $|x| \rightarrow \infty$ . Using variational mountain-pass and concentration-compactness ideas, it is shown that the equation has a nontrivial solution homoclinic to 0.

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### Entire parabolic trajectories as minimal phase transitions

**Gianmaria Verzini**

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**V. Barutello, S. Terracini**

For the class of anisotropic Kepler problems in  $\mathbb{R}^d \setminus \{0\}$  with homogeneous potentials, we seek parabolic trajectories having prescribed asymptotic directions at infinity and which, in addition, are Morse minimizing geodesics for the Jacobi metric. Such trajectories correspond to saddle heteroclinics on the collision manifold, are structurally unstable and appear only for a codimension-one submanifold of such potentials. We give them a variational characterization in terms of the behavior of the parameter-free minimizers of an associated obstacle problem. We then give a full characterization of such a codimension-one manifold of potentials and we show how to parameterize it with respect to the degree of homogeneity.

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### Unbounded solutions for a class of singular hamiltonian systems

**Donglun Wu**

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**Shiqing Zhang**

The existence of unbounded solutions is proved for a class of singular Hamiltonian systems  $\dot{u}(t) + \nabla V(u(t)) = 0$  by taking limit for a sequence of periodic solutions which are obtained by variational methods.

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### The Poincaré-Birkhoff “twist theorem”: some remarks and recent applications to ODEs

**Fabio Zanolin**

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After a brief historical survey of the topic, we present and discuss some variants of the Poincaré-Birkhoff fixed point theorem which look particularly suitable to deal with the study of periodic solutions to non-autonomous planar Hamiltonian system. We also propose some recent applications which, in our opinion, put in evidence how such a classical theorem may be useful in providing some sharp results for second order nonlinear ODEs.

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## Special Session 36: Stochastic Partial Differential Equations and their Optimal Control

Wilfried Grecksch, Martin-Luther-University, Germany

This special session should give a general overview as well about new tendencies in the field of stochastic partial differential equations (spdes) as about optimal control problems for spdes. The invited speakers will discuss issues related to numerics/computations, existence and uniqueness of solution processes for spdes and properties of spdes. Another main goal will be to discuss optimality conditions for optimal control problems with spdes and methods to approximate optimal controls. Especially, spdes and applications are discussed driven by fractional noise processes.

### Portfolio optimization under partial with expert opinions

**Gabih Abdelali**

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**Rüdiger Frey, Ralf Wunderlich**

We investigate optimal portfolio strategies in a market with partial information on the drift. The drift is modelled by continuous Markov chains with finitely many states which are not observable. Information on the drift is obtained from observations on the stocks prices. Moreover Expert Opinions in the form of signals at random discrete time points are included in the analysis. We derive the filtering equation for the return process and incorporate the filter into the state variables of the optimization problem. This problem is studied with dynamic programming method. In particular we propose a policy improvement method to obtain computable approximations of the optimal strategy. Numerical results are presented at the end.

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### Diffusion in heterogeneous domains

**Vo Anh**

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**J.M. Angulo, M.D. Ruiz-Medina**

This work is motivated by the problem of saltwater intrusion in coastal aquifers. These underground aquifers can be a major source of water for irrigation, industrial and town water usage. Due to their proximity to the ocean, excessive demand for groundwater may result in saltwater intrusion with a substantial loss of agricultural land. It is therefore an essential task to develop suitable mathematical models and computational tools for visualisation and prediction of salinity diffusion in aquifers for scenario analysis and management planning. Due to the complex nature of the aquifers, with alternating layers of permeable sandstone and impermeable clay sediments, it is a challenge to understand the role of heterogeneity in the aquifer and in the physical law governing the saltwater flow. We will formulate the

problem in the framework of fractional diffusion in heterogeneous domains. We will outline the existing theory of diffusion on fractals, then move on to a theory of diffusion on multifractals based on the RKHS approach. This latter theory yields a class of models for fractional diffusion with variable singularity order in heterogeneous domains. We will look at the wavelet transforms of these space-time random fields for their statistical estimation and interpolation.

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### A filtering problem for a linear stochastic Schrödinger equation

**Wilfried Grecksch**

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Let  $(V, H, V^*)$  be rigged complex Hilbert spaces and let  $A : V \rightarrow V^*$  be a linear coercive operator. If we choose  $(H^1(G), L^2(G), H^{-1}(G))$  and the Laplacian operator  $\Delta$  in a bounded domain  $G$  with homogeneous Dirichlet boundary conditions then we get an example for  $A$ . All processes are defined on a fixed complete probability space. The state equation is given by

$$dX(t) = iAX(t)dt + g(t)dB(t), \quad t \geq 0 \quad X(0) = X_0,$$

where the solution process is defined by the variational solution. The noise process  $B$  is a Gaussian process with values in a real separable Hilbert space  $K$  and its covariance operator is defined by  $C(t, s)Q$  and  $C$  is a real function with

$$\frac{\partial^2 C}{\partial t \partial s} \in L^2([0, T]^2)$$

and  $Q$  is a trace class operator in  $K$ . The fractional Brownian motion, the subfractional Brownian motion and Liouville fractional Brownian motion are examples for  $B$ . The observation process  $Y$  takes values in a  $n$  dimensional subspace  $H_n$  of  $H$  and has the form

$$dY(t) = G(t)X(t)dt + f_1(t)dB^n(t) + f_2(t)dW(t) \\ Y(0) = 0.$$

The noise  $B^n$  is a  $n$  dimensional part of  $B$  and  $W$  is a Brownian motion with values in  $H_n$ .  $g, f_1, f_2$  are deterministic Hilbert Schmidt operator valued functions and the stochastic differentials are defined in

the sense of *Ito*.

We derive explicit expressions for the optimal filter  $E\{X(t)|Y(s)\}_{s \in [0,t]}$ .

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### Vector calculus on fractals and applications

**Michael Hinz**

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**Michael Roeckner, Alexander Teplyaev**

If a fractal set carries a diffusion (respectively local regular Dirichlet form) the existing analysis on fractals suffices to study many second order PDE. However, until recently the notion of gradient respectively first order derivative had not been understood very well. In this talk we present some new developments concerning a vector calculus on fractals that is flexible enough to have nice applications in PDE and SPDE and to provide some of the basics for related control problems.

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### An optimal control problem for a nonlinear controlled stochastic Schrödinger equation

**Diana Keller**

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First, we consider a nonlinear controlled Schrödinger problem with additive noise, which is given by

$$dX(t, x) = -iAX(t, x) dt + iU(t, x)X(t, x) dt + i\lambda f(X(t, x)) dt + i dW(t, x)$$

for all  $t \in [0, T]$  and all  $x \in (0, 1)$ , where  $A$  is the one-dimensional negative Laplacian,  $\lambda \in \mathbb{R}_+$ ,  $f(X(t, x))$  is a twice continuously differentiable, Lipschitz-continuous, growth-bounded nonlinearity and  $W(t, \cdot)$  is an  $L^2[0, 1]$ -valued  $Q$ -Wiener process. The admissible control  $U(t, x)$  is a stochastic process such that a unique variational solution  $X(t, x)$  exists. For the sake of simplicity, we choose initial and homogeneous boundary conditions in the following way

$$X(0, x) = \varphi(x) \in H^1[0, 1], \quad \forall x \in [0, 1],$$

$$\left. \frac{\partial}{\partial x} X(t, x) \right|_{x=0} = \left. \frac{\partial}{\partial x} X(t, x) \right|_{x=1} = 0, \quad \forall t \in [0, T].$$

Existence, uniqueness and some smoothness properties of the variational solution are discussed. Moreover, we are interested in minimising the objective functional

$$J(U) = \alpha_0 E \int_0^T |X(t, 0) - f_0(t)|^2 dt + \alpha_1 E \int_0^T |X(t, 1) - f_1(t)|^2 dt$$

relative to the control  $U(t, x)$ . Here,  $X(t, x)$  is the solution of the nonlinear stochastic Schrödinger problem belonging to the control,  $\alpha_0, \alpha_1 \in \mathbb{R}$  and  $f_0(t), f_1(t) \in L^2[0, T]$  are given functions. The question of solvability of this control problem is treated as well.

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### On random partial differential equations

**Antje Mugler**

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In addition to stochastic partial differential equations driven by some kind of white noise also partial differential equations with random data especially random coefficients play an important role in a number of applications in engineering. For example the modelling of complex systems like subsurface flows with random permeability can be mentioned here. Also the solution of such random partial differential equations is very often part of uncertainty quantification procedures in this context and has been of growing interest in the last years.

In this talk we present conditions for the existence of a unique weak solution to an illustrative model problem where the random coefficient can be strictly bounded away from zero and above by random variables only. This is an important issue from the application point of view but the classical deterministic approach to solve the problem does not carry over to this case. So we introduce an alternative stochastic Galerkin approach yielding a sequence of approximate solutions converging to the exact solution in the natural topology. Furthermore we present an example where the solutions of the traditional stochastic Galerkin approach, which is widely used in practice nowadays, fails to converge to the exact solution in the natural topology.

This talk is based on joint work with Hans-Jörg Starkloff (Westfälische Hochschule Zwickau).

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### Regularization of ordinary and partial differential equations by noise

**Michael Roeckner**

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It is well known that there are ordinary differential equations (ODE) which have no or many solutions for a given initial condition, but have a unique solution if one adds a sufficiently large noise. We shall first explain this type of "regularization by noise"

on the level of the corresponding Fokker-Planck-Kolmogorov equations both for ODE in finite and infinite dimensional state spaces. Then we shall recall a concrete ODE in finite dimensions given by a merely p-integrable vectorfield which has a unique strong solution when perturbed by a Brownian noise. Finally, we shall present an analogous new result in infinite dimensions, which is applicable to stochastic partial differential equations.

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### A maximum principle for a distributed stochastic optimal control problem

**Christian Roth**

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**W. Grecksch**

We introduce a controlled parabolic Itô equation

$$\begin{aligned} X(t, x) &= X(0, x) + \int_0^t \alpha \Delta_x X(s, x) ds \\ &+ \int_0^t f(s, X(s, x), U(s)) ds \\ &+ \int_0^t \left( X(s, x) g(s), dB^h(s) \right) \end{aligned}$$

for a certain class  $\mathbb{A}$  of admissible controls, where the solution is defined in the sense of [2]. The optimal control problem consists of

$$\min \{J(U, X) : U \in \mathbb{A}\},$$

where

$$J(U, X) = E\Phi(X(t)) + E \int_0^t f(s, X(s), U(s)) ds.$$

An optimality condition of maximum principle type is developed.

### Literature

- [1] F. Biagini, Y. Hu, B. Øksendal and A. Sulem, A stochastic maximum principle for processes driven by fractional Brownian motion, *Stochastic Processes Appl.*, **100(2002)**.  
 [2] C. Roth and D. Julitz, An infinite dimensional quasilinear evolution equation driven by an infinite dimensional Brownian motion, *Festschrift in Celebration of Prof. Dr. Wilfried Grecksch's 60<sup>th</sup> Birthday*.

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### Kolmogorov equations for randomly forced fluids

**Martin Sauer**

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We study so called generalized Newtonian fluids arising as a more general model of the Navier-Stokes equations in mathematical fluid dynamics. A prominent example are the Power Law or Ladyzhenskaya fluids having a power law type structure with exponent  $p$  for the stress tensor. The main features of such equations besides the better description of non-Newtonian behaviour are global uniqueness results depending on the exponent  $p$ . Under stochastic forcing these equations have been discovered only recently and in this talk we will consider them perturbed by additive noise. We prove existence of invariant measures and study the associated Kolmogorov operator in  $L^q$ -spaces of this measure. Under conditions on the exponent  $p$  it is possible to establish a uniqueness result.

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### Generalized polynomial chaos expansion and the solution of random pdes

**Hans-Joerg Starkloff**

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Certain partial differential equations with random data, e.g., random coefficients or forcing terms, can be solved approximately using Hermite or generalized polynomial chaos expansions in connection with stochastic Galerkin methods. In the talk some basic problems related to the use of generalized polynomial chaos expansions for such applications are investigated.

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### Random attractors for multivalued lattice dynamical systems with multiplicative noise

**José Valero**

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**T. Caraballo, F. Morillas**

In this work we study a stochastic lattice dynamical system with multiplicative noise from the point of view of the theory of global random attractors. Using a suitable change of variable the stochastic system is transformed into a random one. The nonlinear term of the system satisfy some dissipative and growth conditions ensuring existence of solutions of the Cauchy problem, but no uniqueness. Hence, we define for it a multivalued random dynamical system and prove the existence of a random global attractor.

The more difficult part in the proof comparing with the single-valued case is to obtain the measurability of the attractor.

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### Ergodic properties of stochastic curve shortening flow

**Max Von Renesse**

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**Abdelhadi Es-Sarhir, Wilhelm Stannat**

We discuss 1+1 dimensional random interface models known as stochastic curve shortening flow. Well-posedness is established in a variational SPDE framework. For the long time behaviour we prove ergodicity using the lower-bound-technique by Peszat/Szarek/Komorowski. Finally we show polynomial stability using a-priori estimates on the invariant measure.

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### Schrödinger equation with noise on the boundary

**Frank Wusterhausen**

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We are interested in existence and uniqueness of distributional solutions for the Schrödinger equation with Neumann noise, i.e.

$$\begin{aligned} i\partial_t u(t, y) - \Delta_y u(t, y) &= 0, & y \in G, t \in [0, T], \\ u(0, y) &= 0, & y \in G, \\ \partial_\nu u(t, y) &= \dot{W}(t, y), & y \in \partial G, t \in [0, T], \end{aligned}$$

where  $G$  is a bounded domain in  $\mathbb{R}^n$  with smooth enough boundary  $\partial G$  and  $\dot{W}$  is an  $L_2$ -valued white noise (white in time), i.e.  $W(t, y) = \sum_{j=1}^{\infty} \gamma_j h_j(y) B^j(t)$ , where  $\gamma \in l_2$ ,  $h_k \in L_2(\partial G)$  with  $\|h_k\|_{L_2(\partial G)} = 1$  for all  $k \in \mathbb{N}$ , and  $B^j(t)$ ,  $t \in [0, T]$ ,  $j = 1, \dots, \infty$ , denote independent one-dimensional standardized Brownian motions. The last equation is understood in the Itô sense.

A function (or distribution)  $u$  is called a distributional solution of the problem above if

$$\begin{aligned} i \int_0^T (u(t, \omega) | \dot{\varphi}(t))_{L_2(G)} dt + \int_0^T (u(t, \omega) | \Delta \varphi(t))_{L_2(G)} dt \\ = - \int_0^T \sum_{j=1}^{\infty} \gamma_j (h_j | \varphi(t))_{L_2(\partial G)} dB_t^j(\omega) \end{aligned}$$

holds for a.e.  $\omega \in \Omega$  and for all  $\varphi \in M$ , where

$$\begin{aligned} M &:= \{\varphi \in C^\infty([0, T] \times G) : \\ &\varphi(T, y) = 0 \quad \forall y \in G, \\ &\partial_\nu \varphi(t, y) = 0 \quad \forall t \in [0, T] \quad \forall y \in \partial G\}, \end{aligned}$$

$(\cdot | \cdot)_{L_2}$  is the inner product in  $L_2$ , and  $\dot{\varphi}$  denotes the derivative of  $\varphi$  with respect to  $t$ . We show that the problem admits a unique distributional solution

$$u \in C^{\alpha, \beta}(0, T; L_2(\Omega; H_2^{-\alpha}(G)))$$

with  $\forall \alpha \in (1/2, 3/2) \quad \forall \beta \in (0, 1/2) : \alpha - 2\beta > 1/2$ .

For the proof we make use of spectral decomposition of the Laplacian with homogeneous Neumann boundary condition.

If we replace the Neumann boundary condition by Dirichlet boundary condition we have the following result:

$$u \in C^{\alpha, \beta}(0, T; L_2(\Omega; H^{-\alpha}(G)))$$

with  $\forall \alpha \in (3/2, 2) \quad \forall \beta \in (0, 1/4) : \alpha - 2\beta > 3/2$ .

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## Special Session 37: Mathematical Models and Computations in Cell and Developmental Biology

**Anna Cai**, The University of New South Wales, Australia  
**Ching-Shan Chou**, Ohio State University, USA  
**Qing Nie**, University of California, Irvine, USA

This minisymposium aims to bring researchers to address recent advances of mathematical modeling on cell and developmental biology. In this minisymposium, researchers will discuss a wide range of complex biological systems which include but not limited to cell polarization, cell signaling pathways, cell-cell interaction to developmental biology. The challenges of modeling these complex systems will be discussed, and more beyond, the new modeling and computational techniques to tackle these problems will also be presented.

### A stochastic density-dependent switch drives spontaneous cell polarization

**Alexandra Jilkine**

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**Sigurd B. Angenent, Lani F. Wu, Steven J. Altschuler**

Cell polarization in yeast is a symmetry breaking process, where a cap of polarity regulator Cdc42 forms to mark where a daughter cell will bud off. A simple positive feedback circuit based on dynamics of Cdc42 shows emergent polarity for intermediate ranges of signaling molecule numbers. Below a critical density of Cdc42, positive feedback robustly maintains an off state; exceeding this threshold switches on the recurrent emergence of highly localized signaling clusters. Cluster formation requires only this minimal positive feedback circuit, and does not require additional mechanisms such as diffusion barriers, spatial cues, or biochemical inhibitors. This mechanism is general, and could be applied to a variety of cellular signaling systems to create clusters in the membrane or cytosol.

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### PDCD5-regulated cell fate decision after UV-irradiation induced DNA damage

**Jinzhi Lei**

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**Changjing Zhuge, Ying Chang, Yanjun Li, Yingyu Chen**

Programmed cell death 5 (PDCD5) is a human apoptosis-related molecule that is involved in both the cytoplasmic caspase-3 activity pathway by regulating Bax translocation from cytoplasm to mitochondria and the nuclear pathway by interacting with Tip60. In this study, we developed a mathematical model of the PDCD5-regulated switching of the cell response from DNA repair to apoptosis after ultraviolet (UV)-irradiated DNA damage. The model was established by combining several hypotheses with experimental observations. Our simulations indicated that the ultimate cell response to DNA damage is

dependent on a signal threshold mechanism, and the PDCD5 promotion of Bax translocation plays an essential role in PDCD5-regulated cell apoptosis. Furthermore, the model simulations revealed that PDCD5 nuclear translocation can attenuate cell apoptosis, and PDCD5 interactions with Tip60 can accelerate DNA damage-induced apoptosis, but the final cell fate decision is insensitive to the PDCD5-Tip60 interaction. These results are consistent with experimental observations. The effect of recombinant human PDCD5 was also investigated and shown to sensitize cells to DNA damage by promoting caspase-3 activity.

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### Mathematical models for phototaxis

**Doron Levy**

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Certain organisms undergo phototaxis, that is they migrate toward light. In this talk we will discuss our recent results on modeling phototaxis in order to understand the functionality of the cell and how the motion of individual cells is translated into emerging patterns on macroscopic scales. This is a joint work with Amanda Galante, Susanne Wisen, Tiago Requeijo, and Devaki Bhaya.

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### Mathematical modeling and computational studies for cell signaling with scaffolds

**Xinfeng Liu**

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**Qing Nie, Lee Bardwell**

I will present a computational analysis of cell signaling in biology and medicine. Scaffold, a class of proteins, plays many important roles in signal transduction. Through studying various models of scaffold, I will show novel regulations induced by its spatial location and switch-like and bistability responses due to scaffold. To efficiently compute the models, we introduce a new fast numerical algorithm incorporated with adaptive mesh refinement

for solving the stiff systems with spatial dynamics.

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### Robust budding site selection and cell polarization in yeast cells

**Wing cheong Lo**

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**Ching-Shan Chou, Hay-Oak Park**

Cell polarity is induced through the localization of specific molecules to proper location of the cell membrane. In this talk, we propose a generic model including membrane bound molecules undergoing polarization, landmark cue and the effect of inhomogeneous distribution of GAPs to study the mechanisms for different polarized site selection patterns: random polarized site selection in the absence of a pre-localized signal, adjacent positioning of axial polarized pattern and bipolar polarization pattern.

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### Quantifying parameters for a mathematical model on the interaction of matrix metalloproteinases and their inhibitors in a wound

**Richard Schugart**

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**Canaan Coppola, Hannah Pennington**

In this work, we present a mathematical model for the interactions of matrix metalloproteinases (MMPs), which degrade the extracellular matrix (ECM), and their inhibitors (TIMPs), and quantify the parameters by fitting the model to data. Measurements of the ratio of MMPs-to-TIMPs may be critical to a successful wound-healing event because an imbalance of these proteins is often found in chronic wounds. Muller et al. measure levels of MMPs and TIMPs in wound fluid. The data were divided into two subgroups – good healers

and bad healers – based on healing rate, and averaged for each subgroup. The parameters in the four-equation model, consisting of MMPs, TIMPs, ECM, and fibroblasts, were fit using the commands “GlobalSearch” and “fmincon” in Matlab. Reasonable parameter values were obtained by minimizing a least-squares functional.

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### Modeling cell-cell and cell-matrix interactions in single and collective motion in 3D

**Muhammad Zaman**

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Cell migration is a fundamental biological process that regulates a spectrum of events during development, wound healing and tumor metastasis. Until now, our approach to study migration has been focused largely on cells cultured or modeled on artificial two-dimensional substrates that are far from in vivo. Consequently, our understanding of cell migration in vivo has been very limited and often naive and inaccurate. Recent developments in both computational and experimental tools have afforded the possibility to study single and collective cell motion in native like 3D environments, that are quantitative and provide a much more realistic picture of the in vivo environment. Our results using multi-scale modeling, integrating molecular dynamics, coarse grained macro-molecular and continuum cellular level approaches suggest an intricate quantitative balance between matrix mechanics, sterics, signaling and migration speed and directionality. We also study the effect of local cell density and cell-cell contacts in regulating matrix reorganization and cellular speed. Our results have shown very good agreement with single and collective cell migration experiments on cancer cells in 3D scaffolds and matrices and provide a framework for understanding the complexity of cell migration in vitro and in vivo.

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## Special Session 38: Bifurcations and Asymptotic Analysis of Solutions of Nonlinear Models

**Jann-Long Chern**, Central University, Taiwan  
**Yoshio Yamada**, Waseda University, Japan  
**Shoji Yotsutani**, Ryukoku University, Japan

The aim of this special session is to exchange recent results, ideas and techniques on bifurcation and asymptotic analysis described by nonlinear elliptic and parabolic PDE models from various fields. In particular, we are interested in the global bifurcation structure for such models. Combinations of numerical simulation and theoretical approach with asymptotic analysis will be very useful to understand the nonlinear phenomena together with underlying structure of solutions. We will give opportunity to both established and junior researchers working in the related area to present their recent results.

### On positive solutions of semilinear elliptic equations with supercritical exponent

**Soohyun Bae**

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I will discuss the existence and the asymptotic behavior of positive solutions of semilinear elliptic equations with supercritical exponent. When coefficient function has some monotonicity, solutions may possess stability property. Slowly decaying solutions appear naturally in semilinear equations with supercritical exponent. I will explain the existence of such solutions and present related questions.

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### Asymptotic behavior of solutions of epidemic models with delays

**Yoichi Enatsu**

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**Yukihiko Nakata**, **Yoshiaki Muroya**, **Huaixing Li**

We consider a global dynamics of epidemic models with delays, in which an immunity loss of infectious diseases of recovered individuals is incorporated. By applying Lyapunov functional techniques in McCluskey (2010) and monotone iterative techniques, we establish new sufficient conditions under which the positive equilibrium is globally asymptotically stable for any delay. Some applications of our approach are also offered. Moreover, for a delayed SIR epidemic model governed by a logistic growth of susceptible individuals in the absence of diseases, we identify a threshold parameter with respect to the basic reproduction number such that the positive equilibrium loses its local stability as the length of the delay increases past a critical value.

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### Dynamical bifurcation of the two dimensional Swift-Hohenberg equation with odd periodic condition

**Chun-hsiung Hsia**

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**Jongmin Han**

In this paper, we study the stability and dynamic bifurcation for the two dimensional Swift-Hohenberg equation with an odd periodic condition. It is shown that an attractor bifurcates from the trivial solution as the control parameter crosses the critical value. The bifurcated attractor consists of finite number of singular points and their connecting orbits. Using the center manifold theory, we verify the nondegeneracy and the stability of the singular points.

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### Global existence in sub-critical cases for 1-D quasilinear degenerate Keller-Segel systems

**Sachiko Ishida**

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**Tomomi Yokota**

We consider the one-dimensional quasilinear degenerate Keller-Segel systems of power type. When the spatial dimension is greater than or equal to 2, the global existence of weak solutions to the systems is shown by Ishida-Yokota (2012) via a slightly complicated approximation. In this talk we derive the same result on the global existence for “1-D” case by using a “simple” approximation.

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### Structures of positive solutions to nonlinear elliptic equations on the hyperbolic space

**Yoshitsugu Kabeya**

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We consider nonlinear elliptic equations on the hyperbolic space and investigate the structures of positive solutions to them. We use a transformation to reduce the equations to the Matukuma type ones on an Euclidean space. This transformation gives us a simple view to the problem. A part of this talk is based on the joint work with C. Bandle (University of Basel).

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### Free boundary problems modeling the spreading of species in symmetric domains

**Yuki Kaneko**

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**Yoshio Yamada**

We consider free boundary problems modeling the spreading of species, where unknown functions are population density and spreading front of the species. Moreover, the dynamical behavior of the free boundary is determined by Stefan-like condition. Such model was first proposed by Du and Lin (2010) and large time behaviors of solutions (spreading and vanishing of species) are completely understood. In this talk, we discuss free boundary problems for reaction-diffusion equations with general nonlinearity in symmetric domains.

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### Non-symmetric low-index solutions for symmetric boundary value problems

**Hans Koch**

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**Gianni Arioli**

We consider a semilinear boundary value problem that is invariant under a nontrivial group of Euclidean symmetries. While earlier work has focused on conditions under which solutions inherit some of the symmetries (a common numerical observation), our goal was to find solutions that have no symmetries at all. In the case where the domain is a square, we give a computer-assisted proof for the existence of a non-symmetric index 2 solution. Some numerical results for the disk will be described as well.

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### Bifurcation structure of steady-states to a reaction-diffusion-advection system in surface chemistry

**Kousuke Kuto**

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**Tohru Tsujikawa**

Hildebrand et al. (1999) have proposed a reaction-diffusion-advection model for pattern formations in the catalytic CO-oxidation on Pt-surface. We consider the stationary problem of the chemical model to give a universal bound for every solution and a sufficient condition for the existence of nonconstant solutions. Furthermore, we obtain the global bifurcation structure of steady-states for a related shadow system. This structure implies that nonconstant steady-states can form a (spontaneous) bifurcation curve connecting boundary-layer states with internal-layer states.

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### On a dynamics of solution with a transition layer to some bistable reaction diffusion equation

**Hiroshi Matsuzawa**

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In this talk, we study a dynamics of solution with a sharp transition layer of a bistable reaction diffusion equation. In particular, we consider the case where bistable nonlinearity in the equation has spatial heterogeneity. If this heterogeneity degenerates on an interval, the dynamics of the transition layer on this interval becomes so called very slow dynamics.

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### Structure and blow up phenomena for plane closed elastic curves

**Minoru Murai**

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**Waichiro Matsumoto, Shoji Yotsutani**

Let  $\Gamma$  be plane closed elastic curve with length  $2\pi$ . We denote arc-length and curvature by  $s$  and  $\kappa(s)$ , respectively. Let  $M$  be the signed area of the domain bounded by  $\Gamma$ . We consider the variational problem: Find a curve  $\Gamma$  such that minimize  $\frac{1}{2} \int_0^{2\pi} \kappa(s)^2 ds$  subject to  $M < \pi$ , and  $\omega M \neq \pi$ , where  $\omega$  is the rotation number.

In this talk, we will discuss the structure and the blow up phenomena of the solution  $\kappa(s)$  to the Euler-Lagrange equation for the above variational problem.

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### The jamming of camphor boats in a circular water channel

Masaharu Nagayama

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Nobuhiko J. Suematsu, Tadashi Ataka

The jamming of camphor boats similar to the traffic jam in a highway was observed in the experiment in a circular water channel. We reproduce the jamming of camphor boats using the mathematical model, and show the occurrence of jamming is Hopf bifurcation.

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### Coexistence problem for a prey-predator model with a protection zone

Kazuhiro Oeda

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We consider a prey-predator model with a protection zone for the prey. We give a necessary and sufficient condition for the existence of positive stationary solutions by using the bifurcation theory. We also discuss the asymptotic behavior of positive stationary solutions as the intrinsic growth rate of the predator goes to infinity. Our results show that the environment inside the protection zone is important for the prey.

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### Bifurcation analysis for the superconducting/normal phase transition of the Ginzburg-Landau system

Tien-Tsan Shieh

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In the talk, I will shortly introduce the Ginzburg-Landau system of superconductivity. When lowering the temperature, superconductors become the superconducting state from the normal state. In order to study this phase transition phenomena, we present a bifurcation and stability analysis on the Ginzburg-Landau system of superconductivity with an applied magnetic field and the de Gennes boundary condition. It is proved there are two different kinds of phase transition from the normal state to the superconducting state: one is jump transition and the other is continuous transition. In particular, we analyse the behavior of solution when the domain is a cylinder and the applied field is parallel to the axis.

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### On a generalized Jacobian elliptic function associated with $p$ -Laplacian

Shingo Takeuchi

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We generalize the Jacobian elliptic function  $\operatorname{sn}(x, k)$  and apply it to a bifurcation problem associated with  $p$ -Laplacian. If the modulus  $k$  equals zero, then the novel function  $\operatorname{sn}_{pq}(x, k)$  coincides with the  $p$ -trigonometric function  $\sin_p x$  or the  $(p, q)$ -trigonometric function  $\sin_{pq} x$  developed by P.Lindqvist, J.Peetre, P.Drabek and R.Manasevich.

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### Structural analysis of solutions to nonlinear systems of elliptic partial differential equations

Yong-Li Tang

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In this talk, we deal with some nonlinear elliptic systems, including sublinear systems, a coupled system in Hamiltonian type, Liouville-type system and simplified Bennett-type system with singular data. Some qualitative properties and complete structures of solutions will be provided by applying the bifurcation theory and linearization approach.

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### On the sharp constant for the weighted Trudinger-Moser type inequality of the scaling invariant form

Hidemitsu Wadade

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Michinori Ishiwata, Makoto Nakamura

We establish the weighted Trudinger-Moser inequality of the scaling invariant form including its best constant and prove the existence of a maximizer for the associated variational problem. The non-singular case was treated by Adachi-Tanaka and the existence of the maximizer is a new result even for the non-singular case. We also discuss the relation between the best constants of the weighted Trudinger-Moser inequality and the Caffarelli-Kohn-Nirenberg inequality in the asymptotic sense.

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### Exact solutions for bifurcation problems of some reaction diffusion systems

**Tohru Wakasa**

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For various nonlinear evolutionary PDEs it is one of central problems to study the global bifurcation structure of stationary/periodic solutions. Various numerical results on global bifurcation diagrams usually shows us a difficulty of theoretical understanding for bifurcation phenomena. In this talk we will introduce some global bifurcation results on some reaction-diffusion systems with use of exact solutions, and will investigate, both theoreticall and numerically, the property of the corresponding linearized operators associated with nontrivial stationary solutions.

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### Structure and stability of stationary solutions to a cross-diffusion equation

**Shoji Yotsutani**

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**Yuan Lou, Wei-Ming Ni**

We are interested in a cross-diffusion equation proposed by Shigesada-Kawasaki-Teramoto in 1979. Limiting equations and their analysis by Lou - Ni (JDE 1996, 1999) play crucial roles. One of the limiting equations is very important to obtain segregated solutions. Lou-Ni-Yotsutani (DCDS, 2004) almost revealed the structure of it. We show the complete structure of it, and also investigate stability problems.

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## Special Session 39: Polynomial Methods for Differential Equations and Dynamical Systems

Stephen Lucas, JMU, USA  
 James Stanley Sochacki, James Madison University, USA  
 Roger Thelwell, JMU, USA  
 Paul Warne, JMU, USA

Since the advent of Taylor Series, polynomial methods have been used to solve differential equations and learn the properties of differential equations. There are many polynomial methods for solving differential equations and understanding dynamical systems. For example, Taylor polynomials, Chebyshev Polynomials and Adomian Polynomials are used to generate approximate solutions. Automatic differentiation can be used with power series and Cauchy products to generate solutions to differential equations. Pade Methods are based on ratios of polynomial functions and products of polynomials and are used to understand singularities in differential equations. Picard Iteration can be used with polynomial differential equations to obtain error estimates and convergence rates. This session intends to bring together researchers in these areas to develop a community and to share ideas and develop new tools and theory for understanding differential equations and dynamical systems and their relationship with polynomials.

### Sparse moment sequences

**Saroj Aryal**

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**Farhad Jafari, Mihai Putinar**

The well-known theorems of Stieltjes, Hamburger and Hausdorff establish conditions on infinite sequences of real numbers to be moment sequences. Further, works by Carathéodory, Schur and Nevanlinna connect moment problems to problems in function theory and functions belonging to various spaces. In many problems associated with realization of a signal or an image, data may be corrupted or missing. Reconstruction of a function from moment sequences with missing terms is an interesting problem leading to advances in image and/or signal reconstruction. It is easy to show that a subsequence of a moment sequence may not be a moment sequence. Conditions are obtained to show how rigid the space of sub-moment sequences is and necessary and sufficient conditions for a sequence to be a sub-moment sequence is established. A deep connection between the sub-moment measures and the moment measures is derived and the determinacy of the moment and sub-moment problems are related. This problem is further related to completion of positive Hankel matrices.

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### Asymptotics of orbits of a kolmogorov type planar vector field with a fixed newton polygon

**Faina Berezovsky**

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Using the Newton polygon technique we show that the orbits of a Kolmogorov type planar vector field, consisting of a finite sum of power terms, have power

asymptotics while tending to the equilibria on the boundary of the Poincare sphere.

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### Systems of polynomial ODE's as a tool for improving the efficiency of numerical methods

**John Bridstrup**

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We show that the efficiency of computational methods can be greatly increased by employing a novel substitution approach to any arbitrary ODE. Parker and Sochacki showed that any ODE can be made into a system of polynomial ODE's, examples will be given and the speed and accuracy of methods such as Runge-Kutta will be tested using this approach. These speeds and accuracies are then compared with the same numerical methods, using the original ODE, as well as with Parker and Sochacki's Modified Picard Method (PSM). The results show that all tested methods are improved by polynomial projection of the ODE's and that PSM is even faster and more accurate for a very large range of problems. A GUI implementing PSM will be demonstrated.

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### Fast generation algorithms for the Adomian polynomials

**Jun-sheng Duan**

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We present different recurrence algorithms for the multivariable Adomian polynomials. These algorithms are comprised of simple recurrence formulas and they are straightforward to implement

in any symbolic software. Recurrence algorithms for the one-variable Adomian polynomials are derived as special cases. In particular, the recurrence process of the reduced polynomials  $C(n, k)$ , where  $C(n, k)$  comprise of the Adomian polynomials  $A_n = \sum_{k=1}^n f^{(k)}(u_0)C(n, k)$ , does not involve the differentiation operation, but significantly only the arithmetic operations of multiplication and addition are involved. The MATHEMATICA program generating the Adomian polynomials based on the new, fast algorithms is designed. We also demonstrate that the Adomian polynomials can be applied to solve the nonlinear fractional differential equations.

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### The unifying view on ordinary differential equations and automatic differentiation, yet with a gap to fill

**Alexander Gofen**

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Among various instruments of approximation representing solutions of Ordinary Differential Equations (ODEs), the Taylor expansions play a unique role. This is because an explicit system of ODEs by itself is a tool for computing n-order derivatives of the solution and therefore delivering the Taylor expansions of the solution at multiple points of the phase space.

This capability by ODEs to generate Taylor expansions relies however on availability of the rules of n-order differentiation and their computational efficiency. Availability and efficiency of the rules of n-order differentiation take place only for a particular sub-class of holomorphic functions. Those are the so called generalized elementary functions, i.e. functions representable as solutions of rational ODEs.

Continuation of (generalized) elementary functions via integration of its ODEs not necessarily expands them into each and every point where these functions exist and are holomorphic. Some entire functions are suspects for being elementary everywhere except isolated unreachable points - the points of their "removable" or "regular" singularity. All the above mentioned and a few other issues are interdependent and complementary to each other. The merger of them into one theory is called the Unifying view on ODEs and AD.

However there is a gap in this otherwise coherent view: an open statement, the Conjecture about the possibility to convert a rational system of ODEs at a regular point into one n-order rational ODE regular at the same point. The Conjecture is important because the question of equivalency of two competing definitions of elementary functions (and a few other open statements) depend on the Conjecture.

The report therefore presents the setting and a few known facts concerning the Conjecture as an invitation for everybody to resolve it and fill the gap in this theory.

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### Different differential equations with the same solution

**Stephen Lucas**

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**J. Sochacki**

We show how different differential equations with the same solution can have different errors when solved using the identical numerical method. This is unexpected, since virtually every discussion of numerical method errors gives them in terms of the solution. We show this behavior with a number of simple examples, including the classic pendulum, where energy should be conserved. Rewriting differential equations in polynomial form is shown to usually be the best option, and we conclude with a discussion of symplectic solvers compared to the Power Series Method.

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### Efficient recurrence relations for univariate and multivariate Taylor series coefficients

**Richard Neidinger**

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The efficient use of Taylor Series depends, not on symbolic differentiation, but on a standard set of recurrence formulas for each of the elementary functions and operations. These relationships are often rediscovered and restated, usually in a piecemeal fashion. We seek to provide a fairly thorough and unified exposition of efficient recurrence relations in both univariate and multivariate settings. Explicit formulas all stem from the fact that multiplication of functions corresponds to a Cauchy product of series coefficients, which is more efficient than the Leibniz rule for nth-order derivatives. This principle is applied to function relationships of the form  $h' = v * u'$ , where the prime indicates a derivative or partial derivative. Each elementary transcendental function corresponds to an equation, or pair of equations, of that form  $h' = v * u'$ . A geometric description of the multivariate operation helps clarify and streamline the computation for each desired multi-indexed coefficient. Several different perspectives on these relationships will be mentioned from a Chinese algorithm in year 1247 to the Differential Transform Method now active in Asian literature.

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**A generalization of Darboux's method****Chara Pantazi**Universitat Politècnica de Catalunya, Spain  
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There is a classical method started by Darboux in 1878 about the computation of a special class of first integrals (called Darboux first integrals) of polynomial differential systems. These Darboux first integrals are the products of powers of polynomials (often called Darboux polynomials). Note that the zero set of the Darboux polynomials define invariant algebraic curves of the corresponding vector field. So, algebraic curves and its multiplicity are the key points in order to construct the Darboux first integrals. More recently this method has been generalized by several authors like Jouanolou, Singer, Schlomiuk, Llibre, Christopher, Zhang among others and is related with some applications concerning limit cycles and bifurcation problems. Several inverse problems in dimension two have also been considered. In this talk we present necessary and sufficient conditions for the existence of Darboux first integrals and we also present a generalization for the nonautonomous case.

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**Solving ODEs using PSM and trees****Philip Parker**James Madison University, USA  
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Using the polynomial projections and power series method (PSM) developed by Parker and Sochacki and others, an implementation has been developed that computes numerical solutions to ODEs to any degree of accuracy using PSM. ODEs are parsed into tree structures, and the trees are traversed with calls to a small library of functions derived for this purpose. The code that solves ODEs using this method will be presented and a discussion of how the form of the algorithm lends to the optimization of PSM will be given.

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**Padé approximants and pole extraction near singular points.****Joseph Rudmin**James Madison University, USA  
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Ed Parker and James Sochacki have rediscovered a powerful way of finding polynomial approximations to systems of differential equations. This Parker Sochacki method always finds the Taylor Series approximation to a given order, if such approximation exists, and gives absolute error limits. For most practical applications, a Padé approximant derived from the Taylor Series provides better fit than the Taylor Series. However, both Taylor Series and Padé Approximants have difficulty modeling poles in the solution. Often one can best model a pole by a change of variable, where the variable explicitly contains the pole. The change of variable can be found from the differential equations by eliminating the highest order feedback loop in the Parker Sochacki approximation, thus simplifying those equations.

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**Mathematical modeling problems that are polynomial ODEs****James Sochacki**James Madison University, USA  
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There are many force problems that can be posed as systems of polynomial ODEs with polynomial energy conditions. Many numerical methods, such as Newton's Method, the Method of Steepest Descent, Inverses of Functions and Pade Approximants can be expressed as systems of polynomial ODEs. Several of these will be presented and it will be discussed how Picard Iteration and/or power series can be used to solve these problems to within a priori error bounds.

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## Special Session 41: New Developments in Qualitative Behavior of Evolutionary PDEs

Ryo Ikehata, Hiroshima University, Japan  
Grozdena Todorova, University of Tennessee, USA

New developments in the theory of evolutionary PDEs will be reported. The topics discussed will include well-posedness, regularity, formation of singularities, stability/instability and asymptotic behavior of solutions to important evolutionary PDE systems. Applications of these problems will also be addressed.

### Uniform decay rate estimates for Schrödinger and Plate equations with nonlinear locally distributed damping

**Marcelo Cavalcanti**

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**Valeria Domingos Cavalcanti, Wellington J. Correa and Cesar A. Bortot**

On a compact  $n$ -dimensional Riemannian manifold  $(\mathcal{M}, \mathbf{g})$ , we establish uniform decay rate estimates for the linear Schrödinger and plate equations subject to an internal nonlinear damping locally distributed on the manifold. Our approach can be also employed for other equations provided that inverse inequality for the linear model occurs. In the particular case of the wave equation, where the well known geometric control condition (GCC) is equivalent to the observability inequality, our method generalizes the results due to Cavalcanti et. al. (Arch. Ration. Mech. Anal. (2010), TRANSACTIONS AMS (2009)) regarding the optimal choice of dissipative regions.

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### Energy decay of a magnetoelastic system in an exterior 3-D domain

**Ruy C. Charão**

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Supported by CAPES (Brazil) (Proc. AEX 7217/12-9)

**J. C. Oliveira, G. Perla Menzala**

In this talk we present results of existence and uniform decay of the total energy of solutions for a system of magneto-elasticity with localized damping “near” infinity in an exterior 3-D domain. The model which we consider is motivated by a phenomenon which appears frequently in nature: The interaction between the strain and electromagnetic fields in an elastic body. For example, this model can be used to investigate the propagation of elastic waves in the presence of Earth’s magnetic field. The system under consideration in this work may be viewed as a coupling between the hyperbolic system of elastic waves and a parabolic system for the magnetic field. Somehow, the coupled system under consideration has a similar structure as the classi-

cal isotropic thermoelastic system. When we try to study the asymptotic behavior of the total energy for the model we are considering in this work, similar difficulties as for the thermoelastic system (in  $n = 2$  or 3 dimensions) will appear.

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### Global existence and sharp decay estimates for the semilinear wave equation with time-dependent damping

**Marcello D’Abbicco**

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**Sandra Lucente, Michael Reissig**

We recently studied the Cauchy problem for the wave equation with time-dependent effective damping. We first derive Matsumura-type estimates for the solution to the linear problem, then we prove the same decay rate with no loss for the semilinear problem. We find the critical exponent  $1 + 2/n$  for the global existence of small data solution, where  $n$  is the space dimension. This exponent is the same obtained in the constant coefficient case by Todorova and Yordanov, and Ikehata and his collaborators. In space dimension  $n > 2$  we use data from weighted energy space. We prove the sharpness of the critical exponent by using a modified test function method.

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### Regularity of solutions for a third order differential equation

**Claudio Fernandez**

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In this talk, which is part of a joint work with V. Poblete and C. Lizama (Chile), we shall discuss well posedness in Lebesgue spaces for the abstract model,

$$\begin{cases} \alpha u'''(t) + u''(t) + \beta Au(t) + \gamma Au'(t) = f(t), t \geq 0; \\ u(0) = u_0, \\ u'(0) = u_1, \\ u''(0) = u_2. \end{cases}$$

that corresponds to a flexible space structure under appropriate initial and boundary conditions. We also study regularity of mild and strong solutions and apply our results showing qualitative properties of the

trajectories in the case of the negative Laplacian operator.

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### Finite time blow-up for damped wave equations with nonlinear memory and space-dependent potential

**Ahmad Fino**

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In this paper, we consider the Cauchy problem in  $\mathbb{R}^n$ ,  $n \geq 1$ , for a semilinear damped wave equation with space-dependent potential and nonlinear memory term. A blow-up result under some positive data in any dimensional space is obtained.

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### On the stabilization of Timoshenko systems with finite memory

**Aissa Guesmia**

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In this work, we consider a one-dimensional Timoshenko systems with different speeds of wave propagation and with only one control given by a viscoelastic term on the angular rotation. For a wide class of relaxation functions and for sufficiently regular initial data, we establish a general decay result for the energy of solution.

This is a joint work with Salim Messaoudi (KFUPM, Dhahran, Saudi Arabia).

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### Energy decay estimates for wave equations with a fractional damping

**Ryo Ikehata**

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**Masato Natsume**

We consider the Cauchy problem in the whole space for wave equations with a fractional damping. We will introduce our recent results concerning the energy decay estimates. The proof is based on the energy method in the Fourier space combined with a device which was introduced by the speaker.

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### Global existence for critical nonlinear massless Dirac equations with null structure in 3D

**Hideo Kubo**

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In this talk I wish to present a result on the Cauchy problem for the nonlinear massless Dirac equation in 3D. It is known that the quadratic nonlinearity is on the critical level for global solvability of the Cauchy problem. We shall show that if the nonlinearity takes some special forms, then the problem has a global solution for small initial data. Moreover, the solution is asymptotically free. The idea of the proof is to exploit the null structure from the special quadratic forms. We remark that this result could not be deduced from the global existence result for the nonlinear wave equations with the null condition.

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### Attractors for a class of Kirchhoff-Boussinesq models with memory

**To Fu Ma**

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This talk is concerned with the existence of finite dimensional attractors for a class of Kirchhoff-Boussinesq models with memory. Roughly speaking one considers an Euler-Bernoulli plate equation with a perturbation of p-Laplacian type and a memory term with past history.

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### Best asymptotic profile for bipolar hydrodynamic model of semiconductors

**Ming Mei**

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**D. Donatelli, B. Rubino, R. Sampalmieri**

In this talk, we consider the bipolar hydrodynamic system of semiconductors which is presented by the coupled Euler-Poisson equations. Traditionally, the asymptotic profiles of the solutions are regarded as the diffusion waves (the solutions to the corresponding (parabolic) porous media equations), and the convergence rates are showed to be algebraic. However, by a deep observation and a heuristic analysis, we recognize that the best asymptotic profiles are the stationary waves (the corresponding steady-state solutions). We further prove that the original solutions converge time-asymptotically to these best asymptotic profile with exponential rates. The adopted approach for proof is the energy method.

This is a joint work with D. Donatelli, B. Rubino and R. Sampalmieri.

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### Uniform resolvent estimates for Helmholtz equation in an exterior domain and their application to scattering problems

**Hideo Nakazawa**

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Uniform resolvent estimates for Helmholtz equations in an exterior domain in  $R^N$  with  $N \geq 2$  is derived by using Hardy type inequalities related to radiation conditions, which are hold for  $N \geq 1$ . From this resolvent estimate, the result by Mizohata and Mochizuki (1966) on the principle of limiting amplitude for dissipative wave equation is improved. Moreover, the smoothing estimate for the related evolution equations (Schroedinger, relativistic Schroedinger or wave (Klein-Gordon) equations) in an exterior domain in  $R^2$  is also proved.

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### Orbital stability of periodic waves

**Fabio Natali**

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**Aloisio Freiria Neves**

Results of existence as well as orbital stability of periodic waves associated with evolutionary partial differential equations will be treated in this talk. Our method establishes such results without knowing an explicit periodic solution and we use a numerical approach. Applications of the theory allow us to determine the first proof of the orbital stability of a family of periodic waves for the 3-Korteweg-de Vries and logarithmic Schroedinger equations.

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### Existence and blow-up of solutions for nonlinear wave equations

**Petronela Radu**

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Several results on wellposedness of solutions for nonlinear wave equations use the structure of the problems that include damping terms to establish local existence in time of solutions in the presence of energy accretive source terms. We will discuss the limitations of these methods and the intricate balance between local existence and regularity of solutions .

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### Global existence and asymptotic behavior of solutions of thermoelasticity of second sound

**Belkacem Said-Houari**

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**Reinhard Racke and Aslan Kasimov**

We consider the one dimensional Cauchy problem in thermoelasticity of second sound, where the heat conduction is given by the so-called Cattaneo law. First, for the linear problem, and based on Fourier's analysis and Lyapunov's functional method, we show that the  $L^2$ -norm of solutions decays with the rate  $t^{-1/4}$ . In addition, by means of a careful spectral analysis, we give sharp description on the decay rates of solutions. In other words, all the decay rates can be improved by  $t^{-1/2}$  for initial data in some weighted spaces. For the nonlinear model, we show a global existence result and obtain polynomial decay rates of the global small solution when the time goes to infinity by employing the large time decay estimates of solutions to the linearized Cauchy problem. Our decay estimates improve some early results.

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### The final problem on the optimality of the general theory for nonlinear wave equations and related topics.

**Hiroyuki Takamura**

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**Kyouhei Wakasa**

The general theory of the initial value problem for fully nonlinear wave equations is to clarify lower bounds of the lifespan, the maximal existence time, of classical solutions in terms of the amplitude of small initial data according to the order of smooth nonlinear terms and space dimensions. All the results had been obtained till 1995. So we have been interested in the optimality of the lower bounds. This can be obtained by blow-up results for model equations. Among such several results, only the case of the quadratic semilinear term in 4 space dimensions has been remained open for more than 20 years. This final problem on the optimality has been known to be the critical case of Strauss' conjecture on semilinear wave equations. In this talk, I will present the final answer for the problem. Also I will discuss its application to systems and introduce another equation in high dimensions which has blowing-up solutions.

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### Higher order expansion of solutions to damped wave equations

**Hiroshi Takeda**

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In this talk, we consider the Cauchy problem for damped wave equations. Based on the precise analysis in the Fourier space, we obtained the higher order expansion of the solution of damped wave equations. Our theorem states that the large time behavior of the solution to damped wave equation is different from that of the heat equation.

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### Generalized diffusion phenomenon in Hilbert space

**Grozdena Todorova**

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**Petronela Radu, Borislav Yordanov**

We show a generalized diffusion phenomenon in Hilbert space. The results have important applications regarding the asymptotic behavior of damped wave equations with variable coefficients in exterior domains. We establish sharp decay estimates which apply to second order hyperbolic equations, fourth order plate equations and hyperbolic systems.

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### Asymptotic profiles for the isothermal Falk-Konopka system of shape memory alloys with weak damping

**Shuji Yoshikawa**

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**Hiroshi Takeda**

We will talk about the results for the weakly damped isothermal Falk-Konopka system which describes the martensitic phase transitions on shape memory alloys. We prove the unique global existence of solution for the Cauchy problem of the system and several asymptotic profiles of the solution. The asymptotic profiles enable us to clarify the relation between higher and lower order terms.

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## Special Session 42: Global or/and Blowup Solutions for Nonlinear Evolution Equations and Their Applications

George Chen, Cape Breton University, Canada  
Ming Mei, McGill University, Canada

This session is devoted to the recent developments in global or/and blowup solutions for nonlinear evolution equations and their applications, include fluid dynamics, delay, localized, non-local, degenerate evolution equations, steady states and their properties.

### Global and blowup solutions for general quasi-linear parabolic systems

**Shaohua Chen**  
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This talk discusses global and blowup solutions of the general quasilinear parabolic system  $u_t = \alpha(u, v)\Delta u + f(u, v, Du)$  and  $v_t = \beta(u, v)\Delta v + g(u, v, Dv)$  with homogeneous Dirichlet boundary conditions. We will give sufficient conditions such that the solutions either exist globally or blow up in a finite time. In special cases, a necessary and sufficient condition for global existence is given. We also discuss a degenerate case.

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### Structure of principal eigenvectors and genetic diversity

**Fengxin Chen**  
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**P. W. Bates**

The main concern of this paper is long-term genotypic diversity. Genotypes are represented as finite sequences  $(s_1, s_2, \dots, s_n)$ , where the entries  $\{s_i\}$  are drawn from a finite alphabet. The mutation matrix is given in terms of Hamming distances. It is proved that the long time behavior of solutions for a class of genotype evolution models is governed by the principal eigenvectors of the sum of the mutation and fitness matrices. It is proved that the components of principal eigenvectors are symmetric and monotonely decreasing in terms of Hamming distances whenever the fitness matrix has those properties.

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### On the behavior of certain nonlinear parabolic equations with periodic boundary conditions

**Jean Cortissoz**  
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Let  $\Omega = [0, L_1] \times \dots \times [0, L_k]$ . In this talk we will

consider the problem

$$\begin{cases} \frac{\partial u}{\partial t} = u^n \Delta u + u^{n+1} & \text{on } \Omega \times (0, T) \\ u(x, 0) = \psi(x) & \text{in } \Omega \end{cases} \quad (1)$$

$n$  a positive integer, under periodic boundary conditions. It is known that if  $\psi > 0$ , then the classical solution to (1) blows up in finite time. We will show that under some assumptions on the Fourier expansion of the initial condition, and on the first nontrivial Laplacian eigenvalue of  $\Omega$ , there are constants  $C_k > 0$  and  $\beta > 0$  such that

$$\left\| u(\cdot, t) - \frac{1}{\text{Vol}(\Omega)} \int_{\Omega} u(x, t) dx \right\|_{C^k(\Omega)} \leq C_k (T - t)^\beta$$

where  $T > 0$  is the blow-up time of the solution to (1). Part of this talk is joint work with Alexander Murcia.

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### Decay property of regularity-loss type for quasi-linear hyperbolic systems of viscoelasticity

**Priyanjana Dharmawardane**  
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In this talk, we consider a quasi-linear hyperbolic systems of viscoelasticity. This system has dissipative properties of the memory type and the friction type. The decay property of this system is of the regularity-loss type. To overcome the difficulty caused by the regularity-loss property, we employ a special time-weighted energy method. Moreover, we combine this time-weighted energy method with the semigroup argument to obtain the global existence and sharp decay estimate of solutions under the smallness conditions and enough regularity assumptions on the initial data.

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**Everywhere regularity for cross diffusion systems involving p-Laplacian: the degenerate case**

**Le Dung**

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Using nonlinear heat approximation and homotopy arguments, we study Hölder regularity of bounded weak solutions to strongly coupled and degenerate parabolic systems.

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**Existence and blow-up results for fast diffusion equations with nonlinear lower order terms**

**Daniela Giachetti**

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We deal with blow-up and existence results for solutions of equations with principal part of fast diffusion type and presenting a reaction term  $g(u)$ . More precisely, as far as the blow-up phenomenon is concerned, we consider the following problem

$$\begin{cases} u_t - \Delta(u^m) = \lambda g(u) & \text{in } \Omega \times (0, \infty), \\ u = 0 & \text{on } \partial\Omega \times (0, \infty), \\ u(x, 0) = u_0(x) & \text{on } \Omega, \end{cases}$$

where  $\Omega \subset \mathbb{R}^N$  is an open bounded set,  $0, 0, g$  nondecreasing and convex and such that

$$\int_0^\infty \frac{d\sigma}{g(\sigma)} < +\infty,$$

and we prove that for  $\lambda$  not too small Fujita property occurs i.e. whatever is the initial datum  $u_0$ , all the solutions blow-up in finite time in the sense that

$$\limsup_{t \rightarrow T^-} \|u(x, t)\|_{L^\infty(\Omega)} = +\infty.$$

On the other hand, as far as the existence is concerned, we will prove existence of global solutions to

$$\begin{cases} u_t - \Delta(|u|^{m-1}u) = g(u) + \mu & \text{in } \Omega \times (0, \infty), \\ u = 0 & \text{on } \partial\Omega \times (0, \infty), \\ u(x, 0) = u_0(x) & \text{on } \Omega \end{cases}$$

essentially covering the complementary situation, i.e.

$$\int_0^\infty \frac{d\sigma}{g(\sigma)} = +\infty,$$

for general data  $\mu$  and  $u_0$ , i.e.  $\mu$  finite Radon measure and  $u_0 \in L^1(\Omega)$ , and for  $\frac{N-1}{N} < m < 1$ . Here we mean  $|u|^{m-1}u = 0$  in the set where  $u = 0$ .

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**An approximation scheme for area-constrained curvature-driven multiphase motions**

**Elliott Ginder**

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**Seiro Omata, Karel Svadlenka**

We develop a numerical method for investigating area-constrained interfacial motions. We show how to reformulate the BMO algorithm in a vector-valued setting, which we then treat by means of a minimizing movement; hence the variational nature of our method permits the inclusion of area constraints, via penalization. As the constrained motions tend to be much slower than motion by mean curvature, the well-known time and grid resolution restrictions of the BMO algorithm become particularly relevant. We thus discuss how to alleviate these restrictions, and we show the numerical results of our method for investigating the viscous motion of bubble clusters.

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**Two-dimensional curved fronts in a periodic shear flow**

**Rui Huang**

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**M. El Smaily, F. Hamel**

In this talk, we consider the traveling fronts of reaction-diffusion equations with periodic advection in the whole plane  $\mathbb{R}^2$ . We are interested in curved fronts satisfying some “conical” conditions at infinity. We prove that there is a minimal speed  $c^*$  such that curved fronts with speed  $c$  exist if and only if  $c \geq c^*$ . Moreover, we show that such curved fronts are decreasing in the direction of propagation, that is, they are increasing in time. We also give some results about the asymptotic behaviors of the speed with respect to the advection, diffusion and reaction coefficients. This is a joint work with M. El Smaily and F. Hamel.

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**On solvability for a class of quasilinear elliptic equations with superlinear growth in weighted spaces**

**Gao Jia**

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This talk will focus on the existence of the solutions for a class of quasilinear elliptic equations.

We are mainly interested in which the nonlinearity possesses the superlinear growth conditions. Our method is based on the Galerkin method, the generalized Brouwer's theorem and a weighted compact Sobolev-type embedding theorem established by V. L. Shapiro.

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### An analysis in the space of BV functions for the equation of motion of a vibrating membrane with a “viscosity” term

**Koji Kikuchi**

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Let  $\Omega$  be a bounded domain in  $\mathbf{R}^n$  with Lipschitz continuous boundary  $\partial\Omega$ . In this talk we investigate the following equation in the space of BV functions:

$$u_{tt} - \operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) - \left(\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)\right)_t = 0, \quad (1)$$

$$u(0, x) = u_0(x), \quad u_t(0, x) = v_0(x), \quad x \in \Omega, \quad (2)$$

$$u(t, x) = 0, \quad x \in \partial\Omega. \quad (3)$$

A function  $u$  is said to be a *BV function* in  $\Omega$  if the distributional derivative  $Du$  is an  $\mathbf{R}^n$  valued finite Radon measure in  $\Omega$ . The vector space of all BV functions in  $\Omega$  is denoted by  $BV(\Omega)$ . It is a Banach space equipped with the norm  $\|u\|_{BV} = \|u\|_{L^1(\Omega)} + |Du|(\Omega)$ . The difficult point is that, for  $u \in BV(\Omega)$ , the operator  $\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)$  is multi-valued. It is usually defined by the use of subdifferential of the area functional. Namely, supposing that  $u \in BV(\Omega) \cap L^2(\Omega)$ , we regard  $-\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)$  as

$$\begin{aligned} \partial J(u) &= \{f \in (BV(\Omega) \cap L^2(\Omega))'\}; \\ J(u + \phi) - J(u) &\geq (f, \phi) \\ \text{for each } \phi &\in BV(\Omega) \cap L^2(\Omega), \end{aligned}$$

where  $J(u) = \int_{\Omega} \sqrt{1+|Du|^2}(\bar{\Omega})$ . We report that, if  $u_0$  is slightly smooth, then there exists a unique solution to (1)–(3).

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### Global solutions of a diffuse interface model for the two-phase flow of compressible viscous fluids in 1-D

**Yinghua Li**

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**Shijin Ding**

In this talk, we consider a coupled Navier-Stokes/Cahn-Hilliard system which describes a diffuse interface model for the two-phase flow of compressible viscous fluids in a bounded domain in one dimension. We prove the existence and uniqueness of global classical solution for  $\rho_0 \in C^{3,\alpha}(I)$  with  $\rho_0 \geq c_0 > 0$ . Moreover, we also discuss the existence of weak solutions and the existence of unique strong solution for  $\rho_0 \in H^1(I)$  and  $\rho_0 \in H^2(I)$ , respectively, satisfying  $\rho_0 \geq c_0 > 0$ .

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### Numerical study for long-time solutions for some hyperbolic conservation laws with nonlinear term

**Chi-Tien Lin**

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In this talk, we study solution behavior for two hyperbolic conservation laws with nonlinear force terms at large time via central-upwind schemes. The most advantage of central-typed scheme is simplicity because no approximate Riemann solver is needed. Central-upwind scheme employs this advantage with less numerical viscosity so that it can be applied to study solution behavior at large time. We start with simulation for an initial-boundary value problem of a 2x2 p-system with nonlinear force term via an central-upwind scheme introduced by Tadmor and Kurganov. We confirm that the solution globally exists and converges to its corresponding diffusion wave, or the solution blows up at a finite time under suitable condition. For convergence case, convergence rates are calculated. We then turn to study solution behavior of an initial-value problem of a 1D Euler-Poisson equation defined on bounded domain. With the help of an improved Kurganov-Tadmorscheme introduced by Kurganov, Noelle and Petrova in 2001, we shall demonstrate that the solution converges to its corresponding boundary value problem.

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### Traveling waves for nonlocal dispersion equation

Ming Mei

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Rui Huang, Yong Wang

In this talk, we study a class of nonlocal dispersion equation with monostable nonlinearity in  $n$ -dimensional space

$$\begin{cases} u_t - J * u + u + d(u(t, x)) = \\ \int_{\mathbb{R}^n} f_\beta(y) b(u(t - \tau, x - y)) dy, \\ u(s, x) = u_0(s, x), \quad s \in [-\tau, 0], x \in \mathbb{R}^n, \end{cases}$$

where the nonlinear functions  $d(u)$  and  $b(u)$  possess the monostable characters like Fisher-KPP type,  $f_\beta(x)$  is the heat kernel, and the kernel  $J(x)$  satisfies  $\hat{J}(\xi) = 1 - \mathcal{K}|\xi|^\alpha + o(|\xi|^\alpha)$  for  $0 < \alpha < 2$ . After establishing the existence for both the planar traveling waves  $\phi(x \cdot \mathbf{e} + ct)$  for  $c \geq c_*$  ( $c_*$  is the critical wave speed) and the solution  $u(t, x)$  for the Cauchy problem, as well as the comparison principles, we prove that, all noncritical planar wavefronts  $\phi(x \cdot \mathbf{e} + ct)$  are globally stable with the exponential convergence rate  $t^{-n/\alpha} e^{-\mu_\tau t}$  for  $\mu_\tau > 0$ , and the critical wavefronts  $\phi(x \cdot \mathbf{e} + c_* t)$  are globally stable in the algebraic form  $t^{-n/\alpha}$ , and these rates are optimal. As application, we also automatically obtain the stability of traveling wavefronts to the classical Fisher-KPP dispersion equations. The adopted approach is Fourier transform and the weighted energy method with a suitably selected weight function. This is a joint work with Rui Huang and Yong Wang.

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### Mathematical and computational aspects of problems involving adhesion, detachment, and collision

Seiro Omata

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We examine the behavior of solutions to free boundary problems expressing the motion of oil droplets and soap bubbles over flat surfaces, as well as bounce-type collision dynamics. Such phenomena are difficult to treat, both analytically and numerically, due to the presence of free boundaries and global constraints (which arise from the presence of volume constraints). We will discuss the problem settings and focus on the development of numerical methods for investigating such phenomena. We will also show the computational results obtained by our methods.

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### On decay estimates for solutions of some parabolic equations

Maria Michaela Porzio

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It is well known that the solution of the heat equation with summable initial datum  $u_0$  satisfies the decay estimate

$$\|u(\cdot, t)\|_{L^\infty} \leq C \frac{\|u_0\|_{L^1}}{t^{\frac{n}{2}}}, \quad t > 0.$$

We show here that decay estimates can be derived simply by integral inequalities. This result allows us to prove this kind of estimates, with an unified proof, for different nonlinear problems, thus obtaining both well known results (for example for the p-Laplacian equation and the porous medium equation) and new decay estimates.

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### Viscosity solutions of a class of degenerate quasilinear parabolic equations

Weihua Ruan

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We study a class of degenerate quasilinear parabolic equations in a bounded domain with a Dirichlet or nonlinear Neumann type boundary condition. The equation under consideration arises from a number of practical model problems including reaction-diffusion processes in a porous medium. Our goal is to establish some comparison properties between viscosity upper and lower solutions and to show the existence of a continuous viscosity solution between them. Applications are given to a porous-medium type of reaction-diffusion model whose global existence and blow up property are sharply different from that of the nondegenerate one.

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### On a class of doubly nonlinear parabolic equations with nonstandard growth: existence, blow-up and vanishing

Sergey Shmarev

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S. Antontsev

The talk addresses the questions of existence and the qualitative behavior of solutions of the homogeneous Dirichlet problem for the doubly nonlinear

anisotropic parabolic equation

$$u_t = \sum_{i=1}^n D_i \left( |D_i(|u|^{m(x)-1}u)|^{p_i(z)-2} D_i(|u|^{m(x)-1}u) \right) + c(z)|u|^{\sigma(z)-2}u.$$

The exponents of nonlinearity  $m(x) > 0$ ,  $p_i(x, t) \in (1, \infty)$ ,  $\sigma(x, t) \in (1, \infty)$  are given functions of their arguments. We prove that the problem admits a strong energy solution in a suitable Orlicz-Sobolev space prompted by the equation and establish sufficient conditions of the finite time blow-up or vanishing.

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### Cauchy problem for the damped singularly perturbed Boussinesq-type equation

**Changming Song**

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We are concerned with the Cauchy problem for the damped singularly perturbed Boussinesq-type equation

$$u_t - u_{xx} - \alpha u_{xxxx} + 2bu_{xxxxx} - \beta u_{xxxxxx} = (u^n)_{xx}$$

in  $\mathbf{R} \times (0, \infty)$ ,

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \mathbf{R},$$

where  $\alpha, \beta > 0$  and  $b > 0$  are real numbers,  $n \geq 2$  is an integer,  $u_0(x)$  and  $u_1(x)$  are the given functions. Under suitable assumptions, we prove that for any  $T > 0$ , the Cauchy problem admits a unique global smooth solution  $u(x, t) \in C^\infty((0, T]; H^\infty(\mathbf{R})) \cap C([0, T]; H^5(\mathbf{R})) \cap C^1([0, T]; H^1(\mathbf{R}))$ .

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### Refined asymptotics for the infinite heat equation with homogeneous Dirichlet boundary conditions

**Christian Stinner**

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**Philippe Laurençot**

We show that the nonnegative viscosity solutions to the infinite heat equation  $\partial_t u = \Delta_\infty u$  with homogeneous Dirichlet boundary conditions converge as  $t \rightarrow \infty$  to a uniquely determined limit after a suitable time rescaling. The proof relies on the half-relaxed limits technique as well as interior positivity estimates and boundary estimates. Moreover, we also study the expansion of the support.

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### Finite-time blow-up in the higher-dimensional Keller-Segel system

**Michael Winkler**

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We study the Neumann initial-boundary value problem for the fully parabolic Keller-Segel system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v), & x \in \Omega, t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega, t > 0, \end{cases} \quad (\star)$$

in a ball  $\Omega \subset \mathbb{R}^n$  with  $n \geq 3$ . This system forms the core of numerous models used in mathematical biology to describe the spatio-temporal evolution of cell populations governed by both diffusive migration and chemotactic movement towards increasing gradients of a chemical that they produce themselves. We demonstrate that for any prescribed  $m > 0$  there exist radially symmetric positive initial data  $(u_0, v_0) \in C^0(\bar{\Omega}) \times W^{1,\infty}(\Omega)$  with  $\int_\Omega u_0 = m$  such that the corresponding solution blows up in finite time. Moreover, by providing an essentially explicit blow-up criterion it is shown that within the space of all radial functions, the set of such blow-up enforcing initial data indeed is large in an appropriate sense; in particular, this set is dense with respect to the topology of  $L^p(\Omega) \times W^{1,2}(\Omega)$  for any  $p \in (1, \frac{2n}{n+2})$ . One focus of the presentation is on the method through which this result is obtained. In contrast to previous approaches, it is based on a more elaborate use of the natural energy inequality associated with  $(\star)$ , involving an estimate of the form

$$\int_\Omega uv \leq C \cdot \left( \left\| \Delta v - v + u \right\|_{L^2(\Omega)}^{2\theta} + \left\| \frac{\nabla u}{\sqrt{u}} - \sqrt{u} \nabla v \right\|_{L^2(\Omega)} + 1 \right),$$

valid with certain  $C > 0$  and  $\theta \in (0, 1)$  for a wide class of smooth positive radial functions  $(u, v) = (u(x), v(x))$ .

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### Global existence and asymptotic behavior of the solutions to the three dimensional bipolar Euler-Poisson systems

**Xiongfeng Yang**

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**Yeping, Li**

In this talk, I will present the Green function of the linearized Euler-Poisson with electric field and frictional damping added to the momentum equations. This method was used to study the optimal decay rate of the evolution PDEs. It was applied to consider the existence of the smooth solution to the three-dimensional bipolar hydrodynamic model when the initial data are close to a constant state.

We found that the electric field affects the dispersion of fluids and reduces the time decay rate of solutions.

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**Existence of monotone traveling waves for a delayed non-monotone population model on 1-D lattice**

**Zhixian Yu**

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In this talk, the existence of monotone traveling waves for general lattice equations with delays will be obtained by a new monotone iteration technique based on a lower solution. The results can be well applied to a delayed non-monotone population model on 1-D lattice and thus the monotone traveling wave will be obtained by choosing a pair of suitable upper-lower solutions, which was left open in some recent works.

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**Longtime dynamics for an elastic waveguide model**

**Yang Zhijian**

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The paper studies the longtime dynamics for a non-linear wave equation arising in elastic waveguide model

$$u_{tt} - \Delta u - \Delta u_{tt} + \Delta^2 u - \Delta u_t - \Delta g(u) = f(x).$$

Under the assumption that  $g$  is of the polynomial growth order, say  $p$ , it proves that (i) when  $1 < p \leq \frac{N+2}{(N-2)^+}$ , the above mentioned model has a global solution in phase space with low regularity  $E_0$ ; (ii) when  $2 \leq p \leq \frac{N}{(N-2)^+}$ , the related solution semi-group  $S(t)$  possesses in  $E_0$  a finite dimensional global attractor  $\mathcal{A}$ , which has  $E_1$ -regularity, and an exponential attractor; (iii) when  $1 \leq p \leq \frac{N+2}{(N-2)^+}$ , the above model possesses a global trajectory attractor.

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## Special Session 43: Stochastic Networks with Applications to Neuroscience

Lee DeVille, University of Illinois, USA  
Georgi Medvedev, Drexel University, USA

Dynamical networks feature a rich variety of spatio-temporal phenomena including synchrony, bursting and avalanches, formation of clusters and waves, intermittency, and chaos. Understanding the mechanisms underlying network dynamics requires a combination of tools from several mathematical disciplines: dynamical systems, stochastic processes, and discrete mathematics. The talks in this special session explore different aspects of network dynamics emphasizing the role of stochasticity and its applications to neuroscience.

**Modeling collective neural activity: when are pairwise maximum entropy methods good enough?**

**Andrea Barreiro**

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**Julijana Gjorgjieva, Fred Rieke, Eric Shea-Brown**

Recent experimental studies find that the activity patterns of many neural circuits are well described by pairwise maximum entropy (PME) models — which require only the activity of single neurons and neuron pairs — even in cases where circuit architecture and input signals seem likely to create a richer set of outputs. Why is this the case? We study spike patterns in a general class of circuits, and attempt to draw general principles about the effects of network architecture and input statistics on the complexity of output spiking patterns. Two significant findings have emerged: in feedforward circuits, responses to unimodal inputs were well described by PME models, while bimodal input signals drove significant departures. Secondly, biophysically motivated recurrence can drive significant departures when added to feedforward circuits.

As a particular application, we investigate the performance of PME models in retinal ganglion cell (RGC) circuits with different architectures and inputs. We find that the distinct filtering properties of parasol cells suppress higher-order interactions by suppressing bimodality in light stimuli, offering a possible mechanism for the remarkable success of PME models in this experimental setting.

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**Dynamics of a stochastic neuronal network model with inhibitory neurons**

**Stephen Berning**

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We will consider extensions and generalizations of the stochastic neuronal network model developed by DeVille et al.; their model corresponded to an all-to-all network of discretized integrate-and-fire excitatory neurons where synapses are failure-prone. It was shown that this model exhibits different metastable phases of asynchronous and synchronous behavior, since the model limits on a mean-field deterministic system with multiple attractors. Our work investigates the effect of adding inhibition and heterogeneity into the model and considers several statistical measures of the dynamics. We will also show that there exists a family of network-supported continuous-time Markov chains that converge to this neuronal network model in a singular limit.

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**Emergence of computation in random networks**

**Stefano Boccaletti**

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**M. Zanin, D. Papo, R. Gutierrez**

One of the open problems in neuroscience is the understanding of how neurons can self-organize to create structures able to perform some meaningful computation. While several theoretical models have been proposed (from Hebbian learning, up to Hopfield networks), they are usually based on regular, or fully connected, initial configurations, which are not found in the early stages of the development of a brain. In this contribution, we will show how a non-trivial computation, i.e., including memory on past states, can be performed by random Erdős-Renyi networks; furthermore, we will show that the probability of the emergence of such feature depends on simple characteristics of the graph, such as the number of nodes and the density of links. These results may represent a new paradigm explaining how natural neural networks, without the help of any external intervention or of any pre-defined architectures, are able to perform the complex computations that we continuously experience.

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**Stochasticity and phase-locking in small neuronal networks**

**Amitabha Bose**

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**Andrea Barreiro, Katka Bodova, Victoria Booth, Badal Joshi, Jung Eun Kim, Jasmin Nirody, Erin Munro**

We investigate stochastic effects in feed-forward neuronal networks. We study a simple network consisting of an oscillator (O) that forces a follower (F) cell to determine the phase-locked solutions of the network. The case of having a deterministic synapse from O to F has been previously studied. We add to the results by explicitly deriving a one-dimensional map involving the PRC of F that reproduces the devil's staircase of solutions. We then show how adding stochasticity to the synapse allows us to continue to use the map formalism to show which portions of the staircase persist. The analysis allows us to give meaning to the term "phase-locking" even in a stochastic setting.

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**Spatially structured networks from sequences****Carina Curto**University of Nebraska-Lincoln, USA  
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Spatially structured networks, such as bump attractor networks, have enjoyed considerable success in modeling a wide range of phenomena in cortical and hippocampal networks. A key question that arises in the case of hippocampal models, however, is how such a spatial organization of the synaptic connectivity matrix can arise in the absence of any a priori topographic structure of the network. Here we demonstrate a simple mechanism by which robust sequences of neuronal activation, such as those observed during hippocampal sharp waves, can lead to the formation of spatially structured networks that exhibit robust bump attractor dynamics.

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**Communicability distance in networks****Ernesto Estrada**University of Strathclyde, Scotland  
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We introduce the concept of communicability distance on the basis of a transformation of the exponential of the adjacency matrix of the network of  $n$  nodes. We prove that the communicability distance is a Euclidean distance. In addition we also prove that the points given rise to the communicability distance can be embedded into a hyper-sphere of dimension  $n-1$  and certain radius determined by the inverse of the communicability distance matrix. We show a few mathematical properties of the communicability distance and compare it with the shortest path distance and the resistance distance. We show that the communicability distance is very useful in identifying the best routes for information traffic in crowded networks. We show some illustrative examples for the case of brain networks.

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**Memory encoding via perturbations of spatially structured networks.****Vladimir Itskov**University of Nebraska-Lincoln, USA  
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Memories in the brain are believed to be encoded via patterns of synaptic strengths of recurrent neuronal networks. It is unclear how exactly this can work however. Given a prescribed list of memories, it is still an unsolved problem how the synaptic weights can be arranged so that exactly those memories are encoded, while avoiding unwanted "spurious" states. The problem is challenging because encoded patterns are usually heavily overlapping; this case is prevalent in many important areas of the brain such as hippocampus. Here we develop a perturbative approach to the memory encoding problem, where memory patterns are encoded via small perturbations of the synaptic weights around a spatially structured low-rank network. We find large classes of sets of overlapping memories that can be encoded exactly as steady states of threshold-linear networks.

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**The geometry of spontaneous spiking in neuronal networks****Georgi Medvedev**Drexel University, USA  
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We study electrically coupled networks of excitable neurons forced by small noise using techniques from dynamical systems and algebraic graph theories. The results are presented from two complementary perspectives of variational analysis of spontaneous network dynamics and slow-fast analysis of synchronization. The former approach yields geometric interpretation of various dynamical regimes including weakly correlated firing, formation of clusters, and complete synchronization; while the latter highlights the contribution of the network architecture to the stability of the synchronous state.

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**Synchrony in stochastic pulse-coupled neuronal network models****Katherine Newhall**Courant Institute, NYU, USA  
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We are interested in the synchronous dynamics of simple pulse-coupled models for neuron dynamics. These time correlations in firing times are seen experimentally. In our model, the size of synchronous firing events depends on the probabilistic dynamics between such events as well as the network structure representing the neuron connections. We presents both analytical results and numerical simulations of these global dynamics.

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**Network structure and synchronization of coupled oscillators****Takashi Nishikawa**Clarkson University, USA  
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Synchronization, in which individual dynamical units keep in pace with each other in a decentralized fashion, depends both on the dynamical units and on the properties of the interaction network. Yet, the role played by the network has resisted comprehensive characterization. In this talk, I will present results that illuminate the intricate relationship between structure and synchronization stability in coupled oscillator networks. Using networks with best complete synchronization, least coupling cost, and maximum dynamical robustness, I will show that "less can be more" in network synchronization: *negative* interactions as well as link *removals* can be used to systematically improve and optimize synchronization properties in both directed and undirected networks. I will also show that having local and/or global *directionality* structure in the network facilitates synchronization.

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### The influence of network structure on neuronal network dynamics

**Duane Nykamp**

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We investigate the influence of network structure on the dynamics of neuronal networks, with a focus on the emergence of synchronous oscillations. Network structure is specified using the framework of second order networks, a network model that captures second order statistics (correlations) among the connections between neurons. We demonstrate that the frequency of a chain motif in the network plays a crucial role in influencing network dynamics, not only modulating the emergence of synchrony but also possibly increasing the range of possible network behaviors.

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### Criticality and dynamic range in network cascading processes

**Juan Restrepo**

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**Daniel Larremore, Edward Ott, Woodrow Shew**

I will present recent work on the effect of network topology on cascading processes. The motivation for our work is a series of recent experiments on cascades of excitation in rat cortical tissue cultures, where it is found that these neural networks maximize their dynamic range (the range of stimulus intensities resulting in distinguishable network responses). We develop a theoretical framework to study the effect of network topology on the response to a stochastic stimulus, and find that the dynamic range is maximized when the largest eigenvalue of the transmission probability matrix is one. In the critical regime with maximum dynamic range, the response of the network is characterized by excitation avalanches with power-law distributions of size and duration. We find that these experimental signatures of criticality are robust to the underlying network structure. Using our theory, we characterize the network topologies that can achieve the largest dynamic range. I will discuss potential applications of these results to other networked systems.

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### Competing spatiotemporal neural codes in the olfaction of the *Manduca sexta* moth

**Eli Shlizerman**

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**Jeffery Riffell, J. Nathan Kutz**

Experiments across different species have shown that perception of odors in the olfactory system is associated with neural encoding patterns. The neurobiological mechanisms responsible for such encoding patterns, their transient dynamics and interactions are yet to be fully understood. We show that a data-driven computational model reduction for the antennal (olfactory) lobe (AL) of the *Manduca sexta* moth reveals the nature of experimentally observed persistent spatial and temporal neural encoding patterns and its associated decision-making dynamics. Utilizing the experimental data we reduce a high-dimensional neural network model of the AL to a decision making model. Analyzing the model we conclude that the mechanism responsible for the robust and persistent appearance of neural codes is a stable fixed point. The model is used to explain, predict and direct experiments when odors are mixed or the structure of the network is altered.

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### Exact mean-field dynamics for a heterogeneous network of globally coupled theta neurons

**Paul So**

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**Tanushree Luke, Ernest Barreto**

The theta neuron is a canonical model for a spiking neuron with Type-I excitability. In this work, we consider a large heterogeneous network of such neurons coupled globally with a pulse-like synapse. Utilizing the recently developed Ott-Antonsen reduction method, we obtained an exact low dimensional mean field result for this network of theta neurons in the thermodynamic limit. Each individual theta neuron is chosen randomly with its parameter drawn from a given time-invariant distribution. By choosing the distribution function to straddle the SNIC (saddle-node on a limit cycle) bifurcation point, this network of theta neurons when uncoupled will naturally segregate into a group of resting but excitable neurons and another group of constantly-spiking neurons. Using the mean field reduction, we analyzed and categorized the range of possible collective behavior for this heterogeneous network of coupled theta neurons as the relative proportion of neurons belonging to the two fundamental groups was varied.

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## Special Session 44: Applications of Chaotic and Stochastic Multiscale Dynamics

**Rafail Abramov**, Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, USA

**Gregor Kovacic**, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, USA

**Ilya Timofeyev**, Department of Mathematics, University of Houston, USA

Multiscale, chaotic and stochastically parameterized processes are common in many areas of contemporary science, such as molecular dynamics, genetics, neuroscience, nonlinear optics, and geosciences, among others. Key topics on applications of chaotic and stochastic dynamics for complex multiscale systems will be brought together in this interdisciplinary session, ranging from fluid dynamics to geophysical sciences, nonlinear optics and computational biology. Examples of applications will include low frequency variability and climate change, parameter estimation in complex unresolved systems, prediction of statistical behavior under external perturbations, applications of multiscale high-dimensional coagulation processes, random dynamics of neuronal networks, stochastic soliton dynamics, and more. Common to all these topics is the mathematical approach used to address them, which will be highlighted in the session.

### A simple linear response closure approximation for slow dynamics of a multiscale system

**Rafail Abramov**

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We propose a method of determining the closed model for slow variables of a multiscale system, which requires only a single computation of appropriate statistics for the fast dynamics with a certain fixed state of the slow variables. The method is suitable for situations with linear, quadratically nonlinear, and multiplicative coupling, and is based on the first-order Taylor expansion of the mean state and covariance matrix of the fast variables with respect to changes in the slow variables, which can be computed using the linear fluctuation-dissipation theorem. We show that the method produces quite comparable statistics to what is exhibited by a complete two-scale model. The main advantage of the method is that it applies even when the statistics of the full multiscale model cannot be simulated due to computational complexity, which makes it practical for real-world large scale applications.

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### Compressed Sensing in Retinal Image Processing

**Victor Barranca**

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Retinal image processing transforms photons into membrane potentials via several nonlinear transformations. This process begins in a large network of photoreceptors and ends in a relatively small ganglion cell network. We posit the loss of visual information despite the decrease in network size is minimized via compressed sensing. Using an idealized mathematical model of the retina and a mean-field analytical

reduction, we demonstrate firing patterns among ganglion cells can be used to reconstruct input images.

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### Low-dimensional descriptions of neural networks

**Andrea Barreiro**

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**Nathan Kutz, Eric Shea-Brown, Eli Shlizerman**

Biological neural circuits display both spontaneous asynchronous activity, and complex, yet ordered activity while actively responding to input. When can model neural networks demonstrate both regimes? Recently, researchers have demonstrated this capability in large, recurrently connected neural networks, or “liquid state machines”, with chaotic activity. We study the transition to chaos in a family of such networks, and use principal orthogonal decomposition techniques to provide a lower-dimensional description of network activity. Finally, we connect our findings to the ability of these networks to perform as liquid state machines.

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### A general method for parametric estimation of stochastic volatility models

**Arjun Beri**

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**Robert Azencott, Ilya Timofeyev**

Modeling dynamics of a physical phenomenon by parametric stochastic differential equation is widespread. In many situations classical techniques of parametric inference given discrete observations do not provide consistent estimates for the drift and diffusion parameters. I will present asymptotic results

for a test case of parametric estimation of a stochastic differential equation. In particular, I consider a stochastic volatility model specified as a system of two-dimensional stochastic differential equation  $(X, Y)$ , where  $Y$  represents the stochastic dynamics of the diffusion coefficient of process  $X$ . Assuming unobservability (or, latency) of the volatility process  $Y$ , parametric estimation of  $Y$  is based only on the available discrete observations from  $X$ . Estimation of parameters characterizing  $Y$  involves an intermediate step of constructing a dataset of empirical estimates for quadratic variation of  $X$ , denoted by  $V_t^\varepsilon = [X]_{t-\varepsilon}^t$ , such that  $V_t^\varepsilon$  converges to  $Y_t$  as  $\varepsilon \rightarrow 0$ . I will present optimal subsampling conditions, verified numerically, for asymptotic consistency of estimators for parameters in  $Y$  based on observations from approximate process  $V_t^\varepsilon$ , and also highlight the general context of estimation under indirect observability, estimation of reduced order model given multiscale (fast-slow) system, and the role of path properties, such as modulus of continuity, in parameter estimation of a limiting model.

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### Finding quasipotential for nongradient SDE's

**Maria Cameron**

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Nongradient Stochastic Differential Equations describe overdamped systems without detailed balance that arise e.g., in chemical reactions far from equilibrium, evolutionary biology, and stochastically modeled computer networks. Nongradient systems allow more complex attractors than stable equilibrium points. The behavior of such a system can be quantified in terms of the quasipotential. It allows to estimate the probability density and transition rates between different attractors, and to find the most likely transition paths between them. I will discuss some theoretical properties of the quasipotential and present a numerical algorithm for computing it on a regular mesh.

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### Nonlinear stochastic inverse models with memory, and prediction of climatic phenomena

**Mickael Chekroun**

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**Dmitri Kondrashov, Michael Ghil, Andrew Robertson**

Recent works on stochastic Navier-Stokes equations and previous approaches coming from Statistical Mechanics such as the Mori-Zwanzig (MZ) formalism, have advocated the idea of representing the higher

(unresolved) modes as functionals of the time history of the low (resolved) modes to deal with the closure problem. In the MZ formalism, these functional dependence arise typically in complicated integral terms obtained by repeated convolutions between decaying memory kernels and the resolved variables. In the case of a lack of scale separation, these memory kernels roughly decay at the same rate than the decorrelation rate of the solution itself; which renders challenging the numerical computations of these integral terms and thus the obtention of an efficient solution to the closure problem via this approach.

By considering a certain class of memory kernels within the MZ formalism, it will be presented a numerically tractable data-driven approach to deal with this problem while allowing the cases where the separation of scales is not necessarily pronounced. The approach will be illustrated for the inverse modeling of two major tropical climatic phenomena: the El Niño-Southern Oscillation (ENSO) and the Madden-Julian Oscillation (MJO). Prediction capabilities of the resulting nonlinear stochastic inverse models will be discussed in each case.

The past noise forecasting method recently developed by Chekroun, Kondrashov and Ghil (PNAS, 108 (29), 2011) will be then presented. It will be shown how this method allows to extend the prediction skill of our MJO and ENSO models by using one hand, information from the estimated path on which the inverse stochastic model lives; and on the other, dynamical features associated with the low-frequency variability captured by these models.

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### Strong collapse turbulence in quintic nonlinear Schroedinger equation.

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**P.M. Lushnikov**

We consider the quintic one dimensional nonlinear Schrödinger equation with forcing and both linear and nonlinear dissipation. Quintic nonlinearity results in multiple collapse events randomly distributed in space and time forming forced turbulence. Without dissipation each of these collapses produces finite time singularity but dissipative terms prevents actual formation of singularity. In statistical steady state of the developed turbulence the spatial correlation function has a universal form with the correlation length determined by the modulational instability scale. The amplitude fluctuations at that scale are nearly-Gaussian while the large amplitude tail of probability density function (PDF) is strongly non-Gaussian with power-like behavior. The small amplitude nearly-Gaussian fluctuations seed formation of large collapse events. The universal spatio-temporal form of these events together with the PDF for their maximum amplitudes define the power-like tail of



PDF for large amplitude fluctuations, i.e., the intermittency of strong turbulence.

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### Stochastic dynamics on networks

**Lee Deville**

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Dynamical systems defined on networks have applications in many fields, including neuroscience, ecology, biophysics, condensed matter theory, etc. In particular, it is interesting to understand if and when networked dynamical systems exhibit synchronous or coherent collective behaviors, but this question has proved surprisingly difficult to answer in many contexts. Yet another question is how stable coherent behavior is to random perturbation, or, conversely, what types of network-level structures can be induced by noise alone. We will examine several applications that fall into this framework and present a few theorems that make this analysis possible.

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### Stochastic models for organized tropical convection

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**Andrew Majda, Yevgeniy Frenkel**

Physical processes associated with clouds and convection have a big impact on large scale circulation and climate variability in the tropics. Current operational climate models capture very poorly the radiative-green house effects and latent heat release due to the formation of clouds and the inherent large scale organization into large (synoptic to planetary) scale convectively coupled waves. To help improve the variability in climate models due to unresolved organized convection, we present and analyze a hierarchy of stochastic and deterministic multcloud cloud models.

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### Dynamics of light interacting resonantly with an active optical medium

**Gregor Kovacic**

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**E. P. Atkins, G. Biondini, I. R. Gabitov, P. R. Kramer**

Resonant interaction of light pulses with a randomly-prepared, lambda-configuration active optical medium is described by exact solutions of a

completely-integrable, random partial differential equation, thus combining the opposing concepts of integrability and disorder. An optical pulse passing through such a material will switch randomly between left- and right-hand circular polarizations. Exact probability distributions of the electric-field envelope variables describing the light polarization and their switching times will be presented. The dynamics of pulses on top of continuous waves will also be discussed.

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### Faster dynamic Monte Carlo via Markov couplings

**Kevin Lin**

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**Jonathan Goodman, Jonathan Mattingly**

Dynamic Monte Carlo methods are widely used in scientific and engineering computations, and are frequently applied to stochastic dynamical systems exhibiting chaotic behavior. In this talk, I will report on recent efforts to accelerate dynamic Monte Carlo calculations using a tool from probability theory, namely Markov couplings. Specifically, I will discuss coupling-based algorithms for two distinct but related problems: sensitivity analysis for stochastic differential equations and variance reduction for nonequilibrium steady-state calculations in statistical physics.

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### A traffic model for pedestrian and its comparison with experimental data

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**Pierre Degond, Cecile Appert-Rolland, Mehdi Moussaid, Guy Theraulaz**

Understanding the various factors which determine pedestrian flow efficiency is an important societal issue (e.g. traffic safety, optimization of traffic flow). With this aim, a simple model has been constructed and calibrated on the basis of experimental recordings. The novelty of the model (2-way Lighthill-Whitham-Richards model) relies on the introduction of a two direction flow depending on co and counter-moving pedestrians. At high density, the model becomes "non-hyperbolic" which leads to the formation of clusters similar to traffic jam observed in high density crowd. A comparison between data and model is performed and we observe good agreement for the apparition of clusters and traveling bands. However, some discrepancies are observed, which are due to the inhomogeneities of the pedestrians comfort walking speeds. To account for these inhomogeneities,

an extension of the model is presented (2-Way Aw-Rascle model). The comfort speed is assimilated from the data by observing the pedestrian velocity at low densities. The resulting model and assimilated initial data are compared to the experimental results and agree quite well.

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### Magnetization reversal in thin film magnetic elements

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**Eric Vanden-Eijnden**

There is considerable interest in understanding thermally induced magnetization reversals in thin film magnetic elements, with application to random access memory storage. We investigate the rare event of thermally induced magnetization reversal in thin film magnetic elements. We include the effect of spin transfer torques, and their influence on the average switching times. To determine the average time for these rare events in the stochastic multi-scaled system, we consider the evolution of the averaged slow variable, the energy, and use a combination of computer simulations and analytical techniques.

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### The neurochemical dynamics of the mammalian sleep-wake regulatory network

**Lisa Rogers**

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The mammalian sleep/wake system is governed by several interacting populations of neurons in and around the hypothalamus. We present here a model of the sleep and wake promoting populations in the VLPO (ventrolateral preoptic nucleus), BF (basal forebrain) and PB/PC (parabrachial nucleus). The model is formed using Morris-Lecar firing dynamics and the chemical kinetics of receptor-neurotransmitter interaction to quantify synaptic input. We also present a novel but simple way of relating firing rates of neuron populations to the corresponding concentrations of neurotransmitter, allowing us to track both electrical and chemical output. Rate and equilibrium constants are obtained using appropriate mammalian data from the BRENDA enzyme database.

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### A multiscale method for epitaxial growth

**Yi Sun**

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**Russel Caflisch, Bjorn Engquist**

We investigate a heterogeneous multiscale method (HMM) for interface tracking and apply the technique to the simulation of epitaxial growth. HMM relies on an efficient coupling between macroscale and microscale models. When the macroscale model is not fully known explicitly or not accurate enough, HMM provides a procedure for supplementing the missing data from a microscale model. Here we design a multiscale method that couples kinetic Monte-Carlo (KMC) simulations on the microscale with the island dynamics model based on the level set method and a diffusion equation. We perform the numerical simulations for submonolayer island growth and step edge evolutions on the macroscale domain while keeping the KMC modeling of the internal boundary conditions. Our goal is to get comparably accurate solutions at potentially lower computational cost than for the full KMC simulations, especially for the step flow problem without nucleation.

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### From stochastic to coarse-grained models of pedestrian traffic

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**A. Chertock, A. Kurganov, A. Polizzi**

Microscopic rules for pedestrian traffic in a narrow street or corridor are discussed and the corresponding stochastic system modeling the pedestrian bi-directional flow is introduced. Mesoscopic and macroscopic PDE models for the pedestrian density are derived. The macroscopic PDE model is a system of conservation laws which can change type depending on the strength of interaction between the pedestrian flows and initial conditions. Behavior of the stochastic and coarse-grained models is compared numerically for several different regimes and initial conditions. Finally, nonlinear diffusive corrections to the PDE model are derived systematically. Numerical simulations show that the diffusive terms can play a crucial role when the conservative coarse-grain PDE model becomes non-hyperbolic.

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### Irreversible Monte Carlo algorithms for efficient sampling

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**Konstantin S. Turitsyn, Michael Chertkov**

Equilibrium systems evolve according to Detailed Balance (DB). This principle guided the development of Monte Carlo sampling techniques, of which the Metropolis-Hastings (MH) algorithm is the famous representative. It is also known that DB is sufficient but not necessary. We construct irreversible deformation of a given reversible algorithm capable of dramatic improvement of sampling from known distribution. Our transformation modifies transition rates keeping the structure of transitions intact. To illustrate the general scheme we design an Irreversible version of Metropolis-Hastings (IMH) and test it on an example of a spin cluster. Standard MH for the model suffers from critical slowdown, while IMH is free from critical slowdown.

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### The effects of time iteration schemes on the climate of the Lorenz 96 model

**Celestine Woodruff**

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**Xiaoming Wang**

We consider the effects of using several different time iteration schemes on the chaotic Lorenz 96 model to predict statistics, which give us the “climate”. This is a simple model, but it exhibits energy-preserving advection, damping, and forcing (some of the same properties that can be found in geophysical models), shares statistical features of weather wave packets of the atmosphere, and is an analogue of Rossby waves. The schemes that we consider are Forward Euler, Backward Euler, 4th order Runge-Kutta, and four new schemes which we call Semi-Implicit 1 (SI1) and Semi-Implicit 2 (SI2), 2nd order Semi-Implicit 1 (SSI1) and 2nd order Semi-Implicit 2 (SSI2). We give theoretical results for stability, with unconditional stability for Backward Euler, SI2, and SSI2. Using a numerical truth, we find that for a small enough time step, all of the methods perform very well. We also see grossly inaccurate solutions and blowup when using Forward Euler for the time step  $h = 10^{-2}$ , which should be a reasonable time step. We show that SI2, which also has energy-preserving properties, is a much better choice for this model and that care needs to be taken when choosing time iteration schemes for climate models in general.

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## Special Session 45: Stochastic and Deterministic Dynamical Systems, and Applications

**Tomas Caraballo**, University of Sevilla, Spain  
**Jose Valero Cuadra**, University Miguel Hernandez (Elche), Spain  
**Maria Garrido-Atienza**, University of Sevilla, Spain

The aim of this session is to offer an overview on recent results concerning the asymptotic behaviour of solutions of stochastic and deterministic partial and ordinary differential equations. The main topics of the session are: existence and properties of pullback attractors for stochastic and non-autonomous equations, stability, stabilization, attractors for equations without uniqueness, dynamics of equations with delay and finite-dimensional dynamics for dynamical systems.

### Lyapunov exponents for autonomous and non-autonomous systems

**Francisco Balibrea**  
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**M. Victoria Caballero**

It is an extended idea to associate positive Lyapunov exponents to instability of orbits in both autonomous and non-autonomous systems and stability to negative values of such exponents. But such issue is not true in general. We will consider separately the two cases stating under what conditions the former statement is true. We will construct one and two dimensional systems to show the differences between the autonomous and non-autonomous situations. Obviously the non-autonomous case is too much more complicated. We will extend the results to  $n$ -dimensional systems of both types.

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### Analytical and numerical results on escape of brownian particles

**Carey Caginalp**  
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A particle moves with Brownian motion in a material, e.g. a unit disc, with reflection from the boundaries except for a portion (called a "window" or "gate") in which it is absorbed. A key question involves the differences between the small, finite step size and the limiting Brownian motion. The stochastic problem of Brownian motion can be transformed into an elliptic PDE with mixed boundary conditions. Our work confirms an asymptotic formula that had been obtained by Chen and Friedman. Furthermore, we obtain an exact solution for a gate of any size.

The main problems are to determine the first hitting time and spatial distribution. Also given is the probability density of the location where a particle hits if initially the particle is at the center or uniformly distributed. Numerical simulations of the stochastic process with finite step size and sufficient amount of sample paths are compared with the exact solution to the Brownian motion (the limit of zero

step size), providing an empirical formula for the divergence. Histograms of first hitting times are also generated. These problems have applications to cell biology and materials science.

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### Asymptotic behavior of linear elliptic problems with Dirichlet conditions on random varying domains

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**Juan Casado-Díaz, Manuel Luna-Laynez**

Given  $(\Omega, \mathcal{F}, P)$  a probability space, we study the asymptotic behavior of the solutions of linear elliptic problems posed in a fixed domain  $O \subset \mathbb{R}^N$ ,  $N \geq 3$ , satisfying Dirichlet boundary conditions on a random sequence of "holes"  $\Gamma_n(\omega) \subset O$ ,  $P$ -a.e.  $\omega$  in  $\Omega$ . Under assumptions about the size and the distribution of the holes, analogously to the classical Cioranescu-Murat "strange term", we show the existence of a new term of order zero in the limit equation. The proof of this result is based on the ergodic theory and Nguetseng and Allaire's two scale convergence method.

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### Asymptotic behaviour of lattice systems perturbed by additive noise

**Tomas Caraballo**  
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In this talk we study the asymptotic behaviour of solutions of a first-order stochastic lattice dynamical system perturbed by noise. We do not assume any Lipschitz condition on the nonlinear term, just a continuity assumption together with growth and dissipative conditions, so that uniqueness of the Cauchy problem fails to be true. Using the theory of multi-valued random dynamical systems we prove the existence of a random compact global attractor.

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**Fractional stochastic porous media equations****Maria Garrido-Atienza**University of Seville, Spain  
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The aim of this talk is to study the existence and uniqueness of solutions of stochastic porous media equations driven by fractional Brownian motion, when the Hurst parameter  $H > 1/2$ . Using the theory of random dynamical systems we also discuss some results on the asymptotic behavior of these solutions.

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**Cahn-Hilliard equations with memory effects****Maurizio Grasselli**Politecnico di Milano, Italy  
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**C.G. Gal, C. Cavaterra**

We introduce a modified Cahn-Hilliard equation proposed by P. Galenko et al. in order to account for rapid spinodal decomposition in deep supercooling glasses or in binary alloys. In this equation the dynamics of the order parameter depends on past history of the chemical potential. Such dependence is expressed through a time convolution integral characterized by a smooth nonnegative exponentially decreasing memory kernel. We intend to present some results on this non-standard equation with special regard to dynamic boundary conditions and long-time behavior of solutions. In particular, we discuss the convergence of solutions when the memory kernel approaches the Dirac mass.

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**A fractional stochastic Schrödinger equation****Wilfried Grecksch**Martin-Luther-University, Germany  
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We consider  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P)$  to be a filtered complete probability space. Let  $(V, (\cdot, \cdot)_V)$  and  $(H, (\cdot, \cdot))$  be separable complex Hilbert spaces, such that  $(V, H, V^*)$  forms a triplet of rigged Hilbert spaces. Let  $K$  be a separable real Hilbert space. We assume  $(W(t))_{t \geq 0}$  to be a  $K$ -valued cylindrical Wiener process adapted to the filtration  $(\mathcal{F}_t)_{t \geq 0}$  and  $(B^h(t))_{t \geq 0}$  to be a  $K$ -valued cylindrical fractional Brownian motion with Hurst index  $h \in (\frac{1}{2}, 1)$  adapted to the filtration  $(\mathcal{F}_t)_{t \geq 0}$ .

We study the properties of the *variational solution*  $X$  of the following stochastic nonlinear evolution

equation of Schrödinger type

$$\begin{aligned} (X(t), v) &= (X_0, v) \\ &- i \int_0^t \langle AX(s), v \rangle ds \\ &+ i \int_0^t (f(s, X(s)), v) ds \\ &+ i \left( \int_0^t g(s, X(s)) dW(s), v \right) \\ &+ i \left( \int_0^t b(s) dB^h(s), v \right) \end{aligned}$$

for a.e.  $\omega \in \Omega$  and all  $t \in [0, T], v \in V$ .

Especially, conditions are given such that the Malliavin derivative of the solution process exists.

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**On the global attractor for the Kazhikhov-Smagulov equations****Juan Gutierrez Santacreu**Universidad de Sevilla, Spain  
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In this talk we will study the existence of the global attractor for the Kazhikhov-Smagulov equation which govern the dynamics of the two fluids having different densities subject to Fick's law:

$$\begin{cases} \partial_t \rho + \mathbf{u} \cdot \nabla \rho - \lambda \Delta \rho = \mathbf{0} & \text{in } \Omega \times (0, \infty), \\ \rho \partial_t \mathbf{u} + (\rho \mathbf{u} \cdot \nabla) \mathbf{u} - \mu \Delta \mathbf{u} + \nabla p - \lambda (\nabla \rho \cdot \nabla) \mathbf{u} - \lambda (\mathbf{u} \cdot \nabla) \nabla \rho = \rho \mathbf{f} & \text{in } \Omega \times (0, \infty), \\ \nabla \cdot \mathbf{u} = 0 & \text{in } \Omega \times (0, \infty), \end{cases}$$

for a bounded two-dimensional domain  $\Omega$ , with initial conditions

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{0}, \quad \partial_n \rho(\mathbf{x}, t) = 0 \quad (\mathbf{x}, t) \in \partial \Omega \times (0, \infty),$$

and the boundary conditions

$$\rho(\mathbf{x}, 0) = \rho_0(\mathbf{x}), \quad \mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}) \quad \mathbf{x} \in \Omega.$$

Here  $\mathbf{u} : \Omega \times (0, \infty) \rightarrow \mathbb{R}^2$  is the velocity vector field,  $p : \Omega \times (0, \infty) \rightarrow \mathbb{R}$  is the pressure scalar field, and  $\rho : \Omega \times (0, \infty) \rightarrow \mathbb{R}$  is the density scalar field. The parameters  $\mu$  and  $\lambda$  are assumed to be constant and represent the kinematic viscosity and a mass diffusion coefficient, respectively.

To simplify the discussion, we will only focus on the evolution of the velocity. For this, we will

take a fixed initial density data  $\rho_0$ . Then we will prove that the velocity trajectories of the weak solutions of the Khazhikov-Smagulov equations are absorbed by a connected, compact invariant set  $\mathcal{A}$  in the natural phase space for the Navier-Stokes problem.

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**Features of fast living: on the weak selection for longevity in degenerate birth-death processes.**

**Hyejin Kim**

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**Yen Ting Lin, Charles Doering**

Deterministic descriptions of dynamics of competing species with identical carrying capacities but distinct birth, death, and reproduction rates predict steady state coexistence with population ratios depending on initial conditions. Demographic fluctuations described by a Markovian birth-death model break this degeneracy. A novel large carrying capacity asymptotic theory confirmed by conventional analysis and simulations reveals a weak preference for longevity in the deterministic limit with finite-time extinction of one of the competitors on a time scale proportional to the total carrying capacity.

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**Some generalizations of the Cahn-Hilliard equation**

**Alain Miranville**

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Our aim in this talk is to discuss a Cahn-Hilliard model with a proliferation term which has, e.g., applications in biology. In particular, we will discuss the asymptotic behavior of the system in terms of finite-dimensional attractors.

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**SDEs driven by fractional Brownian motion: continuous dependence on the Hurst parameter**

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**M.J. Garrido-Atienza**

In this talk, we study the stochastic differential equation

$$dX_t^H = a(X_t^H)dt + \sigma(X_t^H)dB_t^H,$$

$t \in [0, T]$ ,  $X_0^H = x_0 \in \mathbb{R}^d$ , where  $a : \mathbb{R}^d \rightarrow \mathbb{R}^d$  and  $\sigma : \mathbb{R}^d \rightarrow \mathbb{R}^{d,m}$  satisfy standard smoothness assumptions and  $B^H = (B_t^H)_{t \in [0, T]}$  is an  $m$ -dimensional fractional Brownian motion with Hurst parameter  $H \in (0, 1)$  defined on a probability space  $(\Omega, \mathcal{A}, \mathbf{P})$ . Using tools from rough path theory we show that the law of the solution  $X^H = (X_t^H)_{t \in [0, T]}$  depends continuously on  $H \in [1/2, 1)$ , i.e. we have

$$\mathbf{P}^{X^H} \longrightarrow \mathbf{P}^{X^{H_0}} \quad \text{for } H \rightarrow H_0$$

with  $H, H_0 \in [1/2, 1)$ . Moreover, in the case of additive noise we give a stronger pathwise continuous dependence result for  $H \in (0, 1)$  and discuss applications to random attractors of such equations.

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**Localization of solutions to stochastic porous media equations: finite speed of propagation**

**Michael Roekner**

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**Viorel Barbu**

We present a localization result for stochastic porous media equations with linear multiplicative noise. More precisely, we prove that the solution process  $\mathbf{P}$ -a.s. has the property of "finite speed propagation of disturbances" in the sense of [Antontsev/Shmarev, Nonlinear Analysis 2005].

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### The effects of noise on sliding motion

**David Simpson**

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**Rachel Kuske**

Vector fields that are discontinuous along codimension-one surfaces are used as mathematical models of a wide range of physical systems involving a discontinuity or switch such as relay control systems and vibro-impacting systems. Trajectories evolve on discontinuity surfaces whenever the vector field on both sides of the surface points towards the surface. This is known as sliding motion and is formulated by the method of Filippov. To investigate the possible role of background vibrations, parametric uncertainty and model error, we analyze the effect of small, additive, white Gaussian noise on sliding trajectories in a simple two-dimensional, discontinuous vector field. We find that the noise pushes orbits slightly off the discontinuity surface and that this may induce a significant change in the observed dynamics. In addition, for this system we show that the mean of the stochastic solution limits on Filippov's definition as the noise amplitude is taken to zero.

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### Sufficient and necessary criteria for existence of pullback attractors for non-compact random dynamical systems

**Bixiang Wang**

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We study pullback attractors of non-autonomous non-compact dynamical systems generated by differential equations with non-autonomous deterministic as well as stochastic forcing terms. We first introduce the concepts of pullback attractors and asymptotic compactness for such systems. We then prove a sufficient and necessary condition for existence of pullback attractors. We also introduce the concept of complete orbits for this sort of systems and use these special solutions to characterize the structures of pullback attractors. For random systems containing periodic deterministic forcing terms, we show the pullback attractors are also periodic. As an application, we prove the existence of a unique pullback attractor for Reaction-Diffusion equations on unbounded domains with both deterministic and random external terms.

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## Special Session 46: Discrete/Continuous and Nonstandard Analysis

**Kiyoyuki Tchizawa**, Institute of Administration Engineering, Ltd., Japan  
**Imme van den Berg**, University of Evora, Portugal

Our session “discrete/continuous and nonstandard analysis” contains variational problem and linear/nonlinear differential equations and logics itself. Taking intervals for state/time very small in each system, there exists a problem on discrete/continuous and nonstandard analysis. In some cases, we have a solution of these relations, but generally not yet enough. In quantum field, Prof S.Nagamachi gave “minimum length”. We need such an example. There are many problems when discretizing differential equations, for example, wavelet but not only one thing. Even taking a usual limit, we met many singular states, especially for parameters. We are going to discuss such a singular state in our session.

### Smooth models of discontinuous systems

**Luis Albuquerque**

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In this talk we present smooth models of discontinuous systems and make a comparative study of their dynamics and bifurcations using Nonstandard Analysis, once the difference between continuity and S continuity allows the existence of smooth models of quick changing situations, where usually a discontinuous approach is used.

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### The Osgood integral: an extraordinary tool

**Rachid Bebbouchi**

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Initially, Osgood [4] used the integral of Osgood for an unicity criterion to the differential equation  $dy/dx = f(y)$ ,  $f(0) = 0$ . The trivial solution is unique iff this integral goes to the infinite at the origin. Then he can prove the unicity of the trivial solution of  $dy/dx = yLn|y|$ , although the second member is not lipschitzian. Later, Bernfeld [1] shows that all the solutions of  $dy/dx = f(y)$  do not explose iff the same integral goes to the infinite at the infinite and Ceballos-Lira, Macias-Diaz and Villa [3] generalize this Osgood’s test. Finally, we can adapt a result from the Cauchy works [2] as follows: the trivial solution is a singular solution iff the same integral vanishes at the origin. Using non standard analysis, we extend the different criterions to non autonomous differential equations and generalize them.

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### How to find infinitesimals in a big genetic-metabolic model?

**Eric Benoît**

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In the study of metabolism and expression of genes, it appears commonly differential systems of large dimension (dozens of unknowns). To make the most of these models, it is necessary to take account of the orders of magnitude. In fact, we want to modelize the differential system, with non standard (infinitely small or infinitely large parameters) coefficients. The aim of the talk is to discuss the different methods to introduce these infinitesimals in a real system.

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### Error estimation for approximate solutions of SDE

**Shuya Kanagawa**

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The numerical solution of Ito’s stochastic differential equation (SDE) is realized by pseudo-random numbers in terms of an approximate solution on computers. Pseudo-random numbers are defined by some algebraic algorithms such as the linear congruential method, M-sequence, etc. For more details,



see e.g. Knuth (1981) and Fushimi (1989). Since any algorithm has an essential defect for independence and distribution, as Knuth (1981) pointed out, pseudo-random numbers do not behave completely as a sequence of independent random variables with the uniform distribution. Furthermore pseudo-normal random numbers, which are generated from pseudo-random numbers by the Box-Muller method or the Marsaglia method, can not obey the normal distribution exactly. In this note we focus on the distribution of pseudo-random numbers and consider the error estimation of the Euler-Maruyama approximation when the distribution of underlying random variables is different from the normal distribution.

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### Relativistic quantum field theory with a fundamental length

**Shigeaki Nagamachi**

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The relativistic equation of quantum mechanics called Dirac equation

$$i\frac{\hbar}{c}\gamma_{\mu}\frac{\partial}{\partial x_{\mu}}\psi(x) - M\psi(x) = 0,$$

$x_0 = ct, x_1 = x, x_2 = y, x_3 = z$ , contains two constants  $c$  (velocity of light), the fundamental constant in the relativity theory and  $h = 2\pi\hbar$  (Planck constant), the fundamental constant in quantum mechanics. Dimension of  $c$  is  $[LT^{-1}]$  and that of  $h$  is  $[ML^2T^{-1}]$ . W. Heisenberg thought that the equation must also contain a constant  $l$  with dimension  $[L]$ . Then arbitrary dimensions are expressed by the combination of  $c, h$  and  $l$ , e.g.,  $[T] = [L]/[LT^{-1}]$ ,  $[M] = [ML^2T^{-1}]/([L][LT^{-1}][L])$

In 1958, Heisenberg with Pauli introduced the equation

$$\frac{\hbar}{c}\gamma_{\mu}\frac{\partial}{\partial x_{\mu}}\psi(x) \pm l^2\gamma_{\mu}\gamma_5\psi(x)\bar{\psi}(x)\gamma^{\mu}\gamma_5\psi(x) = 0, \quad (1)$$

which is later called the equation of universe. The constant  $l$  has the dimension  $[L]$  and is called the fundamental length. But equation (1) is difficult to solve. So, we consider the following soluble equation having the constant  $l$  with the dimension  $[L]$ :

$$\begin{cases} \square\phi(x) + \left(\frac{cm}{\hbar}\right)^2\phi(x) = 0 \\ \left(i\frac{\hbar}{c}\gamma_{\mu}\frac{\partial}{\partial x_{\mu}} - M\right)\psi(x) \\ + 2\gamma_{\mu}l^2\psi(x)\phi(x)\frac{\partial\phi(x)}{\partial x_{\mu}} = 0 \end{cases} \quad (2).$$

This equation has no solutions in the axiomatic framework of Wightman, that is, the field  $\psi(x)$  is not an operator-valued tempered distribution. But

$\psi(x)$  is an operator-valued tempered ultrahyperfunction.

The equation (2) has a solution in the framework of ultrahyper function quantum field theory which is a generalization of Wightman's tempered field theory. We present the axioms based on tempered ultrahyper functions which have no localization property and discuss their relations to the fundamental length  $l$ : The two events occurred within the distance  $l$  can not be distinguished. Only if the distance between two events is greater than  $l$ , they are distinguished.

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### On relative stability in 4-dimensional canard

**Kiyoyuki Tchizawa**

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This talk gives the existence of a relatively stable duck solution in a slow-fast system in  $R^{2+2}$  with an invariant manifold. It has a 4-dimensional canard having a relatively stable region when there exists the invariant manifold near the pseudo singular node point. It is a revised version of AIMS 2010 and added further results.

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### Transitions between discreteness and continuity of all orders of regularity.

**Imme van den Berg**

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We study discrete functions on regular and irregular infinitesimal grids in one or two dimensions. We consider its difference quotients of higher order and give conditions for them to be infinitely close to the corresponding (partial) derivatives. Important tools are the formula of Faa di Bruno for higher order derivatives and a discrete version of it. Applications include a general method for transitions from difference equations to differential equations, a DeMoivre-Laplace Theorem of higher order and regularity properties of discrete free-boundary problems.

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### A proof-theoretic approach for nonstandard analysis

**Keita Yokoyama**

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Significant tools for nonstandard analysis are the transfer principle, the existence of the standard part, some saturation/enlargement principles, and so on.

Formalizing these, we can construct a formal system for nonstandard analysis. One formal system based on ZFC is given by Edward Nelson in 1977. In fact, most part of nonstandard analysis can be done within a very small fragment of this formal system. In this talk, I will introduce several weak formal systems for nonstandard analysis, and discuss which axioms are essentially needed for nonstandard analysis. This is a proof-theoretic study of nonstandard analysis from the standpoint of so called "reverse mathematics". By this, we can see that several basic theorems of nonstandard analysis are actually equivalent to important axioms of nonstandard analysis. Moreover, I will give a canonical interpretation of nonstandard proofs into elementary proofs without nonstandard methods. This means that, in theory, there is a canonical way to remove nonstandard techniques from a nonstandard proof. By this, we can also compare the length of a proof using nonstandard techniques with a proof without nonstandard analysis.

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## Special Session 47: Dynamics and Games

Alberto Pinto, LIAAD UP, Portugal  
 Michel Benaim, University of Neuchatel, Switzerland

The session is devoted to the development and the diffusion of mathematical ideas and techniques that arise from the analysis and the modelling of systems where agents (whether they be rational players, markets, plants, animals, ecosystems, communication systems, etc) interact dynamically over time. Abstracts should either be motivated by challenging mathematical questions occurring in such systems or provide a rigorous mathematical analysis of models where tools from dynamics and games prove to be useful. Areas covered include differential games, evolutionary games, models of learning and evolution, repeated games, mean field models, etc. and their applications in social, life, physical and computer sciences.

### Anosov and renormalized circle diffeomorphisms

João Almeida

Polytechnic Institute of Bragança, Portugal  
 jpa@ipb.pt

Alberto A. Pinto, David Rand

In this talk we prove a one-to-one correspondence between  $C^{1+}$  smooth conjugacy classes of circle diffeomorphisms that are  $C^{1+}$  fixed points of renormalization and  $C^{1+}$  conjugacy classes of Anosov diffeomorphisms whose Sinai-Ruelle-Bowen measure is absolutely continuous with respect to Lebesgue measure. Furthermore, we use ratio functions to parametrize the infinite dimensional space of  $C^{1+}$  smooth conjugacy classes of circle diffeomorphisms that are  $C^{1+}$  fixed points of renormalization.

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### Anosov diffeomorphisms and golden tilings

João Almeida

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 jpa@ipb.pt

Alberto A. Pinto

In this talk we present the definition of *golden sequence*. These golden sequences have the property of being Fibonacci quasi-periodic and determine a tiling in the real line. We prove a one-to-one correspondence between: (i) affine classes of golden tilings; (ii) smooth conjugacy classes of Anosov diffeomorphisms, with an invariant measure absolutely continuous with respect to the Lebesgue measure, that are topologically conjugate to the Anosov automorphism  $GA(x; y) = (x + y; x)$ ; (iii) solenoid functions. A. Pinto and D. Sullivan developed a theory relating 2-adic sequences (Pinto-Sullivan tilings in the real line) with smooth conjugacy classes of doubling expanding circle maps. The solenoid functions give a parametrization of the infinite dimensional space consisting of the mathematical objects described in the above equivalences.

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### Performance of investment strategies in the absence of correct beliefs

Cisem Bektur

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We study an evolutionary market model with long-lived assets. We show that in the absence of correct beliefs, the strategy which is "closer" to the Kelly rule cannot be driven out of the market. This means that this strategy will either dominate or at least survive. Our techniques are borrowed from the theory of random dynamical systems (RDS).

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### Bayesian-Nash equilibria in a cave psychological model

Helena Ferreira

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We construct a cave psychological model for the theory of Planned Behavior or Reasoned Action. In this model the individuals taste and crowding types follow the shadows of the taste and crowding types of the platonic idealized psychological model, according to a given probability distribution and the individuals know only the expected value of their welfare function. We present sufficient conditions for an individual or group to adopt a certain behavior decision according to the Bayesian-Nash Equilibria. We demonstrate how saturation, boredom and frustration can lead to the adoption of a variety of different behavior decisions and how no saturation can lead to the adoption of a single consistent behavior decision.

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### Nash equilibria in a platonic idealized psychological model

**Helena Ferreira**  
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We construct a platonic idealized psychological model, using Game Theory, for the Theory of Planned Behavior. This model consists of individuals with no uncertainties in their taste and crowding types and welfare function. We propose the Nash Equilibria as one of many possible mechanisms to transform human intentions into behavior decisions. We show that saturation, boredom and frustration can lead to the adoption of a variety of different behavior decisions, as opposed to no saturation, which leads to the adoption of a single consistent behavior decision.

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### Application of queueing theory to emergency care

**James Keesling**  
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Congestion and long waiting times are a serious problem in emergency care in the United States. We are applying Queueing Theory to analyze the activities in an emergency care unit to see what improvements and economies can be obtained.

The study is being done in cooperation with Dr. Joseph Adrian Tyndall, Head of Emergency Care at Shands Hospital at the University of Florida. We will present some preliminary results of this study.

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### The $W^{2,p}$ regularity for solutions of the simplest Isaacs equations

**Jay Kovats**  
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In this talk, we discuss the interior  $W^{2,p}$  regularity of viscosity solutions of the simplest uniformly elliptic Isaacs equations of the form  $F(u_{xx}) := \max_{y \in Y} \min_{z \in Z} \text{tr}[a(y, z)u_{xx}] = 0$  in a domain  $D \subset \mathbb{R}^d$ , where  $Y, Z$  are finite sets. Here,  $\forall y \in Y, z \in Z$ ,  $a(y, z)$  is a symmetric  $d \times d$  matrix, satisfying  $\lambda I_d \leq a(y, z) \leq \Lambda I_d$ , for some constants  $0 < \lambda \leq \Lambda$ . The Isaacs equation originates from the theory of stochastic differential games, and is of interest to pde specialists, since it is the prototypical, fully non-linear elliptic equation which is neither convex nor concave in the Hessian matrix  $u_{xx}$ . Consequently, many of the standard techniques used to obtain regularity results for solutions of concave equations

cannot be used in an obvious way for solutions of Isaacs equations. It is well-known that if  $u \in C(D)$  is a viscosity solution of a uniformly elliptic equation of the form  $F(u_{xx}) = 0$  in  $D$ , then  $u \in W_{loc}^{2,\delta}(D)$  for some  $\delta \in (0, 1)$ . The question is, for solutions of the simplest Isaacs equations described above, will viscosity solutions be locally  $W^{2,2}$ ? We explore this question for  $C^2$  solutions, and give examples.

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### A game theoretical approach to resort prices

**Abdelrahim Mousa**  
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We introduce a resort pricing model, where different types of tourists choose between different resorts. We study the influence of the resort prices on the choices of the different types of tourists. We characterize the coherent strategies of the tourists that are Nash equilibria. We find the prices that lead to the bankruptcy of the resorts and, in particular, their dependence on the characteristics of the tourists.

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### A game theoretical approach to human decision

**Tania Oliveira**  
LIAAD UP, Portugal  
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**Renato Soeiro, Abdelrahim Mousa, Alberto A. Pinto**

We study a dichotomous decision model, where individuals can make the decision yes or no and can influence the decisions of others. We characterize all decisions that form Nash equilibria. Taking into account the way individuals influence the decisions of others, we construct the decision tilings where the axes reflect the personal preferences of the individuals for making the decision yes or no. These tilings characterize geometrically all the pure and mixed Nash equilibria. We show, in these tilings, that Nash equilibria form degenerated hystereses with respect to the replicator dynamics, with the property that the pure Nash equilibria are asymptotically stable and the strict mixed equilibria are unstable. These hystereses can help to explain the sudden appearance of social, political and economic crises. We observe the existence of limit cycles for the replicator dynamics associated to situations where the individuals keep changing their decisions along time, but exhibiting a periodic repetition in their decisions. We introduce the notion of altruist and individualist leaders and study the way that the leader can affect the individuals to make the decision that the leader pretends.

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**The replicator dynamics and human decisions****Tania Oliveira**LIAAD UP, Portugal  
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We add uncertainty to the model previously introduced. We study how these changes in the setup influence the previous results. In particular, we characterize all decisions that form Bayesian Nash equilibria. We look at how this change in the settings affects the way individuals influence the decisions of others and we construct the decision tilings.

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**Random matching Edgeworthian economies trading in the core via a prisoners dilemma****Bruno Oliveira**LIAAD UP, Portugal  
bmpmo@fcna.up.pt

We show that for a specific class of random matching Edgeworthian economies, the expectation of the limiting equilibrium price coincides with the equilibrium price of the related Walrasian economies. This result extends to the study of economies in the presence of uncertainty within the multi-period Arrow-Debreu model, allowing to understand the dynamics of how beliefs survive and propagate through the market.

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**Strategic optimization in R&D investment with uncertainty in the investment****Bruno Oliveira**LIAAD UP, Portugal  
bmpmo@fcna.up.pt

We use d'Aspremont and Jacquemin's strategic optimal R&D investment in a duopoly Cournot competition model to construct myopic optimal discrete and continuous R&D dynamics. We consider the existence of uncertainty in the outcome of the R&D investment. We show that for some high initial production costs, the success or failure of a

firm is very sensitive to small variations in its initial R&D investment strategies.

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**Asynchronous stochastic approximation for learning in stochastic games****Steven Perkins**University of Bristol, England  
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We develop tools to analyse the convergence properties of fictitious play like algorithms in discounted reward stochastic games. To examine the best-response dynamic in stochastic games when asynchronous updating is present the differential inclusions framework for stochastic approximation is required. The asymptotic pseudo-trajectory approach to stochastic approximation of Benaïm, Hofbauer and Sorin is extended for asynchronous stochastic approximations with a set-valued mean field. The asynchronicity of the process is incorporated into the mean field to produce convergence results which remain similar to those of an equivalent synchronised process. In addition, this allows many of the restrictive assumptions previously associated with asynchronous stochastic approximation to be removed. By extending this approach to two timescales we are able to examine the convergence of an actor-critic algorithm for discounted reward stochastic games.

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**Uncertainty effects in resort prices****Alberto Pinto**LIAAD UP, Portugal  
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We add uncertainty to the resort pricing model previously introduced. We study how these changes in the setup influence the previous results. In particular, the coherent strategies of the tourists and the prices that lead to the bankruptcy of the resorts.

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**A distance for belief spaces****Jerome Renault**TSE-GREMAQ, Universite Toulouse, France  
renaultj@math.cnrs.fr**Xavier Venel**

Given a finite set  $K$ , we denote by  $X = \Delta(K)$  the set of probabilities on  $K$  and by  $Z = \Delta_f(X)$  the set of Borel probabilities on  $X$  with finite support. Studying a Markov Decision Process with partial information on  $K$  naturally leads to a Markov Decision Process with full information on  $X$ . We introduce a new metric  $d_*$  on  $Z$  such that the transitions become 1-Lipschitz from  $(X, \|\cdot\|_1)$  to  $(Z, d_*)$ . In this talk, we define and prove several properties of the metric  $d_*$ . Especially,  $d_*$  satisfies a Kantorovich-Rubinstein

type duality formula and can be characterized by using disintegrations. A second talk will contain applications to the existence and the characterization of “Long-term values in Markov Decision Processes and Repeated Games”.

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### Long-term values in Markov decision processes and repeated games

**Jerome Renault**

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**Xavier Venel**

We consider several types of Markov Decision Processes with or without partial observation, and Repeated Games. Each case will be expressed as a “compact non expansive” MDP, where the state space is (pre)compact and the transitions are non expansive for a well-chosen distance. In particular we use an appropriate metric for belief spaces to characterize the limit value in Partial Observation MDP with finitely many states and in repeated games with an informed controller with finite sets of states and actions. Moreover in each case we prove the existence of a generalized notion of uniform value where we consider not only the Cesaro mean when the number of stages is large enough but any evaluation function  $\theta = (\theta_t)_{t \geq 1}$  (weights on stages) when the impatience  $I(\theta) = \sum_{t \geq 1} |\theta_{t+1} - \theta_t|$  is small enough.

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### Evolutionary stability in multiple-move games

**Zibo Xu**

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We consider a basic dynamic evolutionary model (or viewed as a rudimentary learning process) with rare mutation and a best-reply (or better-reply) selection mechanism. We call a state evolutionarily stable if its long-term relative frequency of occurrence is bounded away from zero as the mutation rate decreases to zero. For finite extensive-form games of perfect information where each player can only play at one node, the combined results of Hart and Godeisky show that the backward induction equilibrium becomes in the limit the only evolutionarily stable outcome as the mutation rate decreases to zero and the populations increase to infinity. We give three games where players may play at more than one node along some path. We show that, even when the populations increase to infinity, the backward induction equilibrium or even the whole backward induction equilibrium component is not always the only evolutionarily stable outcome in this model.

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### Project evaluation and hedging in incomplete markets using historical prices

**Jorge Zubelli**

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**Edgardo Brigatti, Max O. de Souza**

The methodology of real options is by now a well accepted one by many corporations, especially for R&D projects. Yet, there are many discussions concerning the validity of applying mathematical finance techniques such as hedging and risk neutral pricing to situations where the decision maker cannot fully replicate his or her business decisions. In this talk we shall present a methodology for project evaluation that makes use of historical data and minimizes certain functionals that quantify risk. We shall also discuss the connection with games and with earlier work of M. Grasselli on the use of utility functions for real option evaluation.

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## Special Session 48: Nonlinear Evolution Equations

**Alex Himonas**, University of Notre Dame, USA  
**Gerson Petronilho**, Federal University of Sao Carlos, Brazil

The theme of this session is nonlinear evolution equations including the NLS equation, the KdV equation, the Camassa-Holm equation, and the Euler equations of hydrodynamics. It will focus on questions of local and global well-posedness, dependence of solutions on the initial data, unique continuation, regularity, and integrability. Such questions are fundamental in both theory and applications.

### Linear instability of periodic traveling waves for nonlinear dispersive models

**Jaime Angulo Pava**  
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**Fabio Natali**

In this talk we present a study on the linear and nonlinear instability of periodic traveling wave associated with some general one-dimensional dispersive models. By using analytic and asymptotic perturbation theory, we establish sufficient conditions for the existence of exponentially growing solutions to the linearized problem and so the linear instability of periodic profiles is obtained. Applications of this approach are concerning with the linear/nonlinear instability of cnoidal wave solutions for the modified Benjamin-Bona-Mahony and the modified Korteweg-de Vries equations. The arguments presented in this investigation has prospects for the study of the instability of periodic traveling wave of other nonlinear evolution equations.

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### Ovsiannikov's theorem for autonomous equations and applications

**Rafael Barostichi**  
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**R. F. Barostichi, A. Alexandrou Himonas, G. Petronilho**

Using a refined version of Ovsiannikov's theorem for autonomous equations we prove local well-posedness of an abstract Cauchy problem in a space of periodic functions that extend analytically in a strip of the complex plane around the x-axis. Then, we apply this result to some important weakly dispersive equations.

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### Nonlinear evolution equation for magneto-convective flow in an active mushy layer

**Dambaru Bhatta**  
 University of Texas-Pan American, USA  
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**Daniel N Riahi**

Here we consider the solidification problem of a binary alloy which is cooled from below. It has been well established by experimentalists that a horizontal mushy layer is formed during the solidification of binary alloys. This study investigates nonlinear behavior of the convective flow in the mushy layer in presence of a magnetic field. The mushy layer is treated as an active porous media with variable permeability. We derive the linear, adjoint and first-order systems and use these solutions to obtain the Landau coefficients which appear in the resulting evolution equation. Numerical results obtained from our computations suggest that there is a slow transition of the flow to a steady state.

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### Illposedness of a weakly dispersive Boussinesq system

**Ming Chen**  
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**Yue Liu**

In this talk we discuss some illposedness issues associated to the Cauchy problem of a long-wave water wave system. Using an abstract wellposedness theory developed by Bejenaru-Tao for semilinear dispersive equations, we find the critical Sobolev index below which the solution map fails to be continuous with respect to initial data. We also derive a criterion for the blow-up of strong solutions. This is a joint work with Yue Liu.

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**Generalized wave maps on the sphere****Daniel da Silva**University of Rochester, USA  
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Wave maps are nonlinear generalizations of the wave equation which have been studied for decades. In this talk, we will consider a generalization of wave maps based on the Adkins-Nappi model of nuclear physics. This model yields nonlinear hyperbolic partial differential equations, for which we consider the question of regularity of solutions. In particular, we will discuss the non-concentration of energy, a preliminary step in establishing a global regularity theory.

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**Continuity properties of the solution map for the generalized reduced Ostrovsky equation****Melissa Davidson**University of Notre Dame, USA  
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It is shown that the data-to-solution map for the generalized reduced Ostrovsky (gRO) equation is not uniformly continuous on bounded sets in Sobolev spaces. Considering that for this range of exponents the gRO equation is well-posed with continuous dependence on initial data, this result makes the continuity of the solution map an optimal property.

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**Solutions of Björling problem for timelike surface and the homogeneous wave equation****Martha Patricia Dussan Angulo**University of Sao Paulo, Brazil  
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We solve the Björling problem for timelike surfaces in  $\mathbb{R}_1^3$  and  $\mathbb{R}_2^4$  by constructing a split-complex representation formula for those surfaces. Our approach includes the construction of split-holomorphic extensions in a natural way using the point of view of solutions to the homogeneous wave equation. Then we also establish Schwarz reflection to obtain split-complex Björling representations in symmetric domains of the split-complex plane.

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**Cauchy problem for some hyperbolic equations of mathematical cosmology****Anahit Galstyan**University of Texas-Pan American, USA  
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The talk is concerned with the waves propagating in the universe modeled by the cosmological models. We will present fundamental solutions of the wave equation in the Einstein-de Sitter spacetime. In this talk we investigate initial value problem for this equation and give the explicit representation formulas for the solutions. The equation is strictly hyperbolic in the domain with positive time. On the initial hypersurface its coefficients have singularities that make difficulties in studying of the initial value problem. In particular, one cannot anticipate the well-posedness in the Cauchy problem for the wave equation in the Einstein-de Sitter spacetime. The initial conditions must be modified to so-called weighted initial conditions in order to adjust them to the equation. We will present also the  $L_p$ - $L_q$  estimates for solutions. This is a joint work with Tamotu Kinoshita (Japan) and Karen Yagdjian (U.S.A.).

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**Analysis of the b-family equation****Katelyn Grayshan**University of Notre Dame, USA  
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We shall consider the Cauchy problem for the b-family of equations with initial data in Sobolev spaces in both the periodic and non-periodic case. In particular, we shall discuss continuity properties of the data-to-solution map. Members of this family are intriguing since they all have peak on solutions. Furthermore, the b-family contains the Camassa-Holm and the Degasperis-Procesi equations, which are integrable.

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**The initial value problem of a periodic KdV equation****Alex Himonas**University of Notre Dame, USA  
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We shall discuss the periodic Cauchy problem for a KdV equation whose dispersion is of order  $2j + 1$ , where  $j$  is a positive integer. The initial data considered range from Sobolev to analytic Gevrey spaces. For  $s \geq -j/2$  well-posedness in  $H^s$  is proved by deriving the bilinear estimates corresponding to the linear part of the KdV equation and using appropriate Bourgain spaces. When the initial data belong to an analytic Gevrey space of order  $\sigma$  then well-posedness is proved by using Foias-Temam-Bourgain type analytic Gevrey spaces.

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**Next generation sequencing and differential gene expression****Curtis Holliman**University of Alabama at Birmingham, USA  
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We give an overview of the bioinformatics associated with next generation sequencing. In particular, we focus on the application of the RNASeq pipeline and consider an experiment involving differentially expressed genes.

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**Well-posedness of the generalized Burgers equation in analytic Gevrey spaces****John Holmes**University of Notre Dame, USA  
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We consider the initial value problem for the generalized Burgers equation and study its well-posedness. In particular, we show that the Cauchy problem is well posed for initial data in a class of Sobolev-Gevrey spaces with index  $r$  greater than or equal to one. This implies that the solution is Gevrey- $r$  in the spacial variable and Gevrey- $2r$  in the time variable.

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**Enhanced lifespan of smooth solutions of a Burgers-Hilbert equation****Mihaela Ifrim**University of California, Davis, USA  
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We consider an initial value problem for a quadratically nonlinear inviscid Burgers-Hilbert equation that models the motion of vorticity discontinuities. We use a normal form transformation, which is implemented by means of a near-identity coordinate change of the independent spatial variable, to prove the existence of small, smooth solutions over cubically nonlinear time-scales. For vorticity discontinuities, this result means that there is a cubically nonlinear time-scale before the onset of filamentation.

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**New ill-posedness results for the boussinesq equation****David Karapetyan**University of Notre Dame, USA  
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We present new ill-posedness results for the nonlinear “good” Boussinesq equation, which improve upon the ones previously obtained in the literature. In particular, it is proved that the solution map is not continuous in Sobolev spaces  $H^s$ , for all  $s$

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**Local existence of solutions of self gravitating relativistic perfect fluids****Lavi Karp**ORT Braude College, Israel  
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The talk will be devoted to the evolution counterpart of Einstein equations which are coupled to the Euler equations. The classical result of Hughes, Kato and Marsden yields a local in time existence and uniqueness for the vacuum Einstein equations in the Bessel potential spaces with regularity index  $s > \frac{5}{2}$ . Their results rely on the technique of quasilinear wave equations, however, this technique is not available when the gravitational fields are coupled with the Euler equations. We have obtained the well-posedness of the coupled system, with the same regularity as in the classic case, by means of a modification of standard approach to the energy estimates for a certain form of first order symmetric hyperbolic systems.

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**Smoothing results for Korteweg de-Vries equations on  $\mathbf{R}$  and  $\mathbf{T}$** **Seungly Oh**University of Kansas, USA  
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We discuss smoothing results for KdV under low-regularity setting. The main technique involved in this discussion is the normal form approach. Let  $u$  be the solution of KdV with the initial data  $u_0$ . In  $\mathbf{R}$ , we obtain the smoothing of 1/2-derivatives for  $u - e^{-t\partial_x^3} u_0$  when the initial data lies in  $H^{-1/2} \cap \dot{H}^{-1}$ . In  $\mathbf{T}$ , we also obtain the smoothing of 1/2-derivatives for  $u - R^*[u_0]$  with  $u_0 \in H^{-1/2}$  where  $R^*[u_0]$  describes the explicit solution to the resonance equation arising from the normal form transformation.

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**Well-posedness and regularity of the periodic gKdV equation****Gerson Petronilho**University Federal of Sao Carlos, Brazil  
gersonpetro@gmail.com**A. Alexandrou Himonas**

In the periodic case it is proved that the generalized Korteweg-de Vries equation (gKdV) is locally well-posed in a class of analytic spaces that are similar to the ones used by Grujić and Kalisch in the non-periodic case. Thus, the uniform analyticity radius of the solution in the space variable does not change as time progresses. We also study regularity in the time variable.

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**Nonuniform continuity of the solution map for CH type equations****Ryan Thompson**University of Notre Dame, USA  
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We consider the Cauchy problem for Camassa-Holm type equations and prove that the data-to-solution map is not uniformly continuous in Sobolev spaces. The main tools to be used in the proof of this result include, but are not limited to, approximate solutions and well-posedness estimates.

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**Generic non self-adjoint Zakharov-Shabat operators****Peter Topalov**Northeastern University, USA  
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I will discuss the generic properties of the spectrum of the Zakharov-Shabat operator with periodic boundary conditions.

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**Global existence of the scalar field in de Sitter spacetime****Karen Yagdjian**University of Texas-Pan American, USA  
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In this talk we present global existence of the small data solutions of the Cauchy problem for the semi-linear Klein-Gordon equation in the de Sitter spacetime. The Klein-Gordon equation arising in relativistic physics and, in particular, general relativity and cosmology, as well as, in more recent quantum field theories, is a covariant equation that is considered in the curved pseudo-Riemannian manifolds.

The latest astronomical observational discovery that the expansion of the universe is speeding supports the model of the expanding universe that is mathematically described by the manifold with metric tensor depending on time and spatial variables. In this talk we restrict ourselves to the manifold arising in the so called de Sitter model of the universe, which is the curved manifold due to the cosmological constant. Unlike the same problem in the Minkowski spacetime, we have no restriction on the order of nonlinearity and structure of the nonlinear term, provided that a physical mass of the field is outside of some interval.

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## Special Session 49: Growth Models and Interface Dynamics

Alexander Nepomnyashchy, Technion - Israel Institute of Technology, Israel

Tatiana Savin, Ohio University, USA

Amy Novick-Cohen, Technion, Israel

The aim of this session is to bring together mathematicians and physicists working on nonlinear growth processes as well as processes occurring on the interfaces. These processes include but are not limited by the Laplacian and elliptic growth, crystal growth and coarsening, growth of solid nuclei in the subdiffusive medium as well as front propagation into the unstable medium.

### Driven free-standing foam films

**Markus Abel**

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**Michael Winkler, Rumen Krastev**

Thin liquid films or foam films are the basic constituents of emulsions, or foams and are important for a wealth of applications ranging from process engineering to everyday articles of use as shampoos or soft drinks. The interface processes are highly interesting and of fundamental interest for physics, physical chemistry, engineering and eventually mathematics. Thin film theory is worked out and understood to a reasonable degree for films on a horizontal or inclined surface. Free-standing, vertical films are different in that they have two free surfaces and consequently the physics of the surfactants play a dominant role. Whereas the equilibrium properties of such free-standing foam films are investigated experimentally and theoretically, transient processes and nonequilibrium dynamics in general are still an area to explore. We present a summary of experimental results for foam films driven by a thermal gradient or electromagnetic forces. Further, we give the basic equations using lubrication theory and explain the relevant dynamics in terms of partial differential equations. A numerical approach using lattice Boltzmann method is presented with first quantitative results. In particular, we present experiments and discuss the propagation of a stable front of so-called black film (on the nanometer scale) into the unstable thicker material.

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### 1D integrable systems and 2D hydrodynamics

**Eldad Bettelheim**

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I will present relations between 2D hydrodynamics and a phase space representation of certain many particle 1D systems of interacting particles. The purpose of establishing such relations is to make use of hidden symmetries of 1D physics in the study of 2D hydrodynamics.

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### Hollow vortices, capillary waves, and double quadrature domains

**Darren Crowdy**

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This talk will describe several newly discovered equilibria for so-called hollow vortex solutions of the two-dimensional Euler equations. A hollow vortex is a finite-area region, with non-zero circulation, in an otherwise irrotational flow. The challenge of finding equilibrium configurations is a free boundary problem; here we employ conformal mapping and free streamline theory to find exact solutions in several cases. It turns out that the solutions have surprising mathematical connections with the physically distinct problem of free surface capillary waves as well as the notion of a “double quadrature domain”.

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### Motion of interfaces governed by the Cahn-Hilliard equation with highly disparate diffusion mobility

**Shibin Dai**

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**Qiang Du**

We consider a two phase system governed by a Cahn-Hilliard type equation with a highly disparate diffusion mobility. It has been observed from recent numerical simulations that the microstructure evolution described by such a system displays a coarsening rate different from that associated with the Cahn-Hilliard equation having either a constant diffusion mobility or a mobility that degenerates in both phases. Using the asymptotic matching method, we derive sharp interface models of the system under consideration to theoretically analyze the interfacial motion with respect to different scales of time  $t$ . In a very short time regime, the transition layer stabilizes into the well-known hyperbolic tangent single-layer profile. On an intermediate time regime, due to the small mobility in one of the phases, the sharp interface limit is a one-sided Stefan problem. On a slower time scale, the leading order dynamics is a one-sided Hele-Shaw problem. The normal velocity of the interface has a small correction that is determined by

the surface diffusion, that is, the surface Laplacian of the mean curvature. As a result, scaling arguments suggest that there should be a crossover in the coarsening rate from  $t^{1/3}$  to  $t^{1/4}$ .

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### Diffusion-generated motion algorithms for multiphase curvature motion

**Matt Elsey**

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**Selim Esedoglu, Peter Smereka**

Many materials, including most metals and ceramics, are composed of crystallites (often called grains), which are differentiated by their crystallographic orientation. Classical models describing annealing-related phenomena for these materials involve multiphase curvature-driven motion. The distance function-based diffusion-generated motion (DFDGM) algorithm is introduced and demonstrated to be an accurate and efficient means for simulating the evolutions described by such models. The DFDGM algorithm makes use of implicit representations of the phases, allowing topological changes to occur naturally. Large-scale simulations of grain growth are presented and are shown to agree well with available theoretical predictions and empirical observations.

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### Capillary-mediated pattern branching

**Martin Glicksman**

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The Gibbs-Thomson temperature distribution can act as a weak thermal field from its tangential gradients along the interface. Provided that the interface conductivity is non-zero, energy conservation for non-equilibrium shapes shows varying rates of deposition and removal of small amounts of capillary-mediated energy. The local freezing rate is retarded slightly where the Stefan energy balance requires that capillary energy is released. Where energy is withdrawn, the local rate of freezing is enhanced. These opposing responses balance if the surface Laplacian of the interface temperature vanishes. The bias in the local freezing rates surrounding this balance point, which is 4th-order in the interface shape, induces deterministic rotation (tilting) of the interface. Rotation couples with the transport field in the melt by changing the local curvature, which eventually produce a side branch. A precision Greens function solver confirms that rotations and branches arise dynamically at locations predicted analytically for various 2-D shapes. Subsequent rotations develop episodically as the tip shape and rotation points co-evolve. A synchronous limit cycle may develop under

conditions not yet fully understood. Noise, per se, and stochastic stability, play no direct roles in this proposed mechanism of pattern morphogenesis.

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### Non-uniqueness of quadrature domains

**Lavi Karp**

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**A. S. Margulis**

The talk will be devoted to the construction of one perimeter family of unbounded quadrature domains with the same source. In the two dimensional plane it is related to the inverse problem of the logarithmic potential for a contact surface. We will consider the constructions both in the plane and in higher dimensions. We shall also discuss the applications to equilibrium measures associated with an admissible weight function.

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### Propagation of singularities of solutions of linear PDE

**Dmitry Khavinson**

University of South Florida, USA  
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We shall discuss applications of Leray's theory of singularities of holomorphic PDEs to the problem of locating the singularities of Laplacian Growth processes.

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### Topological transitions in interface dynamics of evaporating thin films

**Avraham Klein**

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**Oded Agam**

A thin water film on a cleaved mica substrate undergoes a first-order phase transition between two values of film thickness. By inducing a finite evaporation rate of the water, the interface between the two phases develops a lingering instability similar to that observed in the Saffman-Taylor problem. The dynamics of the droplet interface is dictated by an infinite number of conserved quantities: all harmonic moments decay exponentially at the same rate. One can link the interface dynamics to the evolution of a Riemann surface. A typical scenario is the nucleation of a dry patch within the droplet domain. This corresponds to a topological transition of the Riemann surface from genus zero to genus one. We

shall construct solutions of this problem and highlight similarities and differences between them and solutions of Laplacian growth problems.



### Quadrature domains in interface dynamics

**Erik Lundberg**

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We discuss the role of area-quadrature domains in the Laplacian growth or "Hele-Shaw" problem and the appearance of quadrature domains of various types in other problems of fluid dynamics. We give special attention to the lack of explicit examples in the higher-dimensional case.



### Velocity fluctuations of noisy reaction fronts

**Baruch Meerson**

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**Pavel V. Sasorov**

The position of a propagating reaction front fluctuates because of the shot noise coming from the discreteness of reacting particles and stochastic character of reactions. What is the probability that the noisy front moves, during a given time interval, considerably slower or faster than its deterministic counterpart? Can the noise arrest the front motion for some time, or even make it move in the wrong direction? What is the most likely particle density profile of an unusual front realization? I will present a WKB theory that assumes many particles in the front region and, in some cases, answers these questions. The details strongly depend on whether the front propagates into a metastable or unstable state [1,2].

[1] B. Meerson, P.V. Sasorov and Y. Kaplan, Phys. Rev. E **84**, 011147 (2011).

[2] B. Meerson and P.V. Sasorov, Phys. Rev. E **84**, 030101(R) (2011).



### Particle growth in a subdiffusive medium

**Alexander Nepomnyashchy**

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**Vladimir Volpert**

During the last decades, the phenomenon of anomalous diffusion has attracted much attention of researchers. A subdiffusive transport has been observed in numerous physical and biological systems, specifically in gels. We investigate the growth of a solid nucleus due to the subdiffusive transport of a dissolved component towards the nucleus surface. The process is described by a subdiffusive version of the Stefan problem. In planar and spherical cases, exact self-similar solutions of the problem have been found in terms of the Wright function. An instability of the particle growth, which is similar to the Mullins-Sekerka instability of a crystallization front in the case of a normal diffusion, is revealed.



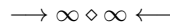
### Stationary solutions of the convective Cahn-Hilliard equation

**Alexander Nepomnyashchy**

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**Michael Zaks**

The convective Cahn-Hilliard equation (CCHE) has a number of physical applications, including the description of phase transitions and faceting by a non-equilibrium crystal growth. Stationary solutions of the CCHE are described by a third-order nonlinear ordinary differential equation, which is not integrable. Nevertheless, a family of explicit solutions can be found by means of multiple-scale expansions. Numerical analysis reveals a complex set of bifurcations leading to the appearance of patterns which include steep kink-like structures and sloppy oscillating fragments.



### Coarsening and the deep quench obstacle problem

**Amy Novick-Cohen**

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**L. Banas, R. Nurnberg**

Phase separation occurs in a wide spectrum of contexts, from galaxy formation to biofilm formation, and many models have been proposed to describe the dynamics. Common to these processes is a linear regime dominated by a "most unstable mode" and a later coarsening regime during which larger

components grow at the expense of the smaller components. We explore some of the features of the dynamics within the relatively simple context of the deep quench (low temperature) obstacle problem. We obtain new analytical bounds on the rate of coarsening, and present results of numerical simulations based on a number of benchmarks. By following the dynamics using a number of different benchmarks, we find that partial scaling can be verified, and that the transition between linear and coarsening behavior is in fact characterized by a number of sequential transitions which call for further analysis.

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### A moment-preserving flow for surfaces and its applications

**Michiaki Onodera**

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Hele-Shaw flow is an incompressible viscous fluid flow in an experimental device which consists of two closely placed parallel plates. An interesting feature of Hele-Shaw flow is that the evolution of a fluid domain under the flow produced by injection of fluid does not change its geometric moments in time, while the area increases. We will introduce a new geometric flow which has an analogous property, that is, the moments of the boundary of the domain are preserved under the flow. Applications to other problems will also be mentioned.

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### Born-Oppenheimer approximation and accuracy of molecular dynamics

**Petr Plechac**

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We present results used for estimating accuracy of molecular dynamics as an approximation of the evolution of heavy nuclei in a many-body quantum system. The presented approach is based on the study of the time-independent Schroedinger equation and thus differs from a more standard analysis derived from time-dependent Schroedinger equation. It gives a different perspective on the Born-Oppenheimer approximation, Schroedinger Hamiltonian systems

and numerical simulations in molecular dynamics at micro-canonical ensemble. Results from joint work with A. Szepessy, H. Hoel and R. Tempone will be presented.

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### Interface dynamics and singularities

**Tatiana Savin**

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We will discuss some models arising in mathematical physics, including the Laplacian growth.

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### Fingering in a channel and tripolar Loewner evolutions

**Giovani Vasconcelos**

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**Miguel A. Duran, Giovani L. Vasconcelos**

Loewner evolutions describes a rather general class of growth processes in two dimensions where a curve starts from a given point on the boundary of a domain  $\mathbb{P}$  in the complex  $z$ -plane and grows into the interior of  $\mathbb{P}$ . More specifically, the Loewner equation is a first-order differential equation for the conformal mapping  $g_t(z)$  from the “physical domain,” consisting of the region  $\mathbb{P}$  minus the curve, onto a “mathematical domain” represented by  $\mathbb{P}$  itself. In this work, a class of Laplacian growth models in the channel geometry is studied using the formalism of tripolar Loewner evolutions, in which three points, namely, the channel corners and the point at infinity, are kept fixed. Initially, the problem of fingered growth, where growth takes place only at the tips of slit-like fingers, is revisited and a class of exact solutions is presented. A model for interface growth is then formulated in terms of a generalized tripolar Loewner equation and several examples are presented. It is shown that the growing interface evolves into a steadily moving finger and that tip competition arises for nonsymmetric initial configurations with multiple tips. Possible extensions, including stochastic tripolar Loewner evolutions, will be briefly discussed.

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## Special Session 50: Mathematical Novelties in Inverse Problems in Imaging Sciences

Alexandru Tamasan, University of Central Florida, USA

The goal of this session is to bring together scientists working on the mathematics of Inverse Problems with applications to Imaging Science. To be presented are the recent advances on the mathematical methods in reconstructing/imaging various physical properties such as electrical conductivity in Electrical Impedance Tomography (EIT), Current Density Impedance Imaging (CDII), or Ultrasound Modulated EIT, or optical properties in Photo-acoustics, or elastic properties in Ultrasound Elastography.

### Geometrical effects of the conductivity on the D-bar method procedure for the electrical impedance tomography

**Toufic El Arwadi**

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Several authors have treated the direct reconstructions methods for the Calderón inverse conductivity problem in dimension two for regular, less regular and discontinuous conductivities. These methods consist on two steps: calculate the scattering transform from Cauchy data, then recover the conductivity from the scattering data. The first step is an ill posed step. For this, many authors have treated several approximations of the scattering transform for regular case (like  $\mathbf{t}^{\text{exp}}$  and  $\mathbf{t}^{\text{B}}$ ) and full regularization strategy in 2009. But only one work treats the case of less regular conductivity. In this talk I will treat the  $\mathbf{t}^{\text{B}}$  approximation for Nachman's proof, and  $S^{\text{a}}$  approximation for Knudsen-Tamasan proof. I will present the geometrical effect of the conductivity on the reconstruction procedure, will present also the stability of  $\mathbf{t}^{\text{B}}$  approximation, relation between  $S^{\text{a}}$  and  $\mathbf{t}^{\text{B}}$  for radial symmetric conductivities and a new numerical scheme for solving the D-bar equation. I will present some numerical examples which justify my theoretical results.

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### Identification of minimum phase preserving operators

**Peter Gibson**

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Minimum phase functions are fundamental in a range of applications, including control theory, communication theory and signal processing. A basic mathematical challenge that arises in the particular context of geophysical imaging is to understand the structure of linear operators preserving the class of minimum phase functions. The heart of the matter is an inverse problem: to reconstruct an unknown minimum phase preserving operator from its value on a limited set of test functions. This entails, as a preliminary step, ascertaining sets of test functions that determine the

operator, as well as the derivation of a corresponding reconstruction scheme. In the present paper we exploit a recent breakthrough in the theory of stable polynomials to solve the stated inverse problem completely. We prove that a minimum phase preserving operator on the half line can be reconstructed from data consisting of its value on precisely two test functions. And we derive an explicit integral representation of the unknown operator in terms of this data. A remarkable corollary of the solution is that if a linear minimum phase preserving operator has rank at least two, then it is necessarily bounded and injective.

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### Some results on the attenuated ray transform

**Nicholas Hoell**

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We discuss several recent results on filtered back-projection inversion formulae for the attenuated ray transform on curves in 2-dimensional settings. The method is based on a particular complexification of the vector fields defining the initial particle transport. This problem first arose in the medical imaging modality SPECT and has more recently proven useful in other contexts such as the unique reconstruction of permittivity and permeability coefficients of a conductive body from boundary measurements.

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### Reinterpretation of the imaginary part of the complex potential In EIT

**Sungwhan Kim**

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This work presents a mathematical analysis of the time harmonic electric potential which is a solution to an elliptic PDE with complex coefficient. In Electrical Impedance Tomography, the complex coefficient is the admittivity distribution of the interested object and hence the corresponding voltage potential is a complex-valued function. In this paper, we are interested in investigating a highly complicated interrelation between the real and imaginary parts

of the complex voltage potential. This investigation is indispensable for complete understanding of the complex elliptic PDE and its applications in EIT. In this work, we show that the imaginary part of the complex potential has a close relation with the difference of its real part and a real-valued voltage potential which is a solution to the standard elliptic PDE with conductivity coefficient. This observation implies that the imaginary part itself is a kind of frequency difference between two potentials at different frequencies and it can be used to image the admittivity changes with respect to frequency from single measurement of the imaginary part unlike typical frequency difference EIT. This sheds new light on the role of the imaginary part of the complex potential.

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### Partial data inverse problems in unbounded domains

**Xiaosheng Li**

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The inverse boundary value problems consist of the recovery of the coefficients of the partial differential equations in a medium from measurements of the solutions on its boundary. In this talk we present the recent developments on such problems in two unbounded domains, a half space and an infinite slab. The available measurements are only on part of the boundary hyperplane(s). The unique determination results will be discussed for the Schroedinger types of equations.

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### A convergent algorithm for the hybrid problem of reconstructing conductivity from minimal interior data

**Amir Moradifam**

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We consider the hybrid problem of reconstructing the isotropic electric conductivity of a body  $\Omega$  from interior Current Density Imaging data obtainable using MRI measurements. We only require knowledge of the magnitude  $|J|$  of one current generated by a given voltage  $f$  on the boundary  $\partial\Omega$ . As previously shown, the corresponding voltage potential  $u$  in  $\Omega$  is a minimizer of the weighted least gradient problem

$$u = \operatorname{argmin} \left\{ \int_{\Omega} a(x) |\nabla u| : u \in H^1(\Omega), u|_{\partial\Omega} = f \right\},$$

with  $a(x) = |J(x)|$ . In this paper we present an alternating split Bregman algorithm for treating such least gradient problems, for  $a \in L^2(\Omega)$  non-negative and  $f \in H^{1/2}(\partial\Omega)$ . We give a detailed convergence

proof by focusing to a large extent on the dual problem. This leads naturally to the alternating split Bregman algorithm. The dual problem also turns out to yield a novel method to recover the full vector field  $J$  from knowledge of its magnitude, and of the voltage  $f$  on the boundary. We then present several numerical experiments that illustrate the convergence behavior of the proposed algorithm. This is a joint work with Adrian Nachman and Alexandre Timonov.

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### Inverse born series for the Calderon problem

**Shari Moskow**

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**Simon Arridge, John Schotland**

We propose a direct reconstruction method for the Calderon problem based on inversion of the Born series. We characterize the convergence, stability and approximation error of the method and illustrate its use in numerical reconstructions.

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### Quantitative thermo-acoustic imaging: an exact reconstruction formula

**Loc Nguyen**

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**H. Ammari, J. Garnier, W. Jing**

The quantitative thermo-acoustic imaging is considered in this talk. Given several data sets of electromagnetic data, we first establish an exact formula for the absorption coefficient, which involves derivatives of the given data up to the third order. However, because of the dependence of such derivatives, this formula is unstable in the sense that small measurement noises may cause large errors. Hence, with the presence of noise, the obtained formula, together with noise regularization, provides an initial guess for the true absorption coefficient. We next correct the errors by deriving a reconstruction formula based on the least square solution of an optimal control problem and show that this optimization step reduces the errors occurring.

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**Inverse problem of determining an absorption coefficient and a speed of sound in the wave equation by the BC method**

**Leonid Pestov**

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We consider inverse dynamical problem for the wave equation with unknown variable speed and absorption in a bounded domain  $\Omega$ . A linear procedure based on the BC method for determining both coefficients is proposed. The procedure includes solution of boundary control problem for states  $u(\cdot, T), u_t(\cdot, T)$  of special kind and determining unknowns from linear integral equations. The time of observation must be greater than optical diameter of  $\Omega$ .

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**Reconstruction strategies in quantitative photoacoustic tomography**

**Kui Ren**

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In quantitative photoacoustic tomography (qPAT) we aim at reconstructing physical parameters of biological tissues from “measured” data of absorbed radiation inside the tissues. Mathematically, qPAT problems can be regarded as inverse problems related to some elliptic partial differential equations. We present in this talk some new reconstruction strategies for inverse problems in qPAT with different types of available data.

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**Existence of a minimizer for the weighted least gradient problem**

**Greg Spradlin**

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**Alexandru Tamasan**

We consider the following problem: Given a bounded smooth domain  $\Omega$  in  $\mathbb{R}^n$ , boundary data  $f \in H^{\frac{1}{2}}(\partial\Omega)$ , and a continuous, positive bounded function  $a : \Omega \rightarrow \mathbb{R}$  bounded away from zero, show that the functional  $\int_{\Omega} a(x) \|D(u)\|$ , where  $u$  has trace  $f$  on  $\partial\Omega$ , has a minimizer. The problem has origins in medical imaging.

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**Viscoelasticity in magnetic resonance elastography**

**Jeong-Rock Yoon**

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Magnetic resonance elastography aims to reconstruct the elasticity of human tissue for early stage cancer detection, using the interior displacement data acquired from MR signal associated with a vibrating transducer. For simplicity, one uses purely elastic model in applications. However, human tissue is viscoelastic that shows energy absorption of the propagating medium, especially in liver. In this talk, we discuss what properties have to be changed under this more realistic model.

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## Special Session 51: Ordinal Symbolic Dynamics and Applications

Jose Maria Amigo, Universidad Miguel Hernandez, Spain  
 Karsten Keller, University of Luebeck, Germany

Ordinal symbolic dynamics is a promising new approach to the investigation of time series and the systems behind them, with the important properties of being conceptually simple and relatively robust with respect to noise. The idea behind it is to consider the order relation between the values of a time series instead of the values themselves. Roughly speaking, a given time series is transformed into a series of order patterns describing the ups and downs of the original series. Then the distribution of ordinal patterns obtained is the basis of the analysis.

Since Bandt and Pompe introduced permutation entropy in their celebrated paper in 2002, the idea has been intensively discussed. For example, there are interesting results and open questions concerning the relation of permutation entropy to conventional ergodic concepts in dynamical systems like Kolmogorov-Sinai entropy. On the other hand, the tools of ordinal symbolic dynamics are being used in real time series analysis with a remarkable success, e.g. for identifying and discriminating different brain states in epilepsy research and in anesthesiology, for heart rate analysis, and for testing independence.

The purpose of the special session is to bring together researchers working on ordinal symbolic dynamics and related topics, in order to discuss new developments in theory and applications, and to celebrate the first ten years of the field!

### Permutation entropy: one concept, two approaches

Jose Maria Amigo  
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The original definition of permutation entropy by C. Bandt and B. Pompe has the theoretical appeal of involving only one infinite process (a limit over the length of the ordinal patterns), partitions being chosen in a canonical way. The relation between the Kolmogorov-Sinai entropy and the permutation entropy for measurable selfmaps of  $R^n$  has been recently studied in depth by K. Keller.

Alternatively, one could mimic the Kolmogorov approach to the conventional metric entropy and use arbitrary partitions instead. As compared to the former approach, the latter involves an additional infinite process, namely, a limit over finer and finer partitions. Along these lines one can introduce permutation entropy even if the state space is not linearly ordered. Not unexpectedly, in this second approach permutation entropy can be shown to coincide with the Kolmogorov-Sinai entropy. In this regard, let us also mention the recent results of T. Haruna and K. Nakajima concerning ordinal symbolic dynamics with a finite alphabet.

Both approaches have their theoretical and practical virtues. In fact, the theoretical results obtained along these two ways are very similar, though not the same. Under which assumptions they coincide, remains an interesting and difficult open problem.

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### Event detection, multimodality and non-stationarity: order patterns, a tool to rule them all?

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Pablo Chamorro, José María Amigó, Francisco B. Rodríguez, Pablo Varona

The control of complex systems involving non-stationary dynamics require very fast and precise methods to identify and correlate events. These methods should combine flexibility and robustness: perfect estimation of all states cannot be performed, but it is necessary to infer enough information to construct efficient actuation laws. Moreover, the presence of noise should be treated conveniently. In this concern, fine-grained orbits could contain too much information to derive an adequate control strategy and, consequently, it could be advisable to discretize them by assigning a proper partition to the phase space. As a matter of fact, it is possible to reconstruct the underlying dynamics of time series upon Markov partitions (or at least generating ones). From a theoretical point of view, the existence of generating partitions is guaranteed for any ergodic process. Nevertheless, the practical counterpart of this result cannot be embodied straightforward. Certainly, different methods have been proposed to obtain the generating partition of a given dynamical system, but this problem is still an open and hard challenge in applied dynamical systems theory. On the other hand, symbolic dynamics is a very useful tool when considering different sources of information, which is the case of multimodal systems. The discretization of time series allows to integrate and compare different dynamics independently of their original codification and accordingly to the basis of information theory.

Order patterns are an alternative to get an approximation of Markov partitions, or at least of gen-

erating ones. The seminal work of Bandt and Pompe in 2002 showed that entropy can be estimated using the ordinal time series of dynamical systems. The so-called permutation entropy can be computed through the histogram of order patterns, which enables the classification of different dynamics and the detection of changes in them. In this vein, the analysis of order patterns has been successfully applied to the uni/multivariate study of EEG signals and Event Related Potentials (ERPs), to the detection of dynamical changes, to the estimation of control parameters, and to the characterization of synchronization and couplings. However, most of previous works on order patterns rely on the offline analysis of long time series. In this work we study the limitations of asymptotic analysis of intra and extra-cellular time series by means of the corresponding order patterns. Specifically we derive different measures from ordinal time series in order to discuss their suitability when real-time estimation and actuation is considered. Additionally, different connectivity measures between ordinal time series are evaluated to establish their adequacy to model interactions in a multimodal setup.

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#### Iteration of differentiable functions under m-modal maps with aperiodic kneading sequences

**Maria Correia**

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**Carlos Ramos, Sandra Vinagre**

We consider an infinite dynamical system obtained by iteration of functions of a class of differentiable functions,  $A$ , under  $m$ -modal maps  $f$ . Using an algorithm, we obtained some numerical and symbolic results related to the frequencies of occurrence of critical values of the iterated functions when the kneading sequences of  $f$  are aperiodic. Moreover, we analyze the evolution as well as the distribution of the aperiodic critical values of the iterated functions.

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#### Analysis of heart rate asymmetry by ordinal patterns

**Grzegorz Graff**

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**Jose M. Amigo, Beata Graff, Agnieszka Kaczkowska, Jaroslaw Piskorski, Krzysztof Narkiewicz, Przemyslaw Guzik**

The problem of recognition of all main features of healthy heart rhythm (so-called sinus rhythm) is still one of the biggest challenges in contemporary cardiology. Recently, there has been observed an

interesting physiological phenomenon of heart rate asymmetry related to unequal contribution of heart rate decelerations and accelerations to heart rate variability. We apply ordinal patterns and permutation entropy to the analysis of ECG time series (RR intervals) and observe further new features of heart rhythm asymmetry. Ordinal patterns analysis might be successfully used for heart rate description in healthy subjects and could provide a reference for identification of pathological states.

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#### Permutation approach to finite-alphabet stationary stochastic processes based on the duality between values and orderings

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**Kohei Nakajima**

The duality between values and orderings is a powerful method to establish equalities between various information theoretic measures and their permutation versions for finite-alphabet stationary stochastic processes. To illustrate its potency, in this talk, we consider the transfer entropy (TE) which is a measure of the direction and magnitude of information flow between two jointly distributed processes. There are two known permutation versions of TE: the symbolic transfer entropy (STE) and the transfer entropy on rank vectors (TERV). We show that the rate versions of both STE and TERV are equal to TE rate for any finite-state finite-alphabet hidden Markov model with an ergodic internal process.

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#### KS entropy and permutation entropy

**Karsten Keller**

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The relationship of KS entropy and Permutation entropy in its original version given by Bandt and Pompe is an interesting open problem. Whereas both entropies are equal for piecewise monotone interval maps as shown by Bandt, Pompe and G. Keller, there is nothing known about coincidence of the entropies for other classes of dynamical systems.

In our talk, we discuss the problem for a generalized concept of permutation entropy, which is based on considering observables on a (multi-dimensional) dynamical system. In particular, we show that KS entropy cannot be larger than Permutation entropy and characterize equality of KS entropy and Permutation entropy on the pure ordinal level. We include Amigo's version of Permutation entropy, which is equivalent to KS entropy, into the discussion.

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### Cardiovascular regulation during sleep quantified by symbolic coupling traces

**Juergen Kurths**

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**Niels Wessel**

Sleep is a complex regulated process with short periods of wakefulness and different sleep stages. These sleep stages modulate autonomous functions such as blood pressure, and heart rate. The method of symbolic coupling traces (SCT) is used to analyze and quantify time-delayed coupling of these measurements during different sleep stages. The symbolic coupling traces, defined as the symmetric and diametric traces of the bivariate word distribution matrix, allow for the quantification of time-delayed coupling. In this paper, the method is applied to heart rate and systolic blood pressure time series during different sleep stages for healthy controls as well as for normo- and hypertensive patients with sleep apneas. Using the SCT, significant different cardiovascular mechanisms not only between the deep sleep and the other sleep stages but also between healthy subjects and patients can be revealed. The SCT method is applied to model systems, compared with established methods, such as cross correlation, mutual information and cross recurrence analysis and demonstrates its advantages especially for non-stationary physiological data. Thereby SCT proves to be more specific in detecting delays of directional interactions than standard coupling analysis methods and yields additional information which can not be measured by standard parameters of heart rate- and blood pressure variability. The proposed method may help to indicate pathological changes in cardiovascular regulation and also effects of continuous positive airway pressure therapy on the cardiovascular system.

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### Symbolic analysis: inference basis for constructing hypotheses

**Mariano Matilla-Garcia**

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**Manuel Ruiz, Jesus Mur, Fernando Lopez**

This paper introduces a novel method for elaborating nonparametric hypothesis tests based on symbolic analysis. The existing tests based on symbolic entropy that have been used for testing central hypotheses in economics, finance and spatial econometrics are now particular cases of this general approach. This family of potential symbolic tests benefits from the use of few assumptions that limit the general applicability of any test. In addition, as a result of the method, we easily construct and provide four new statistics for testing the null of spatiotemporal

independence. Monte Carlo results highlight the well behavior of the proposed test.

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### Information measures to characterize the coupling complexity between dynamical system components

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**Wolfram Bunk, Thomas Aschenbrenner, Jose Maria Amigo**

The concept of transcripts arises naturally when studying relationships between coupled dynamical systems using ordinal symbolic dynamics. Since transcripts belong to the family of ordinal patterns, they also exhibit properties like robustness to noise, computational speed, and a sound theoretical framework. In contrast to other methods of ordinal symbolic dynamics, transcripts explicitly exploit the group theoretical properties of permutations. Using this approach, we propose information measures suitable to characterize the complexity between components of a dynamical systems. The so-called "Coupling Complexity" differs in general from the complexity of the complete system or its individual components. We apply and test the performance of these information measures with numerical and real world data.

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### Local information dynamics via permutation-information theoretic approach

**Kohei Nakajima**

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**Taichi Haruna**

Recently, permutation-information theoretic approach has been used in a broad range of research field. In particular, in a high-dimensional dynamical systems research, it has been shown that this approach can be effectively used to characterize a global property including complexity of their spatiotemporal dynamics. Here, we show that the approach can be also applied to reveal local informational dynamics existing in high-dimensional dynamical systems. J. T. Lizier et al. have recently introduced the concept of local information dynamics, which consists of information storage, transfer, and modification. This concept has been intensively studied with regard to cellular automata and has provided quantitative evidence of several characteristic behaviors observed in the system. In this study, we demonstrate that the application of the permutation-information theoretic approach, which introduces natural symbolization

methods, makes the concept easy to extend to systems that have continuous states. We then apply this approach to coupled map lattices, demonstrating that it can be successfully used as a spatiotemporal filter to stress a coherent structure hidden in the system. In particular, we show that the approach can clearly stress out defect turbulences or Brownian motion of defects from the background. Finally, we demonstrate and discuss scenarios for the application of this approach within the robotics community.

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### Nonlinear signal analysis and classification using ordinal patterns

**Ulrich Parlitz**

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**Sebastian Berg, Stefan Luther**

Ordinal patterns describing amplitude relations in (short) segments of time series have successfully been used in a wide range of applications like detection of determinism in noisy time series, estimation of transfer entropy in epilepsy, or complexity analysis of time series. In this contribution we shall present and discuss applications of ordinal pattern statistics for nonlinear (deterministic) chaotic systems, including synchronization analysis, forecasting, and signal classification. In particular promising applications to ECG data will be presented which show that symbolic dynamics based on ordinal patterns provides a powerful tool for coping with data from life sciences.

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### LE–statistic: a versatile tool in ordinal time series analysis

**Bernd Pompe**

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We introduce a quantity called *LE–statistic*. It is an easily computable functional of two ordinal time series with versatile applications. We demonstrate its usefulness as (i) a statistic in a nonparametric dependence test of two time series, and (ii) a complexity measure of one time series. For chaotic orbits of one–dimensional systems it is related to the Lyapunov characteristic exponent resp. metric entropy.

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### Ordinal pattern distributions

**Mathieu Sinn**

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The distribution of ordinal patterns describing the up and down in the dynamics of a system contains much information on the structure of the system. This fact is illustrated by some examples from dynamical systems and stochastic processes. On one hand we show how simple measures based on the ordinal pattern distribution can be used for the estimation of model parameters, and on the other hand we discuss whether deterministic and stochastic behavior can be distinguished by looking at the ordinal pattern distributions obtained from a system.

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### Conditional entropy of ordinal patterns

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**Karsten Keller, Valentina A. Unakafova**

Ordinal patterns of order  $d$  reflect order relations between  $(d + 1)$  successive points of an orbit of a dynamical system. In our talk, we introduce a conditional entropy measuring the uncertainty of  $(n + 1)$  successive ordinal patterns of some order  $d$  given the first  $n$  ordinal patterns. The use of such conditional entropy provides interesting insights into the problem of the relationship of KS entropy and Permutation entropy. In particular, we provide results that relate these two quantities to conditional entropy of ordinal patterns in some special cases.

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## Special Session 52: Fractional Differential and Integral Equations, Theory and Applications

**Eduardo Cuesta**, University of Valladolid, Spain  
**Mokhtar Kirane**, University of La Rochelle, France  
**Onur Alp Ilhan**, Erciyes University, Turkey

Fractional Differential Equations are met in modeling a great variety of phenomena in Visco-elasticity/damping, Chaos, biology, electronics, chemistry, signal processing, diffusion and wave propagation, percolation, as well as in mathematical economy. This session will focus on the recent developments in the theory of fractional differential equations (such as global existence/blow-up, regularity, long-time behavior, oscillation), their numerical analysis, and recent developments in practical applications.

### On the applications of Volterra equations in image processing and restoration

**Eduardo Cuesta**  
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**M. Kirane, S.A. Malik**

A generalized understanding of PDEs involving fractional time derivatives/integrals leads to Volterra equations which somehow interpolate parabolic and hyperbolic models, therefore enjoying intermediate properties, in fact properties related to diffusion/smoothing of the image. In this talk we provide precise details of this approach and practical results supporting the goodness of this approach.

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### Global existence of the solutions of a class fractional - differential equations with a Legendre derivative

**Svetlin Georgiev**  
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In this talk we consider the problem

$$D^\alpha \left( t^k D^\beta \right) u(x, t) = t^\gamma \Delta u(x, t) \\
+ f(t, x, u, u_{x_1}, u_{x_2}, \dots, u_{x_n}), \quad t > 0, x \in \mathbb{R}^n, \\
\lim_{t \rightarrow 0} u(x, t) = u(x, 0), \quad x \in \mathbb{R}^n,$$

where  $D^\alpha$  and  $D^\beta$  are fractional derivatives in  $t$ , 0

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### Asymptotic stability of abstract dissipative systems with infinite memory

**Aissa Guesmia**  
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We consider in this work the problem of asymptotic behavior of solutions to an abstract linear dissipative integrodifferential equation with infinite memory

(past history) modeling linear viscoelasticity. Under a boundedness condition on the history data, we show that the stability of the system holds for a much larger class of the convolution kernels than the one considered in the literature, and we provide a relation between the decay rate of the solutions and the growth of the kernel at infinity. Additionally, our decay estimates improve, in some particular cases, the decay rates known in the literature. Several particular applications are also given.

These results have been recently published in *J. Math. Anal. Appl.*, 382 (2011), 748-760.

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### Solvability of some partial integral equations in Hilbert space

**Onur Alp Ilhan**  
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An integral equation of contact problem of the theory of visco elasticity of mixed Fredholm and Volterra type with spectral parameter depending on time is considered. In the case where the final value of parameter coincides with some isolated point of the spectrum of Fredholm operator the additional conditions of solvability are established.

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### Solving fractional Riccati differential equations using modified variational iteration method

**Hossein Jafari**  
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**H. Tajadodi, C.M. Khaliq**

In this paper, a modified variational iteration method (MVIM) for solving Fractional Riccati differential equations will be introduced. Also we solve the fractional Riccati differential equation using variational iteration method by considering Adomians polynomials for nonlinear terms. The main advantage of the MVIM is that it can enlarge the convergence region of iterative approximate solutions. Hence, the

solutions obtained using the MVIM give good approximations for a larger interval. Numerical results show that the method is simple and effective.

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### Finite-dimensional behavior in a thermosyphon with a viscoelastic fluid

**Angela Jimenez-Casas**

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We analyze the motion of a Maxwellian viscoelastic fluid [ Faith Morrison, 2001 ] in the interior of a closed loop thermosyphon under the effects of natural convection and a given external heat flux in the diffusion case. This nonlinear model for viscoelastic fluids is, in some sense, a generalization of the previous models [A. Rodriguez-Bernal and E.S. Van Vleck, 1998 and Angela Jimenez-Casas and Alfonso Matias Lozano Ovejero, 2001 between others]. We study the asymptotic properties of the fluid inside the thermosyphon and we use the result and techniques from [J. Yasappan, A. Jimenez-Casas and M. Castro, 2012] to obtain an exact finite dimensional system representing the dynamics on the inertial manifold for this diffusion case.

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### Differences between fractional- and integer-order dynamics

**Eva Kaslik**

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**Seenith Sivasundaram**

The qualitative theory of fractional-order dynamical systems and its applications to the sciences and engineering is a recent focus of interest of many researchers. In addition to natural similarities that can be drawn between fractional- and integer-order derivatives and fractional- and integer-order dynamical systems, very important differences arise as well. Since in many cases, qualitative properties of integer-order dynamical systems cannot be extended by generalization to fractional-order dynamical systems, the analysis of fractional-order dynamical systems is a very important field of research. This talk is devoted to presenting qualitative contrasts between fractional- and integer-order dynamical systems. For example, even though the integer-order derivative of a periodic function is obviously a periodic function of the same period, the fractional-order derivative of a non-constant periodic function cannot be a periodic function of the same period. As a consequence, periodic solutions do not exist in a wide class of fractional-order dynamical systems. Moreover, important differences will be highlighted concerning

the asymptotic stability analysis of fractional- and integer-order dynamical systems, as well. Numerical simulations will also be presented to substantiate the theoretical results.

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### Lyapunov stability for differential systems of fractional order

**Namjip Koo**

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**Sung Kyu Choi, Bowon Kang**

In this talk we introduce the fractional comparison principle. Then we discuss some results about boundedness and Mittag-Leffler stability of solutions of fractional differential systems using Lyapunov-type functions. Also, we give the converse Lyapunov theorem for the Mittag-Leffler stability.

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### A theory of non-local linear drift wave transport

**Sara Moradi**

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**J. Anderson, B. Weysow**

Transport events in tokamak turbulent plasmas often exhibit non-local or non-diffusive action at a distance features that so far have eluded a conclusive theoretical description. A prominent candidate for explaining the suggestive non-local features of plasma turbulence is the inclusion of a fractional velocity derivative in the Fokker-Planck (FP) equation leading to an inherently non-local description as well as giving rise to non-Gaussian Probability Distribution Functions (PDFs) of e.g. densities and heat flux. The non-locality is introduced through the integral description of the fractional derivative and the non-Maxwellian distribution function drives the observed PDFs of densities and heat flux far from Gaussian. The aim of this study is to elucidate the effects of a non-Maxwellian distribution function induced by the fractional velocity derivative in the Fokker-Planck equation. Using fractional generalizations of the Liouville equation, kinetic descriptions have been developed previously. It has been shown that the chaotic dynamics can be described by using the FP equation with coordinate fractional derivatives as a possible tool for the description of anomalous diffusion. Much work has been devoted on investigation of the Langevin equation with Levy white noise, or related fractional FP equation. Furthermore, fractional derivatives have been introduced into the FP framework in a similar manner as the present work but a study including drift waves is still called for.

To this end we quantify the effects of the fractional derivative in the FP equation in terms of a modified dispersion relation for density gradient driven linear plasma drift waves where we have considered a case with constant external magnetic field and a shear-less slab geometry. In order to calculate an equilibrium PDF we use a model based on the motion of a charged Levy particle in a constant external magnetic field obeying non-Gaussian, Levy statistics. This assumption is the natural generalization of the classical example of the motion of a charged Brownian particle with the usual Gaussian statistics. The fractional derivative is represented with the Fourier transform containing a fractional exponent. We find a relation for the deviation from Maxwellian distribution described by  $\epsilon$  through the quasi-neutrality condition and the characteristics of the plasma drift wave are fundamentally changed, i.e. the values of the growth-rate  $\gamma$  and real frequency  $\omega$  are significantly altered. A deviation from the Maxwellian distribution function alters the dispersion relation for the density gradient drift waves such that the growth rates are substantially increased and thereby may cause enhanced levels of transport.

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#### Exponential decay in thermoelastic systems with boundary delay

**Muhammad Mustafa**

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In this paper we consider a thermoelastic system with boundary delay. Under suitable assumption on the weight of the delay, we prove the well posedness of the system and use the energy method to show that the damping effect through heat conduction is still strong enough to uniformly stabilize the system even in the presence of boundary time delay. Our result extends the stability region of the system and improves earlier results existing in the literature.

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#### Almost periodic mild solutions to evolutions equations with stepanov almost periodic coefficients

**Alex Sepulveda**

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**Claudio Cuevas, Alex Sepulveda, Herme Soto**

In this work, we study sufficient conditions for the existence of almost periodic mild solutions to autonomous fractional differential equation, where the fractional derivative is understood in the Riemann-Liouville sense:

$$D_t^\alpha u(t) = Au(t) + D_t^{\alpha-1} f(t, u(t)), \quad t \in \mathbb{X}.$$

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#### Some results to evolutions equations with stepanov-like pseudo-almost periodic coefficients

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**Claudio Cuevas, Alex Sepulveda**

In this work we make use of the properties of Stepanov-like pseudo-almost periodic functions to study the existence of pseudo-almost periodic solutions to fractional differential equation and integro-differential equation with Stepanov pseudo-almost periodic coefficients.

We study the existence of pseudo-almost periodic mild solutions to fractional differential equation:

$$D_t^\alpha u(t) = Au(t) + D_t^{\alpha-1} f(t, u(t)), \quad t \in \mathbb{R}.$$

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## Special Session 53: Greedy Algorithms and Tensor Product Representations for High-Dimensional Problems

Virginie Ehrlacher, Ecole des Ponts ParisTech, France  
Tony Lelièvre, Ecole des Ponts ParisTech, France

The curse of dimensionality remains a major obstacle to numerical simulations in various fields such as quantum chemistry, molecular dynamics or uncertainty quantification for instance. Tensor product representations of multivariate functions are a promising way to avoid this difficulty. Besides, greedy algorithms have been used in many contexts in order to provide satisfactory, but not usually optimal, tensor product representations. The aim of this symposium is to bring together scientists from the greedy algorithms and tensor product representation communities, in order to interact on these subjects.

### Sparse adaptive Taylor approximation algorithms for parametric and stochastic elliptic PDEs

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Albert Cohen, Chris Schwab, Ronald Devore

The numerical approximation of parametric partial differential equations is a computational challenge, in particular when the number of involved parameter is large. We consider a model class of second order, linear, parametric, elliptic PDEs on a bounded domain  $D$  with diffusion coefficients depending on the parameters in an ane manner. For such models, it was shown that under very weak assumptions on the diusion coeicients, the entire family of solutions to such equations can be simultaneously approximated by multivariate sparse polynomials in the parameter vector  $y$  with a controlled number  $N$  of terms. The convergence rate in terms of  $N$  does not depend on the number of parameters, which may be arbitrarily large or countably infinite, thereby breaking the curse of dimensionality. However, these approximation results do not describe the concrete construction of these polynomial expansions, and should therefore rather be viewed as benchmark for the convergence analysis of numerical methods. We present an adaptive numerical algorithm for constructing a sequence of sparse polynomials that is proved to converge toward the solution with the optimal benchmark rate. Numerical experiments are presented in large parameter dimension, which confirm the effectiveness of the adaptive approach.

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### Greedy algorithms for non symmetric linear problems with uncertainty

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Eric Cances, Tony Lelièvre

Greedy algorithms, also called Progressive Generalized Decomposition algorithms, are known to give

very satisfactory results for the approximation of the solution of minimization problems, typically symmetric hermitian uncertain problems with a large number of random parameters. Indeed, their formulation leads to discretized problems whose complexity evolves linearly with respect to the number of parameters, while avoiding the curse of dimensionality. However, naive adaptations of these methods to non-symmetric problems may not converge towards the desired solution. In this talk, different versions of these algorithms will be presented, along with convergence results.

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### Geometric structures in tensor representations

Antonio Falco

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In this talk we discuss about the geometric structures associated with tensor representations based in subspaces. In particular, we use the Grassmann Banach manifold to characterize the manifold of tensors in Tucker format with fixed rank. It allows to extend the dynamical tensor approximation framework to tensor Banach spaces.

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### The POD-Greedy method: convergence rates and applications

Bernard Haasdonk

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Iterative approximation algorithms are successfully applied in parametric approximation tasks. In particular, reduced basis (RB) methods make use of a greedy algorithm for approximating solution sets of parametrized partial differential equations. Recently, a-priori convergence rate statements for this algorithm have been given (Buffa et al. 2009, Binev et al. 2010). When addressing time-dependent parametric problems with RB-methods, the POD-Greedy

algorithm (Haasdonk and Ohlberger 2008) is typically applied. In this algorithm, each greedy step is invoking a temporal compression by performing a proper orthogonal decomposition (POD). Using a suitable coefficient representation of the POD-Greedy algorithm, we recently have shown that the existing convergence rate results of the Greedy algorithm can be extended. In particular, exponential or algebraic convergence rates of the Kolmogorov  $n$ -widths are maintained by the POD-Greedy algorithm.

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### Applications in mathematical finance of a greedy algorithm for solving high-dimensional partial differential equations.

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**Tony Lelievre**

We study an algorithm which has been proposed in Ammar et al. (2007) and analyzed in Cancès et al. (2010) and Le-Bris et al. (2009) to solve high dimensional partial differential equations. The idea is to represent the solution as a sum of tensor products and to compute iteratively the terms of this sum. We will present the application of this non linear approximation method to the option pricing problem. This leads us to consider two extensions of the standard algorithm, which applies to *symmetric linear* partial differential equations: (i) *nonsymmetric linear* problems to value European options, (ii) *nonlinear variational* problems to price American options. For European options, to overcome the fact that we have to solve a non-symmetrical problem we use an implicit-explicit scheme. For American options, we consider the idea proposed in Cancès et al. (2010) of penalizing the constraints of the problem in order to obtain a symmetrical problem. Finally, we study the interest of this method as a variance reduction method. The idea is to use the method to solve the backward Kolmogorov equation associated to a pricing problem in relatively high dimension to solve related pricing problems in very high dimension.

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### Regularized reconstruction with series kernels

**Christian Rieger**  
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The main focus of this talk lies on scattered data approximation using series kernels. The latter are positive definite kernel functions that possess expansions in terms of simple basis functions. In many examples these simple basis functions are elementary tensors products, i.e., tensor products of univariate basis functions. We focus on two aspects: The first aspect is the subtle interplay between the intrinsic (multi-scale, tensor product) structure of the kernel and the choice of data locations in scattered data approximation problems. The second aspect concerns the choice of coefficients with respect to the basis of the trial space. We propose a discrete optimization problem to find the coefficients, based on the well-established compressed sensing techniques. These techniques are known to be greedy in the sense that, under certain conditions, they minimize the number of non-zero coefficients. Convergence results for the function reconstruction are obtained by using sampling inequalities. This is partly based on joint works with M. Griebel and B. Zwicknagl.

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### Optimization in hierarchical tensor formats

**Reinhold Schneider**  
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**Th. Rohwedder**

We will discuss recently introduced hierarchical Tucker representation (Hackbusch, et al.), HT format or tree tensor networks. The particular case of TT-tensors can be written by a matrix product representation (matrix product states (MPS) in quantum information theory). We consider numerical methods solving optimizations problem within a prescribed format, focusing on i)  $L_2$ -approximation, ii) linear equations and iii) eigenvalue problems. We will develop differential equations of gradient flow, and projected gradient methods and an alternating linear scheme, or alternating direction scheme, which is a generalization of an alternating least square (ALs) together with a modification (MALS) which resembles the density matrix renormalization group algorithm (DMRG). Additionally we will discuss a Newton type method. All these methods are local optimization schemes and may suffer from the existence of local minima. Greedy type approaches may be applied for these formats as well.

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## Special Session 54: Dynamics in Complex Networks

**Juan A. Almendral**, Rey Juan Carlos University, Spain  
**Miguel Romance**, Rey Juan Carlos University, Spain

Complex Networks constitutes a theoretical framework that allows modeling the complexity of the systems that can be found in fields as diverse as biology, ecology, sociology or neuroscience. Although Complex Networks became a research line only 10 years ago, the number of disciplines in which their tools are useful is continuously increasing, being at the present moment one of the more active research fields. Its fundamental premise is that the understanding of a real system made of an ensemble of components should consider not only the nature of these units but also the distribution and features of the interactions.

Following the chronology of the Complex Networks development, the studies were initially focused on the topological and structural characteristics of networks, considered like graphs, regardless of the nature of its units. Later, the interest shifted to dynamics in networks, where each node represents now a dynamical unit interacting with the rest of the network according to certain complex distribution of links. This process results in the emergence of certain properties (synchronization, activation, diffusion, propagation) that cannot be deduced from the unit features.

In the last years, the research in this field is focused on the role that network modularity has in the emergence of a specific function. In other words, the study of how the organization in communities that bridge between the local dynamics and the global statistic affects the collective behavior of a system; issue in which not only the community functioning and structure is being debated, but also its detection at different scales. The functioning of many biological (neuronal, metabolic, genetic...), social and technological networks is precisely based on the coordination of parallel processes carried out by different communities. For example, vision depends on the emergence of a collective behavior in two different brain areas, thus perception implies the simultaneous coordination of both processes. How this process coordination takes place and which are its inner (topological and dynamical) mechanisms are still opened questions, although it is already well known that the information processing in the brain combines the distribution (to different points from the network) and segregation (in communities) of this information.

### The integration/segregation phenomena from the complex networks viewpoint

**Juan Almendral**

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**Vanesa Avalos Gaytan, Satu Elisa Schaeffer, Stefano Boccaletti**

We study the integration/segregation problem from the viewpoint of complex networks, but considering that the network topology is not static but there is an adaptive mechanism acting on the links. Our goal is to identify under which conditions network synchronization occurs and what structural properties are present in the network topology when this happens. In particular, we experimentally compute the main descriptive structural properties of the network when it has been modified with the proposed mechanism, and it is elucidated the relationship between these results and the observed synchronization at both the local and global scale. Our main finding is that modularity, a global feature, can naturally emerge in a network when evolving links are considered, that is, by means of dynamical properties at the local scale.

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### Dynamics in a social network surrounding an online political protest

**Rosa Benito**

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**A. Morales, J.C. Losada**

Over the past years, new technologies and specially online social networks have penetrated into world's population at an accelerated pace. In this talk we analyze collected data from the web application Twitter, in order to describe the structure and dynamics of the emergent social networks, based on complexity science. We focused on a Venezuelan protest that took place exclusively by Twitter during December, 2010. We found community structure with highly connected hubs and three different kinds of user behavior that determine the information flow dynamics. We noticed that even though online social networks appear to be a pure social environment, traditional media still holds loads of influence inside the network.

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### Emerging meso-and macroscales from synchronization of adaptive networks

**Stefano Boccaletti**

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Most systems analyzed in the past years using complex networks are characterized by the presence of two important topological features: modular structures and scale-free degree distributions. Yet, an open problem is understanding how these characteristics can spontaneously emerge from the dynamics of the elements composing the system, i.e., without an external intervention. In this talk, we consider a set of interacting phase oscillators, in which the coupling between pairs of them varies according to their synchronization level; also, nodes have limited resources, thus restricting their connectivity. We show that such a competitive mechanism leads to the emergence of a rich modular structure underlying cluster synchronization, and to a scale-free distribution for the connection strengths of the units.

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### Analyzing offline events through the mirror of online social networks

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**A .J. Morales, J.C. Losada, R.M. Benito**

Online social networks, are ideal platforms to exchange and propagate information. Due to the rapidly growing number of users, online social networks have become an increasingly important topic in scientific research. Recent studies have focused on determining the structure and dynamics behind these networks and characterizing user behavior. Following this trend we have studied how important events occurring offline are reflected on these online social networks. To this end, we have built the networks corresponding to the interactions taking place among users.

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### Defibrillation mechanisms on a one-dimensional ring of cardiac tissue

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**A. Simic**

Defibrillation is a medical treatment used to terminate ventricular fibrillation or pulseless ventricular tachycardia. An electrical device via a pair of electrodes delivers controlled amount of electrical energy to the heart in order to reestablish the normal

heart rhythm. First generation of defibrillators applied monophasic shock, in which electrodes did not change polarity during the application of the shock. Later it was found that changing the polarity of the electrodes during the shock leads to better result with less energy applied. Optimal monophasic and biphasic shock release approximately 200 J and 150 J, respectively. It is desirable to use as less energetic shock as possible in order to reduce the damage done to the tissue by the strong electric current. However, to this day, there is no full understanding why biphasic shocks are better than monophasic shocks. To assess this question, we have used a bidomain model for cardiac tissue with modified Beeler-Reuter model for transmembrane currents. Modifications account for anode break phenomena and electroporation effect known to happen during defibrillation. We have studied three different types of protocols for shock application (i.e. monophasic; symmetric biphasic; and asymmetric biphasic shock) in a one dimensional ring of cardiac tissue. The size of the ring was chosen to exhibit a discordant-alternans dynamics. Results of the numerical simulations reveal that monophasic shocks defibrillate with higher rate of success than the two biphasic shock protocols at lower energies. On the contrary for higher shock energies, the biphasic shock are significantly more efficient than monophasic shocks. This latter result confirms the medical common wisdom about defibrillators. Moreover, in this study, we were able to identify and classify the different defibrillation mechanisms that happen in this system. One identifies four different types: direct block, delayed block, annihilation and direct activation. Which defibrillation mechanism prevails depends on the energy level, the current dynamic state of the system and the shock protocol. This study has permitted to uncover and confirm the experimental fact stating that biphasic shocks are more efficient (at high energy) than monophasic shock to defibrillate cardiac tissue.

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### Some structural properties of multilevel networks

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**Miguel Romance**

The concept of multilevel network has been introduced in order to embody some topological properties of heterogeneous -type complex systems which are not completely captured by the classical models. In this talk we will present some metric and structural properties of multilevel networks, a new paradigm for networks with a mesoscaled structure.

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### Analysis of human behaviors from telephone interactions; serendipity measures

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**F. Borondo, W. Creixell, T. Arredondo**

Serendipity is defined as fortunate discoveries made by chance. In this work we explore the idea that topological and dynamical measures of a person's telephone interaction networks could be an indicator about how likely that person is to experience fortunate discoveries. In order to study this phenomenon, a data-set rich in users contextual data was used. We analyse the data obtained in the Reality Mining project at MIT. The data included call logs and nearby Bluetooth devices corresponding to 350000 hours of human behaviour recorded during a nine month period. In that data-set, social network evolution in time can also be observed throughout this period of time.

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### Influence maximization for advertising in multi-agent markets

**Mahsa Maghami**

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The question of how to influence people in a large social system is a perennial problem in marketing, politics, and publishing. It differs from more personal inter-agent interactions that occur in negotiation and argumentation since network structure and group membership often play a more significant role than the content of what is being said, making the messenger more important than the message. In this paper, we propose a new method for propagating information through a social system and demonstrate how it can be used to develop a product advertisement strategy in a simulated market. We consider the desire of agents toward purchasing an item as a random variable and solve the influence maximization problem in steady state using an optimization method to assign the advertisement of available products to appropriate messenger agents. Our market simulation accounts for the 1) effects of group membership on agent attitudes 2) has a network structure that is similar to realistic human systems 3) models inter-product preference correlations that can be learned from market data. The results show that our method is significantly better than network analysis methods based on centrality measures.

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### Robustness in the urban transportation network of Madrid

**Mary Luz Mouronte**

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Madrid has an urban bus network with 204 lines and 4,455 stops and, a subway network with 16 lines and 272 stops. It has also 21 districts with quite different population densities and an unequally distribution of stops. It is possible to improve these networks identifying their critical points where some planning tasks should be done. This research analyzes the robustness of the urban bus and metro networks in the districts of Madrid. Both networks are abstracted in a graph  $G = (N; L)$  where  $N$  are the nodes corresponding to the stops and  $L$  are the links between them. So, we calculate the distribution of the number of pairs of nodes separated by the shortest distance in the original network and in the same network with the most relevant removed nodes; we also estimate the sensitivity in as  $S_{lm} = \frac{1}{N} \frac{d_{lm}}{d}$  where  $d$  is the average path length in the original network, and  $d_{lm}$  correspond to the values of average path length and size of networks with  $m$  removed nodes. We identify the more sensitive districts and those in which the distance distribution changes drastically. The results can help to improve the public transportation network in Madrid.

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### Controlling structural properties of complex networks: centrality measures

**Miguel Romance**

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**Vincenzo Nicosia, Vito Latora, Giovanni Russo and Regino Criado**

The control of properties of complex networks it is a challenge in the study of complex network analysis. In this talk we will present some results related to controlling spectral centrality measures of weighted and unweighted networks by giving an analytical solution to the inverse centrality problem in several cases. We will also show how to modify the centrality scores of the nodes by acting on the structure of a given network. We prove that there exist particular subsets of nodes, called controlling sets, which can assign any prescribed set of centrality values to all the nodes of a graph, by cooperatively tuning the weights of their out-going links. We show that many large networks from the real world have surprisingly small controlling sets, containing even less than 5-10

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## Dynamic community structures analysis

**My Thai**

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**T. N. Dinh**

Most complex networks exhibit a special property: community structures. Identifying community structures can help deeply understand the structures and functions of a network as well as design a robust system with minimum costs. Unfortunately, understanding this structure is very challenging, especially in dynamic complex networks where nodes and links are evolving rapidly. Can we quickly and efficiently identify the network community structure? Can we adaptively update the network structure based on previously known information instead of recomputing from scratch as recomputing from scratch may result in prohibitive computational costs and introduce incorrect evolution phenomena? In this talk, we introduce a general approach to efficiently detect and trace the evolution of communities in an evolving network. Our solution can identify the communities of each network snapshot based on the communities of previous snapshots, thus dynamically updating these communities. We provide a compact network representation that effectively combines the current network structural information with the change of network in next steps. Many existing community detection algorithms can be embedded into our framework to efficiently detect modules and modules' evolution in dynamic complex networks.

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## Network reconstruction from vectors of features

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In the last decade, Complex Networks have proven their potential for the analysis of real world systems. Especially in those cases where information about the relationships between the elements composing the system are not explicitly available, methods for network reconstruction have enabled a better understanding of the underlying structure and dynamics. Yet, most of these techniques only focus on the presence of a *similar* (or correlated) evolution in the dynamics of the composing elements. In this talk, we will explore a new approach for network reconstruction, which allows analyzing sets of *features*, in the search of abnormal relationships between them. Therefore, the focus shifts from *similarities* to *differences*, the natural center of interest in the analysis of data associated to medical and biological systems. The usefulness of this methodology, positioned in the middle of complex systems analysis and data mining, will be proven through several case studies, ranging from the detection of diseases in genetic and metabolic data, up to the identification of the function of unknown genes of plants.

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## Special Session 55: Nonlinear Elliptic and Parabolic Problems

Julian Lopez-Gomez, Universidad Complutense de Madrid, Spain

Nonlinear parabolic problems model a great variety of phenomena in physics, chemistry, biochemistry, engineering, ecology and economics. Quite strikingly, they can model very different natural systems with the same set of mathematical models, establishing very abstract rule bodies for describing and understanding the evolution of nature. Moreover, they provide with excellent mathematical toys for testing and developing new mathematical techniques which later apply to wide areas of mathematics. Consequently, their study is fundamental from a series of different perspectives. Nonlinear elliptic problems provide us with the steady-state solutions of parabolic problems. This session gathers to some of the very best world experts in this field in an attempt for lightening the most relevant advances of the last decade.

### Positive solutions of semilinear boundary value problems of logistic type with nonlinear mixed boundary conditions

Santiago Cano-Casanova

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The main goal of this talk is analyzing the existence, uniqueness or multiplicity and stability of the positive solutions of a very general class of semilinear boundary value problems of logistic type with nonlinear mixed boundary conditions of sub and superlinear type. We will analyze the structure of the set of positive solutions of our problem, depending of the sign of the potential which appears on the boundary condition in front of the nonlinearity, and depending of the relative position with respect to the boundary, of the vanishing set of the potential in front of the nonlinearity in the partial differential equation. Local and global bifurcation, monotonicity and continuation methods and rescaling arguments, are the main technical tools used to obtain the main results.

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### Exporing the evolutionary advantages of quasi-linear dispersal

Robert Stephen Cantrell

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Chris Cosner, Yuan Lou, Chao Xie

In this talk we will discuss how parabolic partial differential equations may be used to study the evolution of dispersal strategies. We will begin with a brief historical overview to set our context. We will then focus on our primary interest, namely exploring the relative advantages of fitness-dependent and random dispersal in a two species competition model. Both species have the same population dynamics, but one species adopts a combination of random and fitness-dependent dispersal and the other adopts random dispersal. In so doing we regard the species as ecologically identical, differing only in their dispersal strategies. The model is realized as a quasi-linear parabolic system that can be regarded as a triangular cross-diffusion system. Global existence of smooth

solutions to the system is established under some conditions. When the single species which combines random and fitness-dependent is considered in the absence of its competitor, we show that a strong tendency of the species to move up its fitness gradient leads to a stable equilibrium that can approximate the ideal free distribution. For the two-species competition model, if one species has a strong tendency to move up its fitness gradient, such approximately ideal free dispersal is evolutionarily advantageous relative to random dispersal. Further, bifurcation analysis shows that the two competing species can coexist when one species has only an intermediate tendency to move up its fitness gradient and the other species has a smaller random dispersal rate. We will conclude our discussion by highlighting a number of open mathematical problems that are prompted by our analysis

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### Global bifurcation of solutions for crime modeling equations

Chris Cosner

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R. S. Cantrell, R. Manasevich

This talk will present results on pattern formation in a quasilinear system of two elliptic equations that has been developed by Short et al. [2] as a model for residential burglary. That model is based on the observation that the rate of burglaries of houses that have been burglarized recently and their close neighbors is typically higher than the average rate in the larger community, which creates hotspots for burglary. The patterns generated by the model describe the location of those hotspots. We prove that the system supports global bifurcation of spatially varying solutions from the spatially constant equilibrium, leading to the formation of spatial patterns. The analysis is based on recent results on global bifurcation in quasilinear elliptic systems derived by Shi and Wang [1]. We show in some cases that near the bifurcation point the bifurcating spatial patterns are stable.

[1]. J. Shi and X. Wang. On global bifurcation for quasilinear elliptic systems on bounded domains, *Journal of Differential Equations*, v.7 (2009), 2788-

2812.

[2]. M.B. Short, M.R. D'Orsogna, V.B. Pasour, G.E. Tita, P.J. Brantingham, A.L. Bertozzi and L.B. Chayes. A Statistical model of criminal behavior, *Math Models and Methods in Applied Sciences*, v.18 (2008), 1249-1267.

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### On homoclinic solutions for singular Hamiltonian systems

**David Costa**

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**H. Tehrani**

We show existence of infinitely many homoclinic orbits at the origin for a class of singular second-order Hamiltonian systems. Variational methods are used under the assumption that the potential satisfies the so-called "Strong-Force" condition.

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### A forward-backward regularization of the Perona-Malik equation

**Patrick Guidotti**

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Since its introduction in 1990 as a tool of image processing, the Perona-Malik equation has attracted a great deal of interest in the mathematical community. Its numerical implementations are characterized by the onset of jump discontinuities, a phenomenon aptly called staircasing. A novel regularization will be presented in this talk which offers a suggestive mathematical explanation of this phenomenon.

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### Diffusion-driven instability for non-autonomous problems

**Georg Hetzer**

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**Anotida Madzvamuse, Wenxian Shen**

Reaction-diffusion systems on domains evolving in time can be studied by transforming the original system into a reaction-diffusion system on a fixed domain with time-dependent diffusion coefficients and non-autonomous reaction terms. If the original system satisfies no flux boundary conditions, a

spatially homogeneous solution can be stable in the absence of diffusion (as a solution of the resulting system of ordinary differential equations), but unstable in the presence of diffusion. I will discuss conditions for this diffusion-driven instability.

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### Single phytoplankton growth on light and nutrient in a water column

**Sze-bi Hsu**

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**Lou Yuan**

In this talk we shall present a mathematical model of single phytoplankton growth on light and nutrient in a water column. The model takes form of nonlocal system of Parabolic PDEs. We shall do the steady states analysis in terms of two bifurcation parameters, the light incidence rate  $I_0$  from the top of the water column and nutrient input rate,  $S_0$  from the bottom. In the  $I_0 - S_0$  bifurcation plane, we have three regions, namely I-limited region, S-limited region on the entire water column and partial S-limited, partial I-limited in the water column.

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### An optimal algebraic invariant to detect any changes of the topological degree

**Julian Lopez-Gomez**

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Through the modern theory of algebraic multiplicities developed by J. Esquinas and C. Mora-Corral under the supervision of the author, one can axiomatize the theory of algebraic multiplicities of eigenvalues of linear operators and construct an optimal invariant detecting any change of the topological degree for general operator pencils. This algebraic-analytic invariant provides us with optimal results in local and global bifurcation theory.

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### On the dependence of the population size on the dispersal rate,

**Yuan Lou**

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**Song Liang**

This talk concerns the dependence of the population size for a single species on its random dispersal rate and its applications on the invasion of species. The population size of a single species often depends on its random dispersal rate in non-trivial manners. Previous results show that the population size is usually



not a monotone function of the dispersal rate. We construct some examples to illustrate that the population size, as a function of the dispersal rate, can have at least two local maxima. As an application we illustrate that the invasion of species depends upon the dispersal rate of the resident species in complicated manners. Previous results show that the total population is maximized at some intermediate dispersal rate for several classes of local intrinsic growth rates. We find one family of local intrinsic growth rates such that the total population is maximized exactly at zero dispersal rate. We show that the population distribution becomes flatter in average if we increase the dispersal rate, and the environmental gradient is always steeper than the population distribution, at least in some average sense.

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**Steady state analysis for a relaxed cross diffusion model**

**Salome Martinez**  
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**Thomas Lepoutre**

The mechanism of cross diffusion has been introduced by Shigesada Kawasaki and Terramoto to model the trend of a species to avoid another one and thereby, possibly segregate. In this pioneer paper, cross diffusion pressure takes the form of a linear dependency to population density in the diffusion coefficient in a reaction diffusion system. In this talk, we will study the following system with cross diffusion and relaxation, in the absence of reaction

$$\begin{cases} \partial_t u - \Delta[a(\tilde{v})u] = 0, & \text{in } \Omega, \\ \partial_t v - \Delta[b(\tilde{u})v] = 0, & \text{in } \Omega, \\ -\delta\Delta\tilde{u} + \tilde{u} = u, & \text{in } \Omega, \\ -\delta\Delta\tilde{v} + \tilde{v} = v, & \text{in } \Omega, \\ \partial_n u = \partial_n v = \partial\tilde{u} = \partial_n\tilde{v} = 0, & \text{on } \partial\Omega, \end{cases}$$

which was proposed in a recent paper by Bendahmane et al. In this situation, it is necessary to incorporate nonlinear cross diffusion to generate segregating behaviour. We will address questions such as the global existence of solutions, existence of nonconstant solutions and its linear stability. This is joint work with Thomas Lepoutre (INRIA).

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**Pointwise estimates for solutions of singular parabolic problems in  $\mathbb{R}^N \times [0, +\infty)$**

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**F.Ragnedda, V.Vespri**

Let us consider the following homogeneous quasi-linear parabolic equation

$$u_t = \operatorname{div}A(x, t, u, Du), \quad (x, t) \in \mathbb{R}^N \times [0, +\infty), \tag{1.1}$$

where the functions  $A := (A_1, \dots, A_N)$  are assumed to be only measurable in  $(x, t) \in \mathbb{R}^N \times [0, +\infty)$ , continuous w.r.t.  $u$  and  $Du$  for almost all  $(x, t)$ . For p-laplacian-type equation we ask  $A$  to satisfy the following structure conditions:

$$A(x, t, u, \eta) \cdot \eta \geq c_0|\eta|^p, \quad |A(x, t, u, \eta)| \leq c_1|\eta|^{p-1}, \quad c_0, c_1 > 0 \tag{1.2}$$

for almost all  $(x, t) \in \mathbb{R}^N \times [0, +\infty)$  and  $(u, \eta) \in \mathbb{R} \times \mathbb{R}^N$  with  $\frac{2N}{N+1} < p < 2$  (supercritical range). Moreover we assume that there exists  $L > 0$  such that

$$\begin{cases} A(x, t, \xi, z_1) - A(x, t, \xi, z_2) \cdot (z_1 - z_2) \geq 0, \\ |A(x, t, \xi_1, z) - A(x, t, \xi_2, z)| \leq L |\xi_1 - \xi_2|(1 + |z|^{p-1}), \end{cases} \tag{1.4}$$

for almost all  $(x, t) \in \mathbb{R}^N \times [0, +\infty)$  and all  $\xi, \xi_i \in \mathbb{R}$  and  $z, z_i \in \mathbb{R}^N, i = 1, 2$ .

We prove that, if  $u$  is a non negative, locally bounded weak solution of

$$\begin{cases} u_t = \operatorname{div}A(x, t, u, Du), & (x, t) \in \mathbb{R}^N \times [0, +\infty), \\ u(x, 0) = \delta(0), & x \in \mathbb{R}^N, \end{cases} \tag{1.5}$$

with  $A$  satisfying (1.2), (1.4),  $p$  in the supercritical range,  $\delta(0)$  the Dirac mass on  $\mathbb{R}^N$ , then there exist positive constants  $\underline{\gamma}, \bar{\gamma}$  depending only upon  $N, p, c_0, c_1$  such that  $\forall x \in \mathbb{R}^N$  and  $\forall t > 0$ ,  $\underline{\gamma}B_p(x, t) \leq u(x, t) \leq \bar{\gamma}B_p(x, t)$ , with  $B_p = t^{-\frac{N}{p}} \left[ 1 + \gamma_p \left( \frac{|x|}{t^{1/\lambda}} \right)^{\frac{1}{p-1}} \right]^{\frac{p-1}{p-2}}$  and  $\lambda = N(p-2) + p > 0$ . These results can be extended to the case of porous medium type equation and to general nonnegative  $L^1$  initial data.

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**Optimization of the first eigenvalue of equations with indefinite weights**

**Giovanni Porru**  
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Reaction-diffusion models for the dynamics and genetics of spatially distributed populations can lead to eigenvalue problems where the weight function may

change sign. Typically, a prediction of persistence or extinction for a population in a reaction-diffusion model, depends on the size of certain parameters relative to the principal eigenvalue of the Laplacian with a spatially varying weight function. If an environment contains some regions that are favorable for a given population but others that are unfavorable then the weight function will change sign. If the overall amounts of favorable and unfavorable habitat are fixed in some sense it is natural to ask which arrangements of those habitats are most likely to lead to persistence and which arrangements are most likely to lead to extinction. That question is of practical importance in the contexts of reserve design and pest control. Mathematically, the question can be formulated as asking which weight functions in a suitable class minimize or maximize the principal eigenvalue. This talk addresses that question in case where the class of weight functions is restricted to be the set of rearrangements of a given function. Suppose that  $\Omega \subset \mathbb{R}^2$  is a smooth bounded domain representing a region occupied by a population that diffuses at rate  $D$  and grows or declines locally at a rate  $g(x)$ , so that  $g(x) > 0$  corresponds to local growth and  $g(x) < 0$  to local decline. Suppose that the exterior of  $\Omega$  is hostile to the population, and that the population is scaled so that the carrying capacity is equal to 1. If  $u(x, t)$  is the population density, the global behavior of the population is described by the equation

$$\begin{aligned} \frac{\partial u}{\partial t} &= D\Delta u + (g(x) - u)u \quad \text{in } \Omega \times (0, T), \\ u &= 0 \quad \text{on } \partial\Omega \times (0, T), \end{aligned}$$

where  $\Delta u$  denotes the spatial Laplacian of  $u(x, t)$ . This equation predicts persistence if and only if  $D < 1/\lambda_g$  where  $\lambda_g$  is the positive principal eigenvalue in

$$\Delta\phi + \lambda g(x)\phi = 0 \quad \text{in } \Omega, \quad \phi = 0 \quad \text{on } \partial\Omega.$$

In the present talk we will consider the question: for weights  $g(x)$  within the set of rearrangements of a given weight function  $g_0(x)$ , which, if any, maximize or minimize  $\lambda_g$ ?

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### Qualitative behavior of a diffusive predator-prey model

**Kimun Ryu**

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This talk describes the study of the qualitative behavior of non-constant positive solutions of a diffusive predator-prey model with a functional response describing behavioral mechanism of certain predator's spatial foraging in linear schools versus school under homogeneous Neumann boundary condition. We investigate the local and global attractor for non-negative time dependent solutions and provide some

sufficient conditions for local stability of the positive constant solutions. Nevertheless, we prove the non-existence and existence of non-constant positive steady-state solutions in the sequel. Ecological implications of the analytical and numerical results are described.

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### An elliptic system with chemotaxis term and nonlinear boundary conditions

**Antonio Suarez**

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**M. Delgado, C. Morales-Rodrigo**

The main goal of this talk is to show some theoretical results concerning to an elliptic system containing a chemotaxis term and nonlinear boundary condition. This model arises from the angiogenesis process, crucial step in the growth tumour. In this process new blood vessels grow from existing ones following the gradient of chemical substances, angiogenic factors. These substances are segregated by the tumour when it attains an specific size. Firstly, we use bifurcation methods to prove the existence of positive solutions and then we give some biological interpretations of the main results. These results are obtained joint Profs. M. Delgado and C. Morales-Rodrigo, Univ. of Sevilla, SPAIN. Supported by MICINN and FEDER under grant MTM2009-12367.

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### Measure valued solutions of the 2D Keller-Segel system

**Yoshie Sugiyama**

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**Stephan Luckhaus, J.J.L. Velazquez**

We deal with the two-dimensional Keller-Segel system describing chemotaxis in a bounded domain with smooth boundary under the nonnegative initial data. As for the Keller-Segel system, the L1 norm is the scaling invariant one for the initial data, and so if the initial data is sufficiently small in L1, then the solution exists globally in time. On the other hand, if its L1 norm is large, then the solution blows up in a finite time. The first purpose of my talk is to construct a time global solution as a measure valued function beyond the blow-up time even though the initial data is large in L1. The second purpose is to show the existence of two measure valued solutions of the different type depending on the approximation, while the classical solution is unique before the blow-up time. For this purpose, we also discuss on the possibility of the aggregation mass greater than  $8\pi$ .

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### Theoretical and numerical analysis of complex bifurcation diagrams related to a class of superlinear indefinite problems

**Andrea Tellini**

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**J. López-Gómez, M. Molina-Meyer, F. Zanolin**

We will present a multiplicity result for large solutions of a class of second order boundary value problems of superlinear indefinite type. Considering one of the parameters involved in the setting of the problem as a bifurcation parameter, we obtain some complex bifurcation diagrams. We will also illustrate the technical problems arising in their numerical computation.

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### Entire solutions for competition-diffusion systems and a priori estimates

**Susanna Terracini**

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**Henri Berestycki, Kelei Wang, Juncheng Wei**

We study the qualitative properties of a limiting elliptic system arising in phase separation for multiple states Bose-Einstein condensates:

$$\begin{cases} \Delta u = uv^2, \\ \Delta v = vu^2, \\ u, v > 0 \quad \text{in } \mathbf{R}^N. \end{cases}$$

We first prove that stable solutions in  $\mathbf{R}^2$  with linear growth must be one-dimensional. Then we construct entire solutions with polynomial growth  $|x|^d$  for any positive integer  $d \geq 1$ . The construction is also extended to multi-component elliptic systems. Finally, we show the connection between the existence/nonexistence of entire solutions and the qualitative properties of the optimal partitions.

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### On the effect of spatial heterogeneity in logistic type elliptic equations with nonlinear boundary conditions

**Kenichiro Umezū**

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This talk is devoted to the study of global bifurcation of positive solutions for some semilinear elliptic equation of logistic type with nonlinear boundary conditions. The purpose of this talk is to understand the role that an indefinite coefficient of the equation

plays in determining the global bifurcation structure. For this we use the implicit function theorem and blow-up arguments to discuss the existence of turning points of the bifurcation component. We argue the nonexistence of positive solutions by stability arguments. By use of the local bifurcation analysis of the reduced equation we obtain the global nature of the bifurcation component.

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### Positive solutions of elliptic equation with Hardy potential

**Lei Wei**

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In this paper, we consider the existence of maximal positive solution and minimal positive solution of

$$-\Delta = \lambda \frac{u}{d^2(x)} - b(x)u^p, \quad x \in \Omega,$$

where  $p > 1$ ,  $d(x) = \text{dist}(x, \partial\Omega)$ ,  $b$  is a nonnegative continuous function over  $\bar{\Omega}$ . We obtain the existence of maximal positive solution and minimal positive solution depending on parameter  $\lambda$  for two different type of  $b$ . For the degenerate case, we infer that the maximum point  $x_\lambda$  of the minimal positive solution  $u_\lambda$  converges to  $\partial\Omega_0$  as  $\lambda \rightarrow \lambda^*$  under suitable conditions.

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### Blow-up rate and uniqueness of singular radial solutions for a class of quasi-linear elliptic equations

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**Chunshan Zhao**

We establish the uniqueness and the blow-up rate of the large positive solution of the quasi-linear elliptic problem  $-\Delta_p u = \lambda u^{p-1} - b(x)h(u)$  in  $B_R(x_0)$  with boundary condition  $u = +\infty$  on  $\partial B_R(x_0)$ , where  $B_R(x_0)$  is a ball centered at  $x_0 \in \mathbb{R}^N$  with radius  $R$ ,  $N \geq 3$ ,  $2 \leq p < \infty$  are constants and the weight function  $b$  is a positive radially symmetrical function. We only require  $h(u)$  to be a locally Lipschitz function with  $h(u)/u^{p-1}$  increasing on  $(0, \infty)$  and  $h(u) \sim u^{q-1}$  for large  $u$  with  $q > p - 1$ .

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**Pairs of nodal solutions for a class of super-sublinear problems****Fabio Zanolin**

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**Alberto Boscaggin**

We consider various boundary value problems associated to the second-order nonlinear ODE

$$-u'' = \lambda f(x, u),$$

where  $f(x, u)/u \rightarrow 0$  as  $u \rightarrow 0$  and  $u \rightarrow \infty$ . Under mild sign conditions on  $f(x, u)$  we obtain multiplicity results of solutions with prescribed nodal properties for certain values of the parameter  $\lambda > 0$ . Our work is related to some classical results by P.H. Rabinowitz (P.H. Rabinowitz: A note on pairs of solutions of a nonlinear Sturm-Liouville problem, *Manuscripta Math.* 11 (1974), 273-282) and some recent contributions (A. Boscaggin and F. Zanolin: Positive periodic solutions of second order nonlinear equations with indefinite weight: Multiplicity results and complex dynamics, *J. Differential Equation* 252 (2012), 2922-2950).

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## Special Session 57: Nonlinear and Dispersive Partial Differential Equations

Netra Khanal, University of Tampa, USA  
Juan-Ming Yuan, Providence University, Taiwan

### Regularity of $\infty$ for elliptic equations with measurable coefficients and its consequences

**Ugur Abdulla**

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This lecture introduces a notion of regularity (or irregularity) of the point at infinity ( $\infty$ ) for the unbounded open set  $\Omega \in R^N$  concerning second order uniformly elliptic equations with bounded and measurable coefficients, according as whether the A-harmonic measure of  $\infty$  is zero (or positive). A necessary and sufficient condition for the existence of a unique bounded solution to the Dirichlet problem in an arbitrary open set of  $R^N$ ,  $N \geq 3$  is established in terms of the Wiener test for the regularity of  $\infty$ . It coincides with the Wiener test for the regularity of  $\infty$  in the case of Laplace equation. From the topological point of view, the Wiener test at  $\infty$  presents thinness criteria of sets near  $\infty$  in fine topology. Precisely, the open set is a deleted neighborhood of  $\infty$  in fine topology if and only if  $\infty$  is irregular.

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### Dispersive blow-up phenomena

**Jerry Bona**

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The lecture will discuss singularity formation due to dispersion. Interestingly, such singularities persist even when nonlinearity is present. Possible applications to rogue-wave formation will also be touched upon.

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### Long-wave limit of periodic solutions of nonlinear wave equations

**Hongqiu Chen**

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In this lecture, we consider some non-linear, dispersive equations posed on the entire real line and the interest here is the relationship between two different types of solutions. One type is in Sobolev spaces defined on  $\mathbb{R}$  and the other is periodic, say of period  $2l$ . For the same equation, if the two types of solutions are close in some sense at some time as  $l \rightarrow \infty$ , say initially, then we show that as  $l \rightarrow \infty$ ,

the periodic solutions converge to the solution in the Sobolev space.

Similarly, solitary-wave solution are often approximated by using periodic data. Our theory is a nice justification.

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### Exploiting bifurcations in waveguide arrays for light detectors

**Natanael Karjanto**

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**H. Susanto**

An array of finite number waveguides driven laterally by injecting light at the outer waveguides is considered. The array is modelled by a discrete nonlinear Schrödinger equation. It has been shown that when the injected light is in the proximity of a bifurcation point, such system can be sensitive to small disturbances, making it possible to act as a light detector. Here, the optimum intensity of the injected light is discussed and an analytical approximation is presented.

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### Complex-valued Burgers and KdV Burgers equations

**Netra Khanal**

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**Juan-Ming Yuan, Jiahong Wu, Bing-Yu Zhang**

Spatially periodic complex-valued solutions of the Burgers equation and KdV-Burgers equations will be discussed. It is shown that for any sufficiently large time  $T$ , there exists an explicit initial datum such that its corresponding solution of the Burgers equation blows up at finite time  $T$ .

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### NLS with quantum potential, the related reaction-diffusion systems and Kaup-Broer system

**Jyh-hao Lee**

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This talk will be presented by two parts. We will review some old results and plan to present some new parts.

At first, we will review some old results about 2 by 2 AKNS system with linear and quadratic spectral parameter and the related evolution equations and the inverse scattering method under the set-up of Beals-Coifman.

Then we will make a brief report on the nonlinear Schrödinger equations with quantum potential, derivative nonlinear Schrödinger equations and the related reaction-diffusion systems. We will discuss some possible applications.

$$i \frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + \frac{\Lambda}{4} |\psi|^2 \psi = s \frac{1}{|\psi|} \frac{\partial^2 |\psi|}{\partial x^2} \psi.$$

A novel integrable version of the NLS equation with quantum potential has been termed the resonant nonlinear Schrödinger equation (RNLS). It can be regarded as a third version of the NLS, intermediate between the defocusing and focusing cases. The critical value  $s = 1$  separates two distinct regions of behaviour. Thus, for  $s < 1$  the model is reducible to the conventional NLS, (focusing for  $\Lambda > 0$  and defocusing for  $\Lambda < 0$ ). However, under the condition, for  $s > 1$ , it is reducible to a reaction-diffusion system, which can be transformed into Kaup-Broer system. In this case, the model exhibits novel solitonic phenomena. The RNLS can be interpreted as an NLS-type equation with an additional quantum potential  $U_Q = |\psi|_{xx}/|\psi|$ . Recently it was shown that RNLS naturally appears in a reduced equation in the plasma physics. Via a Hirota bilinear representation of the Reaction-Diffusion system, here some exact solutions are obtained by this bilinear method. We will also consider a non-Madelung type hydrodynamic representation for resonant nonlinear Schrödinger type equations. New Broer-Kaup type systems of hydrodynamic equations are also derived from the derivative reaction-diffusion systems arising in  $SL(2, \mathbb{R})$  Kaup-Newell hierarchy, represented in the non-Madelung hydrodynamic form. The relation with Kaup-Broer system will be mentioned here. (This is a joint work with O.K.Pashaev.)

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### Global regularity results for the 2D MHD equations with horizontal dissipation and horizontal magnetic diffusion

**Dipendra Regmi**

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We study the global regularity issue concerning the 2D incompressible magnetohydrodynamic (MHD) equations with horizontal dissipation and horizontal magnetic diffusion. We establish a global bound for the  $L^{2r}$ -norm of the horizontal component of the velocity and of the magnetic field for any  $1 \leq r < \infty$  and the bound grows no faster than the order of  $\sqrt{r \log r}$  as  $r$  increases. A global  $L^q$ -bound,  $1 < q \leq 3$ , for pressure is also obtained. In addition, we establish a conditional global regularity in terms of the  $L_t^2 L_x^\infty$ -norm of the horizontal component and the global regularity of a slightly regularized version of the aforementioned MHD equations. This is a joint work with C. Cao and J. Wu

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### An initial boundary value problem for one-dimensional shallow water magnetohydrodynamics in the solar tachocline

**Ming-Cheng Shiue**

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mingcheng.shiue@gmail.com

In this talk we investigate the shallow water magnetohydrodynamics equations in space dimension one with Dirichlet boundary conditions only for the velocity. This model has been proposed to study the phenomena in the solar tachocline. In this talk, the local well-posedness in time of the model is presented by constructing the approximate solutions and showing the strong convergence of the approximate solutions.

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### Recent results on the Serrin-type regularity criteria for Navier-Stokes and related equations

**Kazuo Yamazaki**

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We study the Serrin-type regularity criteria for Navier-Stokes and related equations such as magneto-hydrodynamics and others. Recently a significant amount of effort has been devoted to reduce this criteria from a classical  $L_p$  bound on the gradient of the solution to only its components or partial derivatives. Regularity criteria in the mixed  $L_p$ -space will also be discussed.

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**Fifth-order complex Korteweg-de Vries type equations****Juan-Ming Yuan**

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**Netra Khanal, Jiahong Wu**

We discuss spatially periodic complex-valued solutions of the fifth-order Korteweg-de Vries (KdV) type equations. The aim is at several fundamental issues including the existence, uniqueness and finite-time blowup problems. Special attention is paid to the Kawahara equation, a fifth-order KdV type equation. When a Burgers dissipation is attached to the Kawahara equation, we establish the existence and uniqueness of the Fourier series solution with the Fourier modes decaying algebraically in terms of the wave numbers. We also examine a special series solution to the Kawahara equation and prove the convergence and global regularity of such solutions associated with a single mode initial data. In addition, finite time blow-up results are discussed for the special series solution of the Kawahara equation.

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## Special Session 58: Variational Analysis and Equilibrium Problems

**Patrizia Daniele**, Department of Mathematics and Computer Science - University of Catania, Italy

The aim of the session is to present state-of-the-art and current research directions in variational analysis and in applications to equilibrium problems. Variational analysis encompasses a large area of modern mathematics, including the classical calculus of variations, the theories of perturbation, approximation, subgradient, set convergence, variational inequalities and partial differential equations. Such theories have been applied to various models such as spatial markets, auctions, traffic networks, oligopolistic markets, supernetworks, financial frameworks, electric power supply chains, the Internet, ... This special session will provide a forum for researchers to present current results.

### A variational formulation for dynamic market equilibrium problems with excesses

**Annamaria Barbagallo**

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The aim of the talk is to present the evolutionary variational formulation for market equilibrium problems with production and demand excesses. The equilibrium conditions are expressed according to Cournot-Nash principle and in terms of Lagrange multipliers, and, by means of a recent infinite-dimensional duality theory, the equivalence of both the conditions with an evolutionary variational inequality is proved. The variational formulation allows us to obtain existence and regularity results for the equilibrium solution. Moreover, we introduce a discretization procedure for computing dynamic equilibrium solutions and we provide a numerical example.

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### Generalized Nash Games and Evolutionary Variational Inequalities

**Monica Cojocar**

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**Veronica Gheorghide**

In this work we show how generalized Nash (GN) games with shared constraints can be related to evolutionary variational and quasivariational inequalities. This association can be used for two things: the first is to compute solutions of such class of GN games, the second is to study long term behaviour of a GN game solution set from a dynamical systems perspective, such as stability and reachability. We outline our approach and possible future research avenues.

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### Supply chain networks with corporate financial risks and trade credits under economic uncertainty

**Jose Cruz**

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**Zugang Liu**

The focus of this paper is to provide an analytical framework which can be used to investigate how financial risks affect the values of interconnected supply chain firms from a network perspective, and how financial risks affect the supply chain firms' profitability as well as the cash and credit transactions. In particular, we develop a variational inequality equilibrium model in conjunction with capital asset pricing model (CAPM) and the net present value (NPV) to determine the optimal supply chain prices, profits, and implicit equity values of supply chain firms under financial risks and economic uncertainty. We illustrate the analytical framework with numerical examples which yield interesting managerial implications to the following questions: 1) How do financial risks and economic uncertainty affect the values of interconnected supply chain firms from a network perspective? 2) How do financial risks and economic uncertainty affect the supply chain firms' profitability as well as the cash and credit transactions? 3) How does the effect of financial risks change under different competition scenarios?

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### Electric and economic supply chains: a variational formulation

**Patrizia Daniele**

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First, we present the static electric power supply chain network model with known demands consisting of power generators, power suppliers, power transmitters, and the ultimate consumers. The behavior of the various economic decision-makers associated with the nodes of the network is made explicit. We also derive the governing finite-dimensional variational inequality formulation. Then, we present the time-dependent economic supply chain network



model with three tiers of decision-makers (manufacturers, retailers, and consumers) in the case when excesses of production and excesses of demand of the commodity are present. For such a framework we furnish, using the infinite dimensional duality theory, the equilibrium conditions for the representatives of each tier of the supernetwork, and the time-dependent variational formulation governing the complete supply chain supernetwork.

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### The infinite dimensional Lagrange multiplier rule for convex optimization problems

**Maria Bernadette Donato**

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In this talk an infinite dimensional generalized Lagrange multipliers rule for convex optimization problems is presented and necessary and sufficient optimality conditions are given in order to guarantee the strong duality. Furthermore, an application is presented, in particular the existence of Lagrange multipliers associated to the bi-obstacle problem is obtained.

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### A survey on duality theory in elastic-plastic torsion problem

**Sofia Giuffre**

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**A.Maugeri**

The talk deals with the problem of existence of Lagrange multipliers associated to the elastic-plastic torsion problem. According to R.Von Mises, the elastic-plastic torsion problem of a cylindrical bar with cross section  $\Omega$  is to find a function  $\psi(x)$  which vanishes on the boundary  $\partial\Omega$  and, together with its first derivatives, is continuous on  $\Omega$ ; nowhere on  $\Omega$  the gradient of  $\psi$  must have an absolute value less than or equal to a given positive constant  $\tau$ ; whenever in  $\Omega$  the strict inequality holds, the function  $\psi$  must satisfy the differential equation  $\Delta\psi = -2\mu\theta$ , where the positive constants  $\mu$  and  $\theta$  denote the shearing modulus and the angle of twist per unit length respectively. Aim of the talk is to provide a survey on very recent results on the existence of a Lagrange multiplier associated to the elastic-plastic torsion problem as a Radon measure and as a  $L^\infty$  function. These results are obtained by means of the duality theory.

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### Evolution solutions of network problems: a hybrid dynamical system approach

**Scott Greenhalgh**

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**Monica Cojocaru**

In this talk a new approach for modeling the evolution of networks in states other than equilibrium is presented. Currently, network problems which can be formulated as an evolutionary variational inequality can be modeled in non-equilibrium states, called evolution solutions via a parameterized projected dynamical system. Through the combination of discrete and continuous dynamical systems, namely hybrid dynamical systems, an alternative approach for the modeling of such evolution solutions of networks is constructed. We illustrate examples depicting the advantages/differences of the two approaches and outline stability criteria.

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### A generalized quasi-variational inequality for an economic equilibrium problem

**Monica Milasi**

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In this talk a competitive economic equilibrium model integrated with exchange, consumption and production is presented. In this model the utility functions are assumed concave, proper and uppersemi-continuous. The problem is characterized by means of a suitable generalized quasi-variational inequality. The aim is to give an existence result of the equilibrium by using the variational approach.

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### Two-dimensional approximation of three dimensional piezoelectric membrane shells using gamma convergence

**Sabu Nicholas**

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**J.Raja**

In this paper we consider piezoelectric membrane shells of very small thickness subjected only to mechanical forces and we show that functions which minimize the energy associated with the three dimensional models converge to the function which minimizes the energy of the two dimensional model of elastic membrane shells.

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### Local structure in a class of variational problems

**Stephen Robinson**

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There is a considerable amount of local structure in variational problems that can be posed as finding zeros of  $C^1$  functions restricted to the graphs of maximal monotone operators. Some of this structure exists when the operators act on Hilbert spaces, but considerably more is known when the underlying spaces are finite-dimensional and the operators are the normal-cone operators of polyhedral convex sets. This latter class includes a vast array of useful models including complementarity problems, more general variational inequalities, and reformulations of variational conditions on sets defined by nonlinear constraints. This local structure is useful, for example, in constructing fast solution algorithms or in transforming the problems into versions more suitable for solution. We will give a general explanation of the structural analysis and will relate it to topics such as parametric analysis and computational solution.

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### A quasi-variational approach to the joint implementation of environmental projects

**Laura Scrimali**

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The 1997 Kyoto Protocol prescribes that some industrialized, labeled as Annex I Parties, must reduce their greenhouse gas emissions at least 5 per cent below the 1990 levels for the 2008-2012 period. In this paper we focus on the so-called joint implementation (JI), a mechanism that allows Parties, with emission reduction or limitation commitments, to collect rewards in the form of emission reduction units (ERUs) from an emission-reduction or emission removal project in another Annex I Party, where the abatement costs are lower. We develop a time-dependent pollution control model in which different countries aim to determine the optimal investment allocation in environmental projects and the tolerable pollutant emissions, so as to maximize their welfare. We provide the equilibrium conditions governing the model and derive the characterization in terms of an infinite dimensional quasi-variational inequality problem. The existence of solutions is then investigated.

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### A variational inequality formulation of economic network equilibrium models with nonlinear constraints

**Fuminori Toyasaki**

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**Daniele Patrizia, Wakolbinger Tina**

Variational inequality theory facilitates the formulation of equilibrium problems in economic networks. Examples of successful applications include models of supply chains, financial networks, transportation networks, and electricity networks. Previous economic network equilibrium models that were formulated as variational inequalities only included linear constraints. In this paper, we first highlight with an application from the context of reverse logistics why the introduction of nonlinear constraints is beneficial. We then show mathematical conditions that ensure that the models have unique solutions and we suggest algorithms that can be applied to solve the models.

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### A brief overview on a variational approach for the study of an economic equilibrium problem

**Carmela Vitanza**

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Competitive economic equilibrium problems are presented. These problems are reformulated by means of suitable quasi-variational inequalities. Thanks this characterization, a variational method is presented in order to derive existence and regularity results of equilibria.

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### The influence of technical, market and legislative factors on e-waste flows

**Tina Wakolbinger**

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**Fuminori Toyasaki, Anna Nagurney, Thomas Nowak**

The establishment of efficient reverse logistics for end-of-life products, especially, electrical and electronic equipment waste (WEEE), has been recognized as a crucial building block for sustainable economies. One of the major concerns of many WEEE take-back schemes is whether adequate amounts of WEEE flow into the designed recycling systems. We formulate the e-waste network flow model as a variational inequality problem and analyze how technical, market, and legislative factors influence the total amount of e-waste that is collected, recycled and (legally and illegally) disposed of, the prices that sources of waste, processors and demand markets face, and the profits of collectors and processors.

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## Special Session 61: PDE Models for Biological Pattern Formation

**Thomas Hillen**, University of Alberta, Canada  
**Michael Winkler**, University of Paderborn, Germany

The aim of this session is to discuss various state of the art approaches for the modeling of biological pattern formation, including chemotaxis, cell movement, tumor growth, and spatial ecology. The emphasis lies on the analysis and simulation of PDE-based models.

### Mutations, competition and progression in cancer dynamics

**Marcello Delitala**  
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**Tommaso Lorenzi**

In this talk we present a mathematical model for the development of cancer at the cellular scale. Attention is focused on progression and heterogeneity aspects of carcinogenesis under an evolutionary perspective.

The model relies on a continuous structured formalism and consists of a set of integro-differential equations describing the dynamics of tumor cells under the effects of mutation and competition phenomena, cell proliferation as well as the action of therapeutic agents.

Asymptotic and computational analysis are developed with an exploratory aim and are devoted to study the role that the phenomena under consideration play in cancer evolution. The obtained results suggest that cancer progression selects for highly proliferative clones and point out how some therapeutic agents might act as an additional selective pressure leading to the selection for the most fitting, and then most resistant, cancer clones.

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### Remarks on the global existence in super-critical cases for quasilinear degenerate Keller-Segel systems

**Sachiko Ishida**  
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We consider the global existence in super-critical cases for quasilinear Keller-Segel systems with small initial data. In the non-degenerate case Winkler (2010) established the global existence and boundedness of solutions under the smallness of  $\nabla v_0$ . Recently, Ishida-Yokota (2012) proved the global existence under the smallness of  $\Delta v_0$ . Note that there is a “gap” in differentiability imposed by Winkler and Ishida-Yokota. In this talk we try to fill this gap.

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### Coupled chemotaxis-fluid models

**Alexander Lorz**  
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We consider coupled chemotaxis-fluid models aimed to describe swimming bacteria, which show bio-convective flow patterns on length scales much larger than the bacteria size. This behaviour can be modelled by a system consisting of chemotaxis equations coupled with viscous incompressible fluid equations through transport and external forcing. The global-in-time existence of solutions to the Cauchy problem in two and three space dimensions is established. Precisely, when the fluid motion is described by Stokes equations, we derive free energy functionals to prove global-in-time existence of weak solutions for cell density with finite mass, first-order spatial moment and entropy provided that the potential is weak or the substrate concentration is small. Moreover, with nonlinear diffusion for the bacteria, we give global-in-time existence of weak solutions in two space dimensions.

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### Long time behaviour in some chemotaxis models arising in angiogenesis

**Cristian Morales-Rodrigo**  
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**T. Cieslak, M. Delgado, A. Suarez, J.I. Tello**

The aim of this talk is to study by different methods the long time behaviour of some models arising in angiogenesis. We will show linear as well as nonlinear stability of each semi-trivial steady-states for these models.

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### Global existence of solutions to a parabolic-parabolic system for chemotaxis with logistic source in the higher-dimensional domain

**Etsushi Nakaguchi**

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We study the global existence of solutions to a parabolic-parabolic system for chemotaxis proposed by M.Mimura and T.Tsujikawa [Physica A 230 (1996)] in a three- or higher-dimensional domain. We will show the global existence of solutions under certain restriction on the degradation order of logistic source or on the smallness of initial data. We will also discuss the dynamical system and attractors for the system.

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### Finite-time blowup and global-in-time unbounded solutions to a parabolic-parabolic quasilinear Keller-Segel system

**Christian Stinner**

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**Tomasz Cieslak**

We study radially symmetric solutions to a quasilinear parabolic-parabolic Keller-Segel system in a ball  $B_R \subset \mathbb{R}^n$  with  $n \geq 3$ . Critical nonlinearities had been identified such that in the subcritical case the solution is global in time and bounded while in the supercritical case the solution blows up, but it was not known whether the blowup takes place in finite or infinite time. Assuming a non-decay for the nonlinear chemical sensitivity function, we prove that finite-time blowup occurs for any mass in the whole supercritical case. Moreover, we provide examples showing that in presence of a suitable decay of the sensitivity function some solutions blow up in infinite time. This shows that a non-decay assumption on the sensitivity function is necessary to have finite-time blowup in the whole supercritical case while for decaying sensitivity functions both finite-time and infinite-time blowup can occur. Our proof uses a detailed analysis of the Liapunov functional and generalizes a method which was introduced recently by M. Winkler.

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### Global dynamics in a multi-dimensional chemotaxis-haptotaxis model

**Youshan Tao**

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**Michael Winkler**

This talk addresses a coupled chemotaxis-haptotaxis model of cancer invasion. We discuss the global existence and boundedness of solutions to this model. We

moreover analyze stability and attractivity properties of the nontrivial homogeneous equilibrium state and establish a quantitative result relating the domain of this steady state to the size of the parameter of logistic dampening of cell growth. This talk is based on a joint work with Michael Winkler.

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### Competing effects of attraction vs repulsion in chemotaxis

**Zhian Wang**

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Most of past studies on chemotaxis models deal with attraction and repulsion separately. But in most biological processes (or experiments), the repulsive process usually follows the attractive process for balance in order to accomplish some biological objects. Hence an attraction and repulsion chemotaxis model will be more realistic than a sole attraction or repulsion chemotaxis model in this scenario. In this talk, we shall present the first mathematical results on an attraction-repulsion chemotaxis model and show the interplay of these two opposed biological processes. Some numerical simulations will be shown and various open questions will be presented.

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### Reaching a maximal density threshold in some models of chemotaxis.

**Dariusz Wrzosek**

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We study a quasilinear parabolic system corresponding to models of chemo-taxis in which; 1) there is an impassable threshold for the density of aggregate cells. 2) the diffusion of cells becomes degenerate or singular (fast or super diffusion) when the density approaches the threshold. It is proved that for some range of parameters describing the relation between the diffusive and the chemotactic component of the cell flux there are global-in-time classical solutions which in some cases are separated from the threshold uniformly in time. In the case of fast diffusion existence and uniqueness of weak solutions are proved. For the case of non-degenerate diffusion the existence of solutions which attain the threshold in finite time is proved for the elliptic-parabolic version of the model. The results are contained in the following papers: 1. D. Wrzosek, Model of chemotaxis with threshold density and singular diffusion, *Nonlinear Analysis* 7 (2010) 338-349 .2. Zhi-An Wang, M. Winkler, D. Wrzosek, Singularity formation in chemotaxis systems with volume-filling effect, *Nonlinearity* 24, 3279-3297 (2011) 3. Zhi-An Wang, M. Winkler, D. Wrzosek, Global regularity

vs. infinite-time singularity formation in a chemotaxis model with volume filling effect and degenerate diffusion.

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**On the solvability of generalized degenerate chemotaxis models**

**Tomomi Yokota**

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**Sachiko Ishida**

We consider the solvability of generalized degenerate chemotaxis models. In particular, the solvability for the case of the porous medium-type diffusion has been studied by Sugiyama-Kunii (2006) and Ishida-Yokota (2012). In this talk we try to discuss the solvability for the case of “more generalized” degenerate diffusion by using the maximal regularity.

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## Special Session 62: PDEs and Dynamical Systems, and Their Applications

**Soo Kyung Joo**, Old Dominion University, USA  
**Jinhae Park**, Chungnam National University, South Korea  
**Tuoc Van Phan**, University of Tennessee, USA

Many real life problems have been understood by theories of partial differential equations and dynamical systems. On the other hand, modeling of physical problems give rise to many challenging mathematical problems. In this session, we will learn many recent developments of static and dynamic problems arising from the mechanics, physics, and materials science from the view points of both modeling and mathematical theory.

### Global stability of the normal state of superconductors under the effect of strong electric current

**Yaniv Almog**  
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**Bernard Helffer**

Consider a superconducting wire whose temperature is lower than the critical one. When one flows a sufficiently strong current through the wire, it is well known from experimental observation that the wire becomes resistive, behaving like a normal metal. We prove that the time-dependent Ginzburg-Landau model anticipates this behaviour. We first prove that, for sufficiently strong currents, the semi-group associated with the model, becomes a contraction semi-group. Then, we obtain an upper bound for the critical current where the semi-group becomes stable. We relate this current to the resolvent of the linearized elliptic operator.

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### Analysis of liquid crystals with defects of degree one-half

**Patricia Bauman**  
 Purdue University, USA  
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**Jinhae Park, Daniel Phillips**

We investigate the structure of nematic liquid crystal thin films described by the Landau-de Gennes tensor-valued order parameter with Dirichlet boundary conditions of nonzero degree. We prove that as the elasticity constant goes to zero a limiting uniaxial texture forms with disclination lines corresponding to a finite number of defects, all of degree  $\frac{1}{2}$  or all of degree  $-\frac{1}{2}$

→ ∞ ◊ ∞ ←

### The local structure of the set of steady-states to the 2D incompressible Euler equations of hydrodynamics

**Antoine Choffrut**  
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Steady-states are of great interest in the study of the dynamics of 2D incompressible Euler flows. In this talk I will consider the local structure of the set of steady-states under certain non-degeneracy assumptions. The space of vorticity fields is formally foliated into the coadjoint orbits of the group  $\mathcal{D}_{\text{area}}(\Omega)$  of area-preserving diffeomorphisms of the region  $\Omega$  filled with the ideal fluid. The Euler evolution is a Hamiltonian system on each orbit, with the kinetic energy as Hamiltonian  $H$ . In particular, critical points of  $H$  restricted to the orbits correspond to steady-states. Hence, these should be, locally and under suitable assumptions, in one-to-one correspondence with the coadjoint orbits. In finite dimensions, that is, when one considers a “genuine” Lie group instead of  $\mathcal{D}_{\text{area}}(\Omega)$ , this can be established by a routine application of the classical Implicit Function Theorem. In the case of  $\mathcal{D}_{\text{area}}(\Omega)$ , it seems difficult to give the orbits a satisfactory structure as submanifolds in the space of vorticity fields. Nevertheless, the problem can be approached in an indirect way and an analogue of the finite-dimensional result can be established via the Nash-Moser Inverse Function Theorem. This is joint work with V. Šverák.

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### Recurrences, limit sets and shadowing property in dynamical systems

**Hahng-yun Chu**  
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**Jaeyoo Choy, Se-Hyun Ku, Jong-Suh Park**

We focus on certain dynamic phenomena as recurrences, limit sets, shadowing property and their applications. In this talk, firstly, we discuss the envelope of flows on compact spaces which is a generalized notion of the functional envelope of maps in discrete

case. Next, we deal with the concepts of chain recurrences, attractors and shadowing property in several categories.

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### Asymptotics of Maxwell-Chern-Simons vortices on the unit disc

**Jongmin Han**

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**Namkwon Kim**

In this talk, we are concerned with the asymptotic behavior of the relativistic Maxwell-Chern-Simons(MCS) vortices on the unit disc. We establish the existence of a radial solution of the static MCS equations which is a minimizer of the MCS functional in the class of radial functions. We also derive the asymptotic limit of the radial solution. As a consequence, we obtain a set of elliptic linear equations which is a generalization of the London equation for the Ginzburg-Landau model.

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### A Poincaré–Dulac normal form for Navier-Stokes equations

**Luan Hoang**

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**Ciprian Foias, Jean-Claude Saut**

We study the incompressible Navier-Stokes equations with potential body forces on the three-dimensional torus. We show that the normalization introduced in the paper [Ann. Inst. H. Poincaré Anal. Non Linéaire, 4(1):1–47, 1987] produces a Poincaré-Dulac normal form which is obtained by an explicit change of variable. This change is the formal power series expansion of the inverse of the normalization map. Each homogeneous term of a finite degree in the series is proved to be well-defined in appropriate Sobolev spaces and is estimated recursively by using a family of homogeneous gauges which is suitable for estimating homogeneous polynomials in infinite dimensional spaces.

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### On the singular limit of thin film equations with small Born repulsion force

**Huiqiang Jiang**

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**Xinfu Chen**

We consider dewetting thin film model with both van der Waals and Born repulsion force. We show

that as the Born repulsion force tends to zero, the energy minimizers, passing to a subsequence if necessary, converge to a Dirac mass located on the boundary. The blow up profile of the energy minimizers is identified and the location of spike will be discussed.

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### Local estimates of weak solutions for steady-state non-Newtonian fluid flows

**Kyungkeun Kang**

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**Bum Ja Jin, Jihoon Lee**

We consider steady-state non-Newtonian Stokes and Navier-Stokes equations confined in bounded domains in dimension three. We obtain various estimates of higher derivatives of weak solutions near boundary as well as in the interior, when no-slip boundary conditions are given.

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### Lyapunov functions, attractors and shadowing property in set-valued dynamics

**Se-hyun Ku**

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**Hahng-Yun Chu, Jong-Suh Park**

In this talk, we are concerned with set-valued dynamical systems. In the systems, we obtain some results about Lyapunov functions and attractors, and then we also treat the shadowing property. Moreover, we deal with the Conley's theorem, which is about chain recurrences, on non-compact spaces.

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### On some coupled system with the Navier-Stokes equations

**Jihoon Lee**

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In this talk, we consider some mathematical models coupled with the Navier-Stokes equations. At first, we consider the magnetohydrodynamics equations in 2 and 3 dimensions. We consider the existence of classical solutions and the regularity criterion. Secondly, we consider the Navier-Stokes-Vlasov-Fokker-Planck equations derived from the combustion theory. Thirdly, we consider the Navier-Stokes-Keller-Segel equations which governs the motion of the swimming bacteria. Lastly, we consider some system which is a simplified model of the Ericksen-Leslie system.

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### Global existence and nonrelativistic limit for the Vlasov-Maxwell-Chern-Simons system

**Ho Lee**

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**Hyungjin Huh**

A system of partial differential equations is studied. Considering interaction between the Maxwell-Chern-Simons electrodynamics and the Vlasov matter, the Vlasov-Maxwell-Chern-Simons system is introduced. Global existence of classical solutions is proved, and then its nonrelativistic limit is investigated. When the speed of light tends to infinity, it is proved that solutions of the Vlasov-Maxwell-Chern-Simons system converge to solutions of the Vlasov-Yukawa system.

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### Planar bistable liquid crystal device and dynamics of switching

**Chong Luo**

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**Apala Majumdar, Radek Erban**

A planar bistable liquid crystal device, reported in Tsakonas et al., is modeled within the Landau-de Gennes theory for nematic liquid crystals. This planar device consists of an array of square micron-sized wells. We obtain six different classes of equilibrium profiles and these profiles are classified as *diagonal* or *rotated* solutions. In the strong anchoring case, we propose a Dirichlet boundary condition that mimics the experimentally imposed *tangent boundary conditions*. In the weak anchoring case, we present a suitable surface energy and study the multiplicity of solutions as a function of the anchoring strength. We find that diagonal solutions exist for all values of the anchoring strength  $W \geq 0$  while rotated solutions only exist for  $W \geq W_c > 0$ , where  $W_c$  is a critical anchoring strength that has been computed numerically. We propose a dynamic model for the switching mechanisms based on only dielectric effects. For sufficiently strong external electric fields, we numerically demonstrate diagonal to rotated and rotated to diagonal switching by allowing for variable anchoring strength across the domain boundary.

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$W^{1,p}$  bound and compactness at  $p^* = \frac{np}{n-p}, 1 \leq p < n$

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**Abstract.** In respect of the Rellich-Kondrachov compact embedding theorem, a mathematically simple reason is given as to why the well known counter examples in existence for many years and showing non-compactness at the critical value  $p^* = \frac{np}{n-p}, 1 \leq p < n$ , do not actually survive (become vacuous) for computations in the  $L^{p^*}$ -norm. The Gagliardo-Nirenberg-Sobolev condition (GNS)  $1 - \frac{n}{p} + \frac{n}{p^*} = 0$  is clearly seen to impose a restriction on the sequence index in the definition of the sequence for such counter examples, in so far as the sequence index disappears if the  $L^{p^*}$ -norm is computed without using the gradient, where as, the sequence index need not disappear while using GNS if the  $p^*$ -norm estimation makes use of the gradient of the sequence with the corresponding 'implied value'. We thus obtain a different and natural value for the  $L^{p^*}$ -norm estimate, showing ambiguity in  $L^{p^*}$ -norm computation if the two 'restricted' and 'unrestricted' computations are compared; note that it is indeed compulsory to use the gradient of the sequence and its 'implied value' (in integration) from the pointwise definition of the sequence to check for the  $W^{1,p}$  bound, so computation using the gradient has to be considered 'canonical'. To achieve this, we make use of the strong norm bounds for the Hardy-Littlewood maximal function along with the well known 'potential formula' which expresses a  $C^1$  function of compact support in terms of its gradient. Thus, and very surprisingly, this leaves open the question of compactness at  $p^*$ .

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### Regularity of solutions to the linearized Monge-Ampère equation

**Truyen Nguyen**

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**Cristian Gutierrez**

The linearized Monge-Ampère equation is a second-order degenerate elliptic equation which appears in several applications such as affine differential geometry, complex geometry and fluid mechanics. We study smoothness properties of solutions to the equation. By employing a perturbation argument, we establish interior estimates for the first and second derivatives of its solutions.

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### An analysis of textures in smectic-C films with multiple defects

**Dan Phillips**

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**Sean Colbert-Kelly**

We analyze a model for the elastic energy of planar c-director patterns in a smectic film. Because of boundary conditions and polar fields topological defects form in these patterns. We use a Ginzburg Landau model that allows the director field to have variable length and to vanish at the defect cores. We prove that if the model's G-L parameter is small then low energy states develop degree  $\pm$  one defects that tend to a minimal energy configuration with a limiting far field texture. Our main contribution is that we are able to treat the case of unequal splay and bend elasticity constants. Earlier analytic work for the G-L functional had been limited to the equal constant case.

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### Mathematical analysis and diffusive interface modeling of membrane movements

**Rolf Ryham**

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The role of a biological membrane is to act as a barrier between ionic solutions. One way life controls ionic solution is through ion channels. A second more drastic way is by introducing a hole in the membrane itself. For example, in hemolysis, the osmotic swelling and rupture of a red-blood cell, a single hole forms in the membrane leading to the leak out of the contents of the cell. Similarly, in exocytosis a hole is formed by joining to membrane bilayers. These processes are mathematically challenging to study because they involve physical forces on multiple scales and predicting the time course is more consequential than the equilibrium end states. This talk will show how such complicated fluid mechanical problems yield to quantitative modeling and simulation when using the diffusive interface and energetic approach.

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### Propagation of the advantageous genes in a population with multiple alleles at a locus

**Linlin Su**

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**Roger Lui**

We extended the classical result of Fisher (1937) from the case of two alleles to multiple alleles. Consider a

population living in a homogeneous one-dimensional infinite habitat. Individuals in this population carry a gene that occurs in  $k$  forms, called alleles. Under the joint action of selection and migration, the allele frequencies,  $p_i, i = 1, \dots, k$ , satisfy a system of differential equations. We first showed that under the conditions  $A_1 A_1$  is the most fit among the homozygotes, the system is cooperative, the state that only allele  $A_1$  is present in the population is stable, and the state that only allele  $A_1$  is absent in the population is unstable, there exists a positive constant,  $c^*$ , such that allele  $A_1$  propagates asymptotically with speed  $c^*$  as  $t \rightarrow \infty$ . We then showed that traveling wave solutions connecting these two states exist for  $|c| \geq c^*$ . Finally, we showed that under certain additional conditions,  $c^*$  has an explicit formula. These results allow us to estimate how fast an advantageous gene propagates in a population under selection and migration forces as  $t \rightarrow \infty$  and help to predict the genetic makeup of a population in the long run. This is a joint work with Roger Lui.

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### Dynamic transitions and pattern formations for Cahn-Hilliard model with long-range repulsive interactions

**Shouhong Wang**

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**Honghu Liu, Taylan Sengul, Pingwen Zhang**

In this talk, I shall address the order-disorder phase transition and pattern formation for systems with long-range repulsive interactions. The main focus is on the Cahn-Hilliard model with a nonlocal term in the corresponding energy functional, representing the long-range repulsive interaction. First, we show that as soon as the linear problem loses stability, the system always undergoes a dynamic transition to one of the three types, forming different patterns/structures. The types of transition are then dictated by a nondimensional parameter, measuring the interactions between the long-range repulsive term and the quadratic and cubic nonlinearities in the model. The derived explicit form of this parameter offers precise information for the phase diagrams. Second, we obtain a novel and explicit pattern selection mechanism associated with the competition between the long-range repulsive interaction and the short-range attractive interactions. In particular, the hexagonal pattern is unique to the long-range interaction, and is associated with novel two-dimensional reduced transition equations on the center manifold generated by the unstable modes, consisting of (degenerate) quadratic terms and non-degenerate cubic terms. Finally, explicit information on the metastability and basin of attraction of different disordered/ordered states and patterns are derived as well.

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**Liouville Theorem for higher order elliptic systems****Xiaodong Yan**University of Connecticut, USA  
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We study the following higher order elliptic system in  $R^n$

$$\begin{aligned}\Delta^m u &= v^p \\ \Delta^m v &= u^q\end{aligned}$$

We prove that for  $1/(p+1) + 1/(q+1) > 1 - 2m/n$ , there are no positive radial solutions to the system. If  $p > 1, q > 1$  and  $\max(2m(p+1)/(pq-1), 2m(q+1)/(pq-1)) \geq n - 2m$ , there are no positive solutions to the system. The proof of the radial case uses Rellich identity and the proof in the general case uses estimates on spherical average of solutions.

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**Crystalline surface diffusion****Nung Kwan Aaron Yip**Purdue University, USA  
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We study the surface diffusion of the boundary of a planar polygonal domain in the setting of a crystalline surface energy. It is a motion law mimicking the evolution of a surface in which the surface normal velocity equals the Laplacian of mean curvature. We characterize the velocity in the framework of canonical restriction and study its properties. In particular, the phenomena of facet splitting is analyzed.

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**Kinetic theory and simulations of active nematic suspensions****Ruhai Zhou**Old Dominion University, USA  
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A kinetic model is proposed for dilute or semidilute active nematic suspensions, which generalizes the Smoluchowski equation for rod-like or plate-like nematic polymers. Numerical simulations show the longwave instability of the local concentration field and the long time phenomena of the band structure that forms and breaks periodically.

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## Special Session 63: Infinite Dimensional Dynamics and Applications

**James C. Robinson**, University of Warwick, UK  
**Yuncheng You**, University of South Florida, USA

Many partial differential equations of mathematical physics, particular those of fluid dynamics, can be recast as infinite-dimensional dynamical systems. This allows one to use the qualitative ideas from dynamical systems to try to understand the behaviour of these systems. This may be the time-asymptotic behaviour, studied using the theory of attractors or inertial manifolds, the existence of invariant measures, etc. There is also considerable overlap with the more classical approach to qualitative properties of PDEs - at its most basic, the generation of any dynamical system relies on existence and uniqueness results for the original model. This session aims to cover all these aspects, from the classical PDE theory to applications.

### Markus-Sell's theorem for infinite dimensional asymptotically almost periodic systems.

**David Cheban**

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**Mamma Cristiana**

This talk is dedicated to the study of asymptotic stability of asymptotically almost periodic systems. We formulate and prove the analog of Markus-Sell's theorem for asymptotically almost periodic systems (both finite and infinite dimensional cases). We study this problem in the framework of general non-autonomous dynamical systems. The obtained general results we apply to different classes of non-autonomous evolution equations: Ordinary Differential Equations, Difference Equations, Functional Differential Equations (both with finite retard and neutral type) and Semi-linear Parabolic Equations.

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### Uniform global attractor of the 3D Navier-Stokes equations

**Alexey Cheskidov**

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**Songsong Lu**

We investigate the long time behavior of the 3D Navier-Stokes equations (NSE) with a time-dependent force and obtain the existence and the structure of the weak uniform global attractor. This is done by studying an autonomous evolutionary system without uniqueness, whose trajectories are solutions to the nonautonomous 3D NSE. The established method can also be applied to other nonautonomous partial differential equations.

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### Norm inflation for incompressible magneto-hydrodynamic system in $\dot{B}_{\infty}^{-1,\infty}$

**Mimi Dai**

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**Jie Qing, Maria Schonbek**

We demonstrate that the solutions to the Cauchy problem for the three dimensional incompressible magneto-hydrodynamics (MHD) system can develop different types of norm inflation in  $\dot{B}_{\infty}^{-1,\infty}$ . Particularly the magnetic field can develop norm inflation in short time even when the velocity remains small and vice versa. Another interesting case is that, even with zero initial velocity, the velocity field can develop norm inflation in short time. We construct different initial data to obtain these results using plane waves. The intuition is from the construction method introduced by Bourgain and Pavlović for Navier-Stokes equations.

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### Time-dependent attractors for the oscillon equation

**Francesco Di Plinio**

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**Gregory S. Duane, Roger Temam**

The oscillon equation

$$\begin{cases} \partial_{tt}u(x,t) + H\partial_t u(x,t) - e^{-2Ht}\partial_{xx}u(x,t) \\ \quad + V'(u(x,t)) = 0, \quad x \in (0,1), t \in \mathbb{R}, \\ u(0,t) = u(1,t), \quad t \in \mathbb{R}, \end{cases}$$

where  $V$  is a nonlinear potential, arises from relativistic mechanics as a one-dimensional model for wave propagation in an expanding universe.

We study the asymptotic behavior of the process generated by the oscillon equation, as well as its three-dimensional generalizations, in the ad-hoc framework of pullback attractors in time-dependent spaces. The explicit time-dependence of the phase space allows us to restrict the basin of attraction to the physically meaningful initial data, recover a

suitable notion of dissipation, and construct a finite-dimensional attractor of optimal regularity for the process.

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### Computer assisted rigorous proof of chaos in some infinite dimensional dynamical systems

**Xinchu Fu**

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**Kaihua Wang**

The Conley index, introduced by Charles C. Conley in 1971, has become a powerful tool to study dynamical systems. This is a topological invariant for investigating the structure of isolated invariant sets of continuous or discrete dynamical systems. And recently, the Conley index was generalized to a class of discontinuous systems by Xinchu Fu and Kaihua Wang. However, in most cases, calculating the index, or even finding an index pair, for systems with complicated dynamics, is impossible without exploiting computer assisted methods. Hence many researchers developed the Conley index for multi-valued systems, which are computer representations of the single-valued one. The homotopy invariant property of Conley index ensures that with an appropriate error bound, we can numerically compute the index correctly. This allows us to use the computer to give mathematically rigorous arguments for the existence of chaotic sets. The idea of a computer assisted proof of chaotic dynamics based on topological invariants appeared first in the work of Mischaikow and Mrozek. They used the discrete Conley index to prove the existence of chaotic dynamics for the Henon map and the Lorenz equations. With the development of the algorithms, the Conley index has been successfully applied to detect all kinds of chaotic attractors. Recently, automatic method for the efficient computation of a database of global dynamics of a multi-parameter dynamical system is introduced by Z. Arai et al., which uses Conley index as a main tool. The aim of this talk is to use the numerical method mentioned above to rigorously prove the chaos in some infinite dimensional dynamical systems, e.g., the Ikeda-like map coupled by a network with infinite size. It is shown that the Ikeda-like system with particular parameters has an isolated invariant set containing two periodic points, and a heteroclinic orbit connecting them, then a semiconjugacy between the invariant set and a nontrivial subshift system is established, which implies the system is chaotic. By the homotopy invariant property of the Conley index, all the proof are rigorous.

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### Discrete data assimilation for the 2D Navier-Stokes equations

**Eric Olson**

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**Abderrahim Azouani, Kevin Hayden, Edriss Titi**

We present an algorithm for dynamically constructing an approximate solution  $v(t)$  to the 2D Navier-Stokes equations from observational data of the form  $I_h(u(t_n))$  where  $I_h$  is a suitable interpolation operator and  $t_n$  is a series of discrete points in time. We then show that the difference between the approximate solution and the reference solution tends to zero as  $t$  tends to infinity.

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### Global existence and finite time blow up in a class of stochastic non linear wave equations.

**Rana Parshad**

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**Aslan Kasimov, Belkacem Said-Houari**

We consider a stochastic extension of a class of wave equations with nonlinear damping of viscoelastic type and nonlinear forcing. We show global existence for the stochastic equation, for certain range of parameters. Additionally, when the source term dominates the damping term, and when the initial data are large enough, we show that solutions of the stochastic system blow up in finite time, with positive probability. In the presence of noise, we have also extended the previously known range of initial data corresponding to blow-up. We also comment on asymptotic behavior of solution, in terms of attractors, for the good range of parameters.

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### Embedding of compact invariant sets of dynamical systems on infinite-dimensional manifolds into finite-dimensional spaces

**Volker Reitmann**

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**Sergey Popov**

Takens embedding method and some its generalizations for dynamical systems on Banach spaces use an embedding theorem which is due to B. R. Hunt and V. Yu. Kaloshin. In contrast to this we consider dynamical systems on Banach manifolds for the embedding of compact invariant sets with finite fractal dimension and use an embedding theorem due

to T. Okon. As an example we consider a dynamical system generated by the coupled Maxwell-Dirac equations.

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### 3D Navier-Stokes equations: numerical verification of regularity for bounded sets of initial data

**Witold Sadowski**

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**James C. Robinson, Pedro Marin-Rubio**

The paper concerns three-dimensional Navier-Stokes equations with periodic boundary conditions and zero forcing. As it is very well known global regular solutions to these equations exist for small initial data in  $H^1$ . The problem of regularity of solutions arising from arbitrarily large sets of initial data in  $H^1$  remains open. In the paper we consider the following statement: all initial conditions in a given ball (of arbitrary radius  $R$ ) in  $H^1$  give rise to global regular solutions of the Navier-Stokes equations. We prove that this statement can be verified computationally in a finite time (that is assuming that the statement holds, there exists a numerical algorithm that will verify veracity of the statement in a finite time). This is a joint paper with James C. Robinson and Pedro Marin-Rubio.

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### Separation and bifurcation phenomena for flows interacting with a boundary

**Marco Sammartino**

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**F.Gargano, V.Sciacca**

In this talk we shall review some rigorous and numerical results on the evolution of boundary layers for incompressible Navier-Stokes equation. The possibility of interpreting the phenomena leading to separation of the boundary layer in terms of bifurcation of equilibria will be explored.

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### On representatives of solutions and those properties independent of the chosen representative.

**Nick Sharples**

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**James C. Robinson**

Once a solution  $u \in L^p(0, T; L^q(\mathbb{R}^n; \mathbb{R}^n))$  of a PDE has been found, for example a weak solution of the

Navier-Stokes equations, it is often of interest to switch from the Eulerian to the Lagrangian viewpoint and describe trajectories of this solution by solving  $\dot{X} = u(X, t)$ . We discuss how the trajectories can change when  $u$  is altered on a set of null measure. In particular, we demonstrate that if we alter  $u$  on a set that is ‘small’ in a fractal sense,  $u$  is sufficiently integrable and there exists a generalised flow solution of the ODE then almost every trajectory is unchanged.

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### Global existence for the critical semilinear heat equation

**Mikolaj Sierzega**

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**James C. Robinson**

We will present a method of proving global existence of classical solutions for a critical semilinear heat equation. The result is an extension of the well-known technique based on the construction of supersolutions used in the theory of supercritical nonlinearities. A nonstandard smallness assumption on the initial data will be derived.

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### On the structure of the global attractor for reaction-diffusion equations

**José Valero**

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**A.V. Kapustyan, P.O. Kasyanov**

In this work we study a scalar reaction-diffusion equation for which the non-linear term satisfy some growth and dissipativity assumptions ensuring global existence of solutions for the Cauchy problem, but not uniqueness. We define several multivalued semiflows generated by weak, regular and strong solutions of the equation and study the structure of the global attractor. More, precisely, we prove that the attractor can be characterized as the union of all unstable manifolds of the set of stationary points and of the stable ones as well.

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**Moments and the Navier-Stokes equations****Alejandro Vidal-Lopez**

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**James C Robinson**

We will discuss some results related to the moment problem and statistical solutions of the 3D Navier-Stokes equations

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**Quasi-periodic solutions for 1 dimensional generalized Boussinesq equation****Shi Yanling**

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**Junxiang Xu , Xindong Xu**

In this paper, one-dimensional(1-D)generalized Boussinesq equation:

$$u_{tt} - u_{xx} + (f(u) + u_{xx})_{xx} = 0$$

with hinged boundary conditions is considered, where  $f(u) = u^3$ . It is proved that the above equation admits small-amplitude quasi-periodic solutions corresponding to finite dimensional invariant tori of an associated infinite dimensional dynamical system. The proof is based on an infinite dimensional KAM theorem and partial birkhoff normal form.

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## Special Session 64: Analysis of PDEs and Particle Systems: From Life Sciences, Economics and Materials Science

Toyohiko Aiki, Gifu University, Japan

Nobuyuki Kenmochi, Bukkyo University, Japan

Adrian Muntean, CASA, Eindhoven University of Technology, The Netherlands

The focus of this special session is on the modeling and analysis of continuous and discrete scenarios relevant to selected real-world applications.

Besides well-posedness studies of nonlinear PDEs and particle systems arising from materials and life science, the session also includes presentations on averaging processes in porous media, mathematical homogenization, statistical mechanics of particle systems, large-time behavior of solutions, multiscale dynamics of pedestrian flows.

Particularly, mathematical modeling for macro-economics exposed to extreme conditions (e.g. natural disasters like tsunamis, earthquakes etc.) will be open for discussions.

### One-dimensional concrete carbonation problem with nonlinear Henry's law

Toyohiko Aiki

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Adrian Muntean

In this talk we consider a free boundary problem as a mathematical model for concrete carbonation on a one-dimensional domain  $(0, \infty)$ . The problem is to find a carbonation front  $x = s(t) > 0$  and concentrations  $u$  and  $v$  of carbon dioxide in water and air regions, respectively. In our previous works we consider the linear function  $f(u, v) = a(bv - u)$  as a mathematical description for contribution of Henry's law, where  $a$  and  $b$  are positive constants. Now, we suppose that  $f(u, v) = \phi(bv - u)$ , where  $\phi$  is a continuous and increasing function on  $\mathbb{R}$ . The aim of this talk is to show a large time behavior of the carbonation front.

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### Stochastic nucleation and growth of particle clusters

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The binding of individual components to form composite structures is a ubiquitous phenomenon within the sciences. Within heterogeneous nucleation, particles may be attracted to an initial exogenous site: the formation of droplets, aerosols and crystals usually begins around impurities or boundaries. Homogeneous nucleation on the other hand describes identical particles spontaneously clustering upon contact. Given their ubiquity in physics, chemistry and material sciences, nucleation and growth have been extensively studied in the past decades, often assuming infinitely large numbers of building blocks and unbounded cluster sizes. These assumptions also led to the use of mass-action, mean field descriptions

such as the well known Becker Doering equations. In cellular biology, however, nucleation events often take place in confined spaces, with a finite number of components, so that discrete and stochastic effects must be taken into account. In this talk we examine finite sized homogeneous nucleation by considering a fully stochastic master equation, solved via Monte-Carlo simulations and via analytical insight. We find striking differences between the mean cluster sizes obtained from our discrete, stochastic treatment and those predicted by mean field treatments. We also consider heterogeneous nucleation stochastic treatments, first passage time results and possible applications to prion unfolding and clustering dynamics.

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### Global effect of local anisotropic interactions in crowd dynamics

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Adrian Muntean, Fons van de Ven, Lennart Gulikers, Alexey Lyulin

Many models for describing the behaviour of people in a crowd start from a microscopic point of view, i.e. at the individual's level. These models are mainly inspired by those used in physics, statistical mechanics etc. for studying particles systems (grains, colloids, molecules, etc.). In contrast to particles, people have well-defined front and back sides. Consequently, mutual interactions are highly influenced by the angle under which individuals perceive each other. Up to now, this kind of anisotropy was explored in simulations, but analytic understanding is lacking.

We use a model formulated at the macroscopic level (i.e. the crowd is considered to be a continuum), in which we prescribe the structure of the velocity field. It basically consists of a "desired velocity" which is perturbed by a "social velocity" term, due to the presence of other people. In this talk, I explain our model and the way in which we include

anisotropy due to visual perception. Furthermore, I give a flavour of the large-scale effects that inherently arise.

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### Swarm dynamics and equilibria for a nonlocal aggregation model

**Razvan Fetecau**

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**Y. Huang, T. Kolokolnikov**

We consider the aggregation equation  $\rho_t - \nabla \cdot (\rho \nabla K * \rho) = 0$  in  $\mathbb{R}^n$ , where the interaction potential  $K$  models short-range repulsion and long-range attraction. We study a family of interaction potentials with repulsion given by a Newtonian potential and attraction in the form of a power law. We show global well-posedness of solutions and investigate analytically and numerically the equilibria and their global stability. The equilibria have biologically relevant features, such as finite densities and compact support with sharp boundaries.

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### Abstract theory of the variational inequality and the Lagrange multiplier

**Takeshi Fukao**

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**Nobuyuki Kenmochi**

In this talk, we discuss the well-posedness of the variational inequality of the heat equation with the time-dependent constraint. This kind of problem was treated by E. Ginder (2010) as the heat equation with the time-dependent volume constraint, and the references therein. Using the Lagrange multiplier, the initial and Dirichlet boundary value problem was formulated. The objective is to obtain the abstract theory of the variational inequality and the Lagrange multiplier. Moreover under the assumption weaker than Ginder's one the well-posedness is obtained. Finally, using the relationship between our variational inequality and its Lagrange multiplier, we also discuss the regularity of the solutions.

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### Solvability of the tumor invasion model

**Risei Kano**

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In this talk, we discuss for solvability of the tumor invasion models which is Chaplain-Anderson type having some diffusion. Similar to the results so

far, This result has been used by Quasi-Variational inequalities. Quasi-variational inequalities can be an abstract representation of the system with a strong dependency. Also I talk about the abstract formulations.

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### Revival full model of human and economic activities in disaster regions

**Nobuyuki Kenmochi**

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**Atsusi Kadoya**

After disasters such as earthquake, tsunami, typhoon etc., we have to face a lot of serious problems, for instance, environment destroy for life and economic collapse and dispersion of habitants. It is quite important to recover human and economic activities in the disaster regions as soon as possible, by providing suitable supports for them. In general it takes a long time (10 or 20 years more) to recover the complete environment which is the basis of our life. In any case, we have to restart our life in incomplete circumstances. Moreover, at the same time one has to start adequate reorganization of production systems in order to renew the economical situation in disaster regions. We know many mathematical models dealing with the evolution of human activity and the economic growth, independently each other. However, in disaster regions the both aspects should be treated simultaneously on a correlative connection, although the corresponding model is much more complicated than the usual ones. In this talk we propose a simplified system of ordinary differential equations, which we call the revival full model of human and economic activities in disaster regions.

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### On a mathematical model of moisture transport with a time-dependent porosity in concrete carbonation process

**Kota Kumazaki**

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In this talk we consider a mathematical model of moisture transport in concrete carbonation process in three dimensional case. This model is a diffusion equation with a hysteresis operator indicating the relationship between the relative humidity and the degree of saturation. Also, this equation has a porosity which contains a non-local term in a coefficient of the time derivative of the unknown function and a perturbation. In this talk we prove the existence of a solution of a initial boundary value problem of this model.

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### Solutions to the Kohn-Sham model for heavy atoms

**Michael Melgaard**

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**C. Argaez**

The Density Functional Theory (DFT) of Kohn and Sham has emerged as the most widely-used method of electronic structure calculation in both quantum chemistry, condensed matter physics and materials science.

We study the standard and extended Kohn-Sham models for quasi-relativistic  $N$ -electron Coulomb systems describing heavy atoms; that is, systems where the kinetic energy of the electrons is given by the quasi-relativistic operator

$$\sqrt{-\alpha^{-2}\Delta_{x_n} + \alpha^{-4}} - \alpha^{-2}.$$

For spin-unpolarized systems in the local density approximation, we prove existence of a ground state (or minimizer) provided that the total charge  $Z_{\text{tot}}$  of  $K$  nuclei is greater than  $N - 1$  and that  $Z_{\text{tot}}$  is smaller than a critical charge  $Z_c = 2\alpha^{-1}\pi^{-1}$ . The proof is based on the concentration-compactness approach to locally compact variational problems involving non-local operators.

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### Mathematical modeling for brewing process of Sake and its analysis

**Yusuke Murase**

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**Akio Ito**

”Sake” is brewing liquor with using fermenting technique so-called ”Multiple parallel fermentation”. ”Multiple parallel fermentation” is one of the most difficult way of fermentation. We have a few data about brewing process of Sake because ”Multiple parallel fermentation” is too complicated. We propose a mathematical modeling for brewing process of Sake to analyze the phenomena, and to construct brewing, forecast, and control techniques from mathematical and engineering point of view. We’ll introduce this modeling and discuss existence some solutions for our modeling.

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### Qualitative analysis of homogenization.

**Daniel Onofrei**

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In this talk we will discuss two research avenues for the qualitative analysis of homogenization results: 1. The role of boundary layer correctors in the study of error estimates 2. The use of novel spectral decomposition results for the local representation of multi-scale problems solutions leading to global macro-scale approximation algorithms for the initial micro-scale phenomena. In this talk we will mainly present our ideas in the context of the elliptic homogenization for general problems with non-smooth coefficients and discuss the possible extensions of our results to the time-dependent case and to the case of variable geometries.

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### Elastic properties of a exoskeleton and homogenization of plywood structures

**Mariya Ptashnyk**

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Many biological or industrial composite materials comprise non-periodic microscopic structures, for example fibrous microstructure with varied orientation of fibers in exoskeletons, in polymer membranes and in industrial filters, or space-dependent perforations in concrete. An interesting and important for applications special case of non-periodic microstructures is so called locally-periodic microstructure, where spacial changes of the microstructure are observed on the scale smaller than the size of the considered domain but larger than the characteristic size of the microstructure. The notion of locally-periodic two-scale convergence is introduced. As admissible test functions we consider the functions of two variables such that they periodicity with respect to the second (microscopic) variable may depend on the first (macroscopic) variable. The developed theory is applied to derive effective macroscopic equations for a linear elasticity problem defined in a domain with a plywood structure, a prototypical pattern found in the exoskeleton. The plywood structure is characterised by the superposition of gradually rotated planes of parallel aligned fibres.

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**Continuous dependence of entropy solutions to strongly degenerate parabolic equations with discontinuous coefficients****Hiroshi Watanabe**Salesian Polytechnic, Japan  
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Strongly degenerate parabolic equations are regarded as a linear combination of the time-dependent conservation laws (quasilinear hyperbolic equations) and the porous medium type equations (nonlinear degenerate parabolic equations). Thus, these equations have both properties of hyperbolic equations and those of parabolic equations and describe various nonlinear convective diffusion phenomena such as filtration problems, Stefan problems and so on. In this talk we consider the strongly degenerate parabolic equations with discontinuous coefficients. In particular, we focus our attention on the continuous dependence of entropy solution to such equations.

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**Optimal control problem of positive solutions to second order impulsive differential equations****Noriaki Yamazaki**Kanagawa University, Japan  
noriaki@kanagawa-u.ac.jp**Lingling Zhang, Chengbo Zhai**

In this talk, we focus on optimal control problem of the positive solution to second order impulsive differential equations. We show the existence and uniqueness of positive solutions to our problem for each given control functions. Also, we consider the control problem of positive solutions to our equations. Then, we prove the existence of an optimal control that minimizes the nonlinear cost functional. Moreover we consider the special case of our problem, and then, we given the necessary condition of optimal control.

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## Special Session 67: Applied Analysis and Dynamics in Engineering and Sciences

**Thomas C. Hagen**, University of Memphis, USA  
**Janos Turi**, University of Texas at Dallas, USA

The goal of this session is to bring together mathematicians who work in different areas of applied mathematics and might thus not meet and exchange ideas and points of view. Consequently, the session program addresses a cross section of theoretical, numerical and computational developments and their applications to fluid dynamics, solid mechanics and life sciences. Areas of analytical interest include the theory of linear/nonlinear partial differential equations, the qualitative behavior of solutions, stability and asymptotics, control theoretic issues, and related aspects. The areas of application range from viscoelasticity and water waves to industrial flows and applications in mathematical biology and theoretical physics.

### Minimal norm control asymptotics and numerical approximations for the null controllability of non-standard parabolic-like PDE dynamics

**George Avalos**  
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Semidiscrete finite difference approximation schemes are presented for the null controllability of structurally damped plate equations. The key feature here is that the null controllers being approximated exhibit the asymptotics of the associated minimal energy function. We focus here upon the "nonspectral case"; i.e., the fourth-order elastic component of the dynamics does not necessarily obey (beneficial) hinged boundary conditions.

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### Models for growth of heterogeneous sandpiles via Mosco convergence

**Marian Bocea**  
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The asymptotic behavior of power-law functionals involving variable exponents is studied in the framework of Mosco convergence, leading to the analysis of a class of quasilinear parabolic problems which in the fast/slow diffusion limit models the growth of sandpiles whose critical slopes depend explicitly on the position in the sample. This is joint work with M. Mihailescu, M. Perez-Llanos, and J.D. Rossi.

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### Linear models for fluid-elasticity interactions

**Lorena Bociu**  
 NC State University, USA  
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We consider the non-steady, nonlinear coupling of Navier-Stokes and elasticity. We provide a new linearized model for the system, that takes into account

the curvature of the common interface and shows that the free boundary plays a key role in the analysis of the coupled system and its influence can not be neglected. The clear advantage of our approach is that it highlights the geometrical aspects of the problem, like curvature and boundary acceleration, which are unaccounted for in current linear models.

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### On the stability of closed vortex filaments

**Annalisa Calini**  
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**Thomas Ivey, Stephane Lafortune**

In its simplest form, the self-induced dynamics of a vortex filament in an ideal fluid is described by the Vortex Filament Equation, a completely integrable PDE related to the cubic focusing Nonlinear Schrödinger (NLS) equation by the well-know Hasimoto map. I will describe a general framework for studying the linear stability of solutions of the VFE, based on the correspondence between the VFE and the NLS equation, and on the construction of solutions of its linearization in terms of NLS squared eigenfunctions. Focusing on closed vortex filaments, I will discuss: the linear stability analysis of small-amplitude torus knot solutions of the VFE; criteria for instability of vortex filaments associated with periodic traveling wave solutions of the NLS; and a precise characterization of the linear instabilities of filaments associated with cnoidal NLS potentials, concluding that all knotted filaments in this class are linearly unstable. This work is part of on-going collaborations with Tom Ivey and Stephane Lafortune.

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### The Navier-Stokes flow of a liquid jet with moving free surface

**Shaun Ceci**

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**Thomas C. Hagen**

We analyze a Stokes problem arising in the study of the Navier-Stokes flow of a liquid jet. The analysis is accomplished by showing that the relevant Stokes operator accounting for a free surface gives rise to a sectorial operator which generates an analytic semigroup of contractions. Estimates on solutions are established using Fourier methods. The result presented is the key ingredient in a local existence and uniqueness proof for solutions of the full nonlinear problem.

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### Analysis on stability of solitary-wave solutions for a system of nonlinear dispersive equations

**Hongqiu Chen**

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**Jerry L. Bona**

Considered here is the system

$$\begin{cases} u_t + u_{xxx} + P(u, v)_x = 0, \\ v_t + v_{xxx} + Q(u, v)_x = 0 \end{cases}$$

of coupled KdV-equations introduced by Bona, Cohen and Wang, where  $u = u(x, t), v = v(x, t)$  are functions defined on  $\mathbb{R} \times \mathbb{R}^+$ ,

$$P(u, v) = Au^2 + Buv + Cv^2$$

and

$$Q(u, v) = Du^2 + Euv + Fv^2$$

in which  $A, B, \dots, F$  are real number constants. There are up to three explicit solitary-wave solutions given in terms of hyperbolic secants. Moreover, if the system of linear equations

$$\begin{cases} 2Ba + (E - 2A)b - 4Dc = 0, \\ 4Ca + (2F - B)b - 2Ec = 0 \end{cases}$$

has solutions  $(a, b, c)$  such that  $4ac > b^2$ , then, the stability and instability of the solitary-wave solutions can be checked by simple and straightforward algebra.

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### Numerical optimal unbounded control with a singular integro-differential equation as a constraint

**Shihchung Chiang**

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We consider numerical methods for optimal controls with an integro-differential equation of singular kernel as a constraint. We set the objective to minimize gap between optimal state and some target function for certain period of time. By assuming control to be unbounded, we propose a method of feedback correction which makes correction for optimal control proportionally to the corresponding error until certain criterion is satisfied. There are several advantages about this method such as user-decided accuracy, user-decided number of iterations and time saving. We compare the numerical results with the results by other methods from previous work.

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### Pathogen evolution in switching environments: a hybrid dynamical system approach

**Peter Hinow**

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**Jozsef Farkas, Jan Engelstaedter**

We propose a hybrid dynamical system approach to model the evolution of a pathogen that experiences different selective pressures according to a stochastic process. In every environment, the evolution of the pathogen is described by a version of the Fisher-Haldane-Wright equation while the switching between environments follows a Markov process with a given generator matrix. We investigate how the qualitative behavior of a simple single-host deterministic system changes when the stochastic switching process is added. In particular, we study the exchange of stability between monomorphic equilibria. Our results are consistent with the view that in a fluctuating environment, the genotype with the highest mean fitness will eventually become fixed. However, if the probability of host switching depends on the genotype composition of the population, polymorphism can be stably maintained.

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### Global stability lobes of turning processes with state-dependent delay

**Qingwen Hu**

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**Wieslaw Krawcewicz, Janos Turi**

We obtain global stability lobes of two models of turning processes with inherit non-smoothness due to the presence of state-dependent delays. In the process, we transform the models with state-dependent delays into systems of differential equations with both discrete and distributed delays and develop a procedure to determine analytically the global stability regions with respect to parameters. We find that a spindle speed control strategy can provide essential improvement on the stability of turning processes with state-dependent delay and furthermore we show the existence of a proper subset of the stability region which is independent of system damping. Numerical simulations are presented to illustrate the general results.

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### Non-linear discrete-time singularly-disturbed control dynamical systems and their steady state sets

**Byungik Kahng**

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It is known that Multiple Valued Iterative Dynamical Systems (MVIDS) can be used to model non-linear discrete-time control dynamical systems with disturbance. MVIDS turns out to be a useful tool particularly when the disturbance contains singularity, on which the traditional calculus-based method fails to apply. Under the MVID-modeling, it turns out that the steady state set of the disturbed control dynamical system corresponds to a set called the locally maximal strongly full-invariant set. In this talk, we discuss how to reach the afore-mentioned set in countable steps so that the finite-step approximate control of the steady state set is well-posed. Time permitting, we also discuss the invariant fractal structure that often arises as a byproduct of the singular disturbance.

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### Stability analysis of the delayed oscillator subjected to digital feedback control

**David Lehotzky**

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**Tamas Insperger**

The continuous-time delayed oscillator is considered subjected to digital feedback controllers. The motivation for the analysis is the digital control of regenerative machine tool vibrations. While the governing equation of the open loop system inherently involves a term with continuous-time point delay, the sampling effect of the digital feedback controller presents delayed terms of another type: terms with piecewise constant arguments. Since these latter terms can be represented as terms with a piecewise linearly varying time delay, the resultant system is time-periodic at the sampling period. The semi-discretization method is used to construct approximate stability diagrams in the plane of the system parameters. The combination of the two types of delays results in an intricate stability picture.

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### Conservative time discretization of large, stiff Hamiltonian systems, applied to models of molecular chains and nonlinear optics

**Brenton Lemesurier**

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A variety of problems in biophysics and nonlinear optics lead to the need to solve large, stiff, mildly nonlinear systems of ODEs that have Hamiltonian form, or perturbations of such by slight dissipation or noise. This talk presents a discrete calculus approach to constructing unconditionally stable, time-reversal symmetric discrete gradient conservative schemes for Hamiltonian systems, akin to the methods developed by Simó, Gonzales, et al, together with an iterative scheme for the solution of the resulting nonlinear systems which preserves unconditional stability and exact conservation of quadratic first integrals. Methods for increasing the order of accuracy are also discussed, along with handling of perturbations like noise. Several applications are considered, as time permits. In particular, some new coherent phenomena in molecular chain models of forms related to the discrete nonlinear Schrödinger equation, leading to a novel continuum limit approximation, and solving coupled systems of NLS equations.

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### Computational study of a dynamic contact problem

**Jigarkumar Patel**

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**Janos Turi**

In this paper we describe a computational framework to study the influence of a normal crack on the dynamics of a cantilever beam; i.e., changes in its natural frequency, amplitude, period etc. Due to the opening and closing of the crack during beam vibrations unilateral contact boundary conditions are assumed at the crack location. In the numerical implementation the contact conditions lead to the consideration of a linear complementarity problem. We have introduced a very effective method for solving linear complementarity problem. Numerical experiments are included.

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### Stability analysis of human respiratory system with both central and peripheral control

**Saroj Pradhan**

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**Janos Turi**

Stability analysis of a physiologically correct model of respiratory control where the transport delay from the lung to the central controller ( $\tau_b$ ) is assumed to be always greater than transport delay from the lung to the peripheral controller ( $\tau_p$ ) is carried out. It is seen that the multiple switching between stable and unstable regions occur with respect to changes in transport delays. Effects of variation in other internal and external parameters such as atmospheric pressure, central control gain ( $G_c$ ) and peripheral control gain ( $G_p$ ) on the stability of the human respiratory control is also studied.

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### On the jellyfish joyride: Mathematical analysis of catastrophes in maritime ecosystems

**Florian Rupp**

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**Jürgen Scheurle**

Dramatic increases in jellyfish populations which lead to the collapse of formally healthy ecosystems are repeatedly reported. Due to their devastating effects on fish populations the understanding of the causes for such boomings are of major ecological as well as economical importance. Here, we will study the reduced dynamics of a fish-jellyfish ecosystem

by assuming fish as the dominant predator species. By totally analytic means we completely classify all equilibria in terms of existence and non-linear stability, and give a complete description of this system's non-linear global dynamics. This ansatz complements the study given in [?], where an analogous system is studied which is derived under the assumption of jellyfish being the dominant predator species.

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### Stability of homoclinic orbits of the nonlinear Schrödinger equation

**Constance Schober**

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**Annalisa Calini**

We study the linearization of the Nonlinear Schrödinger (NLS) equation about homoclinic orbits of unstable plane wave solutions with two unstable modes. The family of homoclinic orbits as well as a complete set of solutions of the associated linearized NLS equation can be constructed using Bäcklund transformations. We show that iterating Bäcklund transformations saturates instabilities of the seed solution, making the largest dimensional homoclinic orbits the most stable in the sense of linear stability.

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### Stochastic wave and heat equations with cubic nonlinearity and additive space-time noise in 2D

**Henri Schurz**

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Stochastic wave and heat equations perturbed by cubic nonlinearities and additive  $Q$ -regular space-time white noise are studied. Existence and uniqueness of approximate strong solutions, stability and energy estimates are presented under Dirichlet-type boundary conditions and with  $L^2$ -integrable initial data on bounded rectangular domains  $D$  in  $R^2$ .

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### Critical excitation and control of variational inequalities

**Janos Turi**

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**Alain Bensoussan**

We consider control problems for the variational inequality describing a single degree of freedom elastoplastic oscillator. We are particularly interested in finding the "critical excitation", i.e., the lowest energy input excitation that drives the system between

the prescribed initial and final states within a given time span. The response of the oscillator to an excitation typically can be described as a sequence of alternating elastic and plastic phases where the state variables have to satisfy size and sign constraints. Using the appropriate governing equations in the elastic and plastic regimes, respectively, and continuity conditions between connecting segments we are able to construct a forward process, which combined with an iterative procedure provides approximations of the critical excitation.

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#### Analyzing viscous effects on wave motion

**Jin Wang**

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We study wave motion between two immiscible, incompressible, and slightly viscous fluids. A normal mode analysis is first presented for the linear viscous motion, and results on linear stability, dissipation, and dispersion are discussed. A formal asymptotic analysis is then conducted for the nonlinear progressive waves with slight viscosity. The analytical predictions are verified using numerical simulation, based on a recently developed numerical algorithm for two-phase Navier-Stokes flows, and the wave motion is followed sufficiently in time to quantify the effects of small viscosity.

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## Special Session 68: Analysis and Simulations of Nonlinear Systems

**Wei Feng**, University of North Carolina Wilmington, USA  
**Zhaosheng Feng**, University of Texas-Pan American, USA

The aim of the special session is to address analytic and computational aspects of nonlinear systems and differential equations arising particularly in mechanics and mathematical biology. The session will focus on recent advances in mathematical analysis and simulations of various systems of mixed types (including PDE models, delay equations, and impulsive differential equations), equally with applications to mechanics, population dynamics, wave phenomena, formation of patterns and other physical contexts.

### Pullback random attractor for the FitzHugh-Nagumo system on unbounded domains

**Abiti Adili**  
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**Wang Bi Xiang**

We study the asymptotic behavior of solutions of the FitzHugh-Nagumo system defined on unbounded domains with non-autonomous deterministic as well as stochastic terms. We first prove the pullback asymptotic compactness of solutions and then establish the existence of a unique pullback random attractor for the system. We further show that the random attractor is periodic when the non-autonomous deterministic terms are periodic.

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### Some computational challenges in analyzing global dynamics of certain nonlinear discrete dynamical systems

**Sukanya Basu**  
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We present some computational challenges involved in analyzing global behavior of solutions to certain classes of nonlinear discrete dynamical systems which have applications in mathematical biology and ecology. We also suggest some innovative geometric and computer-based approaches to overcome these challenges.

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### Mathematical modeling with reaction-diffusion-advection systems and its application in biology

**Kanadpriya Basu**  
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 basuk@mailbox.sc.edu  
**Xinfeng Liu**

In this talk, I will discuss about the computational challenges to solve reaction diffusion advection equations and how to overcome these difficulties by designing effective and robust numerical techniques.

Moreover, I will show some interesting applications of reaction diffusion models arising from complex biological systems.

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### A fast finite difference method for fractional diffusion equations

**Treena Basu**  
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**Hong Wang**

Fractional diffusion equations model phenomena exhibiting anomalous diffusion that can not be modeled accurately by the second-order diffusion equations. Because of the nonlocal property of fractional differential operators, the numerical methods have full coefficient matrices which require storage of  $O(N^2)$  and computational cost of  $O(N^3)$  where  $N$  is the number of grid points. In this talk, I will discuss a fast finite difference method for fractional diffusion equations, which only requires storage of  $O(N)$  and computational cost of  $O(N \log^2 N)$  for a one dimensional fractional diffusion equation or  $O(N \log N)$  for a two dimensional fractional diffusion equation while retaining the same accuracy and approximation property as the regular finite difference method. Numerical experiments are presented to show the utility of the method.

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### Local and global well-posedness for KdV-systems

**Jerry Bona**  
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Systems of Korteweg-de Vries type arise in various applications. We study here a class of such systems and obtain local and global well-posedness results together with some associated examples of blow-up in finite time.

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### Hopf bifurcation analysis of a predator-prey dystem with discrete and distributed delays

**Canan Celik Karaaslanli**  
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In this paper, a ratio dependent predator-prey system with both discrete and distributed delays is investigated. First we consider the stability of the positive equilibrium and the existence of local Hopf bifurcations. Then, by choosing the delay time  $\tau$  as a bifurcation parameter, we show that Hopf bifurcation can occur as the delay time  $\tau$  passes some critical values. Using normal form theory and central manifold argument, we also establish the direction and the stability of Hopf bifurcation. Finally, we perform some numerical simulations to support theoretical results.

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### Initial-boundary-value problem of systems of nonlinear dispersive equations

**Hongqiu Chen**  
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**Jerry L. Bona**

Considered here is the system

$$\begin{cases} u_t + u_x - u_{xxt} + P(u, v)_x = 0, & x \in (0, L), t > 0, \\ v_t + v_x - v_{xxt} + Q(u, v)_x = 0, & x \in (0, L), t > 0, \\ u(0, t) = f_1(t), v(0, t) = g_1(t), & t > 0, \\ u(L, t) = f_2(t), v(L, t) = g_2(t), & t > 0, \\ u(x, 0) = u_0(x), v(x, 0) = v_0(x) & x \in (0, L) \end{cases}$$

of coupled BBM-equations posed on bounded domain for  $x$  in the bounded domain  $[0, L]$ , where  $u = u(x, t), v = v(x, t)$  are functions defined on  $(0, L) \times \mathbb{R}^+$ ,  $P(u, v) = Au^2 + Buv + Cv^2$  and  $Q(u, v) = Du^2 + Euv + Fv^2$  in which  $A, B, \dots, F$  are fixed real numbers. We investigate conditions on both boundary and initial data to guarantee the system to be well-posed.

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### Lyapunov stability of elliptic periodic solutions of nonlinear damped equations

**Jifeng Chu**  
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Based on the third order approximation developed by Ortega, we present the formula of the first twist coefficient for a nonlinear damped equation. As examples, we consider the Lyapunov stability of a superlinear damped differential equation and a singular damped differential equations.

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### Existence and global attractivity of positive periodic solution to a Lotka-Volterra model

**Zengji Du**  
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In this talk, a Volterra model with mutual interference and Holling III type functional response is investigated, some sufficient conditions which guarantee the existence and global attractivity of positive periodic solution for the system are obtained, two examples are given to verify the results by using MatLab. We improve the main results of the references.

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### First integral of Duffing-van der Pol oscillator system

**Zhaosheng Feng**  
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In this talk, we restrict our attention to a nonlinear Duffing-van der Pol-type oscillator system by means of the first-integral method. This system has physical relevance as a model in certain flow-induced structural vibration problems, which includes the van der Pol oscillator and the damped Duffing oscillator etc as particular cases. Through applying the first-integral method and the Lie symmetry method, we present a general first integral formula of the Duffing-van der Pol-type oscillator system.

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### Exact solutions of the Burgers-Huxley equation

**Tian Jing**  
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**Zhaosheng Feng**

In this talk, we apply the Lie symmetry reduction method to solve exact solutions of partial differential equations such as the Burgers-Huxley equation and the generalized Fisher equation. Through analyzing the symmetry condition and the resultant determining system, we construct classical and non-classical infinitesimal generators, and obtain exact solutions accordingly. Joint with Zhaosheng Feng

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### The nonlinear Schrödinger equation created by the vibrations of an elastic plate and its dimensional expansion

**Shuya Kanagawa**

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**Ben T. Nohara**

In the course of studying the theory of plates the classical, Kirchhoff plate theory due to Timoshenko (1970) and Ural (1981) in which transverse normal and shear stresses are neglected to study bending, buckling, and natural vibrations of rectangular plates, was first established. The first order shear deformation plate theory extends the kinematics of the classical, Kirchhoff plate theory by relaxing the normality restriction and allowing for arbitrary but constant rotation of transverse normals and finite element models are developed for the precise analysis of the plate characteristics in real problem. In general, the Schrödinger equation governs the spatial and temporal evolution of the amplitude of a wavepacket propagating transversely in any dispersive, lossless medium. The Schrödinger equation governs an envelope created by a wavepacket by Nohara(2003). The nonlinear Schrödinger equation governs the non-linearity of the envelope. Many studies of a wavepacket has been carried out in water waves, fiber-optic communication systems, and some other area as well.

In this paper, we, first, survey the two-dimensional governing equation that describes the propagation of a wavepacket on an elastic plate using the method of multiple scales. Then we expand the governing equation to the multi-dimensional case not only in the sense of mathematical science but also engineering. For example the governing equation for 3-dimensional coordinate  $(x, y, z)$  shows the vibrational dynamics inside the plate. On the other hand 2-dimensional wavenumber  $(k_1, k_2)$  means the vertical wavenumber  $k_1$  and horizontal wavenumber  $k_2$  on the plate, respectively.

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### On the paradoxes of enrichment and pesticides

**Y. Charles Li**

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**Frank Feng, Yipeng Yang**

For the paradox of enrichment (which arose from models of population dynamics), we present a resolution (joint with Frank Feng). For the paradox of pesticides (which arose from experiments), we study mathematical models and present a theory for the resolution (joint with Yipeng Yang).

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### Intravenous glucose tolerance test model and its global stability

**Jiaxu Li**

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**Minghu Wang, Andrea De Gaetano, Pasquale Palumbo, Simona Panunzi**

Diabetes mellitus has become a prevalent disease in the world. Diagnostic protocol for the onset of diabetes mellitus is the initial step in the treatments. The intravenous glucose tolerance test (IVGTT) has been considered as the most accurate method to determine the insulin sensitivity and glucose effectiveness. It is well known that there exists a time delay in insulin secretion stimulated by the elevated glucose concentration level. However, the range of the length of the delay in the existing IVGTT models are not fully discussed and thus in many cases the time delay may be assigned to a value out of its reasonable range. In addition, several attempts had been made to determine when the unique equilibrium point is globally asymptotically stable. However, all these conditions are delay-independent. In this talk, we review the existing IVGTT models, discuss the range of the time delay and provide easy-to-check delay-dependent conditions for the global asymptotic stability of the equilibrium point for a simplified but practical IVGTT model through Liapunov function approach. Estimates of the upper bound of the delay for global stability are given in corollaries. In addition, the numerical simulation in this work is fully incorporated with functional initial conditions, which is natural and more appropriate in delay differential equation systems.

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### Burst synchronization and rhythm dynamics in neuronal networks

**Qishao Lu**

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**Xia Shi**

Physiological experiments have confirmed the existence of synchronous oscillations of neurons in different areas of the brain. Synchronization of neurons plays a significant role in neural information processing. Clinical evidences also point out that the synchronization of individual neurons plays a key role in some pathological conditions like Parkinson's disease, essential tremor, and epilepsies. According to experimental results, we constructed a small world neuronal network to study the effects of the coupling scheme, the intrinsic property of individual neurons and the network topology on burst synchronization and the rhythm dynamics of the network. The results are of fundamental significance to understand information activities in nervous systems.

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### Analytical solutions for the BoitiLeon-Pempinelli equation

**Zhuosheng Lu**

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We apply the classical Lie symmetry reduction method and an constructive algorithm to solve exact solutions of the BoitiLeonPempinelli equation. Infinitesimal generators, Backlund transformation and various types of analytical solutions of the BoitiLeon-Pempinelli equation are obtained.

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### Hyper-spherical harmonics and jumps in financial markets

**Indranil Sengupta**

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**Maria C. Mariani**

The technique of expanding functions in terms of spherical harmonics turns out to be very useful in solving integral equations. This motivates us to apply the same technique in mathematical finance. The governing equation in mathematical finance is mostly a parabolic partial differential equation. We present a technique of solving Heat equation and Black-Scholes equation using the method of spherical harmonics. We also discuss the method related to the Black-Scholes equation in annular domain and some generalized Black-Scholes equations. Finally we solve some integro-differential equation arising in financial models with jumps by using the method of spherical and hyper-spherical harmonics.

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### A mathematical model with time-varying antiretroviral therapy for the treatment of HIV

**Nicoleta Tarfulea**

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We present a mathematical model to investigate theoretically and numerically the effect of immune effectors in modeling HIV pathogenesis. Additionally, by introducing drug therapy, we assess the effect of treatments consisting of a combination of several antiretroviral drugs. A periodic model of bang-bang type and a pharmacokinetic model are employed to estimate the drug efficiencies. Nevertheless, even in the presence of drug therapy, ongoing viral replication can lead to the emergence of drug-resistant virus variances. Thus, by including two viral strains, wild-type and drug-resistant, we show that the inclusion

of the CTL compartment produces a higher rebound for an individuals healthy helper T-cell compartment than does drug therapy alone. We investigate numerically how time-varying drug efficacy due to drug dosing regimen and/or suboptimal adherence affects the antiviral response and the emergence of drug resistance.

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### Investigation of the long-time evolution of localized solutions of a dispersive wave system

**Michail Todorov**

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**Christo I. Christov**

We consider the long-time evolution of the solution of a system with dispersion and nonlinearity which is the progenitor of the different Boussinesq equations. As initial condition we use a wave system comprised by the superposition of two analytical soliton solutions. We use a strongly implicit difference scheme with internal iterations which allows us to follow the evolution of the solution at very long times. We focus on the dynamical behavior of traveling localized solutions developing from critical initial data. The system is rendered into a seven-diagonal band-matrix form and solved effectively by specialized solver with pivoting which is stable to round-off errors even for 2.500.000 points of spatial resolution. The main solitary waves appear virtually non-deformed from the interaction, but additional oscillations are excited at the trailing edge of each one of them. We extract the perturbations and track their evolution for very long times when they tend to adopt a self-similar shape: their amplitudes decrease with the time while the length scales increase. We test a hypothesis about the dependence on time of the amplitude and the support of Airy-function shaped coherent structures which gives a very good quantitative agreement with the numerically obtained solutions. This investigation is supported by the Scientific Foundation of the Bulgarian Ministry of Education, Youth, and Science under Grant DDVU02/71.

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### Entire and blow-up solutions: from semilinear to fully nonlinear elliptic equations

**Antonio Vitolo**

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Semilinear elliptic equations arise in many fields of the applied sciences, in particular in mechanics, engineering and life sciences. We discuss existence, uniqueness and qualitative properties of solutions in the whole space as well as in a bounded domain, possibly explosive on the boundary, for second order elliptic operators with a superlinear growth in the

solution, both in the case of a linear main term and of a fully nonlinear one.

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### Models and applications for minority health Studies

#### Xiaohui Wang

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Minority health status is of big concern to our society. In many cases, researchers seek to understand the effects of predictors (such as demographic measures, social-economic measures, insurance etc.) on health or intervention outcomes. Generalized linear models are used in our studies to explore such relationships. Application examples will be illustrated.

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### Qualitative properties of positive solutions of a class of boundary blow-up problems

#### Lei Wei

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We will consider boundary blow-up problems on an unbounded domain. For the problems, we will study the existence, uniqueness, blow-up rate of positive solutions. Our results extend some results of Professor Julian Lopez Gomez's papers.

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### The effects of singular lines in nonlinear wave equations

#### Lijun Zhang

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In this talk, the dynamical system approach to study traveling wave solutions of nonlinear wave equations is introduced. We pay more careful attentions to the travelling wave solutions of the nonlinear wave equations with nonlinear dispersive terms which corresponding traveling wave system may have singular lines and thus will result in a variety of singular waves. The singular traveling waves of the generalized nonlinear Klein-Gordon model equation are studied as an example to demonstrate the effects of singular lines in nonlinear wave equations.

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### Energy identity for a class of approximate bi-harmonic maps into sphere in dimension four

#### Shenzhou Zheng

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#### Changyou Wang

We considered in dimension four weakly convergent sequences of approximate biharmonic maps into sphere with bi-tension fields bounded in  $L^p$  for some  $p > 1$ . We prove an energy identity that accounts for the loss of Hessian energies by the sum of Hessian energies over finitely many nontrivial biharmonic maps on  $\mathbb{R}^4$ .

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## Special Session 69: Dissipative Systems and Applications

**Georg Hetzer**, Auburn University, USA

**Wenxian Shen**, Auburn University, USA

**Lourdes Tello**, Universidad Politécnica de Madrid, Spain

Dissipative systems arise in many applications. The special session will feature a broad spectrum of research from infinite dynamical systems theory and random dynamical systems to evolutionary partial differential equations and numerical simulation. The scope of applications reaches from reaction-diffusion systems with local and nonlocal dispersal from ecology to climate modeling.

### Attractors of impulsive dissipative semidynamical systems

**Everaldo Bonotto**

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**Daniela Paula Demuner**

We consider a class of dissipative semidynamical systems with impulses and we define some types of attractors for these systems. We present results which relate attractors and dissipative systems (point, bounded and compact). Also, we apply our results for a nonlinear reaction diffusion equation of type  $u' - \Delta u + g(u) = f$  with impulse condition.

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### Some results on Hopf type bifurcation in delayed complex Ginzburg-Landau equations

**Alfonso Casal**

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**Jesus I. Diaz, Jose M. Vegas, Michael Stich**

We consider the complex Ginzburg-Landau equation with feedback control given by some delayed linear terms (possibly dependent of the past spatial average of the solution). We prove several bifurcation results by using the delay as parameter. We start by considering the case of the whole space and, later, that of a bounded domain with periodicity conditions. A linear stability analysis is made with the help of computational arguments (showing evidence of the fulfilment of the delicate transversality condition). In the last section the bifurcation takes place starting from a uniform oscillation and originates a path over a torus. This is obtained by the application of an abstract result over suitable functional spaces.

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### Regularity for strongly coupled parabolic systems via homotopy

**Le Dung**

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We introduce a new technique, the so-called nonlinear heat approximation and BMO preserving homotopy, to investigate regularity properties of BMO weak solutions of strongly coupled nonlinear parabolic systems consisting of more than one equations defined on domain of any dimension.

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### On a cross-diffusion PDE system arising as limit of repelling particle models

**Gonzalo Galiano**

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**Virginia Selgas**

Starting from a stochastic ODE model for two finite populations of particles attracted by the same environmental potential but repelled by intra and inter-population overcrowding, we heuristically identify the limit PDE system of equations satisfied by the population densities when the number of particles tend to infinity. The resulting system is of nonlinear cross-diffusion type, for which we present a proof of existence of solutions based on a finite element approximation. Some numerical demonstrations showing the phenomenon of segregation of populations are also provided.

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### On the local behavior of solutions to thin-film Ginzburg-Landau equations near vortices

**Dmitry Glotov**

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We study the properties of solutions to the thin-film Ginzburg-Landau equations near their zeros and establish that they cannot vanish of infinite order unless identically zero. The method is based on an

extension of a classical result by Hartman and Wintner which allows the introduction of a non-analytic variable thickness function and a non-analytic applied magnetic field.

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### A finite volume scheme for the numerical approximation of a 2D climatological model.

**Arturo Hidalgo**

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In this work we compute the numerical solution of a global climate model including deep ocean effect. The model is based on that proposed by Watts and Morantine but including a coalbedo temperature dependent and nonlinear diffusion at the boundary. One of the main features of the model, which makes the problem of particular interest, is the dynamic and diffusive boundary condition that represents the coupling between ocean and atmosphere. The numerical method used is a finite volume scheme with high order WENO reconstruction in space and third order Runge-Kutta TVD for time discretization.

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### Positive stationary solutions and spreading speeds of KPP equations in locally spatially inhomogeneous media

**Liang Kong**

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**Wenxian Shen**

The current paper is concerned with positive stationary solutions and spatial spreading speeds of KPP type evolution equations with random or nonlocal or discrete dispersal in locally spatially inhomogeneous media. It is shown that such an equation has a unique globally stable positive stationary solution and has a spreading speed in every direction. Moreover, it is shown that the localized spatial inhomogeneity of the medium neither slows down nor speeds up the spatial spreading in all the directions.

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### Faster vs slower diffusers

**King-yeung Lam**

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**Xinfu Chen, Yuan Lou, Wei-Ming Ni**

We study the dynamics of a reaction-diffusion-advection model for two competing species in a spatially heterogeneous environment. We consider

the outcome of the competition between two species which are assumed to have the same population dynamics but different dispersal strategies: both species diffuses by random diffusion and advection along the environmental gradient, but with different diffusion and/or advection rates. We show that when the advection rates are large, then the faster diffuser wins the competition, this is in contrast to the previously known result that the slower diffuser prevails when there is small or no advection (Dockery et. al).

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### On a nonlocal Bernoulli-type problem with unknown measure data

**Juan Francisco Padial**

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Our aim is to study the existence of solutions for a nonlinear nonlocal Bernoulli-type free boundary problem with a unknown measure data. The problem arises in several nonlinear flow laws and physical situation. The elliptic problem was studied by J.I. Diaz, J.F. Padial and J.M. Rakotoson in *On some Bernoulli free boundary type problems for general elliptic operators*, Proceedings of the Royal Society of Edinburgh, 137A (2007), 895-911. Here, one will give an approach for the evolution case. We introduce a semi-implicit time differencing in order to obtain a family of elliptic problems. For each one of this problems, we find weak solution by applying a general mountain pass principle due to Ghoussoub-Preiss for a sequence of approximate nonsingular problems. Finally, apriori estimates allow us to obtain the solution by passing to the limit.

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### On a free boundary problem for a cross-diffusion system

**Sergey Shmarev**

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**G.Galiano, J.Velazco**

We study a free boundary problem for a cross-diffusion system describing the tumor growth. The free boundary is the surface of segregation for the cells of different kind. The method is based on the introduction of a local system of Lagrangian coordinates which renders the free boundary stationary. We prove that the problem admits a weak solution, derive the equation of motion of the free boundary and study its regularity.

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### On a climate energy balance model with continents distribution.

**Lourdes Tello**

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We study a global climate model with continents distribution. The model is based on a long time average energy balance and leads to a nonlinear parabolic equation for the mean surface temperature. This energy balance model is coupled with a deep ocean model in the oceanic areas. We extend some results on the mathematical treatment of the model without land-sea distribution.

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### Inverse problems in lubrication theory

**Jose Ignacio Tello del Castillo**

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We study a system of equations concerning equilibrium positions of journal bearings. The problem consists of two surfaces in relative motion separated by a small distance filled with a lubricant. The shape of the inlet surface is circular, while the other surface has a more general shape. An exterior force  $F$  is applied on the inner cylinder (shaft) which turns with a given velocity. The force applied produces an unknown displacement of the inner cylinder and therefore a new unknown (the displacement) appears in the problem. We will see results concerning the existence of at least one equilibrium of the problem, when  $F$  is constant in time.

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### Approximations of random dispersal operators by nonlocal dispersal operators

**Xiaoxia Xie**

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**Wenxian Shen**

This talk is concerned with the approximations of random dispersal operators/equations by nonlocal dispersal operators/equations. It first proves that the solutions of properly rescaled nonlocal dispersal initial-boundary value problems converge to the solutions of the corresponding random dispersal initial-boundary value problems. Next, it proves that the principal spectrum points of nonlocal dispersal operators with properly rescaled kernels converge to the principal eigenvalues of the corresponding random dispersal operators. Finally, it proves that the unique positive stationary solutions of nonlocal dispersal KPP equations with properly rescaled kernels converge to the unique positive stationary solutions of the corresponding random dispersal KPP equations.

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### Spatial spread and front propagation dynamics of nonlocal monostable equations in periodic habitats

**Aijun Zhang**

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**Wenxian Shen, Aijun Zhang**

This talk is concerned with the spatial spread and front propagation dynamics of monostable equations with nonlocal dispersal in spatially periodic habitats. Such equations arise in modeling the population dynamics of many species which exhibit nonlocal internal interactions and live in spatially periodic habitats. Firstly, we establish a general principal eigenvalue theory for spatially periodic nonlocal dispersal operators. Secondly, applying such theory and comparison principle for sub- and super-solutions, we obtain the existence, uniqueness, and global stability of spatially periodic positive stationary solutions and the existence of a spatial spreading speed in any given direction of a general spatially periodic nonlocal equation. Such features are generic for nonlocal monostable equations in the sense that they are independent of the assumption of the existence of the principal eigenvalue of the linearized nonlocal dispersal operator at 0. Finally, under the above assumption we also investigate the front propagation feature for monostable equations with non-local dispersal in spatially periodic habitats.

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## Special Session 70: Modeling and Dynamics of Infectious Diseases

**Abba Gumel**, University of Manitoba, Canada  
**Tufail Malik**, University of Manitoba, Canada

The mini-symposium will focus on the modeling and dynamics analysis of mathematical models for the spread of emerging and re-emerging infectious diseases in populations. An essential feature of the mini-symposium is the emphasis on the use of state-of-the-art techniques, theories and new applications associated with the use of dynamical systems in modeling the spread of diseases in populations. Current modeling and mathematical challenges will also be discussed.

### Effects of mixed *Plasmodium malariae* and *Plasmodium falciparum* infections on the dynamics of malaria

**Christinah Chiyaka**

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**Zindoga Mukandavire, Prasenjit Das, Farai Nyabadza, Senelani D. Hove Musekwa, Henry Mwambi**

Malaria continues to threaten the lives of people around the world, despite being preventable and treatable. Different factors affect the epidemiology and clinical outcomes of malaria. To investigate importance of mixed species infection, in the spread of malaria, a deterministic model for dual infection of *Plasmodium malariae* and *Plasmodium falciparum* is presented. Qualitative analysis of the model is performed. In addition to the disease free equilibrium, we show that there exists a boundary equilibrium corresponding to each species. The isolation reproductive number of each species is computed as well as the reproductive number of the full model. Conditions for global stability of the disease free equilibrium as well as local stability of the boundary equilibria are derived. The model has an interior equilibrium which exists if at least one of the isolation reproductive numbers is greater than unity. Among the interesting dynamical behaviors of the model, the phenomenon of backward bifurcation where stable boundary equilibrium coexists with a stable interior equilibrium, for a certain range of the associated invasion reproductive number less than unity is observed. Results from analysis of the model show that, when cross-immunity between the two species is weak, there is a high probability of coexistence of the two species and when cross-immunity is strong, competitive exclusion is high. Further, an increase in the reproductive number of species  $i$  increases the stability of its boundary equilibrium and its ability to invade an equilibrium of species  $j$ .

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### Mathematical Analysis of Chikungunya Model with Time Delay

**Salisu Garba**

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Chikungunya is a viral disease that is spread by mosquitoes. The disease, which resembles dengue fever, is spread in Africa, Asia and the Indian sub-continent (and also, in recent decades, in Europe and the Americas). It accounted for over 1.25 million cases in India alone in 2006. In this talk, a deterministic model with time delay (accounting for the incubation period of the disease) is used to study the transmission dynamics of the disease in a given population. Qualitative analyses of the model reveals the presence of the phenomenon of backward bifurcation in the model when the associated reproduction number ( $R_0$ ) is less than unity (this, in turn, makes disease control difficult). Simulations are carried out to investigate the effect of time delay on the disease dynamics (and control). Analysis of the model shows that, the disease will persist, whenever  $R_0 > 1$ . Furthermore, an increase in the length of incubation period, increases the chikungunya burden in the community if a certain threshold quantities, denoted by  $\Delta_b$  and  $\Delta_v$  are positive.

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### Backward bifurcations in disease transmission models

**Abba Gumel**

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Backward bifurcation, a phenomenon where a stable disease-free equilibrium typically co-exists with a stable endemic equilibrium, is known to have important consequences on the effective control or persistence of the disease in a population. The talk discusses some common and new causes of backward bifurcation in Kermack-McKendrick-type deterministic compartmental models for the spread of some emerging and re-emerging diseases.

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### The effects of pre-existing immunity on infectious diseases

**Jane Heffernan**

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Immune system memory (or immunity) is gained as a result of primary infection or vaccination, and boosted by later vaccinations or secondary infections. The components of immune system memory (i.e. T-cells, antibodies) are primed to react and fight quickly against future infections of strains related to those previous experienced by an individual. The existence of immunity, thus affects an individual, but will also affect the spread of a disease in a population by reducing transmission. The extent of the effects of immunity on an individual, and how this translates to the population are not well understood. In this talk we will focus on different aspects of immunity with different diseases as case studies.

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### Modeling recent outbreaks of Dengue Fever in Pakistan

**Adnan Khan**

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**Mudassar Imran, Kanwal Rizvi**

Dengue is a mosquito borne disease prevalent in the tropical and subtropical countries including Pakistan. We evaluate Pakistans dengue situation and compare this to epidemic trends in different countries including Brazil. We propose a deterministic mathematical model incorporating the transmission of different strains of dengue and identify a number of factors related to the spread of the disease in Pakistan. Using data from recent outbreaks we determine the value of basic reproductive number and compare it with the basic reproductive number evaluated in different countries including Brazil.

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### Mathematical insights in evaluating effectiveness of interventions for HIV prevention

**Yang Kuang**

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**Yuin Zhao, Hao Liu, Dobromir Dimitrov**

Mathematical models have been used to simulate HIV transmission and to study the use of chemoprophylaxis among men-who-have sex with men. Often a single intervention outcomes based on cumulative number or fractions of infections prevented, on reduction in HIV prevalence or incidence have been

used to evaluate the effectiveness of PrEP interventions. These indicators express a wide variation over time and often disagree in their forecast on the success of the intervention. We develop a deterministic mathematical model of HIV transmission to evaluate the public-health impact of oral PrEP interventions, compare PrEP effectiveness with respect to different evaluation methods and analyze its dynamics over time. We compare four traditional evaluation methods including relative reduction in HIV prevalence and incidence which are considered to avoid the ambiguity associated with commonly used indicators based on the absolute number of prevented infections. We consider two additional methods which estimate the burden of the epidemic to the public-health system. We also investigate the short term and long term behavior of these indicators and the effects of key parameters on the expected benefits from PrEP use.

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### A data driven spatiotemporal rabies model for skunk and bat interaction in Northeast Texas

**Hao Liu**

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**Rebecca K. Borchering, Mara C. Steinhaus, Carl L. Gardner, Yang Kuang**

We formulate a simple partial differential equation model in an effort to qualitatively reproduce the spread dynamics and spatial pattern of rabies in northeast Texas region with overlapping reservoir species (skunks and bats). Most existing models ignore reservoir species or model them with patchy models by ordinary differential equations. In our model, we incorporate interspecies rabies infection in addition to rabid population random movement. We apply this model to the confirmed case data from northeast Texas with most parameter values obtained or computed from existing literatures. Results of simulations using both the skunk only model and our skunk and bat model demonstrate that the model with overlapping reservoir species more accurately reproduces the progression of rabies spread in northeast Texas.

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**The impact of an imperfect vaccine and Pap cytology screening on the transmission dynamics of Human Papillomavirus and Cervical Cancer.**

**Tufail Malik**

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**J Reimer, A B Gumel, E H Elbasha, S M Mahmud.**

The talk will address the problem of the transmission dynamics of human papillomavirus in a population. A new sex-structured model, which takes into account the associated multiple cervical intraepithelial neoplasia stages, will be used to assess the combined impact of Pap cytology screening and a vaccine on the disease dynamics and the associated dysplasia. Rigorous qualitative analysis will be presented. Simulation results, using a realistic set of parameter values, will also be discussed.

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**Avian influenza: Modeling and implications for control**

**Maia Martcheva**

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At present H5N1 avian influenza is a zoonotic disease where the transmission to humans occurs from infected domestic birds. Since 2003 more than 500 people have been infected and nearly 60% of them have died. If the H5N1 virus becomes efficiently human-to-human transmittable, a pandemic will occur with potentially high mortality. A mathematical model of avian influenza which involves human influenza is introduced to better understand the complex epidemiology of avian influenza and the emergence of a pandemic strain. The model is parameterized based on demographic and epidemiological data on birds and humans. The differential equation system faithfully projects the cumulative number of H5N1 human cases and captures the dynamics of the yearly cases. The model is used to rank the efficacy of the current control measures used to prevent the emergence of a pandemic strain. We find that culling without repopulation and vaccination are the two most efficient control measures each giving 22% decrease in the number of H5N1 infected humans for each 1% change in the parameters. Control measures applied to humans, however, such as wearing protective gear, are not very efficient, giving less than 1% decrease in the number of H5N1 infected humans for each 1% change in the parameters. Furthermore, we find that should a pandemic strain emerge, it will invade, possibly displacing the human influenza virus in circulation at that time. Moreover, higher prevalence levels of human influenza will obstruct the invasion capabilities of the pandemic H5N1 strain. This effect

is not very pronounced, as we find that 1% increase in human influenza prevalence will decrease the invasion capabilities of the pandemic strain with 0.006%.

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**General comments on the construction of mathematical models for predator-prey interactions**

**Ronald Mickens**

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**Ronald E. Mickens**

Mathematical modeling of interacting populations can provide valuable insights into important features of these systems. Further, when combined with relevant mathematical analysis and data, these models may also give rise to a fundamental understanding of the nature of the systems. However, a critical issue is that a priori no universal set of rules exist for the (unique) construction of the models. In spite of this weakness, broad classes of mathematical models predict essentially the same general features for the properties of their solutions. The main purpose of this presentation is to discuss these issues and, in particular, examine the role of dynamic consistency in the modeling construction process. Explicit examples will be given to illustrate some consequences of the application of dynamic consistency.

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**Evaluation of diagnostic test for Lymphatic Filariasis in Papua New Guinea using a mathematical model**

**Anuj Mubayi**

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**Anuj Mubayi, Daniel Tisch, Marian Gidea, Carlos Castillo-Chavez**

Lymphatic Filariasis (LF) is a debilitating disease endemic in developing countries around the world. The prevalence of the LF in Papua New Guinea (PNG) is considered the highest in the world. The most feasible strategy for LF in many countries has been Mass Drug Administration (MDA). But in PNG previous rounds of MDA have not been successful in eliminating LF. Optimal use of such control programs depends on our ability to accurately identify infected individuals after populations have received multiple rounds of MDA. Available diagnostic tests have low and varying efficacy and the value of the tests for monitoring the end points of transmission has not been validated. This study provides a framework to test the value of new and existing diagnostic/monitoring tools as prevalence decreases because of MDW. In our model analysis, we use data

from the ongoing and previous MDA field trials as well as literature review from the PNG. The talk will show results of the impact on the elimination of Lymphatic Filariasis of the errors in five available tests that include microfilarial level-, antigen- and antibody-based tests.

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### Understanding cholera transmission and interventions using mathematical models

**Zindoga Mukandavire**

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Cholera, a preventable disease, remains a global cause of morbidity and mortality, capable of causing periodic epidemic disease. Implementation of appropriate disease control efforts, including vaccination, requires an understanding of transmission dynamics, which may be best quantified by mathematical models. We explore the utility of mathematical models in understanding transmission dynamics of cholera, and in assessing the magnitude of interventions necessary to control epidemic disease

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### Mathematical models of binge drinking, heroin epidemics, anorexia and bulimia

**Giuseppe Mulone**

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We introduce some mathematical models in the Dynamics of Infectious Diseases. In particular, we first develop a two-stage (four component) model for youths with serious drinking problems and their treatment. The youths with alcohol problems are split into two classes, namely those who admit to having a problem and those who do not. We study the stability of the equilibria of a model of heroin epidemics. Then, we propose a mathematical model to study the evolution of the number of anorexic and bulimic people, by analyzing the effect of an education coefficient and the possible influence of media on the spread of eating disorders. Finally, epidemic models with evolution are considered where the diffusion of individuals is influenced by intraspecific competition pressures and are weakly affected by different classes. The nonlinear stability of some of the models is proved by introducing new Liapunov functions in the PDEs model.

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### Impact of enhanced Malaria control on the competition between *Plasmodium falciparum* and *Plasmodium vivax*

**Olivia Prosper**

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**Maia Martcheva**

The primary focus of malaria research and control has been on *P. falciparum*, the most severe of the four *Plasmodium* species causing human disease. However, there is substantial overlap between the spatial distributions of *Plasmodium falciparum* and *Plasmodium vivax*. We developed a mathematical model describing the dynamics of *P. vivax* and *P. falciparum* in the human and mosquito populations and fit the model to clinical case data to understand how improving malaria interventions affects the competition between the two species. We addressed the uncertainty in parameter estimates by performing a parametric bootstrapping procedure. This procedure predicted that *P. vivax* outcompeting *P. falciparum* is the most likely outcome based on our model. Moreover, the predictions of our model are counter to what one expects based on the case data alone. Although the proportion of cases due to *falciparum* has been increasing, the fit of our model to the data reveals that this observation is insufficient to draw conclusions about the long-term competitive outcome of the two species.

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### Robust uniform persistence and competitive exclusion in a nonautonomous multistrain SIR epidemic model with disease-induced mortality

**Paul Salceanu**

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**Azmy S. Ackleh**

We consider a nonautonomous version of the (autonomous) SIR epidemic model of Ackleh and Allen (2003), for competition of  $n$  infection strains in a host population, and focus on new questions, mainly regarding robust uniform persistence of the total population, as well as of the susceptible and infected subpopulations. We show that the first two forms of persistence depend entirely on the rate at which the population grows from the extinction state, respectively the rate at which the disease is vertically transmitted to offspring. We also discuss the competitive exclusion among the  $n$  infection strains, namely when a single infection strain survives and all the others go extinct. Numerical simulations are also presented, to account for the situations not covered by the analytical results. The approach we developed here is general enough to apply to other nonautonomous epidemic models.

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### Global dynamics of cholera models with differential infectivity

Zhisheng Shuai

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Cholera is a bacterial disease that can be transmitted to humans directly by person-to-person contact or indirectly via the environment (mainly contaminated water). An ordinary differential equation model for cholera dynamics is formulated that includes these two transmission pathways with nonlinear incidence, as well as stages of infection and infectivity states of the pathogen. Lyapunov functions and a graph-theoretic approach are used in the model analysis to show that a basic reproduction number gives a sharp threshold determining whether cholera dies out or becomes endemic. A further model that includes temporary immunity using distributed delays is formulated. Numerical simulations show that oscillatory solutions may occur for parameter values taken from the literature on cholera data, and differential infectivity may alter the existence, amplitude and period of oscillations in cholera prevalence.

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### Heterogeneity in the infectiousness of humans in the dynamics of malaria transmission and control.

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Abba Gumel, Chandra Podder

The purpose of this talk is to present an exploration and a discussion of a heterogeneous population-level model for assessing the role of gametocyte density on malaria transmission dynamics. Studies have shown a positive correlation between gametocyte density and infectiousness to mosquito, but this correlation was qualified as loose. In fact, other studies have shown that there are low or non-infectious high-density gametocyte carriers and highly infectious low-density gametocyte carriers with infectious individuals having undetected levels of gametocytes. Hence, the successful transmission of parasites to a mosquito is more about "quality" than quantity and infectiousness cannot be simply explained by parasite density. Therefore, we seek to present and analyze a model that takes into account the heterogeneous nature of infectious humans based on "quality" and quantity which may seriously impact the contact rates and infectiousness of humans to mosquitoes and the overall dynamics of malaria transmission

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### Mathematical analysis of a virus dynamics model with general incidence rate and cure rate

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Khalid Hattaf, Noura Yousfi

The rate of infection in many virus dynamics models is assumed to be bilinear in the virus and uninfected target cells. In this paper, the dynamical behavior of a virus dynamics model with general incidence rate and cure rate is studied. Global dynamics of the model is established. We prove that the virus is cleared and the disease dies out if the basic reproduction number  $R_0 = 1$  while the virus persists in the host and the infection becomes endemic if  $R_0 > 1$ .

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### The impact of temperature on the establishment of Lyme disease Tick Vector Ixodes Scapularis

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Yijun Lou, Venkata R.Duvvuri, Nicholas H.Ogden, Jianhong Wu

A stage-structured periodic deterministic model was formulated to simulate the impact of temperature on the tick (*Ixodes scapularis*) survival and seasonality at Long Point, Ontario, Canada. 7 season-based model coefficients were parameterized using Fourier series analysis by fitting temperature and tick data. We derived the basic reproduction number for the tick population,  $R_0$ , as the number of new female adult ticks produced by an index female adult tick when there are no density dependent constraints acting anywhere in the life cycle of the tick population. We confirmed that, both mathematically and numerically, the tick would go to extinction when  $R_0 < 1$ , and found the successful tick invasion and persistence when  $R_0 > 1$ . A minimum degree-days threshold in a one year to identify whether the tick population extinct or not was found. A global sensitivity analysis based on popular Latin Hypercube Sampling (LHS) sampling method was performed which demonstrated the mean monthly temperature in June, July, August would more significantly sensitive to establish the tick population. Therefore temperature would significantly influence the risk of tick establishment in a habitat.

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## Special Session 72: Special Methods for Solving Systems of Non-linear Differential Equations and their Applications to Sciences and Engineering

Mufid Abudiab, Texas A&M University-Corpus Christi, USA

The purpose of this session is establishing a forum for the most effective special methods in solving systems of non-linear differential equations and their application in sciences and engineering. This forum will provide services for graduate students, post docs, and researcher in the field of non-linear differential equations. Also, it will enhance the view of scientists and engineers about systems of non-linear differential equation (dynamical systems) and provide them with very effective tools for modeling and hence solving a variety of problems in their fields.

### Modeling of the dynamics of client-centered health care

**Mufid Abudiab**

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**Suzanne Beltz**

For the purpose of providing and delivering safe, effective and appropriate health care services, patients need to play an integral role in the informed decision making of the process. The Institute of Medicine has called for transformation of the health care system in the United States and identified client-centered care as one of the major tenets of this transformation. Health care professional use mathematics on a daily basis to execute several medical functions ranging from data collection to regression statistical computations. A variety of mathematical modeling techniques can be utilized to help improve the delivery of health care. In this paper, we use literature, statistical analysis, and mathematical modeling tools to shed light on the main factors that impact the delivery of health care services. Also, we explore the great variation in defining the construct of client-centered care and the lack of tools for measuring the construct. Finally, a conceptual model of the continuous interaction among the client's model of the world, self-care knowledge, self-care assets, self-care resources, and self-care action is presented and analyzed to optimize the way of providing client-centered care.

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### Ratio-dependent predator-prey model with infection in prey population

**Inkyung Ahn**

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**Wonlyul Ko**

We study a predator-prey system with a ratio-dependent functional response when a prey population is infected. First, all nonnegative equilibria are investigated, and then a condition which gives a stability at these equilibria shall be found. Especially, disease-free and biological control states are discussed in view of a biological interpretation. Lastly, under the Neumann boundary condition, the existence of nonconstant positive steady-states are studied. The employed methods are a comparison principle for a parabolic problem and Leray-Schauder Theorem.

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### A spatio-temporal model for tumor-immune interaction and siRNA treatment

**Seunghyeon Baek**

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**Inkyung Ahn**

In this research, a spatio-temporal mathematical model consisting of nonlinear reaction-diffusion equations is examined that describes interactions among immune effectors and tumor cells. The system we consider involves the immuno-suppressive TGF-beta (Transforming growth factor-beta) and TGF-beta-suppressive siRNA. In particular, the density of tumor is assumed to disperse by diffusion together with directed movement towards more sufficient nutrition. Also the concentration of siRNA is assumed to have the oriented movement of cells in response to the one of TGF-beta due to the chemotaxis of TGF-beta. Tumors can evade immune surveillance by secreting immuno-suppressive factors such as TGF-beta. TGF-beta inhibits the activation of the effector cells and stimulates tumor growth by promoting angiogenesis. Our main goal is to investigate the spatial effects of tumor-immune interactions, as well as siRNA treatment which can control the tumor effectively by suppressing TGF-beta production.

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### Global existence to the Navier-Stokes equations through a diffusive upper solution.

**Guy Bernard**

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A global existence result is presented for the Navier-Stokes equations filling out all of three-dimensional Euclidean space. The initial velocity is required to satisfy a smallness condition in the form of an upper bound (bell-like) rational function. The method of proof is based on elementary symmetry transformations of the Navier-Stokes consisting of dilations and contractions of space, time, and velocity. The presented demonstration is a step method. A solution to the Navier-Stokes equations is shown to exist in a small time interval through a self-similar-like upper solution modeling the diffusive nature of the principal part of the Navier-Stokes equations (i.e. a heat equation). The local solution is then extended to an even larger time interval by transforming the N-S equations through a symmetry transformation which reduces this first extension to the first local solution. This process is repeated iteratively to produce a global solution. The symmetry transformations utilized exploit the diffusive and self-similar-like nature of the upper solution.

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### A unique positive solution to a system of nonlinear elliptic equations

**Diane Denny**

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We study a system of nonlinear elliptic equations that arises from a predator-prey model. Previous related work proved uniqueness when a positive parameter in the system of equations is sufficiently large. We prove the existence of a unique positive solution to this system of equations for all positive values of this parameter.

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### Solutions and conservation laws of a coupled Kadomtsev-Petviashvili system

**Chaudry Khalique**

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In this talk we study the coupled Kadomtsev-Petviashvili system

$$\begin{aligned} (u_t + u_{xxx} + 3uu_x + 3ww_x)_x + u_{yy} &= 0, \\ (v_t + v_{xxx} + 3vv_x + 3ww_x)_x + v_{yy} &= 0, \\ \left( w_t + w_{xxx} + \frac{3}{2}(uw)_x + \frac{3}{2}(vw)_x \right)_x + w_{yy} &= 0, \end{aligned}$$

which has applications in many scientific fields. Exact solutions will be obtained using the Lie symmetry method along with the simplest equation method of the underlying system. In addition the conservation laws will be derived using the multiplier method.

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### Conservation laws of some evolution equations via non variational approach

**Muhammad Danish Khan**

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**Imran Naeem**

We consider (1+1)-dimensional third order non linear modified Benjamin Bona Mahony (mBBM) equation and generalized regularized long wave (GRLW) equation which have their importance in many branches of physics. These equations arise in wave phenomena in fluid dynamics, elastic media, optic fibres and plasma etc. and are thus important. We compute the conservation laws of these equations via partial Lagrangian approach. Since this approach works only for those differential equations whose higher order terms are of even order so first we increase their order by assuming dependent variable to be the derivative of new variable by setting either  $u = v_x$  or  $u = v_t$ , where  $u$  and  $v$  are old and new dependent variable respectively. The conservation laws for mBBM and GRLW equations are computed in both cases and it is found that the results obtained for both equations are very useful in their solution process.

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### The enzyme-catalysed reaction-diffusion system

**Wonlyul Ko**

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In this talk, we discuss the biochemical reaction-diffusion model with Michaelis-Menten type response velocity function, which captures the feature of an enzyme-catalysed reaction from glycolysis, and we mainly study the corresponding steady-state under the homogeneous Neumann-boundary condition. In particular, we provide some results on the existence and nonexistence of nonconstant stationary solutions.

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### The qualitative study on a new special analytical method for solving wide classes of non-linear differential equations-reality, potential

**Habibolla Latifizadeh**

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**Faramarz Tahamtani, Esmail Hesameddini**

Nonlinear phenomena play a crucial role in applied mathematics, physics and engineering. Much effort was paid on existence, uniqueness and stability of the solution of the problem. Concerning this important topic over the last decades several analytical/approximate methods have been developed to solve nonlinear ordinary and partial differential equations. So at this paper a new analytical iterative procedure have been establish and proved. It may be concluded that the method is very powerful and efficient technique in finding an acceptable solution for wide classes of nonlinear problems. Also, it can be noted that there are many advantage of this proposed method that is based on Laplace transform, the main advantages are the fast convergence to the solutions, does not require discretizations of space and time variables, no need to solve nonlinear system of equations as in finite element methods and finite difference methods, then, no necessity of large computer memory. So to illustrate the afore-said claims some nonlinear differential equations that has some applications in fields applied mathematics and engineering is analyzed. The obtained results are in good agreement with the existing ones in open literature and it is shown that the technique introduced here is robust, efficient and easy to implement.

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### Approximate solutions for 1-D premixed flame propagation model

**Christina Martinez**

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**Devanayagam Palaniappan, Lea-Der Chen**

The one-dimensional boundary value problem of the premixed flame propagation in a condensed medium leads to a system of highly nonlinear ordinary differential equations. In this presentation, some explicit approximate solutions obtained via matched asymptotic expansion will be shown. A brief discussion of our numerical solutions to this model will be provided. Comparison of our results with the existing solutions shows an excellent agreement. Applications of our results in the context of transport mechanisms will also be demonstrated.

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**Analysis of a system of reaction-diffusion PDE model****Devanayagam Palaniappan**Texas A&M University-Corpus Christi, USA  
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Reaction-diffusion systems are a set of semi-linear parabolic partial differential equations (PDEs). Such mathematical models can be used to describe several dynamical processes of chemical and non-chemical nature. In this presentation, we discuss a model that is applicable in biology, especially, in the determination of infected and noninfected cells. A direct similarity method - introduced originally by Clarkson and Kruskal (CK-method) for Boussinesq equation is modified and utilized in the reduction of pdes to ordinary differential equations(ODEs). We present an analysis of the reduced system of ODEs and show exact solutions for certain boundary-value problems with specified rate constants. A brief discussion of the application of our results in the biological context will be also provided.

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**A reaction-diffusion model of Bovine Viral Diarrhea Virus (BVDV) infection****Sami Shahin**Southwest Minnesota State University, USA  
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BVDV (Bovine Viral Diarrhea Virus Infection) is an infection that affects many populations of cattle. A severe infection can be linked to many different causes such as the state of immune system, physiological status, and other diseases. Most BVDV infections are caused by non-cytopathic viruses. Severely infected animals can go through a brief shift in the number of circulating leukocytes. Due to virus multiplication, contaminated macrophages change its functions and cause the depletion of B and T type lymphocytes. In this paper, a PDE model of four reaction-diffusion equations that represents the basics of the contact between immune cells and the virus will be introduced. The system will be solved numerically and analytical via the upper-lower solution method.

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**Polynomial ODEs for conservation laws****James Sochacki**James Madison University, USA  
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Burger's equation without diffusion is the prototype for conservation laws and can be derived from Euler's equation for gas dynamics. Euler's equation for gas dynamics can be obtained from the Navier-Stokes equation which includes diffusion. Burger's equation with diffusion can be obtained from the Navier-Stokes equation. In this talk we discuss these relations through ordinary differential equations and show how these ordinary differential equations can be used to derive polynomial solutions to these partial differential equations and conservation laws in general. We demonstrate these ideas for the acoustic equations that can be derived from the Euler equations and highlight interesting consequences for these equations.

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**Quasilinear differential equations in exterior domains and application****Nicolae Tarfulea**Purdue University Calumet, USA  
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In this talk we address the existence and uniqueness of weak solutions to a class of quasilinear elliptic equations with nonlinear Neumann boundary conditions in exterior domains. An important model case related to the initial data problem in general relativity is presented. Correlated to our results, we discuss the existence and uniqueness of the conformal factor for the Hamiltonian constraint in general relativity.

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**Numerical solutions for weak scattering in a turbulent fluid****Christopher Trombley**Texas A&M Corpus Christi, USA  
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A mathematical model for optical scattering in a turbulent fluid is presented. Navier-Stokes equations are adapted for the turbulent fluid region while the scattered field is modeled using the Helmholtz equation. This set of nonlinear partial differential equations with the appropriate boundary conditions are solved numerically with the aid of the finite element method. The scattered amplitude is then given, which can be used to reduce errors in radio signals in a turbulent fluid.

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## Special Session 73: Mathematical Models for Upwelling Ocean Currents and Related Phenomena

David Rivas, CICESE, Mexico  
 Sherry Scott, Marquette University, USA  
 Anna Ghazaryan, Miami University-Oxford, USA

In this mini symposium, we bring together a multidisciplinary group of mathematicians and scientists who use different perspectives to model fluid dynamics of upwelling and downwelling in ocean flow. Of a particular interest are related phenomena such as polynyas which are large persistent regions of open water where one would expect ice. Topics include a variety of techniques and aspects involved in the modeling of these phenomena.

### The effect of bathymetric profile on the structure of coastal ocean upwelling

**Paul Choboter**  
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**J. S. Allen, R. M. Samelson**

Pedlosky's (1978) dynamical model of coastal ocean upwelling is elegantly simple, yet complex enough to possess rich and subtle dynamics. This model continues to generate new insights into the dynamics of upwelling. Solutions to the Pedlosky model are compared with numerical simulations to study the effects of the bathymetric profile on the structure of velocity and density during upwelling. When the bathymetric profile has positive curvature, the deep onshore-directed flow is surface-intensified. When the bathymetric profile has negative curvature, such as over a continental shelf break, the deep onshore flow is bottom-intensified. For negative curvature of sufficient magnitude, the dynamical model solutions exhibit features of an obstacle problem, which is a free-boundary problem that arises in the study of elliptic partial differential equations. These features make the problem challenging to solve, either analytically or numerically. Recent progress toward solving the problem is discussed.

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### Locating coherent structures in turbulent flows using the geodesic theory of transport barriers

**Mohammad Farazmand**  
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**George Haller**

We use the recently developed geodesic theory of transport barriers to locate a variety of Lagrangian Coherent Structures (LCSs) in two-dimensional turbulent flows. We review the numerical challenges in the implementation of the theory, and describe a numerical algorithm that addresses these challenges. The algorithm is in turn illustrated on direct numerical simulations of decaying and forced Navier–Stokes

turbulence. In particular, we identify hyperbolic barriers (generalized stable and unstable manifolds) and elliptic barriers (Lagrangian vortex boundaries) in the flow. The latter barriers enclose coherent vortices that are more robust and live longer than typical vortices in turbulence. We also identify a systematic difference in the size of Lagrangian eddies in forced and decaying turbulence.

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### Using Lagrangian coherent structures to understand coastal water quality

**Laura Fiorentino**  
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**M.J. Olascoaga, F.J. Beron-Vera, A Reniers, Z Feng**

The accumulation of pollutants near the shoreline can result in low quality coastal water levels with negative effects on human health. To understand the water quality of a particular beach requires the study of the Lagrangian circulation in the vicinity of the beach in question. Here we demonstrate the utility of dynamical systems methods in achieving such an understanding. The specific dynamical system notion considered is that of Lagrangian Coherent Structure (LCS). Hidden in the currents, the LCSs constitute the centerpieces of patterns formed by fluid particle trajectories. As such, the LCSs act as skeletons of the Lagrangian circulation. Particular focus is placed on Hobie Beach, a recreational marine beach located in Virginia Key in Miami, Florida. According to studies of water quality, Hobie Beach is characterized by high microbial levels. Possible sources of pollution in Hobie Beach include human bather shedding, dog fecal matter, and sand efflux at high tides. Consistent with the patterns formed by satellite-tracked drifter trajectories, the LCSs extracted from simulated currents reveal a Lagrangian circulation which favors the retention near the shoreline of pollutants released along the shoreline, which can help explain the low quality water levels registered at Hobie Beach.

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**Sea ice processes in Antarctic polynyas****Kenneth Golden**University of Utah, USA  
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The largest polynya observed in the Antarctic was the Weddell Polynya of 1975-77. The Antarctic Zone Flux Experiment (ANZFLUX) in 1994 was conducted to explore the dynamic region of the Southern Ocean where this polynya appeared. Large vertical heat fluxes from convective overturning and strong winter storms subject the sea ice pack to powerful forces that affect its growth and decay, as well as the robust ecosystems living within the ice. We will discuss various processes that are active in this region and the Southern Ocean at large, such as snow ice formation, nutrient replenishment, brine drainage, and enhanced thermal exchanges due to brine convection. Moreover, we will describe recent progress in the mathematics of sea ice that was spawned by our findings from ANZFLUX and subsequent expeditions.

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**Seasonality of the circulation on the West Florida Shelf****Yonggang Liu**University of South Florida, USA  
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Seasonal variations of the circulation on the West Florida Continental Shelf (WFS) are described using sustained (1998-2009) observations of velocity from an array of moored acoustic Doppler current profilers, plus various ancillary data. A robust seasonal circulation cycle is found, which varies in a dynamically sensible way across the shelf. Over most of the inner shelf these seasonal variations are primarily in response to local wind forcing, through Ekman-geostrophic spin-up, as previously found for the synoptic scale variability. Thus the inner shelf circulation is predominantly upwelling favorable during fall to spring months (October-April) and downwelling favorable during summer months (June-September). Seaward from about the 50 m isobath, where the buoyancy forcing becomes of increasing importance, the distinctive inner shelf seasonal variations blend into a continuum of variability. Over the outer shelf and near the southwestern end of the WFS, the seasonal circulations are obscured by the deep ocean effects of the Gulf of Mexico Loop Current and its eddies.

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**The connection between the Loop Current excursions and the Florida Red Tides****Grace Maze**University of Miami, USA  
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Red tides are found all over the world, however they are not all caused by the same organism. In the Gulf of Mexico the red tides are caused by a slow growing dinoflagellate, *Karenia brevis*. *K. brevis* is found in background concentration over most of the Gulf, but tends to produce the most intense and frequent blooms on the West Florida Shelf (WFS). *K. brevis* releases a suite of neurotoxins called brevetoxins. These toxins cause large kills in marine life and is also hazardous to humans who come in contact with the aerosolized toxin or eat contaminated shellfish.

Lots of work has been done to try to determine the physical mechanisms responsible for the red tides since the biological growth and reproduction rates are not enough to account for the rapid increase in concentration associated with a bloom. However, none of the research presented to date has come up with a definitive answer that accounts for the majority of recorded blooms in history.

This presentation will examine the influence of the position of the Loop Current and different wind regimes on the concentration of *K. brevis* on the WFS in conjunction with some of the theories of red tide initiation.

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**Lagrangian transport analysis of the surface ocean circulation in the Gulf of Mexico****Maria Olascoaga**RSMAS/UM, USA  
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Analysis of drifter trajectories in the Gulf of Mexico (GoM) has revealed the existence of a region on the southern portion of the West Florida Shelf (WFS) that is not visited by drifters that are released outside of the region. This so-called “forbidden zone” (FZ) suggests the existence of a persistent cross-shelf transport barrier on the southern portion of the WFS. Seven-year-long records of surface currents produced by a Hybrid-Coordinate Ocean Model simulation of the GoM are used to compute Lagrangian coherent structures (LCSs), which reveal the presence of a persistent cross-shelf transport barrier in approximately the same location as the boundary of the FZ. The location of the cross-shelf transport barrier follows an oscillation, being closer to the coast when the Loop Current is in its northernmost location. The analysis also suggests the existences of other two regions with similar isolated characteristics on the Texas-Louisiana Shelf and the Yucatan Shelf. Implications of the results for the dispersal of pollutants, such as oil, in the GoM are discussed.

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### Coastal variability and Lagrangian circulation in Todos Santos Bay and off Baja California during Spring-Summer 2007

**David Rivas**  
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The circulation in Todos Santos Bay area (31.88N) and off Baja California is studied for period Spring-Summer 2007, when an intense toxic algal bloom occurred. High-resolution, numerical model simulations were carried out to study dynamical features along the coast of the northern Baja California (BC) Peninsula and within Todos Santos Bay (TSB), and also to be used in a three-dimensional Lagrangian analysis which provides information about the origin and distribution of the waters present in the Bay during the occurrence of the toxic bloom. The regional dynamics reproduced by the model include the poleward propagation of coastal trapped waves along the BC coast, the so-called Ensenada front, and a persistent cyclonic eddy formed northwest of TSB. The Lagrangian results show that these last two features drive the paths followed by the water parcels found in TSB in Spring 2007. Most of those waters come from locations west of TSB (even beyond the model's domain), stay within the Bay by about one month, and ultimately scatter south-southwestward along the BC coast.

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### Individual trajectory complexity methods & an upwelling flow example

**Sherry Scott**  
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**A. Ghazaryan, D. Rivas, I. Rypina, A. Wert**

We consider the analysis of fluid flows in terms of the complexity of the individual trajectories. Specifically, we explore the complexity of the fluid particle trajectories using two measures - the correlation dimension and the ergodicity defect. The goal is to use these measures to reveal structures resembling Lagrangian coherent structures in the flow. We test the technique with upwelling flow data from the Oregon Coast during 2005. As these measures use properties of individual trajectories, and not separation rates between closely spaced trajectories, they may have advantages for the analysis of typical float and drifter data sets in which trajectories are widely and non-uniformly spaced.

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### Identification and tracking of coherent Agulhas Current rings

**Yan Wang**  
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**F. J. Beron-Vera, G. Haller, M. J. Olascoaga, G. J. Goni**

We introduce a Lagrangian method, referred to as "geodesic method", for objectively identifying and tracking oceanic mesoscale eddies in altimetry sea-surface height (SSH) datasets. The geodesic method derives from the geodesic theory of transport barriers in two-dimensional flows by Haller and Beron-Vera (2012). Traditional Eulerian methods as the Okubo-Weiss parameter-based method and Chelton et al.'s (2011) SSH-based automated eddy identification procedure are also applied in the same region for comparison. Our findings indicate that unlike traditional Eulerian methods, the geodesic method is capable of correctly locating and tracing coherent Agulhas Current rings. Implications of the results for the computation of transport are discussed.

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### An overview of models for the opening of coastal polynyas

**Andrew Willmott**  
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**Miguel Morales Maqueda**

Polynyas, polar oceanic regions with little sea ice cover during the winter, are recurrent features found in certain oceanic locations. They support rich marine ecosystems and they are important sites for the formation of dense water that contributes to the maintenance of the global thermohaline circulation.

During the past 20 years a mathematical framework for describing the opening, and closing, of coastal polynyas has been developed based on equations describing the thermodynamic production of sea ice and its subsequent transport offshore/onshore. These "flux models" are amenable to analytical treatment under certain assumptions. All flux models predict that under steady forcing (i.e. ice production rate and offshore wind stress) a coastal polynya will open to a steady-state width. This presentation will report on recent research that shows this behaviour is an artefact of a long-standing assumption in flux models about the behaviour of sea ice near the polynya edge. In this presentation a new model for the opening of a coastal polynya will be presented that relaxes this assumption. Whether or not a polynya reaches a steady-state width is shown to be dependent upon the rheological description of the sea ice.

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**Geostrophic adjustment in a polar basin****Andrew Willmott**National Oceanography Centre, England  
ajwil@noc.ac.uk**Maria Luneva, Miguel Morales Maqueda**

The geostrophic adjustment of homogeneous fluid in a circular basin with idealised topography is addressed using a numerical ocean circulation model and analytical process models. When the basin is uniformly rotating, the adjustment takes place via excitation of boundary propagating waves, and when topography is present, via topographic Rossby waves. In the numerically derived solution, the waves are damped due to bottom friction, and a quasi-steady geostrophically balanced state emerges that subsequently spins-down on a long-time scale. It is demonstrated that the adjusted states emerging in a circular basin with a step escarpment or a top hat ridge, centred on a line of symmetry, are equivalent to that in a uniform depth semi-circular basin, for a given initial condition. These quasi-steady solutions agree well with linear analytical solutions for the latter case in the inviscid limit.

On the polar plane, the high latitude equivalent to  $\beta$  - plane, no quasi-steady adjusted state emerges from the adjustment process. In the intermediate time-scales, after the fast Poincaré/Kelvin waves are damped by friction, the solutions take the form of steady-state adjusted solutions on the f-plane. On the longer timescales, planetary waves control the flow evolution and the dynamics of these waves will be discussed.

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## Special Session 75: Heteroclinic Cycles: Theory and Applications

**Peter Ashwin**, University of Exeter, UK  
**Pascal Chossat**, CNRS University of Nice, France  
**Reiner Lauterbach**, University of Hamburg, Germany

Heteroclinic cycles were first studied in the 1980's to explain aperiodic switching between different steady-states observed in certain hydrodynamical experiments. These flow invariant objects are non generic, however they can be structurally as well as dynamically stable when symmetries or other structures are imposed on the system. Symmetry-breaking bifurcation theory is typically used to study them. Since their discovery, further applications have been found as well as new mechanisms to create such cycles. Typical questions relate to notions of stability, understanding of nearby dynamics and the effects of forced symmetry breaking.

The aim of the workshop is to collect together various approaches and up-to-date results. We aim to bring together mathematicians and other scientists to increase interactions between the more theoretical aspects and applications. Recent progress in experimental and computational neuroscience and other life sciences lead us to expect that new applications of heteroclinic cycles will show up in these domains.

We also aim to bring together established researchers and younger scientists, in order to generate new ideas and to provide input for new developments.

### Bifurcation of robust heteroclinic cycles in spherically invariant systems with $\ell = 3, 4$ mode interaction

**Philippe Beltrame**

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**Pascal Chossat**

Bifurcation with spherical symmetry and mode interaction is known to produce robust heteroclinic cycles between axisymmetric steady-states as it occurs for the mode interaction with representation of  $SO(3)$  of degrees  $l=1$  and  $2$  in the context of the onset of Rayleigh-Bénard convection. The existence of such heteroclinic cycles for the mode interaction between two consecutive modes  $(l, l+1)$  was proved generically. However the  $(3, 4)$  mode interaction is an exception of this last analyze. Moreover it may correspond to the onset of convection of an experiment performed recently in the International Space Station designed in order to provide a system with (nearly) spherical symmetry (GeoFlow project). Motivated initially by this experiment, we have analyzed the occurrence of robust heteroclinic cycle for this  $(3, 4)$  mode interaction. This case is highly complex but, applying the methods of equivariant bifurcation theory, we have shown the existence of (generalized) robust heteroclinic cycles involving, not only axisymmetric states, but also and principally states with cubic symmetry. These objects are observable in the numerical simulations of the dynamics on the center manifold for parameter values relevant as well for geophysics and as for the GeoFlow project.

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### Robust heteroclinic cycles in delay-differential equations

**Pietro-Luciano Buono**

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Robust heteroclinic cycles in delay-differential equations can arise naturally on finite-dimensional centre manifolds; in this case, the theory is straightforward. I will discuss an example of a heteroclinic cycle in a symmetric delay-differential equation which does not lie in a finite-dimensional submanifold and robust with respect to a specific class of perturbations. These are called Inherently Infinite Dimensional Cycles. The example is a generalization of the Guckenheimer-Holmes cycle seen as a delay-coupled system of three cells. I will also present some numerical simulations and some open questions.

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### Stability and dynamics along a heteroclinic network

**Sofia Castro**

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The literature on heteroclinic networks has introduced several notions of stability that relate to attraction properties of the network. At the same time, different types of complex behaviour near the network have been studied. These are often associated to the existence of some type of switching at nodes or along connections of the network. The object of this talk is the discussion of the relation between different types of stability and admissible types of dynamics that may be observed. This will be illustrated by means of a Rock-Scissors-Paper game with two players and a problem in convection.

Some of the results presented in this talk were obtained jointly with M. Aguiar, I. Labouriau and O. Podvigina.

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### Characteristic features of the heteroclinic networks with a child-cycle

**Tsuyoshi Chawanya**

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Heteroclinic networks with various types of structure can be found in dynamical systems with a certain class of constraints, including heteroclinic networks with depth larger than one, i.e. network with connections from fixed point saddle to saddle-like heteroclinic cycle/network (child-cycle/network). In this talk, we will mainly concentrate on the origin of effective nonlinearity that induces complex phenomena like the coexistence of infinitely many sequence of attractors in the neighborhood of such heteroclinic networks.

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### Bifurcation of a heteroclinic network in a problem of pattern formation in the Poincaré disk

**Gregory Faye**

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**Pascal Chossat**

We consider the spontaneous formation of steady solution of a PDE or integro-differential equation set on the Poincaré disk  $\mathbb{D}$ , which is invariant under the action of the octagonal lattice group  $\Gamma$ . This problem was introduced as an example of spontaneous pattern formation in a model of image feature detection by the visual cortex where the features are assumed to be represented in the space of structure tensors. Under "generic" assumptions the bifurcation problem reduces to an ODE which is invariant by an irreducible representation of the group of automorphisms  $\mathcal{G}$  of the compact Riemann surface  $\mathbb{D}/\Gamma$ . The irreducible representations of  $\mathcal{G}$  have dimension one, two, three and four. In this presentation, we show that for one of the four-dimensional representation there is a generic bifurcation of a heteroclinic network connecting equilibria with two different orbit types. We also present some results about the stability of this heteroclinic network together with some numerical simulations.

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### Heteroclinic cycles in complex systems

**Michael Field**

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We describe recent work on robust heteroclinic cycles in large asymmetric adaptive networks of approximately identical cells and as well as in hybrid networks mixing deterministic and random dynamics. We also describe some examples using a SLOGALS architecture (SLOppy Globally Asynchronous Locally Synchronous).

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### Dynamics of codimension one homoclinic cycles

**Juergen Knobloch**

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We discuss the dynamics near codimension one homoclinic cycles in flows that are equivariant under actions of the group  $D_m$ . We show how suspensions of subshifts of finite type appear in the unfolding. The descriptions are provided in terms of the geometry of the homoclinic cycle. The analysis is not complete for a certain type of  $D_m$  cycles. We show how those cycles can be constructed, and we present a more complete picture of the dynamics based on numerical simulations.

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### Asymptotic stability of robust heteroclinic cycles

**Maciej Krupa**

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**Ian Melbourne**

In this talk I present the results on asymptotic stability of heteroclinic cycles obtained in joint work with Ian Melbourne. In our first article we identified a class of cycles for which we could find an optimal condition for stability. However our condition was not optimal for another class of cycles studied by Hofbauer and Sigmund and by Field and Swift. These authors obtained a different optimal condition using a transition matrix technique. In a later work we combined the two approaches, generalizing both of them. Finally in a recent work, we obtained a further generalization of the transition matrix method

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### On relations between the stability index and attraction properties of heteroclinic cycles

Alexander Lohse

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In this talk I will present recent results on the relationship between the stability index for heteroclinic cycles that Peter Ashwin and Olga Podvigina defined in their paper "On local attraction properties and a stability index for heteroclinic connections" and different stability properties of the cycle. The index quantifies the extent of the basin of attraction of the cycle in a small  $\varepsilon$ -ball around it. It is related to stability properties of the cycle, but not always in an obvious way. For example, if there is a point on the cycle where the index exists and is greater than  $-\infty$ , then the cycle is essentially asymptotically stable (e.a.s.). A similar result can be shown for predominant asymptotic stability (p.a.s.) of the cycle in the case that the index exists everywhere and is greater than some constant  $c > 0$ . On the way to proving these results we establish a more general statement, namely that any cycle with a set of positive measure in its basin of attraction is e.a.s. One crucial idea for the proof of this was developed by Olga Podvigina.

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### Synaptic cellular automaton for description the sequential dynamics of excitatory neural networks

Vladimir Nekorkin

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Alexey S. Dmitrichev, Dmitri V. Kasatkin

Neurophysiological experiments have indicated that some neural processes are accompanied by transient short-time activity of individual neurons or small groups of neurons. In such a process, a sequence of activity phases of different neurons emerges successively in time. It was shown that this behavior can be related to the existence of a collection of metastable invariant sets joined by heteroclinic trajectories in the phase space (heteroclinic network) and thus can be thought of as a process of successive switching among these metastable sets. There are some models in which the heteroclinic network can be rigorously established. However, it is not always a simple problem. We have proposed an approach of the reduction of continuous sequential dynamics of excitatory neural networks to a discrete one, in the form of a cellular automata (CA) on the graph of connections. In our approach the main role is played by the dynamics of synapses but not by the specific features of neurons. The CA represents a network of synapses with a finite number of states which alternate with each other, according to some

fixed rules. We illustrate our approach on an example of network of Morris-Lecar neurons coupled by chemical synapses with short-term plasticity.

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### Stability and bifurcations of heteroclinic cycles of type Z

Olga Podvigina

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Dynamical systems, equivariant under the action of a non-trivial symmetry group, can possess structurally stable heteroclinic cycles. We consider stability properties of a class of structurally stable heteroclinic cycles in  $\mathbb{R}^n$ , which we call heteroclinic cycles of type Z. It is well-known that a heteroclinic cycle, that is not asymptotically stable, can attract nevertheless a positive measure set from its neighbourhood. We call such cycles fragmentarily asymptotically stable. Necessary and sufficient conditions for fragmentary asymptotic stability are expressed in terms of eigenvalues and eigenvectors of transition matrices. Finally, we discuss bifurcations occurring when the conditions for asymptotic stability or for fragmentary asymptotic stability are broken.

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### Resonance of robust heteroclinic networks

Claire Postlethwaite

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Vivien Kirk, Alastair Rucklidge

It is well known that heteroclinic cycles and networks can exist robustly in systems with symmetry. Resonance bifurcations are one way in which heteroclinic cycles can change stability. Such bifurcations occur when an algebraic condition on the eigenvalues of the equilibria in the cycle is satisfied, and generically are accompanied by the birth or death of a long-period periodic orbit. Although resonance bifurcations of heteroclinic cycles have been extensively studied, very little is known about resonances of heteroclinic networks. In this talk, I will describe new work on understanding resonance bifurcations of heteroclinic networks. In a network, at least one unstable manifold is two-dimensional; I will describe a technique to account for all the trajectories on these manifolds. We find that the sub-cycles of the network undergo resonance bifurcations as might be expected if they were isolated from the network. There is an additional resonance point due to the structure of the network at which the periodic orbits bifurcating from the different sub-cycles of the network interact.

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**Heteroclinic bifurcations near non-reversible homoclinic snaking****Thorsten Riess**University of Konstanz, Germany  
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Non-reversible homoclinic snaking is a scenario where the bifurcation curve of a codimension-one homoclinic orbit to a hyperbolic equilibrium exists on a snaking curve in parameter space. We consider systems without any particular structure and give analytical and geometric statements when such a scenario is to be expected. The numerical analysis of heteroclinic bifurcations near the non-reversible homoclinic snaking is presented, in particular equilibrium-to-periodic heteroclinic connections.

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**Heteroclinic phenomena****Alexandre Rodrigues**Sciences Faculty Oporto University, Portugal  
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For systems with symmetry, it is possible to find robust heteroclinic cycles and networks, which may be seen as the skeleton for the understanding of complicated dynamics. In this talk, we discuss the geometric behaviour of trajectories in a neighbourhood of a special class of heteroclinic networks, near which we observe chaotic dynamics. More precisely, we present two persistent phenomena which we call switching and double cycling, and some dynamical consequences.

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**Timing control of networks with switching dynamics****Irma Tristan**University of California San Diego, Mexico  
itristanlopez@ucsd.edu**Nikolai Rulkov, Ramon Huerta, Mikhail Rabinovich**

Timing control is a fundamental principle of cognition and behavior. The idea that intrinsic dynamics of the neuronal networks allows them to generate the different time intervals for the spatiotemporal encoding is supported by theoretical and experimental studies. Still the mechanisms by which this temporal control is achieved remain unclear. We have focused on the properties of inhibitory neuronal network sequential dynamics since they are a substantial factor on the generation of a rhythm in central pattern generators. Here we consider the new mechanism that is related to the multi-neuronal network dynamics - winnerless competition (WLC) between three unidirectionally inhibitory clusters of Hodgkin-Huxley modeled neurons inducing closed limit cycles that remind the heteroclinic cycles included the saddle cycle. Findings indicate a sensitive dependence of the sequential timing on the effective synaptic strength of the connections between clusters. For our particular purpose, neurons within the clusters were not connected supporting the idea that such connectivity is not necessary for timing control of the network. Role of the cluster size on the control cycling activity timing is discussed.

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## Special Session 76: On PDEs from Biology

Alexander Lorz, University Pierre et Marie Curie - Paris 6, France

Partial differential equations are an extremely effective and powerful tool in mathematical biology. This session will focus on PDE-based models in a wide range of areas including, but not limited to, chemotaxis, population dynamics, collective behavior, self-propulsion and aggregation.

### Selection-mutation dynamics

**Pierre-Emmanuel Jabin**  
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We present several integro-differential nonlinear models that describe the evolution of a population structured by a quantitative trait. The interactions between traits occur from competition, for example for resources whose concentrations depend on the current state of the population; mutations are also taken into account. Complex concentration phenomena arise in the limit of strong selection and small mutations. We also describe several modifications taking the effect of small populations into account.

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### Asymptotic dynamics in structured populations endangered by global warming and habitat shrinking

**Tommaso Lorenzi**  
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**Giorgio Restori**

This talk deals with a class of differential equations relying on a continuous selection/mutation formalism and describing the dynamics of species exposed to the selective pressures exerted by the surrounding environment. In particular, we present a model for the dynamics of a population structured by two phenotypic parameters related to the sensitivity of individuals to global warming and habitat shrinking. The results of asymptotic analysis and numerical simulations are presented. They highlight how climate change and soil consumption induced by human activities could qualitatively modify the evolutionary dynamics of species, eventually pushing them to the edge of extinction.

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### Self-propulsion in viscous fluids through shape deformation

**Marco Morandotti**  
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**Gianni Dal Maso, Antonio DeSimone**

I will present a model for micro-swimmers in viscous fluids, both plain and particulate. Given the Reynolds number is very low, Stokes' and Brinkman's equations can be used to govern the velocity and the pressure of the surrounding, infinite fluid. Imposing a no-slip boundary condition, allows to relate the deformation of the swimmer to the fluid velocity field, while self-propulsion is the constraint through which we can reduce, via an integral representation of the viscous forces and momenta, the equations of motion for the swimmer to a system of six ODEs. Under mild regularity assumptions, an existence and uniqueness theorem for the motion is proved. Eventually, I will focus on the case of a flagellum swimming in a viscous fluid. In this case, the equations of motion are derived from an approximate theory, and optimality results are discussed. This is partially joint work with Gianni Dal Maso and Antonio DeSimone (SISSA, Trieste, Italy).

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### Hydrodynamic models of self-organized dynamics

**Sebastien Motsch**  
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**Pierre Degond, Jian-Guo Liu, Vladislav Panferov**

We present the derivation and analysis of hydrodynamic models for systems of self-propelled particles subject to alignment interaction and attraction-repulsion. By introducing appropriate scalings, we show that the non-local effects of the alignment and attraction-repulsion interactions can be kept in the hydrodynamic limit and result in extra pressure, viscosity terms and capillary force. The systems are shown to be symmetrizable hyperbolic systems with viscosity terms.

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### Spatio-temporal chaos in models for chemotaxis

**Kevin Painter**

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**T. Hillen**

Chemotaxis, the directed migration of cells or organisms in response to external chemical signals, is a widely utilised navigational aid throughout biology. In certain instances, for example bacterial populations and the slime mold *Dictyostelium discoideum*, chemotaxis is a driving mechanism behind the self-organisation of a dispersed population into a self supported aggregate.

The Keller-Segel equations have been widely employed in the modelling of chemotactic populations, with their capacity for symmetry breaking a key factor behind their success. In this talk I shall concentrate on the wide variety of spatio-temporal patterns generated and their ability to demonstrate spatio-temporal chaos. I will conclude with a discussion of some potential applications, including in tumour invasion and bacterial organisation.

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### On global well-posedness for reaction-advection-diffusion systems for chemotaxis with growth and crime patterns

**Nancy Rodriguez**

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There are many seemingly unrelated phenomena that can be modeled with similar PDE systems. In particular, in this talk I will briefly discuss a connection between crime patterns and chemotaxis. In fact, we note that these two phenomena can be modeled by system that are very alike. I will then present some global existence results for a class of reaction-advection-diffusion models, which were originally developed as basic models for crime-patterns. However, they can be seen as a chemotaxis systems with a source. These models have two mechanisms to present finite time blow-up, I will discuss the importance of both mechanisms.

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### Continuum limits for discrete models of collective behavior

**Jesus Rosado Linares**

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We review some of the individual based models for collective behavior of agents, called swarming. These models based on ODEs exhibit a complex rich asymptotic behavior in terms of patterns, that we show numerically. Moreover, we comment on how these particle models are connected to partial differential equations to describe the evolution of densities of individuals in a continuum manner and on the stability issues of these PDE models.

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### Individual based and mean-field modelling of direct aggregation

**Marie-Therese Wolfram**

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**Martin Burger, Jan Haskovec**

We introduce two novel models of biological aggregation, based on randomly moving particles with individual diffusivities depending on the perceived average population density in their neighbourhood. In the first-order model the location of each individual is subject to a density-dependent random walk, while in the second-order model the density-dependent random walk acts on the velocity variable, together with a density-dependent damping term. The main novelty of our models is that we do not assume any explicit aggregative force acting on the individuals; instead, aggregation is obtained exclusively by reducing the diffusivity in response to higher perceived density. We formally derive the corresponding mean-field limits, leading to nonlocal, possibly degenerate diffusions. Then, we carry out the mathematical analysis of the first-order model, in particular, we prove the existence of weak solutions and show that it allows for measure-valued steady states. We also perform linear stability analysis and identify conditions for pattern formation. Moreover, we discuss the role of the nonlocality for well-posedness of the first-order model. Finally, we present results of numerical simulations for both the first- and second-order model on the individual based and continuum levels of description.

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**On inverse problems for some structured population PDEs**

**Jorge Zubelli**  
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Structured population models in biology lead to integro-differential equations that describe the evolution in time of the population density taking into account a given feature such as the age, the size, or the volume. These models possess interesting analytic properties and have been used extensively in a number of areas. After giving an introduction to this subject, we will discuss the inverse problem. In this part, we consider a size-structured model for cell division and address the question of determining the division (birth) rate from the measured stable size distribution of the population. We formulate such question as an inverse problem for an integro-differential equation posed on the half line. We develop firstly a regular dependency theory for the solution in terms of the coefficients and, secondly, a regularization technique for tackling this inverse problem which takes into account the specific nature of the equation. The work presented here was developed jointly with P. Maia (UW), M. Doumic (INRIA) and B. Perthame (UPMC)

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## Special Session 77: The Navier-Stokes Equations and Related Problems

Sarka Necasova, Mathematical Institute, Academy of Sciences, Prague, Czech Republic

Reimund Rautmann, Paderborn University, Germany

Werner Varnhorn, Institute of Mathematics - Kassel University, Germany

The Navier-Stokes equations represent the fundamental equations in mathematical fluid dynamics. Recently researchers from many countries around the world have developed important new results to the theory of these important equations and their applications: Existence proofs for weak or even strong solutions for time-dependent as well as for stationary boundary value problems of the fully nonlinear Navier-Stokes equations or for the linearized Stokes equations, considered with a wide variety of boundary conditions, uniqueness classes in the frame of suitably adapted abstract spaces, questions of asymptotic behavior and stability of solutions, and of maximum regularity, and convergence results with vanishing viscosity. A further important part of research is the interaction of fluid flow with moving bodies or particles or with additional heat flow, leading to enlarged dynamical systems which combine the Navier-Stokes equations with other equations of evolution. The strong progress in theory opens the way to efficient numerical schemes for problems in technology and medicine. The aim of our special session will be to bring together leading researchers from all parts of the world and from different working directions as mentioned above, and to initiate exchange of ideas as well as future cooperations.

### The $L^\infty$ -Stokes semigroup in exterior domains

Ken Abe

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Yoshikazu Giga

Analyticity of semigroups is one of fundamental properties for semigroups associated with parabolic equations. In this talk we study analyticity of the Stokes semigroup in spaces of bounded functions both in bounded and unbounded domains. In particular, we consider the Stokes system in an exterior domain as a typical example of an unbounded domain. Even the existence of solution is nontrivial in that case. For the proof of analyticity of the Stokes semigroup, we appeal to a priori  $L^\infty$ -estimates which is derived from a blow-up argument.

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### $L^p$ -theory for Stokes and Navier-Stokes equations with non-standard boundary conditions

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Nour El Houda Seloula

We consider here elliptical systems as Stokes and Navier-Stokes problems in a bounded domain, eventually multiply connected, whose boundary consists of multi-connected components. We investigate the solvability in  $L^p$  theory, with  $1 < p < \infty$ , under the non standard boundary conditions

$$\begin{aligned} \mathbf{u} \cdot \mathbf{n} &= g, & \operatorname{curl} \mathbf{u} \times \mathbf{n} &= \mathbf{h} & \text{or} \\ \mathbf{u} \times \mathbf{n} &= g, & \pi &= \pi_* & \text{on } \Gamma. \end{aligned}$$

The main ingredients for this solvability are given by the Inf-Sup conditions, some Sobolev's inequalities

for vector fields and the theory of vector potentials satisfying

$$\boldsymbol{\psi} \cdot \mathbf{n} = 0, \quad \text{or} \quad \boldsymbol{\psi} \times \mathbf{n} = \mathbf{0} \quad \text{on } \Gamma.$$

Those inequalities play a fundamental key and are obtained thanks to Calderon-Zygmund inequalities and integral representations. In the study of elliptical problems, we consider both generalized solutions and strong solutions that very weak solutions.

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### Boundary regularity for the steady Stokes type flow with shear thickening viscosity

Hyeong-ohk Bae

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Jorg Wolf

We work on the boundary regularity for weak solutions to the stationary Stokes type equations with shear dependent viscosity. Using a weighted estimate near the boundary, we obtain the Holder continuity of the solution for the shear thickening fluid without the convection term.

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### An obstacle problem for capillary surfaces

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G.P. Galdi, M. Kyed

Let  $G := \Omega \times \mathbb{R}^+$ ,  $\Omega \subseteq \mathbb{R}^2$  a bounded domain, be a cylinder that is partly filled with liquid;  $\mathcal{B}$  is a rigid body that is floating on it, and the interface between the fluid and the air above is described as a graph of a real function  $u$ .

If we assume that the shape of the interface is governed by surface tension then the unknowns of the problems, which are the function  $u$  and the position of  $\mathcal{B}$ , are determined by a variational problem for the energy  $\mathcal{E}$  of the configuration.

$\mathcal{E}$  consists of the interfacial energy which is proportional to the area of the graph of  $u$ , the adhesion energy, which is proportional to the wetted part of the boundary of  $\mathcal{B}$  and of the cylindrical boundary  $\partial\Omega \times \mathbb{R}^+$  as well as the gravitational energies of the fluid and of the floating body.

Because of the presence of  $\mathcal{B}$  the capillary surface  $\Sigma$  is bounded by some curve on  $\partial\Omega \times \mathbb{R}^+$  as well as a contact line  $\Gamma$  on  $\partial\mathcal{B}$ ; therefore in our formulation of the variational problem the body  $\mathcal{B}$  acts as an obstacle for  $u$ .

We show the existence of a minimizer and investigate some of its properties, in particular the regularity. This is based on the first variation of the energy which also gives a variant of Archimedes' principle that includes the forces exerted on  $\mathcal{B}$  by  $\Sigma$ .

#### References

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#### Maximum modulus estimate in nonstationary Stokes system

**Hi Jun Choe**  
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**TongKeun Chang**

A maximum modulus estimate for the nonstationary Stokes system in smooth domain is found. The singular part and regular part of Poisson kernel are analyzed. The singular part is a gradient potential related to only normal component of the boundary data. Furthermore, the normal velocity near the boundary is bounded if the boundary data is bounded. If the normal component of the boundary data is Dini-continuous and the tangential component of the boundary data is bounded, then the maximum modulus of velocity is bounded in whole domain

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#### Some properties of solutions to liquid crystal systems

**Mimi Dai**  
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**Jie Qiang, Maria Schonbek**

In this talk, I will present some work on Liquid Crystal systems. The existence of global weak solutions and regularity of solutions to the nematic liquid crystals with non-constant fluid density were established. We also established a long time behavior result for the regular solutions to the liquid crystal system with constant density, providing small initial condition.

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#### Pointwise decay of incompressible flows around rigid bodies

**Paul Deuring**  
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We consider pointwise spatial decay of flows around rigid bodies moving steadily in an incompressible viscous fluid.

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#### Well-posedness of the Euler equations in planar convex domains

**Francesco Di Plinio**  
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**Madalina Petcu, Roger Temam**

Let  $\Omega \subset \mathbb{R}^2$  be a bounded convex domain. We consider the well-posedness of the Euler equations

$$\begin{cases} \partial_t \mathbf{u}(x, t) + (\mathbf{u} \cdot \nabla) \mathbf{u}(x, t) + \nabla p(x, t) = \mathbf{f}(x, t), \\ x \in \Omega, t \in (0, T) \\ \nabla \cdot \mathbf{u}(x, t) = 0, \quad x \in \Omega, t \in (0, T) \end{cases}$$

with impermeability boundary conditions. For divergence-free initial data  $\mathbf{u}_0 = \mathbf{u}(t=0) \in H^1(\Omega)$ , we show the existence of a weak solution

$$\mathbf{u} = \mathbf{u}(x, t) \in L^\infty((0, T), H^1(\Omega)^2) \cap W^{1,2}((0, T); L^p(\Omega)^2)$$

to the Euler equations. Under the additional assumptions that  $\Omega$  is a polygonal-type domain, and that  $\text{curl } \mathbf{u}_0 \in L^\infty(\Omega)$ , the obtained solution  $\mathbf{u}$  is unique, and possesses the additional regularity

$$D\mathbf{u} \in L^\infty((0, T), \text{Exp}L^1(\Omega)).$$

Even for smooth domains, these results improve on the classical theorem of Kato; they also extend to

polygonal-type domains the uniqueness theorem of Yudovich, thus partly improving results by Taylor, and Gérard-Varet and Lacave.

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### On the steady equations for compressible radiative gas

**Ondřej Kreml**

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**Šárka Nečasová, Milan Pokorný**

We study the equations describing the steady flow of a compressible radiative gas with newtonian rheology. Under suitable assumptions on the data which include the physically relevant situations (i.e. the pressure law for monoatomic gas, the heat conductivity growing with square root of the temperature) we show the existence of a variational entropy solution to the corresponding system of partial differential equations. Under additional restrictions we also show the existence of a weak solution to this problem.

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### Weak solutions for the motion of a self-propelled deformable structure in a viscous incompressible fluid

**Sarka Necasova**

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**M. Tucsnak, T. Takahashi**

We deal with a model of swimming where we consider that the muscles of the creature are enough to perform a given deformation. Using this model we investigate the self-propelled motion in a viscous fluid. We study the existence of weak solution. The proof is based on a penalization method.

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### On a stationary compressible flow with slip-inflow boundary conditions

**Tomasz Piasecki**

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In my talk I am going to discuss the issue of existence of stationary solutions to the Navier-Stokes system describing the flow of a compressible fluid in a cylindrical domain. On the boundary we prescribe inhomogeneous slip conditions on the velocity. I will consider both the barotropic flow and the complete system with thermal effects. In both cases we show the existence of a solution in a vicinity of a given

special solution such as a constant flow with nonzero velocity or a Poiseuille-like profile. The main problem to face in the proof is the lack of compactness in the continuity equation, I will discuss different ways to overcome this problem, such as elliptic regularization, successive approximations or reformulation of the problem in a kind of Lagrangian coordinates. The results have been obtained together with Piotr B. Mucha and Milan Pokorný.

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### Hopf-Galerkin approach to vorticity transport & diffusion.

**Reimund Rautmann**

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To the initial boundary value problem of vorticity transport & diffusion in a smoothly bounded 3-dimensional domain we present a convergent Hopf-Galerkin scheme including error estimates locally in time.

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### Maximal regularity on cross-sections implies maximal regularity on a cylinder

**Juergen Saal**

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**Tobias Nau**

The proof of maximal regularity for general boundary value problems on domains with non-compact boundaries relies typically on intricate localization procedures. In the talk presented, I will show that for the  $L^p$ -approach such procedures can be avoided, if the domain (and the corresponding differential operator) is cylinder like, i.e., of the form  $\Omega = \Omega_1 \times \Omega_2$  and maximal regularity is known on  $\Omega_1 \subset \mathbb{R}^k$  and  $\Omega_2 \subset \mathbb{R}^m$ . For domains of this type there is a much more elegant approach based on operator-valued functional-calculus techniques. The approach has also some advantages in comparison to localization procedures, as e.g. it also applies to Lipschitz cross-sections. The methods apply to Stokes and Navier-Stokes equations as well, as will be demonstrated in the talk, too. This is a joint project with Tobias Nau at the University of Konstanz.

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**On questions of decay for solutions to liquid crystals**

**Maria Schonbek**  
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**M. Dai, J. Qing**

I will discuss the asymptotic behavior for solutions to a nematic liquid crystals system in the whole space of three dimensions. The fluid under consideration has constant density and small initial data.

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**The fractional Laplacian in an exterior domain**

**Tomas Schonbek**  
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**Leonardo Kosloff**

We study the fractional Laplacian in the complement of an obstacle and use our result to obtain results about solutions of some fluid dynamics equations in exterior domains. The work relies heavily on a generalization of the Fourier transform due to Ikebe and Ramm, and the characterization of the fractional Laplacian as a local operator due to Caffarelli and Silvestre.

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**Fluid-structure interaction problems in hemodynamics**

**Adelia Sequeira**  
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Blood flow interacts mechanically with the vessel wall, giving rise to pressure waves propagating in arteries, which deform under the action of blood pressure. In order to capture these phenomena, complex fluid-structure interaction (FSI) problems must be considered, coupling physiologically meaningful models for both the blood and the vessel wall. From the theoretical point of view, this is extremely difficult because of the high non-linearity of the problem and the low regularity of the displacement of the fluid-structure interface. So far, mathematical results have been obtained only in simplified cases. In this talk, simulations of the mechanical interaction between blood flow and vessel walls will be shown, based on a partitioned approach. A 3D FSI model in a compliant vessel is used to describe the pressure wave propagation. The 3D fluid is described through the Navier-Stokes equations (or a shear-thinning generalized Newtonian model) and the structure by a 3D hyperelastic model. In order to cope with the spurious reflections due to the truncation of the computational domain, several absorbing boundary

conditions are analyzed. This work has been done in collaboration with A. Moura, J. Janela, and A. Gambaruto.

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**On the motion of a fluid-rigid ball system at the zero limit of the rigid ball radius**

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**Takéo Takahashi**

We investigate the limit of a coupled system “rigid ball + Navier-Stokes liquid” when the radius of the ball goes to 0. Dashti and Robinson [Arch. Ration. Mech. Anal., 2011] solved this problem in dimension two, in the absence of rotation. Our aim is to improve their result, considering the two-dimensional case when the disk is also spinning and the general three-dimensional case.

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**(Almost) everything you always wanted to know about the Helmholtz decomposition but were afraid to ask.**

**Maria Specovius-Neugebauer**  
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The Helmholtz decomposition of  $L^p$ -spaces (and in its wake the definition of the Stokes operator) is a crucial tool in the theory of Navier-Stokes equations. This contribution collects results on this topic with a particular emphasis on why certain arguments do not work in particular situations.

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**Compressible Navier-Stokes equations with slip boundary conditions in time dependent domains**

**Jan Stebel**  
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**Eduard Feireisl, Ondřej Kreml, Šárka Nečasová, Jiří Neustupa**

We consider the compressible (barotropic) Navier-Stokes system on time-dependent domains, supplemented with slip boundary conditions. Our approach is based on penalization of the boundary behaviour, viscosity, and the pressure in the weak formulation. Global-in-time weak solutions are obtained.

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### Extensions of Serrin's condition for weak solutions of the Navier-Stokes equations

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**Reinhard Farwig, Hermann Sohr**

We consider weak solutions of the non-stationary Navier-Stokes equations in a smoothly bounded three-dimensional domain and develop some extensions of Serrin's uniqueness and regularity condition for this case.

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### On a new 3D model for incompressible Euler and Navier-Stokes equations

**Shu Wang**

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In this talk, we investigate some new properties of the incompressible Euler and Navier-Stokes equations by studying a 3D model for axisymmetric 3D incompressible Euler and Navier-Stokes equations with swirl. The 3D model is derived by reformulating the axisymmetric 3D incompressible Euler and Navier-Stokes equations and then neglecting the convection term of the resulting equations. Some properties of this 3D model are reviewed. Finally, some potential features of the incompressible Euler and Navier-Stokes equations such as the stabilizing effect of the convection are presented. (The research was supported by National Basic Research Program of China (973 Program, 2011CB808002), the NSFC (11071009) and PHR-IHLB (200906103))

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### Chemically reacting mixtures

**Ewelina Zatorska**

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Development of rigorous mathematical theory for reactive flows is of fundamental need for purposes of many practical applications. In most of them, one has to deal with multicomponent mixtures undergoing reactions that are, in general, completely reversible. In this talk I will summarize the recent results on the existence of solutions to the full Navier-Stokes system coupled with the species mass balance equations and present the open problems.

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## Special Session 78: Multiple Time Scale Dynamics With a View Towards Biological Applications

**Mathieu Desroches**, INRIA (Paris-Rocquencourt Centre), France  
**Maciej Krupa**, University of Le Havre/INRIA, France  
**Alexandre Vidal**, University of Evry, France

Bursting is a well known mechanism of oscillation occurring in slow/fast systems. More recently the canard phenomenon has been identified as providing a complementary route to sensitive oscillatory dynamics. This special session will cover topics related to special orbits of multiple time scale systems (bursting, canards, torus canards, MMOs) and their relation to each other. The talks will present theoretical results and their applications in life sciences (neuronal dynamics, endocrinology, epidemiology, etc).

### Torus canards in $R^3$

**Nick Benes**

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**A.M. Barry, T.J. Kaper, M.A. Kramer, J. Burke**

Torus canards are a higher-dimensional generalization of the classical canard orbits familiar from planar systems and arise in fast-slow systems of ordinary differential equations in which the fast subsystem contains a saddle-node bifurcation of limit cycles. Torus canards are trajectories that pass near the saddle-node and subsequently spend long times near a repelling branch of slowly varying limit cycles. They can arise in computational models of neurons in a narrow region between bursting and fast-spiking behavior, similar to the way traditional canards separate small amplitude orbits from relaxation oscillations. It has been shown in an elementary third-order system that the relative speed of the two subsystems affects the importance of torus canards. In the regime of fast rotation, the torus canards behave much like their planar counterparts. In the regime of slow rotation, the phase dependence creates rich torus canard dynamics and dynamics of mixed mode type.

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### Canard explosions in the templator model

**Morten Brons**

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The templator is a nonlinear differential equation model for a self-replicating chemical system. During one of the reaction steps adimer acts as a catalyst (template) for its own production. Mixed mode oscillations and canard explosions have been found numerically in the model. We show how the model can be recast as a singular perturbation problem with different definitions of the small parameter in different parameter regimes. Asymptotic determination of the canard point agrees very well with the numerical result. We also discuss how the canard point may be determined without explicitly determining a small parameter.

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### Canards and Hopf mechanisms in a model for perceptual rivalry

**Rodica Curtu**

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**Jonathan Rubin**

Stimulus tuning in a firing rate model for perceptual rivalry leads to mixed-mode oscillations (MMOs), a temporal pattern featuring alternating small- and large-amplitude oscillations. Key ingredients for the generation of MMOs are mutual inhibition, slow negative-feedback in the form of adaptation, and nonlinearity of the gain function. By exploiting a normal form calculation, we show that MMOs occur due to the interaction of canard and singular Hopf mechanisms as the stimulus strength approaches a critical regime.

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### Canards, inflection and excitability threshold

**Mathieu Desroches**

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**Martin Krupa, Serafim Rodrigues**

In this talk, we will revisit the "inflection line method" introduced in the early 1990s to characterize canard explosions geometrically and study their dependence on the time scale ratio  $\varepsilon$ . We will then apply this idea to the context of planar neuronal dynamics, where we will show that the inflection set provide an approximation to the excitability threshold in both integrator and resonator type neuron models.

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### Canards and MMOs in coupled oscillator systems

**Maciej Krupa**

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**Aziz Alaoui, Benjamin Ambrosio**

We present a study of mixed-mode oscillations in systems of coupled oscillators with one slow and one fast dimension. We use the combination of the slow/fast and coupled oscillator structures to reduce some of the dimensions. For two coupled oscillators we can reduce the problem to the classical folded node/folded saddle node case. For more than two coupled oscillators we use the recently developed approach of Wechselberger to generalize the analysis. We also consider some degenerate cases.

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### Shaping bursting by electrical coupling and noise

**Georgi Medvedev**

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We study a randomly perturbed slow-fast system of differential equations modeling electrical activity in networks of pancreatic beta cells. Our analysis reveals the interplay of the intrinsic properties of the cells, network topology, and noise in shaping network dynamics. In particular, we describe a transition from irregular spiking to synchronized bursting under the variation of the strength of coupling. We also provide a detailed analysis of synchronization in this model. Analytical results are illustrated by numerical simulations of conductance-based networks.

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### Canards in slow-fast piecewise-linear planar systems

**Enrique Ponce**

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**M. Desroches, E. Freire, S. J. Hogan, P. Thota**

In this talk, we will present recent results on the behavior of canard solutions in the context of planar piecewise-linear systems. We will focus on Liénard type systems, which is a sufficiently big class of interesting systems from the point of view of applications. We will show how periodic orbits of canard type appear in this context through a Hopf-like event and how his explosive evolution is related to a bifurcation at infinity. This will allow us to draw a parallel with the smooth case.

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### Transitional isochron portraits in biological models with multiple time-scales

**William Sherwood**

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Phase response is an important functional property of many oscillatory biological systems, and is frequently a key determinant of network behaviors such as synchronization. We have found that the phase response properties of multiple time-scale dynamical systems, e.g. neuronal and predator-prey models, depend heavily on the specific characteristics of their fast and slow subsystems, especially bifurcation structure. Computation of isochrons, manifolds of equivalent asymptotic phase, gives a global portrait of a system's intrinsic phase response properties (i.e. for perturbations of arbitrary strength and shape). In our analysis, we link isochronal curvature, phase response sensitivity, and time-scale separation. We illustrate our findings in neuronal and ecological models transitioning between activity modes, e.g. between spiking and bursting regimes.

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### Bifurcations of bursting polyrhythms in plausible 3-cell motifs

**Andrey Shilnikov**

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**Jeremy Wojick, Robert Clewley**

Motifs of three coupled cells are a common network configuration including models of biological central pattern generators. We describe a novel computational approach to reduce detailed models of central pattern generators to equation-less return mappings for the phase lags between the constituting bursting interneurons. Such mappings are studied geometrically as the model parameters, including coupling properties of inhibitory and excitatory synapses, or external inputs are varied. Bifurcations of the fixed points and invariant circles of the mappings corresponding to various types of rhythmic activity are examined. This reveals the organizing centers of emergent poly-rhythmic patterns and their bifurcations, as the asymmetry of the synaptic coupling is varied. These changes uncover possible biophysical mechanisms for control and modulation of motor-pattern generation. Our analysis does not require knowledge of the equations that model the system, and so provides a powerful new approach to studying detailed models, applicable to a variety of biological phenomena beyond motor control.

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**The dynamics underlying pseudo-plateau bursting in a pituitary cell model**

**Wondimu Teka**

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**Joel Tabak, Theodore Vo, Martin Wechselberger, Richard Bertram**

Pituitary cells of the anterior pituitary gland secrete hormones in response to patterns of electrical activity. Several types of pituitary cells produce short bursts of electrical activity which are more effective than single spikes in evoking hormone release. These bursts, called pseudo-plateau bursts, are unlike bursts studied mathematically in neurons (plateau bursting) and the standard fast-slow analysis used for plateau bursting is of limited use. Using an alternative fast-slow analysis, with one fast and two slow variables, we show that pseudo-plateau bursting is a canard-induced mixed mode oscillation. Using this technique, it is possible to determine the region of parameter space where bursting occurs as well as salient properties of the burst such as the number of spikes in the burst. The information gained from this one-fast/two-slow decomposition complements the information obtained from a two-fast/one-slow decomposition.

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**Mixed-mode oscillations in a model of hormone secretion: insight into the variability of GnRH surge-to-pulse transition**

**Alexandre Vidal**

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The secretion of Gonadotropin Releasing Hormone (GnRH) by specific hypothalamic neurons plays a major role in the neuroendocrine control of the reproductive function in female mammals. The periodic back-and-forth transitions between the pulsatile secretion phase and the pre-ovulatory surge are among the most crucial mechanisms underlying this control, but they remain not completely understood on the biological level. I will present a model of the GnRH secretion and show how Mixed-Mode Oscillations arise in the surge-to-pulse transition. Often MMO solutions are quite regular, i.e. the number of small oscillations does not vary too much. I will show solutions that have very high variability of small oscillations in a same orbit. Such solutions exist in the GnRH model even when the singular parameter goes to 0.

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## Special Session 79: Numerical Methods Based on Homogenization and on Two-Scale Convergence

Emmanuel Frenod, University of South Brittany, France

The aim of this Special Session is to take stoke of Numerical Methods for solving Partial Differential Equations that manage Multi-Scale Phenomena, Oscillations and Heterogeneities by incorporating concepts coming from Homogenization Theory and Two-Scale Convergence.

At the present time, there are several research program exploiting this kind of ideas. They concern Hyperbolic, Elliptic and Parabolic PDE. The application fields are Environmental Sciences, Fluid Dynamics, Elasticity, Tokamak Physic, ...

One of the goal of this special session is to gather people working in different teams of different countries and having in mind different applications in order to exhibit and synthesize what is common between the different fields.

### Reduced basis finite element heterogeneous multiscale method

**Yun Bai**

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**Assyr Abdulle**

In this talk, we introduce a new multiscale method for elliptic homogenization problems, that combines the finite element heterogeneous multiscale method (FE-HMM) with reduced basis (RB) techniques based on offline-online strategy [A. Abdulle, Y. Bai, submitted to J. Comput. Phys., 2011]. The FE-HMM, relies on a large number of micro problems with increasing degrees of freedom to achieve optimal convergence rates [A. Abdulle, SIAM, Multiscale Model. Simul., 2005]. In contrast the RB-FE-HMM needs only a small number of micro problems selected by a rigorous a posteriori error estimator, computed accurately in an offline stage. Suitable interpolations of the pre-computed micro solutions are used in an online stage to compute the macro solution in a very efficient way, specially for high order macro methods or three dimensional multiscale computations. Adaptive FE-HMM [A. Abdulle, A. Nonnenmacher, Comput. Methods Appl. Mech. Engrg., 2011] is also shown to benefit from the RB approach. A priori error estimates of the RB-FE-HMM and numerical examples illustrating the performance of our approach will be discussed.

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### Neutrino transport in core collapse supernovae by the Isotropic Diffusion Source Approximation

**Heiko Berninger**

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**Emmanuel Frénod, Martin J. Gander, Matthias Liebendörfer, Jérôme Michaud, Nicolas Vasset**

Simulations of core collapse supernovae require models that couple hydrodynamics of the background

matter with radiative transfer of neutrinos. Full coupling of hydrodynamic and Boltzmann equations in 3D will probably remain computationally too costly even on supercomputers of the next generation. This is mainly due to regimes described by the Boltzmann equation in which high density of neutrinos, i.e., small mean free paths prevail. Therefore, one seeks approximations of the Boltzmann equation that capture the main processes of neutrino transport, e.g., in these regimes, while being computationally cheaper. A basic physical observation is that trapped neutrinos in high density regimes behave like diffusive particles whereas neutrinos in low density regimes are practically freely streaming particles. In both cases, the Boltzmann equation can be reduced considerably. This observation leads to the idea of considering the distribution function of the neutrinos to be decomposed additively into a trapped and a streaming particle component and to describe the behavior of these components by reduced equations. This is the underlying idea of the Isotropic Diffusion Source Approximation (IDSA) by Liebendörfer et al. The major challenge of this approximation is to find an appropriate coupling of the reduced equations. In this talk, we will give an introduction into the IDSA in spherical symmetry, both from a physical and a mathematical perspective. In particular, we will provide a justification of the IDSA by asymptotic analysis applying Chapman–Enskog and Hilbert expansions. We will also address the discretization and the numerical solution of the IDSA and introduce a solution technique for a full Boltzmann model that involves time splitting and finite volumes. Numerical results that compare the IDSA with the full Boltzmann model in 1D will conclude the talk.

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### Location problems by shape and topological optimization

**Seck Diaraf**

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In this talk, we are going to present works on shape and topological optimization. This consists on studying location problems in order to optimize a criterion. We point out links between heterogeneity/homogeneity of domains and the achievement of the optimality of the considered criteria. As applications, we quote the crystal photonics and the identification of the interface with two fluids to get optimal design of a material, the placement of obstacle in a material so as to minimize the first eigenvalue of the Laplacian operator.

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### Some numerical simulations on sand transport

**Ibrahima Faye**

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We develop a numerical method to simulate sand transport in tidal area. This method is based on two scale convergence method due to G. Allaire and G. Nguetseng and permits to get a new model which approaches the initial model. Using the equation modeling sand transport and its scaling, we get a model of short term dynamics of dunes and its to scale limit. We give some numerical results obtained by the two scale numerical method and we compare the evolution of the solutions of the initial model and its two scale limit at different time .

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### Synthetic introduction to homogenization based numerical methods

**Emmanuel Frenod**

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This talk is a synthetic introduction to the Special Session. I will introduce basic concepts of Homogenization and Two Scale Convergence, for non-specialists. Then, I will explain how those concepts are used to set out efficient numerical methods to tackle problems where several scales are present. Among those problems, I will evoke questions in elliptic, parabolic and hyperbolic pde with oscillating coefficients or boundary conditions. I will also talk about the way those concepts are used for analyzing data with heterogeneities or in the context of optimal shape design.

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### Reduced basis method in the numerical homogenization of a nonlinearly coupled system: Application to nuclear waste storage

**Zakaria Habibi**

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**Antoine Gloria, Thierry Goudon, Stella Krell**

We are interested in the homogenization of a nonlinearly coupled system of PDEs describing radionuclide transport in a periodic porous media modelling a repository of nuclear waste storage. The resulting effective model is a nonlinearly coupled system of PDEs posed in a homogeneous domain with homogenized coefficients evaluated by solving so-called cell problems. Due to the coupling, the homogenized (or effective) coefficients always depend on the slow variable (as a parameter  $x$ ), even in the simple case when the porosity is taken purely periodic. Therefore, the determination of these coefficients is the most important part of the computational time for the numerical simulation of such problems. We propose a new numerical algorithm based on Reduced Basis techniques, which significantly improves the computational performances. Some features of this work are that the homogenized coefficients are unbounded, the parameter  $x$  belong to  $\mathbb{R}^d$  which is not a compact set and the dependence of these coefficients upon  $x$  is not affine, while the reduced basis method is designed to simply deal with bounded coefficients with affine dependence upon a parameter belonging to a compact set. Eventually, we provide some 3D numerical results in order to show the computational advantages of our method.

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### Filtering partially observed multiscale systems with heterogeneous multiscale methods based reduced climate models

**John Harlim**

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In this talk, I will discuss a fast reduced filtering strategy for assimilating multiscale systems in the presence of observations of only the macroscopic (or large-scale) variables. This reduced filtering strategy introduces model errors in estimating the prior forecast statistics through the Heterogeneous Multiscale Methods (HMM) based reduced climate model as an alternative to the standard expensive direct numerical simulation (DNS) based fully resolved model. More importantly, this approach is not restricted to any analysis (or Bayesian updating) step from various ensemble-based filters. In a regime where there is a distinctive separation of scales, high filtering skill is obtained through applying the HMM alone with any desirable analysis step from ensemble Kalman filters. When separation of scales is not

apparent as typically observed in geophysical turbulent systems, an additional procedure is proposed to reinitialize the microscopic variables to statistically reflect pseudo-observations that are constructed based on the unbiased estimates of the macroscopic variables. Specifically, these pseudo-observations are constructed off-line from the conditional distributions of the microscopic forcing to the macroscopic dynamics given the macroscopic variables with the method-of-moments estimator. This HMM based filter is comparable to the more expensive standard DNS based filter on a stringent testbed the two-layer Lorenz' 96 model in various regimes of scale gap, including the not so apparent one. This high filtering skill is robust in the presence of additional model errors through inconsistent pseudo-observations and even when macroscopic observations are spatially incomplete.

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### Error control for heterogeneous multiscale approximations of nonlinear monotone problems

**Patrick Henning**

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**Mario Ohlberger**

In this talk, we introduce and analyse a heterogeneous multiscale finite element method (HMM) for monotone elliptic operators with rapid oscillations [Henning, Ohlberger, *On the implementation of a heterogeneous multiscale finite element method for nonlinear elliptic problems*, Proceedings of the DUNE-User Meeting 2010, 2012]. We first present a macroscopic limit problem for the oscillating nonlinear equations in a general heterogeneous setting and then show the convergence of the HMM approximations to the solution of the macroscopic limit equation. On the basis of this, we derive an optimal a-posteriori error estimate for the  $L^2$ -error between the HMM approximation and the solution of the macroscopic limit equation [Henning, *Heterogeneous multiscale finite element methods for advection-diffusion and nonlinear elliptic multiscale problems*, PhD Thesis, University of Münster, 2011]. The a-posteriori error estimate is obtained in a general heterogeneous setting with scale separation without assuming periodicity or stochastic ergodicity. The applicability of the method and the usage of the a posteriori error estimate for adaptive local mesh refinement is demonstrated in numerical experiments. The experimental results underline the applicability of the a-posteriori error estimate in non-periodic homogenization settings.

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### Two-scale asymptotic-preserving particle-in-cell method for a Vlasov-Poisson system

**Sever Hirstoaga**

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The aim of this work is to test on a simplified model the Two-Scale Asymptotic-Preserving Schemes. The model is a two dimensional in phase space Vlasov-Poisson equation with a small parameter, which induces high frequency oscillations in the solution. The aim of the model is to be used for a long time simulation of a beam in a focusing channel. This work was already done in the case where the solution is approximated by the two scale limit. The goals are first to improve this approximation, by going further, to the first order one, and secondly, to replace this approximation by an exact decomposition, using the macro-micro framework. This last approach will permit to treat the case of a not necessary small parameter.

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### Variance reduction methods in stochastic homogenization

**Frederic Legoll**

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**X. Blanc, R. Costeauc, C. Le Bris**

The simulation of random heterogeneous materials is often very expensive. For instance, in a homogenization setting, the homogenized matrix is defined from the so-called corrector function, that solves a partial differential equation set on the entire space. This is in contrast with the periodic case, where the corrector function solves an equation set on a single periodic cell. As a consequence, in the stochastic setting, the numerical approximation of the corrector function is a challenging computational task. In practice, the corrector problem is solved on a truncated domain, and the exact homogenized matrix is recovered only in the limit of an infinitely large domain. As a consequence of this truncation, the approximated homogenized matrix turns out to be stochastic, whereas the exact homogenized matrix is deterministic. One then has to resort to Monte-Carlo methods, in order to compute the expectation of the (approximated) homogenized matrix within a good accuracy. Variance reduction questions thus naturally come into play, in order to increase the accuracy (e.g. reduce the size of the confidence interval) for a fixed computational cost. In this work, we show that we can apply the classical technique of antithetic variables and get an approximation of the homogenized matrix with a smaller variance, for an equal computational cost. We will demonstrate, both theoretically and numerically, the efficiency of the approach.

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### An efficient higher-order heterogeneous multiscale method for elliptic problems and related issues

**Pingbing Ming**

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**Ruo Li, Fengyang Tang**

In this talk, I will propose an efficient heterogeneous multiscale finite element method based on a local least-squares reconstruction of the effective matrix using the data retrieved from the solution of cell problems posed on the vertices of the triangulation. The method achieves high order accuracy for high order macroscopic solver with essentially the same cost as the linear macroscopic solver. Optimal error bounds are proved for the elliptic problem. Numerical results demonstrate that the new method significantly reduces the cost without loss of accuracy.

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### Numerical simulations of confinement for paralic ecosystems

**Antoine Rousseau**

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**Jean-Philippe Bernard, Emmanuel Frénod**

In this work we present a modelling procedure in order to compute the confinement field of a lagoon. We improve existing models in order to account for tide oscillations in any kind of geometry such as non-rectangular lagoons with a non-flat bottom. The confinement can be defined at various scales (ocean, interior sea, large bay, lagoons) and we shall introduce a suitable multi-scale numerical method for the computation of confinement.

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### Multiscale geometric integration of deterministic and stochastic systems

**Molei Tao**

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**Houman Owhadi, Jerrold E. Marsden (deceased)**

In order to accelerate computations, improve long time accuracy of numerical simulations, and sample statistics distribution by dynamics, we develop multiscale geometric integrators. The talk will be focused on the description of FLOW AVERAGING INTEGRATORS (FLAVORs), which apply to general multiscale stiff ODEs, SDEs, and PDEs. These integrators employ coarse integration steps that do not resolve the fast timescale in the dynamics; nevertheless, they capture the correct effective contribution of the fast

dynamics — in fact, we show that FLAVORs converge in a sense called two-scale flow convergence (F-convergence). Distinct from existing approaches, an identification of the underlying slow variables (or process) is not required, and intrinsic geometric structures (e.g., symplecticity, conservation laws, and invariant distribution) can be preserved by the multiscale simulation. These new properties are due to that FLAVORs average flow maps instead of vector fields.

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### Design of a Schwarz coupling method for a dimensionally heterogeneous problem

**Manel Tayachi**

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**Eric Blayo, Antoine Rousseau**

When dealing with simulation of complex physical phenomena and in order to avoid heavy numerical simulations, one can reduce a complex model in some locations and replace it by simplest ones - usually obtained after simplifications. Such simplifications in the model may involve a change in the geometry and the dimension of the physical domain. In that case, one deals with dimensionally heterogeneous coupling. We will present in this talk the case of 2-D Laplace equation with non symmetric boundary conditions coupled with a corresponding 1-D Laplace equation. We will first show how to obtain the 1-D model from the 2-D one by integration along one direction and after asymptotic analysis, by analogy with the link between shallow water equations and the Navier-Stokes system. Then, we will present an efficient Schwarz-like iterative coupling method. We will discuss the choice of boundary conditions at coupling interfaces. We will prove the convergence of such algorithms and give some theoretical results related to the choice of the location of the coupling interface, and the control of the error between a global 2-D reference solution and the 2-D coupled one. These theoretical results will be illustrated numerically.

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### Numerical homogenization with non-separable scales

**Lei Zhang**

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Consider homogenization of divergence form operators with  $L^\infty$  coefficients which allows non-separable scales, in the sense of approximating the solution space with a finite dimensional space. Our method does not rely on concepts of ergodicity or scale-separation, but on the property that the solution space of these operators is compactly embedded in

$H^1$  if source terms are in the unit ball of  $L^2$  instead of the unit ball of  $H^{-1}$ . Approximation spaces are generated by solving elliptic PDEs on localized sub-domains with source terms corresponding to approximation bases for  $H^2$ . The  $H^1$ -error estimates show that  $\mathcal{O}(h^{-d})$ -dimensional spaces with basis elements localized to sub-domains of diameter  $\mathcal{O}(h^\alpha \ln \frac{1}{h})$  (with  $\alpha \in [1/2, 1)$ ) result in an  $\mathcal{O}(h^{2-2\alpha})$  accuracy for elliptic, parabolic and hyperbolic problems. The proposed method can be naturally generalized to vectorial equations (such as elasto-dynamics).

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## Special Session 80: Advances in the Numerical Solution of nonlinear evolution equations

Mechthild Thalhammer, University of Innsbruck, Austria

The intention of this special session on "Advances in the numerical solution of nonlinear evolution equations" is to gather mathematicians and theoretical physicists, interconnected through their field of application, the analytical tools, or the numerical methods used. The scope of topics includes but is not limited to Schrödinger type equations, highly oscillatory equations, parabolic problems, and adaptive integration methods for partial differential equations.

### Defect and local error of exponential splitting schemes

**Winfried Auzinger**

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**Othmar Koch, Mechthild Thalhammer**

Linear or nonlinear evolution equations with a right-hand side split up into two parts can often be solved in an efficient way by splitting up the problem in each integration step. We discuss an approach for analyzing the error of exponential splitting schemes. The scheme is associated with an approximate flow, which can be characterized in detail. The defect of the scheme with respect to the given problem is now a well-defined quantity. The (local) order of the scheme can be characterized by the asymptotic behavior of the defect, and local defect integration is the theoretical basis for a posteriori error estimation. Several aspects of this approach are discussed. As an example, Schrödinger type equations are considered.

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### Some mathematical results on time-dependent density functional theory

**Eric Cancès**

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**Gabriel Stoltz**

Time-dependent Density Functional Theory (TDDFT) is a widely used method in chemistry and materials science to model electronic excited states and non-adiabatic electron dynamics. In the case of a finite molecular system containing  $N$  electrons, TDDFT models read as coupled systems of  $N$  nonlinear Schrödinger equations. In a perfect crystal, the number of electrons is infinite, and the electronic state of the system must be described by a density matrix, that is a self-adjoint operator on  $L^2(\mathbb{R}^3)$ . After recalling the basics of TDDFT, I will present a TDDFT model for crystals with local defects and show how it can be used to compute the macroscopic time-dependent polarizability of a crystal.

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### Projected explicit Lawson methods for the integration of Schrödinger equation

**Begona Cano**

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We will prove that explicit Lawson methods, when projected onto one of the invariants of nonlinear Schrödinger equation (norm) are also automatically projected onto another invariant (momentum) for many solutions. As this procedure is very cheap and geometric because two invariants are conserved, it offers an efficient tool to integrate many regular solutions of this equation till long times. Numerical comparisons with splitting methods will also be shown.

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### A simple moving mesh method for blow-up problems

**Shaohua Chen**

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**Lauren DeDieu**

We introduce a simple moving mesh method in which the mesh is generated directly from the physical domain and add higher order difference terms to make the mesh moves more smoothly and orthogonally. Then, we apply this method to heat equations with both single blow-up point and blow-up at space infinity.

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### Exponential integrators for nonlinear Schrödinger equations over long times

**David Cohen**

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**Ludwig Gauckler**

Near-conservation over long times of the actions, of the energy, of the mass and of the momentum along the numerical solution of the cubic Schrödinger equation with small initial data is shown. Spectral discretisation in space and one-stage exponential integrators in time are used. The proofs use modulated Fourier expansions.

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### High-order numerical methods for the stationary Gross-Pitaevskii equation

**Ionut Danaila**

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We compute vortex states of a rotating Bose-Einstein condensate by numerically solving the stationary Gross-Pitaevskii (GP) equation in 2D and 3D. Different types of methods are used: (i) direct minimization of the GP energy functional using Newton methods or steepest descent methods based on Sobolev gradients and (ii) imaginary time propagation of the wave function. Advantages and drawbacks of each method are summarized and convergence properties are presented. We also show that a high spatial accuracy scheme is compulsory to accurately capture configurations with quantized vortices. We present numerical setups using 6th order finite difference schemes and finite elements with mesh adaptivity that were successfully used to compute a rich variety of difficult cases with quantized vortices.

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### High-order accurate Runge-Kutta discontinuous Galerkin methods for a two-dimensional nonlinear Dirac model

**Xia Ji**

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The aim of this paper is to develop high-order accurate Runge-Kutta local discontinuous Galerkin methods approximations of two dimensional nonlinear Dirac equation. The DG methods discussed here are a class of the finite element methods, which adopt completely discontinuous piecewise polynomial space for the numerical solutions and the test functions in the spatial variables, coupled with explicit and nonlinearly stable high order Runge-Kutta time discretization. The propagation of single dromion and the interaction of two dromions are studied. We also prove three continuum conservation laws of the 2D NLD model and an entropy inequality, i.e. the total charge non-increasing of the semi-discrete RKDG methods, which are demonstrated by various numerical examples.

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### Efficient time integration in nonlinear acoustics

**Barbara Kaltenbacher**

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**Mechthild Thalhammer**

The propagation of high intensity ultrasound as used in a large number of applications ranging from cleaning or welding to medical therapy is described by nonlinear wave equations. Among these models are the classical Westervelt and Kuznetsov equation but also more recent modified and/or generalized models. A particular challenge due to nonlinearity and the presence of different wave lengths is efficient and robust time integration. For this purpose, a promising approach are operator splitting techniques exploiting the intrinsic structure of the equations. In this talk we will discuss possible additive or exponential splitting approaches, address their convergence, and show the results of first numerical tests.

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### On the causality of real-valued semigroups and diffusion

**Richard Kowar**

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We show that a process modeled by a strongly continuous real-valued semigroup (that has a space convolution operator as infinitesimal generator) cannot satisfy causality. We present and analyze a causal model of diffusion that satisfies the semigroup property at a discrete set of time points  $M := \{\tau_m \mid m \in \mathbb{N}_0\}$  and that is in contrast to the classical diffusion model not smooth. More precisely, if  $v$  denotes the concentration of a substance diffusing with constant speed, then  $v$  is continuous but its time derivative is discontinuous at the discrete set  $M$  of time points. It is this property of diffusion that forbids the classical limit procedure that leads to the noncausal diffusion model in Stochastics. Furthermore, we show that diffusion with constant speed satisfies an inhomogeneous wave equation with a time dependent coefficient.

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### A micro-macro parareal algorithm with application to singularly perturbed ordinary differential equations

**Frederic Legoll**

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We introduce a parareal algorithm for the time-parallel integration of micro-macro systems. We apply this algorithm to a prototypical example of such a micro-macro model, namely singularly perturbed ordinary differential equations. The system we consider includes some fast and some slow variables, the limiting dynamics of which (in the limit of infinite time scale separation) is known. The algorithm first computes a cheap but inaccurate macroscopic solution using a coarse propagator (by only evolving the slow variables according to their limiting dynamics). This solution is iteratively corrected by using a fine-scale propagator (simulating the full microscopic dynamics on both slow and fast variables), in the parareal algorithm spirit. We provide a numerical analysis of the convergence of the algorithm, the efficiency of which is illustrated by representative numerical experiments. Joint work with T. Lelievre and G. Samaey.

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### Some recent mathematical contributions to multiscale modelling for polymeric fluids

**Tony Lelievre**

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In this talk, I will present two recent works concerning multiscale models for polymeric fluids.

First, I will present a numerical closure procedure that we recently proposed in [1] to get, from a microscopic model, a closed macroscopic model. This procedure is related to the so-called quasi-equilibrium approximation method, and can be seen as a justification of this approach.

Second, I will discuss the longtime behaviour of some models for rigid polymers (liquid crystals models). Such models are interesting since their longtime behaviour may be quite complicated, including convergence to periodic in time solutions. I will explain how such convergence can be proven using entropy techniques, see [2].

References: [1] V. Legat, T. Lelievre and G. Samaey, A numerical closure approach for kinetic models of polymeric fluids: exploring closure relations for FENE dumbbells, *Computers and Fluids*, 43, 119-133, (2011). [2] L. He, C. Le Bris and T. Lelievre, Periodic long-time behaviour

for an approximate model of nematic polymers, <http://arxiv.org/abs/1107.3592>.

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### Optimal bilinear control of Gross-Pitaevskii equations

**Daniel Marahrens**

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**Michael Hintermüller, Peter A. Markowich, Christof Sparber**

We present a mathematical framework for optimal bilinear control of nonlinear Schrödinger equations of Gross-Pitaevskii type. We prove existence of an optimal control and derive the first-order optimality system. Finally we propose a numerical solution method based on a Newton type iteration and present numerical simulations of several coherent quantum control problems.

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### Favourable space and time discretisations for low-dimensional nonlinear Schrödinger equations

**Mechthild Thalhammer**

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In this talk, I shall address the issue of efficient numerical methods for the space and time discretisation of low-dimensional nonlinear Schrödinger equations such as systems of coupled time-dependent Gross-Pitaevskii equations arising in quantum physics for the description of multi-component Bose-Einstein condensates. For the considered class of problems, a variety of contributions confirms the favourable behaviour of pseudo-spectral and higher-order exponential operator splitting methods regarding efficiency and accuracy. However, in the absence of an adaptive local error control in space and time, the reliability of the numerical solution and the performance of the space and time discretisation strongly depends on the experienced scientist selecting the space and time grid in advance, I will exemplify different approaches for the reliable time integration of Gross-Pitaevskii systems on the basis of a local error control for splitting methods. A convergence result for full discretisations by higher-order time-splitting and different pseudo-spectral methods (Fourier, Hermite, Sine) provides the theoretical basis.

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## Special Session 81: Analysis and Simulation of Multi-Scale Problems

**Xiao-Ping Wang**, Department of Mathematics, Hong Kong University of Science and Technology, China  
**Yang Xiang**, Department of Mathematics, Hong Kong University of Science and Technology, China

Multiscale phenomena occur in a diverse range of science and engineering problems. Tremendous progress have been made in recent years in analysis, algorithm design and applications for various multi-scale problems. This special session brings together experts in both analysis and numerical methods to report their recent develops in this exciting field.

### Numerical simulations of the suspended particle in a shear flow with slipping

**Yana Di**

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The talk will study the evolution of an ellipsoid particle in an incompressible, Newtonian shear flow by considering the fluid slipping at solid surface. A continuum hydrodynamic model is constructed, using phase-field diffuse-interface modeling for fluid-solid interface. Fluid slipping at solid particle surface is incorporated into the model by a decrease in the shear viscosity in the interfacial region. Numerical simulations will be given to show the effect of the fluid slipping on the orientational motion of the ellipsoid particle.

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### Lattice Boltzmann method for Helmholtz equation

**Jie Liao**

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**Radjesvarane Alexandre**

Many physical phenomena and engineering problems may have their origins at molecular scales, although they need to interface with the macroscopic scales. The difficulty arises in bridging the results of these models across the span of length and time scales. The lattice Boltzmann method attempts to bridge this gap.

We will present a lattice Boltzmann scheme for solving non homogeneous Helmholtz equation. This massively parallel LB scheme is easy to implement, the computation at each site is determined only by local parameters, and can be easily adapted to solve multiple scattering problem with many scatterers or wave propagation in non homogeneous medium.

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### A new approach to energy bounds for heterogeneous media

**Liping Liu**

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In this paper we present a new method of deriving microstructure-dependent bounds on the effective properties of general heterogeneous media. We first define and calculate the higher-order polarization tensors for multiphase heterogeneous media, and next derive a differential inequality on the energy with the initial condition given by the polarization tensors. Using the comparison theorem we obtain bounds on the energy induced by the inhomogeneities. These new bounds, taking into account of the average Eshelby tensors for homogeneous problems, are much tighter than the microstructure-independent bounds such as the classic Hashin-Shtrikman bounds on one hand, and on the other hand, recover the classic bounds by minimizing or maximizing the bounds over all possible average Eshelby tensors. Also, these bounds are applicable to non-well-ordered composites and multifunctional composites. It is anticipated that this new approach will be useful for the modeling and optimal design of a variety of heterogeneous media.

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### Well-posedness of a generalized Peierls-Nabarro model

**Pingbing Ming**

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**Yang Xiang**

In this talk, I will present the wellposedness of a generalized Peierls-Nabarro model recently proposed by us in a natural energy space. The global well-posedness of the classic Peierls-Nabarro model is a by-product of this result.

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### A seamless multiscale method and its application to complex fluids

**Weiqing Ren**

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I will present a seamless multiscale method for the study of multiscale problems. The multiscale method captures the macroscale behavior of the system with the help of a microscale model. The macro model provides the necessary constraint for the micro model, and the micro model supplies the missing data (e.g. the constitutive relation or the boundary conditions) for the macro model. The macro and micro models evolve simultaneously using different time steps, and they exchange data at every step. The micro model uses its own appropriate (micro) time step. The macro model uses a macro time step but runs at a slower pace than required by accuracy and stability considerations in order for the micro dynamics to have sufficient time to adapt to the environment provided by the macro state. The method has the advantage that it does not require the reinitialization of the micro model at each macro time step or each macro iteration step. I will discuss the algorithm of the multiscale method, the error analysis, and its application to complex fluids.

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### Coarse-graining Kohn-Sham density functional theory

**Phanish Suryanarayana**

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Defects, though present in relatively minute concentrations, play a significant role in determining macroscopic properties. Even vacancies, the simplest and most common type of defect, are fundamental to phenomena like creep, spall and radiation ageing. This necessitates an accurate characterization of defects at physically relevant concentrations, which is typically in parts per million. This represents a unique challenge since both the electronic structure of the defect core as well as the long range elastic field need to be resolved simultaneously. Unfortunately, accurate ab-initio electronic structure calculations are limited to a few hundred atoms, which is orders of magnitude smaller than that necessary for a complete description. Thus, defects represent a truly challenging multiscale problem. Density functional theory developed by Hohenberg, Kohn and Sham (DFT) is a widely accepted, reliable ab-initio method for computing a wide range of material properties. Traditional implementations of DFT solve for the wavefunctions, a procedure which has cubic-scaling with respect to the number of atoms. This places serious limitations on the size of the system which can be studied. Further, they are not amenable

to coarse-graining since the wavefunctions need to be orthonormal, a global constraint. To overcome this, we have developed a linear-scaling method for DFT where the key idea is to directly evaluate the electron density without solving for the individual wavefunctions. Based on this linear-scaling method, we have developed a numerical scheme to coarse-grain DFT derived solely based on approximation theory, without the introduction of any new equations and resultant spurious physics. This allows us to study defects at a fraction of the original computational cost, without any significant loss of accuracy. We demonstrate the efficiency and efficacy of the proposed methods through examples.

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### Efficient numerical methods for the phase field simulation of moving contact line problem

**Xiaoping Wang**

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In this talk, I will describe a newly developed phase field model for two phase fluid flow based on Cahn Hilliard Navier Stokes equation with generalized Navier boundary condition. Then I will describe some efficient numerical methods for the model including adaptive mesh methods. Several numerical results will then be presented.

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### Some uniqueness problems in free boundary problems associated with degeneracy

**Peiyong Wang**

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It is known the uniqueness of a solution in a free boundary problem does not always hold (see a manuscript by G.Lu and the author). Degeneracy of a partial differential equation presents another aspect of the uniqueness problem. The author will address this problem and first present the results with degeneracy of the partial differential equation. Then he will turn to the degenerate situation and consider both aspects simultaneously.

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### A continuum model for the dynamics of dislocation arrays

**Yang Xiang**

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**Xiaohong Zhu, Shuyang Dai**

We derive a continuum model for the dynamics of a dislocation array that consists of dislocations in different slip planes. In the continuum model, the dislocation array is represented by a continuous surface, of which there are many dislocations in a unit area at the scale of the continuum model. The continuum model is derived rigorously from the discrete model of the dynamics of the constituent dislocations in the array using asymptotic analysis. The obtained continuum model contains an integral over the dislocation array surface representing the long-range interaction of dislocations, and a local term that comes from the line tension effect of dislocations. The size-dependent effect due to dislocation line tension is accurately incorporated in the continuum model. We also present a numerical implementation method based on the level set representation of the surfaces the fast Fourier transform method for the long-range interaction.

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### Some mathematical analysis for wetting on chemically patterned surfaces

**Xianmin Xu**

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Wetting phenomena describe how liquid drops stay and move on solid surfaces. They are common in nature and industrial applications. In this talk, we will introduce our recent analysis for wetting on chemically patterned surfaces using some mathematical models. We mainly concern how the macroscopic properties of wetting, such as apparent contact angles and contact angle hysteresis could be affected by the microscopic information of solid surfaces. We would also like to show the application of our analysis in a two-phase fluid problem.

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### Minimizers and Meissner states for nonself-dual Chern-Simons-Higgs energy

**Xiaodong Yan**

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**Dan Spirn**

We prove existence of minimizers of nonself-dual Chern-Simons-Higgs energy. The minimizer is vortexless below the first critical field and vortex appears when  $h_{ex}$  exceeds first critical field. We also prove existence of Meissner state solution for  $h_{ex}$

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## Special Session 82: Multi-component Integrable Systems, Solitons, and Nonlinear Waves

**Stephen Anco**, Department of Mathematics, Brock University, Canada  
**Yue Liu**, Department of Mathematics, University of Texas at Arlington, USA  
**Changzheng Qu**, Department of Mathematics, Ningbo University, China

The aim of this session broadly is to discuss recent developments in integrable systems and evolution equations related to nonlinear waves. Topics of special focus will be generalizations of Camassa-Holm and Hunter-Saxton equations, multi-component NLS and KdV-type equations, analysis of breaking waves, soliton solutions, bi-Hamiltonian structures, and connections with geometry.

### Multi-component soliton equations

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**Stephen Anco**

In this talk we derive new multi-component soliton equations of mKdV-type, NLS-type and Sine-Gordon type. The method uses a general construction of bi-Hamiltonian integrable systems from inelastic curve flows in symmetric spaces applied to 3 examples of hermitian spaces:  $SU(n+1)/U(n)$ ,  $SO(n+2)/SO(n) \times SO(2)$ ,  $SO(2n)/U(n)$ . The resulting integrable systems in these spaces exhibit unitary invariance.

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### Inelastic curve flows in 2-3 dimensional Minkowskian space

**Kivilcim Alkan**

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**Stephen Anco**

The purpose of this talk to derive integrable systems from inelastic curve flows in 2 and 3 dimensional Minkowskian space by using Hasimoto variables. We introduce a Lorentzian version of a moving parallel frame and show that its structure equations encode the Hasimoto variables in natural way. For timelike/spacelike curves in the Minkowskian plane, we obtain the defocusing mKdV equation and its bi-Hamiltonian structure. For null curves, we find Burgers equation. For timelike curves in 3 dimensional Minkowskian space, we have derive the complex defocusing mKdV and the NLS equations whereas for spacelike curves, we find similar equations with complex numbers replaced by hyperbolic numbers.

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### Multi-component soliton equations from geometric curve flows

**Stephen Anco**

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I will survey some recent work on deriving multi-component soliton equations (and their integrability structure) from geometric curve flows.

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### $C^3$ ill-posedness of the gravity-capillary problem

**Ming Chen**

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**Jeremy L. Marzuola, Daniel Spirn, J. Doug Wright**

We prove via explicitly constructed initial data that solutions to the gravity-capillary wave system in  $\mathbb{R}^3$  representing a  $2d$  air-water interface immediately fail to be  $C^3$  with respect to the initial data if the initial data  $(h_0, \psi_0) \in H^{s+\frac{1}{2}} \otimes H^s$  for  $s < 3$ .

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### The Cauchy problem for the Novikov equation

**Alex Himonas**

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We shall discuss well-posedness of the initial value problem for a class of weakly dispersive nonlinear evolution equations, including the Camassa-Holm, the Degasperis-Procesi, and the Novikov equation. The focus will be continuity properties of the data-to-solution map in Sobolev spaces. This talk is based on work in collaboration with Carlos Kenig, Gerard Misiolek and Curtis Holliman and Katelyn Grayshan.

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### On small quasi-periodic perturbation of two-dimensional hyperbolic-type degenerate nonlinear systems

**Xu Junxiang**

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This paper considers a class of two-dimensional nonlinear quasi-periodic systems with small perturbations. The unperturbed system has zero as a hyperbolic-type degenerate equilibrium point. If the quasi-periodic frequency satisfies the Diophantine conditions, by KAM iteration we prove that it can be reduced to a suitable normal form with zero as an equilibrium point by a nonlinear quasi-periodic transformation. Thus, we obtain a small quasi-periodic solution for the perturbed system.

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### Wave breaking and global existence for the generalized periodic two-component Hunter-Saxton system

**Byungsoo Moon**

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**Jeremy L. Marzuola, Daniel Sporn, J. Doug Wright**

In this paper, we study the wave-breaking phenomena and global existence for the generalized two-component Hunter-Saxton system in the periodic setting. We first establish local well-posedness for the generalized two component Hunter-Saxton system. We obtain a wave-breaking criterion for solutions and results of wave-breaking solutions with certain initial profiles. We also determine the exact blow-up rate of strong solutions. Finally, we give a sufficient condition for global solutions.

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### Integrability, wave breaking and peakons for a modified $\mu$ -Camassa-Holm equation

**Changzheng Qu**

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**Yue Liu, Ying Fu**

In this talk, a modified  $\mu$ -Camassa-Holm equation with cubic nonlinearity is introduced. It is an integrable equation in the sense that it admits Lax-pair and bi-Hamiltonian structure. The formation of singularities and the existence of peaked traveling-wave solutions for the equation are investigated. It is shown to admit a single peaked soliton and multi-peak on solutions, of a similar character of the  $\mu$ -Camassa-Holm equation. Singularities of the solutions can occur only in the form of wave-breaking,

and a wave-breaking mechanism for solutions with certain initial profiles is described in detail. Sufficient conditions for blow up of solutions to initial value problem are also given.

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### Generalized Euler-Poincaré equations on Lie groups and Homogeneous spaces, orbit invariants and applications

**Feride Tiglay**

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**Cornelia Vizman**

We develop the necessary tools, including a notion of logarithmic derivative for curves in homogeneous spaces, for deriving a general class of equations including Euler-Poincaré equations on Lie groups and homogeneous spaces. Orbit invariants play an important role in this context and we use these invariants to prove global existence and uniqueness results for a class of PDE. This class includes Euler-Poincaré equations that have not yet been considered in the literature as well as integrable equations like Camassa-Holm, Degasperis-Procesi, CH and DP equations, and the geodesic equations with respect to right-invariant Sobolev metrics on the group of diffeomorphisms of the circle.

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### Painlevé and Lax-pair tests for the integrability of a two-component nonlinear Schrödinger equations with variable coefficients and its nonautonomous solitons

**Emmanuel Yomba**

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In the literature, we have found that the Painlevé and the Lax-pair conditions of integrability are the same for some models and that those conditions are different for some other models. Having this in mind, we apply the Painlevé test and the Lax-pair test on the generalized two-component Nonlinear Schrödinger (NLS) equations with variable coefficients and the external potentials with the goal to see in which category we can classify it. The results obtained so far show that this system fails the Painlevé test but it passes the Lax-pair test. By employing the homogeneous balance principle and the F-expansion technique, we construct abundant exact traveling wave and abundant solitary wave-like solutions including bright-dark-, bright-bright-, and dark-dark-like solitary pairs. The solutions are obtained by choosing special forms of the gain/loss and dispersion terms. More precisely, we allow those coefficients to be modulated by Hermite-Gaussian functions of different orders. Our analytical results suggest a way of controlling the dynamics of solitary

wave-like solutions by an appropriate time modulation of those coefficients.

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**The existence of ground states for quasilinear asymptotically periodic Schrodinger equations**

**Fubao Zhang**

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**Junxiang Xu ,Hui Zhang**

In this talk we will establish the existence of ground states for quasilinear asymptotically periodic Schrodinger equations in  $R^n$ . The proof is based on the methods of Nehari manifold and concentration compactness principle.

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**Semi-discrete AKNS system: Hamiltonian structures and applications**

**Dajun Zhang**

Shanghai University, Peoples Rep of China  
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**Yufeng Cao, Wei Fu, Zhijun Qiao**

The Ablowitz-Ladik (AL) spectral problem is known as a discrete version of the Ablowitz-Kaup-Newell-Segur (AKNS) spectral problem, but it can not directly lead to semi-discrete AKNS hierarchy. In the talk I will start with integrable aspects of the AL hierarchy. By suitable combination we get the semi-discrete AKNS system and its symmetries, conservation laws and Hamiltonian structures. These results will go to the correspondence of the continuous AKNS system in continuous limit. Like in the continuous case, the semi-discrete AKNS system admits reductions and we can get the KdV, modified KdV and nonlinear Schrödinger equations in semi-discrete version. We will mainly focus the semi-discrete KdV equation and investigate its tri-Hamiltonian structures. Then using Hamiltonian operators we may reach to a semi-discrete model which is related to the negative order KdV equation and Camassa-Holm equation, but still many questions are open...

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**On continuous limits theory and integrability for a semidiscrete system**

**Zuo-nong Zhu**

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**Zhu Zuonong**

In this talk, we discuss continuous limits theory and integrability for a semidiscrete system including the Lax pairs, conservation laws, the Darboux transformation and soliton solutions.

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## Special Session 84: Theory, Numerics and Applications of Quasi-Periodic and Almost Periodic Schrodinger Operators

Charles Fulton, Florida Institute of Technology, USA

### Computation of the spectral density function for periodic potentials

**Charles Fulton**

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In this talk we will discuss a new numerical algorithm for computation of spectral density function over the stability intervals associated with periodic potentials (Mathieu equation, in particular) on the half line  $[0, \infty)$ . The algorithm is based on a new characterization of the spectral density function, which enables the shooting with piecewise trigonometric/ hyperbolic splines to be done over a single period. This represents an improvement in speed and accuracy over the algorithm used in the SLEDGE software package (compare ACM TOMS 22 (1996), 423-446). Some ideas for extension to the almost Mathieu equation will be discussed. This is joint work with Steven Pruess.

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### Computation of the spectral density function for periodic potentials

**Charles Fulton**

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We give a characterization of the spectral function associated with one-sided tridiagonal Jacobi matrices, and some numerical approximations for the absolutely continuous band spectra of discrete periodic potentials.

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### Spectral theory of extended Harper's model

**Christoph Marx**

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Extended Harper's model arises as a natural generalization of Harper's (or almost Mathieu) operator, when allowing the tight-binding electron to hop to both nearest and next-nearest neighboring lattice sites. In this talk we present a complete spectral picture of the model, holding for Lebesgue a.e. frequency. Most interestingly, it will be shown that in the self-dual regime, the model exhibits a collapse from purely absolutely to purely singular continuous spectrum induced by a (lack of) symmetry in the

next-nearest neighbor couplings. This symmetry induced collapse is not at all present in the "classical" Harper's model, thus constitutes one good reason to go beyond nearest neighbor interaction. The talk is based on joint work with S. Jitomirskaya.

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### Quasiperiodic operators with rough potentials

**Rajinder Mavi**

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We discuss discrete Schrodinger operators in the regime of positive Lyapunov exponent. Many properties of the operator in this case are known for analytic potentials. We discuss a technique to extend some of these results to the case of Lipschitz continuous potentials.

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### Positivity of the Lyapunov exponent from crude estimates on the density of the states

**Mira Shamis**

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**T. Spencer**

We prove a (sharp) lower bound on the Lyapunov exponent for several classes of Schrödinger operators, for all energies outside an exceptional set which is (super) exponentially small in the coupling constant. The main application is to the Schrödinger operator corresponding to the Standard Map. This is joint work with T. Spencer.

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### Electronic transport in golden-mean and silver-mean labyrinth tilings

**Stefanie Thiem**

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The electronic transport properties are studied for two- and three-dimensional labyrinth tilings constructed from a class of quasiperiodic chains, in which the atoms are coupled by weak and strong bonds aligned according to the golden-mean and silver-mean sequences. The numerical results of the wave packet dynamics reveal anomalous diffusion for

these systems. Using a renormalization group approach and perturbation theory it is possible to show that the underlying quasiperiodic structure of the labyrinth tiling and its electronic transport properties are related in the regime of strong quasiperiodic modulation.

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**Perturbations of finite gap Jacobi matrices beyond the Szego class**

**Maxim Zinchenko**

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In this talk I will discuss a perturbation result for finite gap Jacobi matrices and the associated orthogonal polynomials that goes beyond the settings of the Szego class.

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## Contributed Session 01: Equations and Qualitative Analysis

### Stability of equilibria for the $\mathfrak{so}(4)$ free rigid body

**Petre Birtea**

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**Ioan Casu, Tudor S. Ratiu, Murat Turhan**

The stability for all generic equilibria of the Lie-Poisson dynamics of the  $\mathfrak{so}(4)$  rigid body dynamics is completely determined. It is shown that for the generalized rigid body certain Cartan subalgebras (called of coordinate type) of  $\mathfrak{so}(n)$  are equilibrium points for the rigid body dynamics. In the case of  $\mathfrak{so}(4)$  there are three coordinate type Cartan subalgebras whose intersection with a regular adjoint orbit give three Weyl group orbits of equilibria. These coordinate type Cartan subalgebras are the analogues of the three axes of equilibria for the classical rigid body in  $\mathfrak{so}(3)$ . In addition to these coordinate type Cartan equilibria there are others that come in curves. For each case of nonlinear stability previously found, for the  $\mathfrak{so}(4)$  free rigid body, we construct a Lyapunov function. These Lyapunov functions are linear combinations of Mishchenko's constants of motion.

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### Triple collision dynamics in chaotic photoionization of planar helium

**Narknyul Choi**

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**Min-Ho Lee, Chang Woo Byun, Gregor Tanner**

Recently photoionization cross sections were accurately calculated for the 1-dimensional collinear helium model, which show chaotic fluctuations as a function of the total energy. Based on the underlying classical mechanics of the two-electron atom, we found that the fluctuations can be characterized by the closed triple collision orbits starting and ending at the triple collision. Extension of our theory to the planar helium atom is straightforward without any serious modification. However, quantum calculations of the photoionization cross sections of planar helium is a challenge, especially for higher energies close to the double ionization threshold where the classical-quantum correspondence holds. We present accurate results of the quantum calculation for planar helium and interpret them in terms of quantities associated with the classical mechanics of two-electron atoms.

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### Existence and uniqueness of solution to an integral boundary value problem for impulsive fractional functional differential equations with infinite delay

**Jaydev Dabas**

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**Archna Chauhan**

In this work, we prove the existence and uniqueness of solutions to an impulsive fractional functional integro-differential equations with an integral boundary condition. We prove our result by applying some well known fixed point theorems.

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### Decay and destruction of invariant Tori in volume preserving maps

**Adam Fox**

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**James Meiss**

Invariant tori play a prominent role in the dynamics of symplectic maps. These tori are especially important in two dimensional systems where they form a boundary to transport. Volume preserving maps also admit families of invariant rotational tori, which will restrict transport in a  $d$  dimensional map with one action and  $d - 1$  angles. These maps most commonly arise in the study of incompressible fluid flows, however can also be used to model magnetic field-line flows, granular mixing, and the perturbed motion of comets in near-parabolic orbits. Although a wealth of theory has been developed describing tori in symplectic maps, little of this theory extends to the volume preserving case. In this talk we will explore the invariant tori of a 3 dimensional quadratic, volume preserving map with one action and two angles. A method will be presented for determining when an invariant torus with a given frequency is destroyed under perturbation, based on the stability of approximating periodic orbits.

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**Stability of fixed points for periodic Hamiltonian systems**

**Luis Franco**

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**Ernesto Perez-Chavela**

In this talk we study the stability of equilibria for periodic Hamiltonian systems with one and a half degrees of freedom. We focus on systems coming from the second Newton's Law and we show that equilibria are unstable solutions when the force depends on time periodically and at the equilibria is increasing. We give conditions to determine when the equilibria have hyperbolic structure. We show some examples exhibiting the powerful of the above result.

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**Global asymptotic behavior of some nonlinear nonautonomous difference equations**

**Vlajko L. Kocic**

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We study the global asymptotic behavior of some classes of nonlinear nonautonomous difference equations with and without delay. The main emphasis was placed on special class of periodic difference equations. In particular, the questions of boundedness, existence of unbounded solutions, oscillations, and extreme stability are addressed. For periodic systems we also study the existence and stability of periodic solutions, attenuation and resonance of periodic cycles. Examples include some well-known nonautonomous population models.

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**Existence and uniqueness of bounded solution for nonlinear functional differential equation with anticipation and retardation**

**Karthikeya Sameer Kumar Mamillapalle**

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**Gnana Bhaskar Tenali**

We discuss the existence of unique bounded solution for nonlinear first order functional differential equations with anticipation and retardation (FDEAR)

$$u'(t) = f(t, u(t - h_1), u(t), u(t + h_2)), \quad t \geq t_0,$$

$$u(t) = a(t), \quad t \in [t_0 - h_1, t_0]$$

where  $h_1 > 0$  and  $h_2 > 0$  employing the iterative technique.

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**Almost periodicity in hereditary systems of second order**

**Andrea Cristina Prokopczyk Arita**

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In this work we study the existence of asymptotically almost periodic solutions for a class of second order abstract neutral differential equations with unbounded delay described in the form

$$\frac{d^2}{dt^2} [x(t) + g(t, x_t)] = Ax(t) + f(t, x_t), \quad t \in [0, \infty) \quad (1)$$

$$x_0 = \varphi \in \mathcal{B}, \quad (2)$$

$$x'(0) = \xi \in X, \quad (3)$$

where  $A$  is the infinitesimal generator of a strongly continuous cosine family of bounded linear operators on a Banach space  $(X, \|\cdot\|)$ , the history  $x_t : (-\infty, 0] \rightarrow X$ ,  $x_t(\theta) = x(t + \theta)$ , belongs to an abstract phase space  $\mathcal{B}$  defined axiomatically and  $f, g$  are suitable functions.

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**An analytical approach to the stability of horizontally sheared flow**

**Pai Song**

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**Iordanka Panayotova, John McHugh**

We investigate the stability of a shear flow in a stratified fluid. The flow is assumed to be inviscid and Boussinesq and the base state density gradient is vertical with constant Brunt-Vaisala frequency. The shear is taken as horizontal, where the base-state velocity has uniform direction and its magnitude depends on the transverse horizontal coordinate,  $U(y)$ . Unlike vertical shear flows, this combination of horizontal shear with vertical stratification is inherently three-dimensional and Squire's theorem is inapplicable. Analytical normal-mode solutions are obtained for an entire class of velocity profiles using the Riccati transform. For other velocity profiles the results are determined by numerical methods. Sensitivity of the stability characteristics and their qualitative features are investigated for free-shear flows, such as the hyperbolic tangent velocity profile, and jet-like flows, such as a Gaussian profile.

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**On the stability of solutions of a class of neutral differential equations with multiple deviating arguments**

**Cemil Tunc**

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In this paper, using the Lyapunov-Krasovskii functional approach, some novel stability criteria are given for all solutions of a class of nonlinear neutral differential equations to tend zero.

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## Contributed Session 02: ODEs and Applications

### Monte - Carlo Galerkin approximation of fractional stochastic integro-differential equation

**Abdallah Badr**

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**Hanan Salem El-Hoety**

A stochastic differential equation, SDE, describes the dynamics of a stochastic process defined on a space-time continuum. This paper reformulates the fractional stochastic integro-differential equation as a SDE. Existence and uniqueness of the solution to this equation is discussed. A numerical method for solving SDEs based on the Monte - Carlo Galerkin method is presented.

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### Geometry of invariant surfaces of Lotka-Volterra systems

**Stephen Baigent**

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Three-dimensional strongly competitive Lotka-Volterra (LV) systems have globally attracting invariant surfaces known as carrying simplices. Using methods similar to those used for studying the evolution of interfaces, we study the geometry of these invariant surfaces by following the evolution of an initially planar surface under the LV flow. The evolving surface converges to the carrying simplex. By monitoring the evolution of the surface's 2nd fundamental form and normal map, we give examples where the entire carrying simplex is convex, concave or saddle-like. We also discuss how the geometry of the carrying simplex relates to the stability of fixed points of the flow, and how our results extend to more general Kolomogorov systems.

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### Split-Lyapunov stability of Lotka-Volterra system

**Stephen Baigent**

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**Zhanyuan Hou**

The Split Lyapunov method was introduced by M-L Zeeman and E.C. Zeeman to study global stability of interior fixed points of competitive Lotka-Volterra systems. Their approach uses the carrying simplex, an invariant manifold that attracts all points except

the unstable origin. Here we describe two advancements: (i) We extend the Split Lyapunov method to deal with non-competitive systems, and (ii) we also deal with the stability of boundary fixed points. Moreover, our method does not rely upon carrying simplices, but does rely on permanence and partial permanence. We also show briefly how these stability results for strongly competitive systems - where the carrying simplex is known to exist - are related to the Gaussian curvature of the carrying simplex.

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### New class of exact solutions for the equations of motion of a chain of $n$ rigid bodies

**Dmitriy Chebanov**

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One of the classical directions of investigations in the theory of motion of a system of several coupled rigid bodies is concerned with particular cases of integrability of the equations of motion of the system. In comparison with the Euler and Poisson equations describing the dynamics of a single rigid body about a fixed point, the analytical study of mathematical models of a system of hinge-connected rigid bodies is a much more complicated problem due to the fact that the increase in the number of bodies constituting the system leads to the increase of both the number of mechanical parameters characterizing the system and the number of differential equations describing its motion. In this paper we construct a new class of nonstationary exact solutions for the equations of motion of a classical model of multibody dynamics - a chain of  $n$  heavy rigid bodies that are sequentially coupled by ideal spherical hinges. We establish sufficient conditions for the existence of the solutions and show how the motion equations can be integrated in the case when these conditions are fulfilled. The new class generalizes most of the exact solutions to the problem of chain's motion known so far.

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### Existence and uniqueness of linear functional differential equations with anticipation

**Duane Chin-quee**

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**Gnana Bhaskar Tenali**

We consider functional differential equations of the form  $\dot{u} + Mu = r$ ,  $t \geq t_0$ ,  $t_0 \in \mathbb{R}$ . The anticipation operator  $M$  is a linear mapping  $M : C(\mathbb{R}, \mathbb{E}) \rightarrow B(\mathbb{R}, \mathbb{E})$  where  $\mathbb{E}$  is a Banach Space and  $B$  is the linear space of

all measurable functions  $f : \mathbb{R} \rightarrow \mathbb{E}$  that are Bochner-Integrable on each compact interval. We establish an existence and uniqueness result with prescribed boundary conditions. An illustrative example will be presented.

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### Geometrical dissipation for dynamical systems

**Dan Comanescu**

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**Petre Birtea**

On a Riemannian manifold  $(M, g)$  we consider the  $k + 1$  functions  $F_1, \dots, F_k, G$  and construct the vector fields that conserve  $F_1, \dots, F_k$  and dissipate  $G$  with a prescribed rate. We study the geometry of these vector fields and prove that they are of gradient type on the regular leaves corresponding to  $F_1, \dots, F_k$ . By using these constructions we show that the cubic Morrison dissipation and the Landau-Lifschitz equation can be formulated in a unitary form. The stability problem in the presence of the geometric dissipation is also presented.

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### Equality problems in a class of conjugate means

**Judita Dascal**

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**Pál Burai, Zoltán Daróczy**

A common generalization of the well-known arithmetic, geometric, harmonic, quasi-arithmetic and other classical means is the so-called L-conjugate mean introduced by Daróczy and Páles and it has the form

$$\varphi^{-1}(p\varphi(x) + q\varphi(y) + (1 - p - q)\varphi(L(x, y))),$$

where  $\varphi$ , called a generating function, is a continuous strictly monotone function defined on an interval  $I$ ,  $L$  is a fixed strict mean and  $p, q$  are parameters from the interval  $]0, 1]$ . When characterizing a class of mean values, it is natural to ask under what necessary and sufficient conditions do two functions generate the same mean. This problem is called the equality problem in a given class of mean values. We will solve the equality problem of L-conjugate means of two variables, when  $L$  is a quasi-arithmetic mean. The investigations lead to solving composite functional equations and differential equations, sometimes making use of the software *Maple*<sup>®</sup>14 to simplify the computation.

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### The parametrization method of research and solving of boundary value problems for integro-differential equations

**Dulat Dzhumabaev**

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In report a linear two-point boundary value problem for Fredholm integro-differential equation is considered. The method bases on dividing interval and introducing of additional parameters is proposed. The necessary and sufficient conditions of solvability and unique solvability considered two-point boundary value problem are established.

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### Criterion of well-posed solvability of linear semi-periodical boundary value problem for system of loaded hyperbolic equations

**Zhazira Kadirbayeva**

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**Dulat Dzhumabaev**

The report is devoted to finding of necessary and sufficient conditions for well-posed solvability of linear semi-periodical boundary value problem for system of loaded hyperbolic equations with mixed derivative. In addition to the original problem appropriate family of periodical boundary value problems for systems of ordinary differential equations is considered. In terms of families of periodical boundary value problems the criterion of well-posed solvability of initial semi-periodical boundary value problem is obtained.

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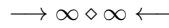
### Constrained mechanics and idealized models for aquatic locomotion

**Scott Kelly**

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The geometric structure underlying the mechanics of finite-dimensional systems subject to nonholonomic constraints has been detailed in recent years, particularly in the context of systems exhibiting symmetries, and control strategies exploiting this structure have been applied successfully to problems in wheeled robotic locomotion. Many features of such systems are paralleled in idealized models for aquatic locomotion in which propulsive vortex shedding is driven by the concurrent enforcement of velocity constraints (like Kutta conditions) and conservation laws. We examine aspects of this parallelism and their implications for motion control, focusing on single-input

locomotion systems in which velocity constraints couple equations for the evolution of rotational momentum and translational momentum.

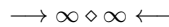


**A fitted numerical method to solve a mathematical model describing TB dynamics**

**Kailash C. Patidar**

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Mathematical models described by the autonomous systems of nonlinear ordinary differential equations arising in biology are sometimes so complex that they cannot be solved analytically and one has to rely on efficient numerical integrators. However, conventional numerical methods like Euler, Runge-Kutta and even some popular MATLAB ode solvers fail to solve these nonlinear systems in the sense that they generate oscillations, chaos, and false steady states. In this talk, we will discuss the design and implementation of a new class of fitted finite difference methods to solve a mathematical model describing Mycobacterium tuberculosis transmission dynamics. The dynamics of this model are studied numerically using the qualitative theory of dynamical systems. We analyze this method for stability. Furthermore, we show that this method also preserves positivity of the solution which is one of the essential requirements when modeling epidemic diseases. To show the power of the proposed method, numerous comparisons are made with the method that are commonly used by other researchers in the field.



**Limit cycle existence and uniqueness in elementary piecewise linear continuous systems**

**Enrique Ponce**

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**J. Llibre, M. Ordoñez**

Some techniques to show the existence and uniqueness of limit cycles, typically stated for smooth vector fields, are extended to continuous piecewise-linear differential systems. New results are obtained for systems with three linearity zones without symmetry and having one equilibrium point in the central region. We also revisit the case of systems with only two linear zones giving shorter proofs of known results.



**Uniform asymptotic expansions of solutions of a class of singularly perturbed boundary value problems**

**Susmita Sadhu**

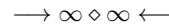
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**J.B. McLeod**

We will consider a class of singularly perturbed boundary value problems (BVPs):

$$\varepsilon y'' + 2y' + f(y) = 0, \quad y(0) = 0, \quad y(A) = 0,$$

where  $f \in C^2[0, \infty)$  is a positive function satisfying certain conditions. It can be shown that the BVP admits at most two solutions depending on  $A$ . The main goal of this talk is to rigorously prove a uniform asymptotic expansion of the “smaller” solution using an integral equation method, whenever the problem admits two solutions. To achieve our goal, we will prove an existence result that will ensure a uniform bound on the “smaller” solution and that would lead us to the asymptotics.



**Criteria for determining the limit point case and limit circle case for singular Sturm-Liouville differential operators**

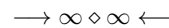
**Kishor Shinde**

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**S. M. Padhye**

We extend a result on the criteria for determining the limit point and limit circle case for Sturm-Liouville differential operators. This forms a comparison theorem for limit point and limit circle case for Sturm-Liouville differential operators. These results can be used to analyze limit point case and limit circle cases at 0 for the function  $q(x) = \frac{k}{x^p}$ , for all  $k$  and  $p$ . We determine the values of  $k$  and  $p$  such that the Sturm-Liouville differential operator  $\tau u = -u'' + qu$  is in limit point case or limit circle case at zero, where  $q(x) = \frac{k}{x^p}$ .  $\tau$  is in the limit circle case when (i)  $p < 0$  and for all  $k$  (ii)  $p = 0$  and for all  $k$  (iii)  $p > 0$  and  $k \leq 0$  (iv)  $p = 2$  and  $0 < k < \frac{3}{4}$  (v)  $0 < p < 2$  and for all  $k > 0$  and  $\tau$  is in the limit point case when (i)  $p = 2$  and  $k \geq \frac{3}{4}$  (ii)  $p > 2$  and  $k > 0$ .





**Diffusion approximations for metapopulation models****Andrew Smith**University of Queensland, Australia  
a.smith23@uq.edu.au**Ross McVinish, Phil Pollett**

A metapopulation is a population that occupies several geographically distinct patches. Stochastic models are often employed in order to capture the randomness inherent in these populations. However, realistic models, which account for patch-specific dynamics and arbitrary spatial arrangement of patches, can be complicated and are often intractable. For this reason, we propose to approximate model behaviour using a diffusion process. We adopt a technique developed by Thomas Kurtz, which enables us to identify an appropriate approximating deterministic dynamical system. This system is then analysed to give qualitative results that relate back to the original stochastic model. Conditions under which the population persists, or otherwise becomes extinct, are presented.

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## Contributed Session 04: Modelling and Math Biology

### Oscillations in epidemic models: the role of infection and recovery times

**Guillermo Abramson**

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**Sebastian Goncalves, Marcelo F. C. Gomes**

Traditional epidemic models consider that individual processes occur at constant rates. That is, an infected individual has a constant probability per unit time of recovering from infection after contagion. This assumption certainly fails for almost all infectious diseases, in which the infection time usually follows a probability distribution more or less spread around a mean value. We show a general treatment for an SIRS model in which both the infected and the immune phases admit such a description. The general behavior of the system shows transitions between endemic and oscillating situations that could be relevant in many real scenarios. The interaction with the other main source of oscillations, seasonality, will also be discussed.

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### Optimal vaccine procurement strategy for smallpox epidemic

**Burcu Adivar**

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**Ebru Selin Selen**

In this study, epidemiological modeling is used to develop optimal order policy for vaccine requirements for an anticipated epidemic or pandemic attack. Based on a compartmental model for the dispersion of smallpox virus, we consider vaccination as the main control policy in addition to hospitalization and quarantine. Solution to a set of ordinary differential equations is used to estimate the need for vaccines for two different population sizes. Assuming zero initial stock level for smallpox vaccines, we propose a minimum cost vaccine procurement strategy by determining optimal order quantity and order timing to stop the dispersion of epidemic as early as possible. Dynamic programming is used to solve the single commodity inventory model under deterministic time varying demand rate.

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### Can activation of latently infected cells reduce the size of the HIV reservoir?

**Stanca Ciupe**

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**Jon Forde, Joseph Volpe**

While antiretroviral drugs can drive HIV to undetectably low levels in the blood, eradication is hindered by the persistence of long-lived, latently infected memory CD4 T cells. Immune activation therapy aims to eliminate this latent reservoir by reactivating these memory cells, exposing them to removal by the immune system and the cytotoxic effects of active infection. In this paper we develop a mathematical model that investigates the use of immune activation strategies while limiting virus and latent class rebound. Our model considers infection of two memory classes, central and transitional CD4 T cells and the role that general immune activation therapy has on their elimination. Further, we incorporate ways to control viral rebound by blocking activated cell proliferation through anti proliferation therapy. Using the model we provide insight into the control of latent infection and subsequently into the long term control of HIV infection.

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### The stability analysis and impact of predator mortality rate on age-structured models

**Michael Cowen**

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**Wei Feng, Shana Johnson**

This paper analyzes the effects of an age-structured prey-predator system where the prey has two stages, juvenile and adult. Three different models are used to evaluate the benefits of this structure with regards to predator mortality rate and stability of the system. We assessed how various parameters for prey growth rate and death rate affected each model and we determined necessary conditions for stability in all cases. The focus of this paper is to find the conditions necessary to ensure asymptotic stability of the equilibrium point where both the predator and prey can co-exist. More specifically, we demonstrate how the importance of predator mortality rate changes in each system.

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### Dynamics of evolution in two-patch ecological models

**Theodore Galanthay**

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**Samuel M. Flaxman**

In most habitat selection models in theoretical ecology, one assumes that organisms tend to move towards areas that locally maximize organisms' fitness. It remains unclear, though, in the presence of realistic constraints such as errors in habitat assessment and various costs, which general movement strategies might actually result in higher fitness and thus be termed adaptive. We study a single-species, two-patch habitat selection model and compute, analytically, a set of optimal information-use strategies. These strategies use both fitness-based and habitat-based information. We demonstrate that these strategies are evolutionarily stable and convergent by applying tools from adaptive dynamics and basic linear stability theory. Furthermore, we show that in the presence of certain types of costs, an organism can maximize its fitness by ignoring information about fitness in favor of acting upon more proximate habitat cues.

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### Mathematical model for mutation acquisition in tumorigenesis

**Alexandra Jilkine**

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**Ryan Gutenkunst**

Most tissues consist of three classes of cells: stem cells, transit-amplifying progenitor cells, and differentiated cells. Many tumors also have a hierarchical organization, with the bulk of the tumor composed of relatively differentiated short-lived cells with a limited replicative potential. Tumors are thought to be maintained by a small subpopulation of cancer stem cells (CSC), which have the capacity to proliferate indefinitely, and drive tumor growth. It is unclear whether CSCs originate from stem cells or from de-differentiated mature cells. We consider a hybrid stochastic deterministic model of mutation acquisition in stem cells and their progeny. We study the effects of competition between cells both at the stem cell level (in a stochastic model) and the progenitor level (in an age structured PDE model), as well as the effects of de-differentiation of progenitor cells to stem-cell like state. We give estimates on the necessary division and mutation rates to maintain a stable cohort of mutant transit-amplifying cells due to progenitor mutations alone. However, to obtain unlimited growth, de-differentiation from progenitor to stem cell state is essential. Interestingly, effects of

de-differentiation only become important once homeostasis, which limits the number of cells in the stem cell pool, is lost.

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### Further analysis on high order singularities of LKE model

**Cuiping Li**

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**Xiaohan Li, Chaozhe He**

In this paper, a novel two-dimensional Lotka-Volterra type model that incorporates chemical heterogeneity of the grazer-producer system is further studied. Given the differential property of the digestion rate function, we verified the type of high order equilibriums in this model and discuss the possible number of inner equilibriums in the model. A sufficient condition to support the existence of the limit cycle in the system was also given using Hopf bifurcation theory.

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### Hierarchical inductive process modeling and analysis

**Youri Nelson**

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**Stuart Borrett, Wei Feng**

Understanding the Phytoplankton dynamic in the Ross Sea Polynya may yield useful knowledge in the search for solving the worlds rising carbon dioxide levels. Modeling such dynamics is a very lengthy and tedious process that can be helped with the use of computational tools like HIPM. This system relies on knowledge that is already available, in the shape of time series data and process library, to construct and then evaluates these models. In this research models were ranked by sum of squared error, from lowest to highest. The lowest being the best t model. Some of the questions that arise from the use of HIPM are about the amount and value of the time series provided to the software, from which we formulated two hypotheses. Will having more time series better the output of the system? Will time series for different variables provide different quality of output? Through 31 experiments and mathematical analysis, we began to answer these questions. The computational result showed us that our first hypothesis does not always hold true, which is thought to be because of the way the fit is measured. On the other hand the mathematical analysis showed us many variations, over all the experiments, in the zooplankton equation structure which can be indication that the process library needs to be better defined and that the system needs to take into consideration not only

Phaeocystis antarctica phytoplankton species but also diatoms. This thesis provides the start to an answer for this hypothesis but further research is still needed.

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### The importance of stochasticity and safety nets in breaking disease-induced poverty traps

**Calistus Ngonghala**

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**Mateusz Plucinski, Matthew Bonds**

Deterministic and stochastic models are applied to investigate feedback between infectious diseases and poverty. It is shown that economic development in deterministic models requires significant external changes to the initial conditions, or a change in the parametric structure of the system. Therefore, vicious cycles of disease and poverty (disease-induced poverty traps) arising from deterministic models lead to more limited policy options. In contrast, it is demonstrated that there is always some probability that a population will escape or fall into a poverty trap in stochastic models. A “safety net”, defined as an externally enforced minimum level of health or economic condition can guarantee ultimate escape from the poverty trap, even when it is set within the basin of attraction of the poverty trap or implemented only as an economic or health care intervention. Based on the analysis of the stochastic model, the following two economic and public health intervention questions are answered: (i) Is it more effective to provide health care or income/income generating resources to enable populations to escape from poverty traps? (ii) How long will it take a population that is caught in a poverty trap to become developed when the initial conditions are reinforced by safety nets?

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### A note on age-character-dependent model in population dynamics

**Olawale Obabiyi**

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We consider a linear time continuous, age and character dependent model of 1-sex population such that our character variable is body weight at birth or future body weight with age factor. We use a priori estimate and contraction mapping principle to establish existence of a weak solution.

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### State and parameter estimation for nonlinear models

**Ulrich Parlitz**

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**Jan Schumann-Bischoff, Stefan Luther**

We present and compare efficient methods for estimating variables and parameters of systems of ordinary differential equations by adapting the model output to an observed time series from the (physical) process described by the model. In particular, optimization based methods are considered where the optimization method exploits the particular structure of the relevant cost function [1]. We apply these data assimilation methods to (chaotic) time series generated by different types of dynamical systems, including low and high dimensional chaos, delay systems, and biological cell models.[1] J. Schumann-Bischoff and U. Parlitz, State and parameter estimation using unconstrained optimization, Phys. Rev. E 84, 056214 (2011)

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### Mathematical model for mutation acquisition in tumorigenesis

**Laurent Pujo-Menjouet**

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Transmissible spongiform encephalopathies, or prion diseases, are a group of fatal neurodegenerative disorders of humans and animals. The pathogenic process is typically associated with conformational conversion of a cellular protein, called prion or PrPC, to a misfolded isoform, called PrPSc. The “protein-only” model asserts that this rogue PrPSc represents the infectious prion agent, self-propagating by binding to PrPC and inducing its conversion to the abnormal PrPSc. This scenario was quantitatively described as a nucleation-dependent amyloid polymerization. However, inconsistent results follow from this theory in comparison to in vitro polymerization experiments. Indeed although the dynamics of polymerization resemble a simple nucleus-dependent fibrillogenesis, neither the initial concentration dependence nor off-pathway hypothesis fit with completely experimental results. In order to reconcile the experimental results with the nucleus dependent polymerization, we have postulated the existence of an on-pathway step that takes place before nucleation. Here we show that micelles were formed leading to an amyloid competent isoform of the prion protein (i.e. PrP\*) necessary to engage the nucleation and then amyloid polymerization. To analyse the consequences of this proposition, we developed a quantitative model with an explicit description of the

microscopic processes, and we compared to experimental data with the predicted results. A detailed analysis of the lag phase under several conditions has been done to validate a micelle on-pathway as an explanation of amyloid dynamic. We recall here the original theoretical model and some of the main results, we show then the inconsistencies with the experiment results. We propose the new model taking the new biological assumptions of prion formation into account and compare it to experimental data. The model consists of systems of non linear differential and partial differential equations describing polymerization and fragmentation of polymers.

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### Modelling the effects of flagellar hook compliance on bacterial motility

**Henry Shum**

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**Eamonn Gaffney**

Individual-level models of swimming micro-organisms vary in complexity from a single point singularity in the fluid to collections of hundreds of beads interconnected by elastic springs and arranged to resemble the shape of the modelled organism. We explore a model with an intermediate number of degrees of freedom for a bacterium propelled by turning a helical flagellum with a rotary motor. We use the boundary element method to accurately resolve the flow field around the cell body and flagellum, both of which are assumed to be rigid structures. Building on early models, which maintained a fixed axis of rotation of the flagellum relative to the cell body, we introduce an elastic connection between the two structures corresponding to the flagellar hook. The hook is modelled as a Kirchhoff rod and analysis of this system reveals distinct phases of swimming behaviour, with biologically relevant consequences, depending on the hook stiffness and motor frequency.

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### Numerical continuation of equilibria of cell population models with internal cell cycle

**Charlotte Sonck**

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**C. Sonck, M.A. Kirkilionis, W. Govaerts**

Mathematical modelling of the cell cycle has been a subject of study for a few decades and J.J. Tyson and B. Novák, as prominent leaders, have developed several widely studied models.

Our goal is to incorporate these realistic models in structured cell population ODE models to study the behaviour of the cells at population level and its

dependence on the nutrient level. An equilibrium of this cell population model corresponds to a constant distribution of the mass of cells born per unit of time.

Numerically, the idea is to obtain the equilibrium as the fixed point of a map. We implement this map in our code as the output of a large collection of integrations over age for cells born with a given mass, followed by its implications for the consumption of nutrient. A found equilibrium can then be continued under parameter variation. This allows us to study the influence of natural parameters such as growth rate of the cells and the concentration of the nutrient.

I will give the results of our computations so far, starting with a model with a fairly simple choice for the cell cycle mechanism, and will give a look-out to further steps and challenges.

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### The randomness of gene expression

**Adam Stinchcombe**

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**Charles S. Peskin, Daniel Tranchina**

The process by which the genetic code comes to life is a fundamentally stochastic process. In order to begin to quantify this randomness, this work models transcription using a population density approach. In the model, a single gene of interest fluctuates stochastically between an inactive state, in which transcription cannot occur, and an active state, in which discrete transcription events occur; and the individual mRNA molecules are degraded stochastically in an independent manner. The random dwell times in the inactive and active states are independent random variables drawn from any specified distributions. This problem can be reduced to a pair of integral equations for the unknown probability densities immediately after a gene switch. Previously, this sort of model with exponential dwell times has been successful in explaining experimental estimates of the distribution of random mRNA copy number within a population of isogenic cells. I will present efficient numerical methods for computing steady-state mRNA distributions, an analytic formula for the mRNA autocovariance function, and a procedure for model identification based on laboratory data. It is hoped that these theoretical advancements will lead to a better understanding of stochastic gene expression, in theory and experimentally.

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### Modeling the nonlocal dispersal of invasive plant species in heterogeneous landscapes

**Christopher Strickland**

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**Patrick Shipman, Gerhard Dangelmayr**

Mathematical models for the spread of invading organisms typically utilize population growth and dispersal dynamics in an attempt to predict an expected value of the population distribution at some point in the future. These models often ignore uncertainty in initial conditions, neglect ecological heterogeneity in the landscape, and even misrepresent the underlying stochastic growth and dispersal processes they are supposed to represent. Assuming the underlying stochastic process for the population dynamics is a contact birth process, we derive a deterministic model for the probability of species presence as a function of time and space. By abandoning any attempt to model the size or internal population dynamics of a population, our model focuses on the more realistic goal of species presence prediction resulting in an equation derived directly from the stochastic process that naturally incorporates heterogeneity in the landscape, as well as uncertainty in initial conditions.

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### Complex macroscopic behavior in systems of phase oscillators with adaptive coupling

**Dane Taylor**

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**Per Sebastian Skardal, Juan G. Restrepo**

A major difficulty in the study of complex systems (e.g., neural processing and cell function) is overcoming the common disconnect between simple microscopic and complex macroscopic dynamics referred to as emergence. Utilizing recent dimension-reduction techniques for large systems of coupled phase oscillators exhibiting bistability, we analyze complex macroscopic behavior arising when the coupling is allowed to evolve slowly as a function of either macroscopic or local system properties. For example, we observe macroscopic excitability and intermittent synchrony (i.e., as observed for the classical order parameter) in a system of time-delayed Kuramoto oscillators with Hebbian and anti-Hebbian learning. We highlight the robustness of our analysis by considering systems with increasing complexity, including time-delayed oscillators with adaptive network structure and community interaction, as well as a system with bimodal frequency distribution.

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## Contributed Session 06: Control and Optimization

### Dual representations of cones and functions on mixed domains

**Murat Adivar**

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In this study, we establish a platform which enables us to study convexity and duality properties of the cones and functions on discrete and mixed domains.

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### Analytical methods in optimal control of HIV treatment

**Ellina Grigorieva**

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**Khailov E.N., Korobeinikov A.**

We formulate nonlinear control models of HIV dynamics withing a host which include nonlytic and lytic immune response and different types of antiviral therapy. The ODE models describe the dynamics of HIV infection and have the following phase variables: infected and uninfected T helper cells count, free virus population, and concentration of the medication. The models reflect differences in multi-drug therapy, virus remission and drug resistance. The medication intake in these models is assumed to be controlled. Though many HIV control models have been suggested and investigated, qualitative analytical methods suitable for constructing the optimal control for complex nonlinear models are very rare. In this talk we present analytical methods for finding the optimal controls for a variety of objective functions and models. These methods allow to reduce a two point boundary value problem for the Maximum Principle to a problem of the finite-dimensional optimization. Computer simulations will demonstrate behavior of the control model for different parameters.

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### Optimal control in the treatment of Retinitis Pigmentosa

**Luis Melara**

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**E. Camacho, M.C. Villalobos, S. Wirkus**

Numerous therapies have been implemented in an effort to stop the debilitating progression of the degenerative eye disease Retinitis Pigmentosa (RP), yet none have provided satisfactory long term stoppage of the degeneration of photoreceptors. The recent discovery of the protein termed rod-derived cone viability factor (RdCVF) has provided researchers with a potential therapy that could slow the secondary wave of cone death. In this work, we build on an existing mathematical model of photoreceptor interactions in the presence of RP and incorporate therapy via RdCVF. Our results show that an optimal control exists for the administration of RdCVF. In addition, our numerical solutions show the experimentally observed rescue effect that the RdCVF has on the cones.

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### Reliability modeling and analysis of a desalination plant system

**Syed Mohd Rizwan**

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The paper presents a reliability modeling and analysis of an evaporator/system of a desalination plant. The desalination plants are being used to purify the sea water for domestic usage. In the present work, seven years maintenance data on an evaporator is collected and then a robust model is developed by embedding the real failure situations of the evaporator as depicted in the data. The model reflects the various possible state transitions of the system under consideration. There are many reasons for evaporator failure which have been reflected in the model for analysis. Optimized reliability indices of the evaporator/system have been obtained using the model which reveals the evaporator effectiveness. The profit model for the evaporator has also been developed in order to evaluate the optimum profit of the evaporator. The semi-Markov processes and regenerative point techniques are used in the entire analysis.

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## Contributed Session 07: Scientific Computation and Numerical Algorithms

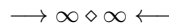
### Time-splitting scale-selective numerical scheme for atmospheric modeling

**Andrei Bourchtein**

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**Ludmila Bourchtein**

The atmosphere is a complex system supporting processes of different space and time scales. Accordingly, the complete 3D mathematical models of the atmosphere contain multi-scale solutions with fast and slow components. It is well-known that the fastest atmospheric waves are the acoustic ones, which do not contain any significant part of the atmospheric energy. The slower gravity waves are more energy valuable, while relatively slow advective processes and Rossby waves carry the main part of the atmospheric energy. Since differential approximations, which filter out fast waves, introduce distortions to the main physical modes, the problem of stiffness of the complete mathematical models of atmospheric dynamics should be addressed in design of numerical scheme. In this study, a semi-implicit finite difference scheme is proposed for the nonhydrostatic atmospheric model based on Euler equations. The fast acoustic and gravity waves are approximated implicitly, while slow advective terms and Rossby modes are treated explicitly. Stability analysis of the scheme shows that the time step is restricted only by the maximum velocity of advection. The performed numerical experiments show computational efficiency of the designed scheme and accuracy of the predicted atmospheric fields.

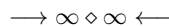


### A method to solve fractional differential equations

**Elif Demirci**

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Fractional order differential equations (FDE) are generalizations of integer order differential equations and became an important tool in mathematical modeling, in recent years. Several solution techniques for FDE's were studied in earlier works. We give a new method to solve first order FDE's using a transformation. This new method has many advantages like its speed, simplicity and applicability. Some examples are also given to show that this technique works properly. Finally, an application of the method is given to solve an epidemic model.



### Complexity analysis of a winding number algorithm by iterated function methods

**Juan-Carlos Diaz-Martin**

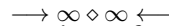
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**Juan-Luis García Zapata**

The winding number of plane curves is a geometric quantity often used in the context of complex analysis to locate the zeros and poles of a rational function. It is usually applied to study the stability of linear systems or as two-dimensional bisection procedure for approximating function zeros. We present an analysis of computational complexity of an algorithm for calculating the winding number, using dynamical systems techniques.

The algorithm is based on an adaptive insertion process of interpolation points in the given curve  $\Delta$  (Ying-Katz process). This process may be viewed as the iteration, in the space of polygonal curves (not finite-dimensional)  $\mathbb{C}^* = \bigcup_{i=1}^{\infty} \mathbb{C}^i$ , of the application  $f_{\Delta} : \mathbb{C}^* \rightarrow \mathbb{C}^*$  carrying  $P = (\Delta(s_0), \Delta(s_1), \dots, \Delta(s_n))$  to  $f_{\Delta}(P) = (\dots, \Delta(s_i), \Delta(\frac{s_i+s_{i+1}}{2}), \Delta(s_{i+1}), \dots)$  if the points  $\Delta(s_i)$  and  $\Delta(s_{i+1})$  are in not consecutive quadrants in the plane, and  $f_{\Delta}(P) = P$  otherwise. The fixed points of  $f_{\Delta}$  are the sampling polygons of  $\Delta$  with vertices in consecutive quadrants.

We show that the iteration of  $f_{\Delta}$  reaches a fixed point in a finite number of iterations because there is a termination function, related to the distance from the origin of the curve. We also demonstrate the correctness of the algorithm from certain property that is invariant along the orbits of  $f_{\Delta}$ .



### Constructing extensions of nonstandard finite difference schemes

**Katharine Gurski**

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We consider both the nonstandard finite difference schemes of Mickens and the higher order methods of Erdogan and Ozis. The latter's method formulates a method to calculate some of the denominator functions and nonlocal approximations that characterize Micken's schemes. We extend this analysis by presenting a construction for nonlinear functions of the form

$$\frac{d^2y}{dt^2} = f(t, y, y')$$

and systems of differential equations. We present analysis and numerical results to demonstrate both the effectiveness of the technique and when this method is best used.





**On functions having the fixed point property****Paula Kemp**

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In this paper, necessary and sufficient conditions are given for a family of functions with the fixed point property to be equivalent to the Axiom of Choice.

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**Uncertainty quantification with probabilistic cellular automata****Dominic Kohler**

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**Johannes Müller, Utz Wever**

Development of real-time tools for uncertainty propagation is of increasing interest in industrial applications such as monitoring of infrastructure grids. However, it faces major challenges due to the complexity of the underlying models. To meet these challenges we suggest to source out large parts of the computations into a preprocessing procedure. Our ideas are demonstrated by the propagation of initial value uncertainties under deterministic PDE dynamics. In particular we introduce a discretization of PDEs which is based on a variant of the set oriented method. As preprocessing it translates PDEs to completely discrete (in time, space and state) dynamical systems: probabilistic cellular automata. These automata are much simpler than the usual models and become accessible to very efficient simulation techniques. The goal is to use an automaton to approximate the evolution of a system's probability density in real-time. It is observed that the solution of the approximating system converges to the exact solution for refinement of the discretization's resolution. To test our results we consider the reactive transportation of chemical pollutants in drinking water. We compare our method to Wiener's polynomial chaos approach as implemented in conventional solvers.

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**A discontinuous Galerkin least-squares finite element method for solving Fisher's equation****Runchang Lin**

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Fisher's equation is a semilinear parabolic reaction-diffusion equation, which was introduced to investigate the dispersion of a mutant gene through a population in a one-dimensional habitat. Fisher's equation admits traveling wave solution, for which it is challenging to obtain reliable numerical approximation. In particular, for equations under strong

reactions, the solutions evolve into shock-like waves, which is a classical difficult problem in numerical computation. In this paper, we introduce a discontinuous Galerkin least-squares finite element method for solving Fisher's equation, which produces accurate and stable solutions in uniform meshes. Numerical results are provided to confirm the efficiency of the method. A comparison study of several numerical methods is also included.

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**Conformal conservation laws and geometric integration for damped Hamiltonian PDEs****Brian Moore**

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**Laura Norena and Constance Schober**

Conformal conservation laws are defined and derived for a class of multi-symplectic equations with added dissipation. In particular, the conservation laws of energy and momentum are considered, along with those that arise from linear symmetries. Numerical methods that preserve these conformal conservation laws are presented in detail, providing a new framework for proving a numerical method exactly preserves the dissipative properties considered. The conformal methods are compared analytically and numerically to standard conservative methods, which includes a thorough inspection of numerical solution behavior for linear equations. The semi-linear wave equation and nonlinear Schrodinger equation with added dissipation, as well as an elliptic boundary value problem, are used as examples to demonstrate the results.

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**Determining important parameters in the dynamics of a three-compartment model of abiotic nutrient pool, autotroph and detritus****Eucharia Nwachukwu**

University of Port Harcourt, Nigeria

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**Enu N. Ekaka-a**

We conduct a systematic parameter ranking for an existing three-compartment model of abiotic nutrient pool, autotroph and detritus. We commence by describing this important model and highlighting a few characterizations of the complexities in the model. Using existing set of parameter values and assuming initial conditions, we were able to conduct this rigorous parameter ranking. Our present method permits us to deduce interesting conclusions about the relative importance of individual parameters within the model. This method also permits us to gain some insights into the ability of the model to show what happens within the African ecological system. Considering alternative parameter values,

we test the behavior of this model which has allowed us to identify a deficiency in one part of the model. For this complex model (a nonlinear system of ordinary differential equations with a large number of parameters), we observe in this group of six parameters that the experimental time is ranked as an important parameter than the harvesting rate of autotroph biomass that is removed from the system.

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### Sensitivity analysis of a partially coupled system of differential equations without delay

**Eucharia Nwachukwu**

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**Enu N. Ekaka-a**

For a class of non-linear partially coupled differential equations of first order, we propose a numerical sensitivity analysis based on the behavior of the solution trajectories due to a variation of a model parameter when other parameters are fixed. The numerical algorithms were written and corresponding cumulative percentage changes are calculated. Numerical results are presented and discussed. We observe in this rigorous sensitivity analysis over a time interval that the experimental time,  $T$  and the ratio of nutrient mass to total biomass in autotroph and detritus,  $\Upsilon$  are the most sensitive and the least sensitive parameters respectively. We will expect this contribution to provide insights in understanding the role of parameter estimation in this important scientific problem.

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### Fully fuzzy systems of linear equations

**Purnima Pandit**

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The systems of linear equations arise in various fields of Engineering and Sciences like Electrical, Civil, Economics, and Dietary etc. Many times in real applications it is difficult to obtain the precise values of the parameters involved in the systems rather an interval in which their values fall can only be estimated by expert knowledge. In such situation it may be convenient to represent such parameters can be represented by fuzzy numbers (refer [1]). Our

work extends the results for the solution of linear algebraic equations with fuzzy numbers (refer [2]) to the system of linear equations with fuzzy parameter using matrices. The fuzzy system is first converted into crisp using  $\alpha$ -cuts. From the solution of converted crisp system, the solution of fuzzy system can be obtained if they satisfy the conditions given in our main result.

References:

1. L.A. Zadeh, Fuzzy sets, Information and Control, 8 , 338-353 (1965)

2. Klir G., Yuan B., Fuzzy Sets and Fuzzy Logic Theory and Applications, Prentice Hall, (1997)

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### Modeling the thermal conductance of phononic crystal plates

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**Stefanie Thiem**

We present a method to model the phonon thermal conductance in porous phononic crystal plates. The goal is to optimize the figure of merit for materials, which is the primary criterion for the efficiency of a thermoelectric device. Values of about three or higher allow for the construction of thermoelectric generators based on the Seebeck effect, which are more efficient than conventional electrical generators. The quantity to be optimized for the phononic crystals slabs is the Landauer-Büttiker thermal conductance, which can be achieved by varying the geometrical structure concerning width and thickness of the sample as well as pore size, shape and density. To obtain the Landauer-Büttiker thermal conductance we have to determine the phonon mode distribution by solving the elastic equations of motion for the considered systems. Making use of the periodicity of the system, the equations of elastic wave propagation can be Fourier transformed. This yields a generalized eigenvalue problem for every vector  $\mathbf{k}$  in the Brillouin zone. Making use of parallelization we can efficiently compute the dispersion relation and the corresponding phonon mode distribution in a straightforward way.

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## Contributed Session 09: PDEs and Applications

### Solving certain PDE's using a generalized Hankel transform

**Ismail Ali**

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We introduce a generalized form of the Hankel transform, and study some of its properties. A partial differential equation associated with the problem of transport of a heavy pollutant (dust) from the ground level sources within the framework of the diffusion theory is treated by this integral transform. The pollutant concentration is expressed in terms of a given flux of dust from the ground surface to the atmosphere.

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### Applications of mixed boundary conditions and Sturm Liouville equations in atherosclerosis

**Igor Balsim**

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**David S. Rumschitzki, Shripad Joshi**

We have solved the problem of the existence and uniqueness of the solutions for the pressure of the blood flow of Laplace's equation with the Dirichlet and Robin mixed boundary conditions. This mathematical problem is given as a model of the steady state diffusion of a large tracer molecule or molecular aggregate from the blood, across the vessel lining, comprised of a monolayer of endothelial cells that are tightly bound to one another, and into the vessel wall. The geometry of the problem is a rectangle in the Euclidean plane that is divided into two rectangles: media, and intima with the two solutions. By expanding the boundary conditions in terms of the eigenfunctions of Bessel functions used to represent the solution and applying the given boundary conditions, we found a relationship between the coefficients of these expansions. By using Brower's fixed point theorem for an operator equation we showed that the vector of coefficients of the solutions converges in the Banach space which proved the existence of an unique solution. Our proof supports a surprising result that was confirmed experimentally that the convergence rate increases as the size of the leaky juncture increases.

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### Topological methods for boundary value problems involving discrete vector $\phi$ -Laplacians

**Dana Bereanu**

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**Cristian Bereanu**

In this talk, using Brouwer degree arguments, we present some existence results for nonlinear problems of the type

$$-\nabla[\phi(\Delta x_m)] = g_m(x_m, \Delta x_m) \quad (1 \leq m \leq n-1),$$

submitted to Dirichlet, Neumann or periodic boundary conditions, where  $\phi(x) = |x|^{p-2}x$  ( $p > 1$ ) or  $\phi(x) = \frac{x}{\sqrt{1-|x|^2}}$ , and  $g_m : \mathbb{R}^N \rightarrow \mathbb{R}^N$  ( $1 \leq m \leq n-1$ ) are continuous nonlinearities satisfying some additional assumptions. This is a joint work with Cristian Bereanu. C. Bereanu, D. Gheorghe, Topological methods for boundary value problems involving discrete vector  $\phi$ -Laplacians, *Topol. Methods Nonlinear Anal.* 38 (2011), 265-276.

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### Poincare Sobolev equations in the hyperbolic space

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**K. Sandeep**

We have studied the a priori estimates, existence/nonexistence of radial sign changing solution, and the Palais-Smale characterisation of the problem  $-\Delta_{B^N} u - \lambda u = |u|^{p-1}u$ ,  $u \in H^1(B^N)$  in the hyperbolic space  $B^N$  where 1

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### On solvability of balance equations for atmosphere dynamics

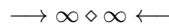
**Andrei Bourchtein**

Pelotas State University, Brazil  
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**Ludmila Bourchtein**

Numerical atmosphere modeling includes computational solution of the PDEs expressing the laws of continuum media in the rotating reference frame. The governing equations support both relatively slow energy dominant processes and fast gravity and acoustic waves of small amplitude. Solution of such

a stiff system requires, in particular, definition of appropriate initial conditions, which usually are not adjusted dynamically, that gives rise to fast oscillations of great amplitude, which are not observed in the real atmosphere. An adjustment of the initial data for the atmospheric models usually leads to a set of diagnostic PDEs representing balance relations. One general problem in solving such diagnostic relations is non-ellipticity of the PDEs for some real atmospheric conditions that does not allow to formulate well posed boundary value problems. For example, the nonlinear balance equation by Charney is of the Monge-Ampere type and as such it is non-elliptic for given pressure function in the regions where anticyclonic activity is rather strong. In this study, we present ellipticity conditions for more complex differential systems of nonlinear adjustment. Based on these results, we show distribution of non-elliptic regions in the gridded data of the actual atmospheric fields for different forms of the balance equations.



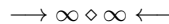
**Global regularity for the initial value problem of a 2-D Kazhikhov-Smagulov type model**

**Xiaoyun Cai**

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**Liao Liangwen, Yongzhong Sun**

We prove global-in-time existence of regular solution to the initial value problem of a 2-D Kazhikhov-Smagulov type model for incompressible nonhomogeneous fluids with mass diffusion.



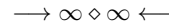
**Regularization of ill-posed evolution problems in Hilbert space**

**Matthew Fury**

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In recent literature, many results have been proved concerning the regularization of the backwards heat equation, the classic ill-posed problem. Such initial value problems may not have unique solutions for sufficiently many initial data and or solutions may not depend continuously on the initial data. The regularization of the backwards heat equation and other ill-posed problems then involves approximating a known solution of the problem for a given initial value by the solution of an approximate well-posed problem with the same initial data. In this paper, we generalize this process to prove regularization for abstract evolution problems in a Hilbert space  $H$  of the form  $du/dt = A(t, D)u(t) + h(t)$ ,  $0 \leq s \leq t \leq T$  with initial value  $u(s) = x$ , where  $h(t)$  is an  $H$ -valued function and  $A(t, D)$  is an operator depending on both

$t \in [0, T]$  and a positive, self-adjoint operator  $D$  in  $H$ . We use operator theory to prove the regularization and apply the results to the inhomogeneous backwards heat equation with a time-dependent diffusion coefficient and other higher-order partial differential equations in  $L^2(\mathbb{R})$  where  $D = -\Delta$ .

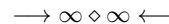


**Nonlinear hyperbolic balance laws coupled with ordinary differential equations**

**Mauro Garavello**

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Systems composed by PDEs and ODEs can be used for the description of complex phenomena, whose evolution has both macroscopic and microscopic behaviors. Similar multiscale models are used, for example, in the simulation of the blood flow in the human body: the main parts of the circulatory system are modeled by PDEs, while the remaining parts by ODEs. Other possible applications of such models are: traffic flows, supply chain, particles inside fluids. In the talk we present a nonlinear hyperbolic system of balance laws coupled with a system of ordinary differential equations. More precisely, the ordinary differential equations influence the solution to the balance laws by means of the boundary term and, at the same time, the balance laws modify the vector field of the ordinary differential equations. We discuss existence and well posedness of the Cauchy problem of this coupled system. This is a joint work with R. Borsche and R. M. Colombo.

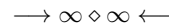


**Some results in tomography**

**Nicholas Hoell**

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We present recent work on the inverse problem of analytically inverting the attenuated x-ray transform on curves in two-dimensional regions of Euclidean space and more generally, simple Riemannian manifolds. The work is based on recasting the problem as a Beltrami equation with analytic dependence on a parameter to find a suitable integrating factor for the corresponding stationary transport equation. The problem originated in the medical imaging modality SPECT and has also recently arisen in the unique reconstruction of permittivity and permeability parameters of a conductive body made from external measurements.



### Temporal dispersion along uniform flow in one-dimensional porous media

**Dilip Kumar Jaiswal**

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**Atul Kumar, R. R. Yadav**

The use of water has increased year to year and the consumption rate varies among countries. Over exploitation of water resources without commensurate recharge has resulted in fall in groundwater table. Further, leaching of pollutants from disposal garbage sites, pesticides and fertilizers into the aquifer has degraded groundwater quality. In the present study, analytical solutions are obtained for temporal dispersion along uniform flow velocity in a one-dimensional semi-infinite domain. Initially the domain is not solute free. It is combination of exponentially increasing function of space variable and ratio of zero order production and first decay which are inversely proportion to dispersion coefficient. Retardation factor is also considered. Using Laplace transform technique to obtained the solutions. Numerical examples are given with different graphs.

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### The boundary Harnack principle for second order elliptic equations with unbounded drift

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**Mikhail Safonov**

We derive the Carleson type estimates and the boundary Harnack principle for positive solutions of non-divergence, second order elliptic equations  $Lu = a_{ij}D_{ij}u + b_iD_iu = 0$  in a bounded domain  $\Omega \subset \mathbb{R}^n$ . We assume that  $b_i \in L^n(\Omega)$ , and  $\Omega$  is a twisted Hölder domain (THD) of order  $\alpha \in (0, 1]$ . In addition,  $\Omega$  satisfies a strong regularity condition if  $\alpha \in (0, 1/2]$ , and a weak regularity condition if  $\alpha \in (1/2, 1]$ .

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### Analytical approach of one - dimensional solute transport through inhomogeneous semi-infinite porous domain for unsteady flow: dispersion being proportional to square of velocity

**Atul Kumar**

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**Dilip Kumar Jaiswal, R. R. Yadav**

In the present work, analytical solution of one-dimensional solute transport for a pulse type point source for uniform nature along the unsteady flow

through inhomogeneous semi-infinite porous medium along longitudinal directions. According to Scheidger (1957), dispersion is considered directly proportional to the square of velocity as the linear spatially dependent function defining the inhomogeneity and temporally dependent function. It is expressed in degenerate form. Initially the domain is solute free. The input condition is considered pulse type and introduced at the origin of the domain. Other condition is considered flux type at the end of the domain. Certain new independent variables are introduced through separate transformation to eliminate the variable coefficients of Advection Diffusion Equation (ADE) into constant coefficients. Then Laplace transform technique (LTT) is used to get the analytical solution of ADE. It is illustrated for exponentially decreasing and increasing time dependent functions and concentration profiles are shown graphically.

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### Evaluation of the stochastic modeling on options

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**Zhijuan Mao, Hongkun Zhang, Zhian Liang, Jinguo Lian**

Modern option pricing techniques are often considered among the most mathematically complex of all applied areas of financial engineering. In particular these techniques derive their impetus from four milestone of option pricing models: Bachelier model, Samuelson model, Black-Scholes-Merton model and Levy model. In this paper we evaluate all related option pricing models based on these milestones, by comparing the corresponding stochastic differential equations and option pricing formulas. In addition we also include some simulations to make the comparisons more transparent.

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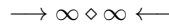
### Group classification of nonlinear equations on different surfaces

**Imran Naeem**

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A complete group classification for the  $(1 + n)$ -dimensional Klein-Gordon equation when  $n = 2, 3$  is presented. Then using these results we extend the classification to the general case when  $n$  is arbitrary. It is shown that there is a class of functions which is independent of the number of independent variables for which the group classification exists. Symmetry generators, up to equivalence transformations,

arecalculated for each  $f(u)$  when the principal Lie algebra extends. Moreover the groupclassification of the  $(1 + 2)$ -dimensional Klein-Gordon equations on the sphere (S2)and torus (T2) are discussed. Classifications of these symmetry algebras are obtainedup to conjugacy classes and similarity reduction for each class is given. The effect off(u) and the underlying space on the infinitesimal generators is also discussed. Wealso comment on the group classification of the  $(1 + n)$ -dimensional Klein-Gordonequation on  $S_n$  and  $T_n$ .



**A complex Noether approach for derivation of conservation laws for partial differential equations in complex field**

**Rehana Naz**

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We study scalar complex partial differential equations with two real independent variables which admit variational formulations. Such a complex partial differential equation, via a complex split of the dependent variable, splits into a system of two real partial differential equations. The decomposition of the Lagrangian of the complex partial differential equation in the real domain yields two real Lagrangians for the split system. The complex Maxwellian distribution equation, transonic gas flow equation, Maxwellian tail, wave equation with dissipation and Klein-Gordan equation are considered. The Noether symmetries and gauge terms of the split system corresponding to both Lagrangians are constructed by the Noether approach. We compare the Noether symmetries and gauge terms of the split system with the split Noether-like operators and gauge terms of the given complex partial differential equation in the real domain. We conclude that the split Noether-like operators and gauge terms of the complex partial differential equation are not in general the same as the Noether symmetries and gauge terms of the split system of real partial differential equations. They are the same when all the Noether symmetries of the complex partial differential equation has either pure real or pure imaginary form. Furthermore, the split conserved vectors of the complex partial differential equation are the same as the conserved vectors of the split system of real partial differential equations in the case of coupled systems.



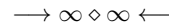
**Attractors for semilinear parabolic problems with concentrated and oscillating terms on the boundary**

**Marcone Pereira**

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**Gleiciane S. Aragão, Antônio L. Pereira**

In this talk we analyze the dynamics of the flow generated by a nonlinear parabolic problem when some reaction and potential terms are concentrated in a neighborhood of the boundary. We assume that this neighborhood shrinks to the boundary as a parameter  $\epsilon$  goes to zero. Also, we suppose that the “inner boundary” of this neighborhood presents a highly oscillatory behavior. Our main goal here is to show the continuity of the family of attractors with respect to  $\epsilon$ . Indeed, we use abstract results from [A. L. Pereira and M. C. Pereira, *Continuity of attractors for a reaction-diffusion problem with nonlinear boundary conditions with respect to variations of the domain*, Journal of Differential Equations **239** (2007), 343-370] to extend results from [A. Jiménez-Casas and A. Rodríguez-Bernal, *Asymptotic behaviour of a parabolic problem with terms concentrated in the boundary*, Nonlinear Analysis: Theory, Methods & Applications **71** (2009), 2377-2383] to a parabolic problem in which the “inner boundary” of  $\omega_\epsilon$  presents a highly oscillatory behavior, and assuming hyperbolicity of the equilibria of the limit problem, we also obtain results on the lower semi-continuity of the attractors.

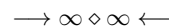


**Energy decay of Klein - Gordon - Schrödinger type with linear memory term**

**Marilena Poulou**

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This paper is concerned with the existence, uniqueness and uniform decay of the solutions of a Klein-Gordon-Schrödinger type system with linear memory term. The existence is proved by means of the Faedo-Galerkin method and the asymptotic behavior is obtained by making use of the multiplier technique combined with integral inequalities.



**Note on “common fixed point results for non-commuting mappings without continuity in cone metric spaces”**

**Hamidreza Rahimi**

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**Ghasem Soleimani Rad**

In 2007, Huang and Zhang introduced the concept of the cone metric space, replacing the set of real numbers by an ordered Banach space, and they showed some fixed point theorems of contractive type mappings in cone metric spaces. The concept of w-distance in complete metric spaces was defined by Kada et al. in 1996. Then Cho et al. Wang and Guo defined a concept of the c-distance in a cone metric space, which is a cone version of the w-distance and proved some fixed point theorems in ordered cone metric spaces. In this talk, we investigate to a common fixed point theorem by using the generalized distance in a cone metric space. Our theorems extend some results of Abbas and Jungck [M. Abbas, G. Jungck, Common fixed point results for noncommuting mappings without continuity in cone metric spaces, J.Math. Anal. Appl. 341 (2008) 416-420] and Cho et al. [Y.J. Cho, R. Saadati, S.H. Wang, Common fixed point theorems on generalized distance in ordered cone metric spaces, Comput. Math. Appl.61 (2011) 1254-1260].

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**On ratio-dependent predator-prey systems with disease in the prey**

**Kimun Ryu**

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In this talk, we consider ratio-dependent predator-prey systems with disease in the prey under Neumann boundary conditions. we investigate the properties of nonnegative solutions to the reaction-diffusion systems. Furthermore, we provide sufficient conditions for the existence of non-constant positive steady-state solutions.

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**Spectrally matched optimal grids for receiver-targeted PDE problems**

**Adnan Sabuwala**

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In this talk, we will present an introduction to the construction of non-linear spectrally matched optimal grids for solving receiver-targeted PDE problems such as in geophysical exploration. This involves a

clever rational approximation of the NtD (Neumann to Dirichlet) map. The error convergence rate at the receiver locations will be investigated for a 2-d isotropic and a 1-d anisotropic problem. We will look at an application of this method to Laplace’s equation on a rectangle with unbounded spectrum for the Neumann data will be illustrated along with numerical evidence.

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**A homogenization problem for a thin domain with a highly oscillating boundary**

**Ricardo Silva**

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”A homogenization problem for a thin domain with a highly oscillating boundary” We combine methods from linear homogenization theory to obtain convergence rates for solutions of the Poisson Equation with Neumann boundary conditions posed in an one parameter family of two dimensional domains with highly oscillating boundary and which collapse on one dimensional set as the parameter goes to 0.

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**Dynamics in the nonlinearly excited 6th-order phase equation**

**Dmitry Strunin**

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Excitable media behaving in oscillatory manner can often be simulated by a single active-dissipative partial differential equation for the phase of oscillations. The equation reduces the complex phenomenon into a relatively concise form, examples being the Kuramoto-Sivashinsky equation describing unstable flame fronts and the Nikolaevskiy equation describing resonances in seismic waves in fluid-saturated rocks. The both equations incorporate excitation by a linear mechanism. This mechanism has been studied quite extensively by now, whereas the nonlinear mechanism remains under-explored. In the present work we study three-dimensional dynamics in the nonlinearly excited phase equation where dissipation is represented by the 6th-order spatial derivative. Numerical solutions are obtained for the phase evolving in a rectangular spatial domain. Regular and chaotic modes of the evolution are discussed.

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### Approximate analytical solution of reaction-diffusion Brusselator system with fractional time derivative

**Canan Unlu**  
istanbul university, Turkey  
unlucanan@hotmail.com

**Canan Unlu, Sunil Kumar, Yasir Khan, Ahmet Yildirim**

In this letter, we used homotopy perturbation method(HPM) to obtain approximate analytical solutions of reaction-diffusion Brusselator system with fractional time derivative. Fractional reaction-diffusion Brusselator system has important applications in chemical reaction-diffusion processes. We present numerical results with graphically and we see that the homotopy perturbation method is an effective and convenient method to solve fractional-order reaction diffusion Brusselator system.

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### Influence of gravity and initial stress on the Torsional wave propagation in a Substratum over a dry sandy gibson half space

**Sumit Vishwakarma**  
Indian School of Mines, Dhanbad, India  
sumo.ism@ismu.ac.in  
**Shishir Gupta**

The propagation of Torsional surface waves in an inhomogeneous anisotropic substratum lying over a dry sandy gravitating gibson half space in the presence of initial stress have been studied analytically and computed numerically. The dispersion equation has been derived in a closed form using Whittaker's function and its derivative. The influence of various inhomogeneity parameters on the propagation of Torsional surface wave has been described under the effect of gravity and compressive initial stress by means of graphs. The influence of Biot's gravity parameter and sandy parameter has also been shown on the Torsional surface wave propagation. Dispersion equations are in perfect agreement with the standard results when derived for some particular cases. It is observed that the presence of gravity field always allow the Torsional surface wave to propagate. It has also been concluded that the Torsional surface wave propagates more smoothly in the layer when the lower gibson half space is elastic in comparison to dry sandy gibson half space. Further, anisotropy has also much effect in enhancing the velocity of Torsional surface wave. Graphical user interface (GUI) software has been developed using MATLAB to generalize the effect of various parameter discussed.

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### Sharp blow-up for semilinear wave equations with non-compactly supported data

**Kyouhei Wakasa**  
Future University Hakodate, Japan  
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**Hiroyuki Takamura, Hiroshi Uesaka**

In this talk, I will introduce a correction of Asakura's observation on semilinear wave equations with non-compactly supported data by showing a sharp blow-up theorem for classical solutions. We know that there is no global in time solution for any power nonlinearity if the spatial decay of the initial data is weak, in spite of finite propagation speed of the linear wave. Our blow-up theorem clarifies the final criterion on such a phenomenon. I will discuss the assumption on the data in various forms.

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## Contributed Session 10: Bifurcation and Chaotic Dynamics

### Gursey instantons under the quantum fluctuation in phase space

**Fatma Aydogmus**

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**K. Gediz Akdeniz**

Gursey Model which has been assumed in 1956 as a possible basis for a unitary description of elementary particles is only possible four dimensional conformally invariant nonlinear pure spinor model. Recently, the behaviors of Gursey instantons in phase space were investigated depending on coupling constant. In this presentation, we shall investigate the behaviors of Gursey instantons in phase space under the quantum fluctuation to provide the better understanding quantum dynamics of spinor type instantons in vacuum.

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### Numerical normal forms for limit cycles

**Virginie De Witte**

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**W. Govaerts, Yu. A. Kuznetsov, F. Della Rossa, H.G.E. Meijer**

In this talk we concentrate on the computation of the normal form coefficients for codim 2 bifurcations of limit cycles. Our formulation is independent of the dimension of the phase space and involves solutions of certain linear boundary value problems. In the Matlab numerical bifurcation software MatCont these BVPs are discretized by orthogonal collocation with piecewise polynomials. A great advantage of our method is that we are able to avoid the computation of Poincaré maps and their derivatives. The formulas allow us to distinguish between various bifurcation scenarios near codim 2 bifurcations of limit cycles. We discuss the theoretical derivation of the normal form coefficients, their implementation in MatCont and their practical use by means of examples.

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### The study of the standard families of unfoldings of the nilpotent saddle of codimension 2 and 3 whose x-axis is invariant.

**Remy Magloire Etoua**

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In this talk we study the standard families of unfoldings of the nilpotent saddle of codimension 2 and

3 whose x-axis is invariant. The goal of our study is to make a systematic analysis of the bifurcations of these models (which leads to the determination of their global bifurcation diagram): saddle-node bifurcations, Hopf bifurcation of codimension 1 or 2, heteroclinic bifurcation of codimension 1 or 2.

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### Bifurcations of piecewise smooth flows: perspectives, methodologies and open problems

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The theory of bifurcations in piecewise smooth flows is critically surveyed. The focus is on results that hold in arbitrarily (but finitely) many dimensions, highlighting significant areas where a detailed understanding is presently lacking. The clearest results to date concern equilibria undergoing bifurcations at switching boundaries and limit cycles undergoing grazing and sliding bifurcations. After discussing fundamental concepts such as topological equivalence of two piecewise smooth systems, discontinuity-induced bifurcations are defined for equilibria and limit cycles. Conditions for equilibria to exist in  $n$ -dimensions are given, followed by the conditions under which they generically undergo codimension-one bifurcations. The extent of knowledge of their unfoldings is also summarized. Codimension-one bifurcations of limit cycles and boundary-intersection crossing are described together with techniques for their classification. Codimension-two bifurcations are discussed with suggestions for further study.

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### Multi-state and multi-stage synchronization of Hindmarsh-Rose neurons with excitatory chemical and electrical synapses

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The new phenomena of the multi-state synchronization of Hindmarsh-Rose (HR) neurons with both excitatory chemical and electrical synapses over the complex network are analytically studied. The regions for coupling strengths to achieve local synchronization are explicitly obtained. Such regions are characterized by the second largest eigenvalue  $\lambda_2$  of the electrical connection matrix and the number  $k$  of chemical signals each neuron receives. The dynamics

of the multi-state synchronization includes the coexistence of stable regular bursting and periodic/steady-state behaviors. These are in contrast with coupled oscillator systems or coupled map lattices where only single-state synchronization is found. It should also be noted that if the parameters of HR neurons are chosen resulting in an irregular (chaotic) bursting, then the coexistence state would contain chaotic attractor. Our method employed here is quite general. For instance, it can be immediately applied to other coupled nervous systems such as FitzHugh-Nagumo and Morris-Lecar nervous systems. The analytical tools and concepts needed include coordinate transformations, matrix measures, monotone dynamics and time averaging estimates.

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### Bifurcation and asymptotic analysis of a coupled gyroscope system

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**Huy Vu, Nathan Davis, Visarath In, Patrick Longhini**

A critical component of many navigation systems are the gyroscopes, a device used for detecting rotation rates and orientation. Gyroscopes are subject to material imperfections and manufacturing defects, potentially reducing their ability to detect signals and to minimize phase drift. This work explores the idea of performance enhancement by combining the input-output response of an ensemble of gyroscopes coupled in a directed ring fashion. Bifurcation and asymptotic analysis show the existence of a wide range of behaviors, including: chaos, quasi-periodicity, and of particular interest, a stable synchronization state in which all gyroscopes vibrate with the same phase and amplitude. In this state, the sum output of the sensing modes is significantly larger than any individual sensing mode, which leads to substantial reductions in phase drift caused by material imperfections, fabrication variations, and system noise.

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### A fold-Hopf-like bifurcation in piecewise linear continuous differential systems with symmetry

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**Enrique Ponce, Elisabet Vela**

Three-dimensional symmetric piecewise linear differential systems near the conditions corresponding to the fold-Hopf bifurcation are considered. By introducing one small parameter, we study the bifurcation of limit cycles in passing through its critical value, that is, when the three eigenvalues of the linear part

at the origin are at the imaginary axis of the complex plane. The simultaneous bifurcation of three limit cycles is proved. Conditions for stability of these limit cycles are provided, and analytical expressions for their period and amplitude are obtained. Finally, we apply the theoretical results of this work to a generalized version of Chua's circuit, showing that the fold Hopf bifurcation takes place for a wide range of the parameters.

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### Focus-center-limit cycle bifurcation in discontinuous planar piecewise linear systems without sliding

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A family of planar discontinuous piecewise linear systems with two linearity zones is considered. By using some changes of variables and parameters, a Liénard canonical form is obtained. This canonical form has seven parameters. In the particular case analyzed with focus-focus dynamic and without sliding, a reduced canonical form with only four parameters is obtained. Under certain hypotheses the existence of a limit cycle bifurcation is assured. Analytic expressions for the amplitude, period and characteristic multiplier of the bifurcating limit cycle are provided. To illustrate the appearance of this bifurcation in real world applications, a Wien bridge oscillator without symmetry is analyzed using the provided theoretical results.

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### Diffusions in chaotic billiards

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Lorentz gas is a simple model of diffusive billiards in studying the transport process of electrons in an ionized metal. Since the key aim in statistical mechanics is to characterize the diffusion matrix that appear in the CLT. We study 2-d periodic Lorentz gas in the presence of a twist force on the scatterers. In this system, particle orbits are still geodesics between collisions, but do not reflect elastically when reaching the boundary. When the horizon is finite, i.e. the free flights between collisions are bounded, the resulting current  $J$  is proportional to the strength of the twist force. We also prove the existence of a unique SRB measure, for which the Pesin entropy formula and Young's expression for the fractal dimension are valid. The classical CLT is verified and the diffusion matrix is calculated according to a formula involves the twist function.

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## Poster Session

### Calculating the width parameter of the potential energy surface of some chemical reactions using simple quantum mechanics

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A simple computational procedure is used for defining the width parameter of the potential energy surface of different elementary interchange chemical reactions.

Concerning the computational theoretical basis, a simplified quantum mechanical treatment is presented for the phenomena of the reactive scattering of an atom A and a diatomic molecule BC giving rise to a free atom C together with a diatomic molecule AB.

The mathematical formula derived allows the reduced masses to vary smoothly with the reaction coordinate "u". This property is extremely important, since it is the actual nature in which the reduced masses of reactants change into those of the products.

Also the treatment under consideration uses Eckhart potential, which truly and completely describes the reaction process. This potential allows the reactants to start at the initial state asymptote  $-\infty$ , with a certain initial potential energy  $v_{-\infty}$ , ascending smoothly over a definite potential barrier, descending again smoothly, and finally ending with a certain constant final potential level  $v_{+\infty}$  at the far final state asymptote  $+\infty$ .

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### Classical Hybrid Monte-Carlo Simulation of the Interconversion of Hexabromocyclododecane stereoisomers

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**Marcus Weber, Roland Becker, Vedat Durmaz, Robert Koeppen**

The interconversion of the six main stereoisomers of the flame retardant hexabromocyclododecane (HBCD) is investigated by means of statistical thermodynamics using classical force-fields. (+)-alpha-, (+)-beta- and (+)-gamma-HBCD interconvert by swapping of absolute configurations on the three different (BrHC ??? CHBr)-moieties. The approach avoids saddle-point energy computations, but relies on classical thermodynamic simulation and pursues three consecutive steps. First, the application of classical hybrid Monte-Carlo simulations for quantum mechanical processes is justified. Second, the

problem of insufficient convergence properties of hybrid Monte-Carlo methods for the generation of low temperature canonical ensembles is solved by an interpolation approach. Third, it is shown how free energy differences among stereoisomers are derived and how they can be used for the computation of interconversion rates. The simulation results confirm the experimentally observed interconversion rates and correctly identify alpha-HBCD as a thermodynamical sink in the oscillating mixture of stereoisomers.

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### A heuristic mathematical model for the core circadian clock in *Neurospora crassa*.

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**Sookkyung Lim, Christian Hong**

Circadian rhythms are cycles of about 24 hours, which provide temporal information to various biological processes such as sleep/wake cycle. These rhythms have been observed in many organisms from cyanobacteria up to humans. The molecular mechanisms of circadian rhythms are similar from *Neurospora crassa* to mammals. Therefore, a comprehensive analysis of its circadian rhythm will facilitate our understanding of the underlying properties of the human circadian rhythms. The *Neurospora crassa* circadian clock is a molecular mechanism consisting of complex gene regulations (activation, inhibition), post-transcriptional and post-translational regulations that create a time delayed negative feedback loop. An initial mathematical approach is presented to model the core of this clock. Parameter estimation, bifurcation and stability analysis is presented to determine the dynamical features of this mathematical model.

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### Linear Volterra Stieltjes integral equations in the sense of the Perron integral and an application to functional differential equations

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We prove an existence result for linear Volterra Stieltjes integral equations with Perron-Stieltjes type integrals. As an application, we consider a linear functional differential equations perturbed by a Perron integrable function and we get an existence result for this equation.

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**A Class of Solutions for the Generalized Langevin Equation Driven by Stable Processes**

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**Fabiano F.T. dos Santos**

Random motions of particles are generally modelled as diffusions driven by some noise process and solutions to the resulting stochastic differential equations are searched. When white noise is detected the Langevin equation may arise as a suitable model. On the other hand, in modelling a large variety of complex systems, one encounters anomalous diffusive phenomena. In this case, Generalized Langevin Equation (GLE) driven by stable processes with infinite variance might well be considered. Assuming stability index 1



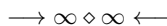
**A Reaction-Diffusion Model for Cell Populations in the Colonic Crypt**

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**Gilberto Schleiniger, Bruce Boman**

The human colon contains microscopic finger-like invaginations called crypts that are responsible for continuous epithelial cell reproduction. Inside the crypt, stem cells accumulate near the base and migrate up the crypt walls while proliferating and differentiating, until finally exiting through the top of the crypt as a completely differentiated and non-proliferating cell. Once the crypt establishes a specific cell organization, it splits into two new crypts. In a cancerous colon, we see an abundance of stem cells throughout the entire crypt, which leads to abnormal crypt fission and eventually polyp-like adenoma. Our goal is to model the normal cell regulation inside the crypt using a reaction-diffusion model in a 2-D cylindrical domain, and then perturb the system to gain insight into how it becomes disorganized. We consider three cell populations: stem cells, proliferating cells, and differentiated cells. The non-linear reaction terms describe cell production, while the diffusion terms describe the migratory effect throughout the crypt.



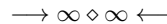
**Equilibrium in a two-oscillator system coupled to a chaotic environment**

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This study explores the dynamics underlying the coupling of two oscillators to classical low-dimensional chaos. In particular, we examine the case in which the oscillators are close in frequency. It is shown that, given sufficient coupling, resonant oscillators can exchange energy and equilibrate to nonequipartition values. The off-resonant cases studied were shown to have equilibrium momentum distributions close to Gaussian. We found additionally that the Rough and kinetic temperatures relaxed in time in a quite similar manner.



**Predator-Prey Relationship in a Closed Habitat**

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In this paper we study Predator-Prey dynamics by utilizing, as a basis for investigation, the Haefner-Holling Model of Prediction in Simple Communities. The interactivity and stability of a moose and wolf relationship will be analyzed mathematically. The general form of this model will be reviewed and applied to an environmental system that is represented by three sub-systems: the habitat, the moose population, and the wolf population. Looking at seven specific variables: moose reproduction rate, moose habitat carrying capacity, moose/wolf encounter rate, total time available for moose to forage, time required for wolf to harvest a moose, conversion rate of victims to new predators and per capita wolf death rate, predictions of future population stability will be examined. A look into a hypothetical situation will conclude the investigation to insure integrity of the model.



### Effects of Optimal Antipredator Behavior of Prey on Predator-Prey Dynamics: The Role of Refuges

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**Brandy Jones**

The purpose of this paper is to give some mathematical analysis on the predator-prey models where prey has anti-predator behavior with the use of refuges. The models examined were introduced by Vlastimil Krivan in his paper, "Effects of Optimal Anti-predator Behavior of Prey on Predator-Prey Dynamics: The Role of Refuges", and were based on the Lotka-Volterra and Holling Type II functional responses. Analysis on local and global stability for equilibrium solutions will be performed on proposed values for given variables, and constraints for stability will be studied. We also demonstrate numerical simulation and discuss the ecological implications of our results.

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### Sufficient Conditions for Optimality of Bang-bang Controls in Compartmental Models for Cancer Chemotherapy with Blocking Agents

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In mathematical models for cancer chemotherapy, cell-cycle specificity is an important aspect that allows for a more precise modeling of the actions of drugs. In this poster, a cell-cycle specific compartmental model for the growth of cancer cells that combines a cytotoxic (killing) agent with a cytostatic (blocking) agent is considered. The blocking agent is used to synchronize the transitions of the cell through the cell-cycle and thus leads to a more effective action of the killing agent. We analyze the model as an optimal control problem with the objective to minimize a weighted average of the cancer cells in the various compartments over a fixed therapy horizon, both over the course of therapy and at the end of the therapy horizon, while keeping the side effects of the drugs under control. For this model, candidates for optimal controls are bang-bang controls corresponding to full dose treatment intervals with rest periods in between. However, the optimality of these controls, particularly at the switching times, has to be verified. In this poster an algorithm will be formulated that allows us to verify the overall optimality of bang-bang trajectories.

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### From Chaotic to Hyperchaotic Rossler model: Periodic Orbits analysis.

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**Roberto Barrio, Sergio Serrano.**

An hyperchaotic dynamical system is defined by two positive Lyapunov exponents. This is only possible for at least a four dimensional system as for example the Rössler (1979) hyperchaotic model where  $\dot{x} = -(y + z)$ ,  $\dot{y} = x + ay + w$ ,  $\dot{z} = b + xz$  and  $\dot{w} = -cz + dw$  with  $a=0.25$ ,  $b=3.0$ ,  $c=0.5$  and  $d=0.05$ . Unlike the well-known 3D chaotic model, this system has been barely studied. Applying our periodic orbits (POs) search strategy we found 2, 3, 5 and 10 POs of multiplicity one to four with a mean period of 6.5, 13.2, 19.9 and 26.7, respectively. To find them we have carried on a systematic search using the Stability Transformation (ST) method of Schmelcher and Diakonov (1997) combining with the Newton method. The combination of this two methods will improve the efficiency of the ST method since near the solution the Newton method converges quadratically. For the sake of completeness we have also studied the periodic orbits collection found in the traditional 3D Rössler model and we have investigated the differences between the chaotic and hyperchaotic Rössler systems.

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### Bistable Fronts in Discrete Inhomogeneous Media

**Brian Moore**

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Solution behavior of a bistable differential-difference equation with inhomogeneous diffusion is considered in depth. Employing a piece-wise linear nonlinearity, often referred to as McKean's caricature of the cubic, front solutions are constructed which correspond, in the case of homogeneous diffusion, to monotone traveling front solutions or stationary front solutions in the case of propagation failure. A general form for these fronts is given for essentially arbitrary inhomogeneous discrete diffusion, and conditions are given for the existence of solutions to the original discrete Nagumo equation. The specific case of one defect is considered in depth, giving a complete understanding of propagation failure and a grasp on changes in wave speed. Insight into the dynamic behavior of these front solutions as a function of the magnitude and relative position of the defects is obtained with the assistance of numerical results.

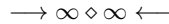
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**Bifurcation and Stability Analysis of a Prey-Predator Aystem with a Reserved Area**

**Debasis Mukherjee**

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This paper analyzes a prey-predator system with a reserved area. The predator functional response is taken to be of Holling type II .It is assumed that the habitat is divided into two zones, namely free zone and the other is reserved zone where predation is prohibited. The local and global stability analysis have been carried out. When the carrying capacity of the environment crosses a critical value, we determine that strictly positive equilibrium enters into Hopf bifurcation. We obtain conditions which influence the persistence of all the populations. Numerical simulation with a hypothetical set of data has been done to support the analytical findings.



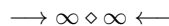
**Optimal Control for Multi-Drug Chemotherapies for Cell-Cycle Specific Models**

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**Urszula Ledzewicz, Heinz Schättler**

In mathematical models for cancer chemotherapy, cell-cycle specificity is an important aspect that allows for a more precise modeling of the actions of drugs. In this poster, a cell-cycle specific compartmental model for the growth of cancer cells that combines a cytotoxic (killing) agent with a recruiting agent is considered. This kind of therapy is particularly relevant for leukemia which is characterized by long s?jour times of the cancer cells in the dormant compartment. We analyze the model as an optimal control problem with the objective to minimize a weighted average of the cancer cells in the various compartments over a fixed therapy horizon, both over the course of therapy and at the end of the therapy horizon, while keeping the side effects of the drugs under control. The model will be considered without and with linear (one- or two-compartment) pharmacokinetic models on the drugs. The effect that the inclusion of these pharmacokinetic models has on the structure of solutions - optimal dosage protocols - will be analyzed. It is shown that in either case optimal controls are bang-bang and numerical examples of optimal solutions will be given.



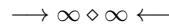
**Analysis of a System of Differential Equations: Population and Harvesting Dynamics of the North Carolina Red Drum**

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**Sofya Zaytseva**

This paper focuses on the population and harvesting dynamics of the North Carolina Red Drum. The model used for analysis is a two dimensional non-linear model. The purpose of this ODE model is to assess damage on the Red Drum stock size prior to year 2000, and to return the stock size to a healthy number but avoiding overpopulation. There are nine parameters used in this model. Some of the parameters were approximated using sensitivity analysis, while some were gathered using data from 1990. Three different cases were tested, using various parameters. It was found that in each case, there were two sets of real equilibrium points, although only one of them was asymptotically stable. This model has proved to be an efficient way to regulate fishing effort in North Carolina. In 2009, the state of North Carolina fisheries was taken out of the 'over fished' category and placed under the 'recovering' category.

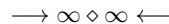


**Recursiveness in impulsive semidynamical systems**

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We consider a class of impulsive semidynamical systems and we study recursiveness properties for these systems. We present results which relate minimal sets, recurrent sets and almost periodic sets.



**Optical Biosensors with Arrays of Reacting Zones****Matt Zumbrum**University of Delaware, USA  
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Optical biosensors are widely used for measuring reaction rate constants and for understanding surface-volume reaction dynamics. Early biosensors included a single reacting zone for the study of a single reaction. New devices include arrays of reacting zones in a single flow channel, allowing for the study of up to four hundred reactions at once. Real time measurements of bound reactants in a reacting zone are taken and averaged to obtain a sensogram of the bound state. We discuss a model for ligand depletion and bound state evolution over arrays of reacting zones and extend previous work to arrays of circular reacting zones and arrays having reacting zones with different association and dissociation rates. The effect of these extensions is quantified and compared with previous results.

 $\rightarrow \infty \diamond \infty \leftarrow$

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Abernathy, Kristen (SS 16)	73
Abernathy, Zachary (SS 16)	73
Abramov, Rafail (OrgSS 44, SS 44)	186,186
Abrams, Daniel (SS 25)	111
Abramson, Guillermo (CS 4)	317
Abudiab, Mufid (OrgSS 72, SS 72)	272,272
Acosta-Humanez, Primitivo (SS 23)	100
Adili, Abiti (SS 68)	259
Adivar, Burcu (CS 4)	317
Adivar, Murat (CS 6)	322
Ahmed, Ahmed (SS 82)	305
Ahmed, Nasiruddin (OrgSS 15, SS 15)	66,66
Ahn, Inkyung (SS 72)	272
Aiki, Toyohiko (OrgSS 64, SS 64)	250,250
Aizicovici, Sergiu (SS 32)	142
Akagi, Goro (SS 1, SS 15)	4,66
Akhmatskaya, Elena (OrgSS 5, SS 5)	24,24
Albuquerque, Luis (SS 46)	195
Ali, Ismail (CS 9)	326
Alkan, Kivilcim (SS 82)	305
Allshouse, Michael (SS 27)	120
Almeida, João (SS 47)	198
Almendral, Juan (OrgSS 54, SS 54)	222,222
Almocera, Lorna (SS 18)	81
Almog, Yaniv (SS 62)	241
Alsahhaf, Ma`arib (PS)	334
Alseda, Lluís (OrgSS 23, SS 23)	100,100
Altenberg, Lee (SS 6)	28
Aluie, Hussein (SS 30)	133
Alvin, Lori (SS 23)	100
Ambrose, David (OrgSS 34, SS 34)	152,152
Amigo, Jose (OrgSS 51, SS 51)	213,213
Amrouche, Chérif (SS 77)	286
Amy Poh, Ai Ling (SS 9)	39
An, Aijun (SS 9)	39
Anco, Stephen (OrgSS 82, SS 82)	305,305
Andrae, Karsten (PS)	334
Angulo Pava, Jaime (SS 48)	202
Anh, Vo (SS 36)	160
Arciero, Julia (SS 25)	111
Arioli, Gianni (SS 10)	43
Arnaudon, Marc (SS 24)	106
Arroyo, David (SS 51)	213

Aryal, Saroj (SS 39)	170
Ashwin, Peter (OrgSS 75)	279
Auzinger, Winfried (SS 80)	299
Avalos, George (SS 4, SS 67)	17,254
Aydogmus, Fatma (CS 10)	332

## B

Badr, Abdallah (CS 2)	313
Bae, Hyeong-Ohk (SS 77)	286
Bae, Soohyun (SS 38)	166
Baek, Seunghyeon (SS 72)	272
Bai, Yun (SS 79)	294
Baigent, Stephen (CS 2)	313
Balagué, Daniel (SS 3)	12
Balauriya, Sanjeeva (SS 27)	120
Balibrea, Francisco (OrgSS 23, SS 23, SS 45)	100,100,191
Balseiro, Paula (SS 24)	106
Balsim, Igor (CS 9)	326
Baptista, Antonio (SS 5)	24
Barbagallo, Annamaria (SS 58)	235
Barbaro, Alethea (SS 3)	12
Barletta, Giuseppina (SS 32)	142
Barostichi, Rafael (SS 48)	202
Barranca, Victor (SS 44)	186
Barreiro, Andrea (SS 43, SS 44)	183,186
Barwell, Andrew (SS 23)	101
Basanta, David (SS 31)	137
Basu, Kanadpriya (SS 68)	259
Basu, Sukanya (SS 68)	259
Basu, Treena (SS 68)	259
Batt, Jurgen (SS 18)	81
Bauman, Patricia (SS 62)	241
Bebbouchi, Rachid (SS 46)	195
Bektur, Cisem (SS 47)	198
Bell, Jonathan (SS 25)	112
Beltrame, Philippe (SS 75)	279
Bemelmans, Josef (SS 77)	286
Benaim, Michel (OrgSS 47)	198
Benes, Nick (SS 78)	291
Benito, Rosa (SS 54)	222
Benoît, Eric (SS 17, SS 46)	78,195
Bereanu, Cristian (SS 26)	117
Bereanu, Dana (CS 9)	326
Berezovsky, Faina (SS 39)	170
Beri, Arjun (SS 44)	186
Bernard, Guy (SS 72)	272
Bernhardt, Christopher (SS 23)	101
Berning, Stephen (SS 43)	183



Berninger, Heiko (SS 79)	294	Calini, Annalisa (SS 67)	254
Beron-Vera, Francisco (SS 13, SS 27)	55,120	Calleja, Renato (SS 13)	55
Bertozzi, Andrea (PL, OrgSS 3, SS 28)	1,12,125	Calvo-Jurado, Carmen (SS 45)	191
Bertram, Richard (SS 14)	59	Camassa, Roberto (SS 19)	84
Best, Janet (SS 25)	112	Cameron, Maria (SS 44)	187
Bettelheim, Eldad (SS 49)	206	Cances, Eric (SS 80)	299
Bezuglyi, Sergey (SS 23)	101	Candela, Anna Maria (SS 32)	142
Bhakta, Mousomi, (CS 9)	326	Candito, Pasquale (SS 32, SS 35)	143,156
Bhatta, Dambaru (SS 48)	202	Cano, Begona (SS 80)	299
Bilinsky, Lydia (SS 14)	59	Cano-Casanova, Santiago (SS 55)	226
Birtea, Petre (CS 1)	310	Cantrell, Robert Stephen (OrgSS 6, SS 16, SS 55)	28,73,226
Biswas, Animikh (SS 28, SS 30)	125,133	Cantrijn, Frans (OrgSS 24)	106
Bizhanova, Galina (SS 12)	52	Caraballo, Tomas (OrgSS 45, SS 45)	191,191
Bloch, Anthony (SS 24)	106	Carl, Siegfried (OrgSS 32, SS 32)	142,143
Block, Louis (SS 23)	101	Casal, Alfonso (SS 69)	264
Boccaletti, Stefano (SS 43, SS 54)	183,223	Castro, Sofia (SS 75)	279
Bocea, Marian (SS 67)	254	Cavalcanti, Marcelo (SS 4, SS 41)	17,173
Bociu, Lorena (OrgSS 4, SS 4, SS 67)	17,17,254	Ceci, Shaun (SS 67)	255
Bona, Jerry (SS 11, SS 57, SS 68)	48,232,259	Celik Karaaslanli, Canan (SS 68)	260
Bonanno, Gabriele (OrgSS 32, SS 32, SS 35)	142,142,156	Chang Lara, Hector (SS 6)	28
Bonotto, Everaldo (SS 69)	264	Charão, Ruy (SS 41)	173
Borondo, F. Javier (SS 54)	223	Chauviere, Arnaud (SS 31)	137
Borondo, Florentino (SS 13)	55	Chawanya, Tsuyoshi (SS 75)	280
Boronski, Jan (SS 23)	101	Cheban, David (SS 63)	246
Boscaggin, Alberto (SS 35)	156	Chebanov, Dmitriy (CS 2)	313
Bose, Amitabha (SS 43)	183	Chekroun, Mickael (SS 11, SS 44)	48,187
Bosia, Stefano (SS 2)	7	Chemetov, Nikolai (SS 34)	152
Bourchtein, Andrei (CS 7, CS 9)	323,326	Chen, Duan (SS 31)	137
Boyer, Franck (SS 2, SS 34)	7,152	Chen, Fengxin (SS 18, SS 42)	81,177
Bozorgmagham, Amir (SS 27)	120	Chen, George (OrgSS 42)	177
Bragard, Jean (SS 54)	223	Chen, Hongqiu (SS 57, SS 67, SS 68)	232,255,260
Branicki, Michal (SS 20)	88	Chen, Ming (SS 48, SS 82)	202,305
Branstator, Grant (SS 20)	88	Chen, Shaohua (SS 42, SS 80)	177,299
Bridstrup, John (SS 39)	170	Chen, Thomas (SS 21)	92
Brons, Morten (SS 78)	291	Chen, Yan-Yu (SS 8)	35
Brown, Michael (SS 27)	121	Chen, Zhiping (SS 9)	39
Brubaker, Nicholas (SS 29)	129	Cheng, Bin (SS 11)	48
Bucci, Francesca (SS 4)	17	Cherfils, Laurence (SS 2)	8
Bunimovich, Svetlana (SS 14)	59	Chern, Jann-Long (SS 33, OrgSS 38)	147,166
Buono, Pietro-Luciano (SS 75)	279	Cheskidov, Alexey (SS 30, SS 63)	133,246
Burda, Martin (SS 5)	24	Chhay, Marx (SS 24)	106
Burlica, Monica-Dana (SS 15)	66	Chhetri, Maya (SS 16)	73
Burra, Lakshmi (SS 35)	156	Chiang, Shihchung (SS 67)	255
Butler, Dagny (SS 16)	73	Chiba, Hayato (SS 17, SS 33)	78,147
<hr/>			
<b>C</b>			
<hr/>			
Caginalp, Carey (SS 45)	191	Chin-queue, Duane (CS 2)	313
Caginalp, Gunduz (OrgSS 2, SS 2)	7,7	Chinni, Antonia (SS 32, SS 35)	143,157
Cai, Anna (OrgSS 37)	164	Chiyaka, Christinah (SS 70)	267
Cai, Xiaoyun (CS 9)	327	Chkifa, Moulay Abdellah (SS 53)	220
Caicedo Casso, Angelica (PS)	334	Choboter, Paul (SS 73)	275
		Choe, Hi Jun (SS 77)	287
		Choffrut, Antoine (SS 62)	241
		Choi, Narknyul (CS 1)	310
		Chong, Dazhi (SS 9)	39
		Chossat, Pascal (OrgSS 75)	279
		Chou, Ching-Shan (OrgSS 37)	164

Chrayteh, Houssam (SS 11)	49	Delitala, Marcello (SS 61)	238
Christov, Ivan (SS 4)	18	Dellnitz, Michael (PL, OrgSS 10)	1,43
Chu, Hahng-Yun (SS 62)	241	Dell'Oro, Filippo (SS 2)	8
Chu, Jifeng (SS 68)	260	Demirci, Elif (CS 7)	323
Chung, Yeojin (SS 44)	187	Denny, Diane (SS 72)	273
Cipriano, Fernanda (SS 12)	52	Desroches, Mathieu (SS 17, OrgSS 78, SS 78)	78,291,291
Ciupe, Stanca (CS 4)	317	Dettmann, Carl (SS 27)	121
Cohen, David (SS 80)	299	Deuring, Paul (SS 77)	287
Cojocaru, Monica (SS 58)	235	DeVille, Lee (SS 10, OrgSS 43, SS 44)	43,183,188
Collegari, Rodolfo (PS)	334	Dharmawardane, Priyanjana (SS 42)	177
Colombo, Leonardo (SS 24)	107	Di, Yana (SS 81)	302
Comanescu, Dan (CS 2)	314	Di Bella, Beatrice (SS 32)	143
Correia, Maria (SS 23, SS 51)	102,214	Di Plinio, Francesco (SS 63, SS 77)	246,287
Cortazar, Carmen (SS 26)	117	Diaraf, Seck (SS 79)	295
Cortissoz, Jean (SS 42)	177	Dias, Juliana (SS 19)	84
Cosner, Chris (OrgSS 6, SS 6, SS 55)	28,28,226	Diaz-Martin, Juan-Carlos (CS 7)	323
Costa, David (SS 26, SS 55)	117,227	Dogan, Abdulkadir (SS 22)	95
Coti Zelati, Michele (SS 2)	8	Dolaptchiev, Stamen (SS 20)	88
Coville, Jerome (SS 6)	29	Domingos Cavalcanti, Valéria (SS 4)	18
Cowen, Michael (CS 4)	317	Donato, Maria Bernadette (SS 58)	236
Criado, Regino (SS 54)	223	Dong, Hongjie (SS 28)	125
Crowdy, Darren (SS 49)	206	Dorea, Chang (PS)	335
Cruz, Jose (SS 58)	235	Doumbe, Brice (SS 2)	8
Cruzeiro, Ana Bela (SS 24)	107	Du, Yihong (SS 1, SS 33)	4,147
Cuesta, Eduardo (OrgSS 52, SS 52)	217,217	Du, Zengji (SS 22, SS 68)	95,260
Curto, Carina (SS 43)	184	Duan, Jun-Sheng (SS 39)	170
Curtu, Rodica (SS 78)	291	Duan, Lixia (SS 25)	112
		Dumortier, Freddy (OrgSS17, SS 17)	78,78
		Dung, Le (SS 42, SS 69)	178,264
		Dussan Angulo, Martha Patricia (SS 48)	203
		Dzhumabaev, Dulat (CS 2)	314
		D'Abbicco, Marcello (SS 41)	173

---

## D

---

D'Agui, Giuseppina (SS 32, SS 35)	143,157
D'Orsogna, Maria (SS 3, SS 64)	12,250
da Silva, Daniel (SS 48)	203
Dabas, Jaydev (CS 1)	310
Dai, Shibin (SS 34, SS 49)	152,206
Dai, Mimi (SS 63, SS 77)	246,287
Dalbono, Francesca (SS 35)	157
Damanik, David (OrgSS 21, SS 21)	92,92
Dambrine, Julien (SS 2)	8
Danaila, Ionut (SS 80)	300
Daniele, Patrizia (OrgSS 58, SS 58)	235,235
Darve, Eric (SS 10)	43
Dascal, Judita (CS 2)	314
Davidson, Melissa (SS 48)	203
Davila, Juan (OrgSS 6, SS 6, SS 29)	28,29,129
Day, Judy (SS 25)	112
De Gaetano, Andrea (SS 14, SS 31)	60,138
de la Llave, Rafael (OrgSS 13)	55
de Leon, Manuel (OrgSS 24, SS 24)	106,107
De Maesschalck, Peter (OrgSS 17, SS 17)	78,78
De Witte, Virginie (CS 10)	332
DeAngelis, Donald (SS 6)	29
del Rio, Rafael (SS 21)	92

---

## E

---

Ehrlacher, Virginie (OrgSS 53, SS 53)	220,220
Eisenberg, Marisa (SS 31)	138
El Arwadi, Toufic (SS 50)	210
Elia, Cinzia (SS 10)	43
Elsey, Matthew (SS 49)	207
Emerick, Brooks (PS)	335
Enatsu, Yoichi (SS 38)	166
Enderling, Heiko (SS 31)	138
Enguiçca, Ricardo (SS 26)	117
Esenturk, Emre (SS 2)	8
Esquivel-Avila, Jorge (SS 15)	67
Estrada, Ernesto (SS 43)	184
Etoua, Remy Magloire (CS 10)	332
Evers, Joep (SS 15, SS 64)	67,250

## F

Fabbri, Roberta (SS 10, SS 21)	44,92
Fagan, William (SS 6)	29
Falco, Antonio (SS 53)	220
Fang, Jian (SS 8)	35
Farazmand, Mohammad (SS 30, SS 73)	134,275
Fariello, Ricardo (PS)	335
Faye, Gregory (SS 75)	280
Faye, Ibrahima (SS 79)	295
Feireisl Eduard (PL, OrgSS 7)	1,32
Felli, Veronica (SS 29)	129
Feng, Wei (OrgSS 68)	259
Feng, Wenying (SS 9, SS 22)	40,95
Feng, Zhaosheng (SS 14, OrgSS 68, SS 68)	60,259,260
Fernandez, Claudio (SS 41)	173
Fernandez, Oscar (SS 24)	107
Ferreira, Helena (SS 47)	198
Fetecau, Razvan (SS 3, SS 64)	12,251
Fewster, Nicholas (SS 35)	157
Fialho, Joao (SS 22)	95
Field, Michael (SS 75)	280
Figliolia, James (PS)	335
Filippakis, Michael (SS 15)	67
Fino, Ahmad (SS 41)	174
Fiorentino, Laura (SS 73)	275
Flaxman, Samuel (SS 6)	29
Forde, Jonathan (SS 14)	60
Foss, Mikil (SS 4)	18
Fox, Adam (CS 1)	310
Franco, Luis (CS 1)	311
Frenod, Emmanuel (OrgSS 79, SS 79)	294,295
Friedman, Avner (PL)	2
Frigon, Marlene (SS 22)	96
Froyland, Gary (SS 10, SS 27)	44,121
Fu, Xinchu (SS 63)	247
Fukao, Takeshi (SS 64)	251
Fulton, Charles (OrgSS 84, SS 84)	308,308
Fury, Matthew (CS 9)	327

## G

Gal, Ciprian (SS 2, SS 4)	9,18
Galenko, Peter (SS 2)	9
Galanthay, Theodore (CS 4)	318
Galiano, Gonzalo (SS 69)	264
Galstyan, Anahit (SS 48)	203
Gan, Shaobo (SS 35)	157
Gao, Yinghui (SS 25)	113
Garavello, Mauro (CS 9)	327

Garba, Salisu (SS 70)	267
Garcia Huidobro, Marta (OrgSS 26, SS 26)	117,117
Garnier, Jimmy (SS 6)	30
Garrido-Atienza, Maria (OrgSS 45, SS 45)	191,192
Garrione, Maurizio (SS 35)	157
Gaudiello, Antonio (SS 4)	19
Gautero, Francois (SS 23)	102
Gay Balmaz, Francois (SS 24)	107
Georgiev, Svetlin (SS 52)	217
Ghazaryan, Anna (SS 17, SS 18, OrgSS 73)	79,82,275
Ghosh, Bijoy (OrgSS 25)	111
Giachetti, Daniela (SS 42)	178
Giannakis, Dimitrios (SS 19, OrgSS 20, SS 20)	84,88,89
Gibson, Peter (SS 50)	210
Gidea, Marian (SS 13)	55
Gie, Gung-Min (SS 12)	52
Ginder, Elliott (SS 42)	178
Giuffre, Sofia (SS 58)	236
Gkioulekas, Eleftherios (OrgSS 30, SS 30)	133,134
Glatt-Holtz, Nathan (SS 11, SS 28)	49,125
Glicksman, Martin (SS 49)	207
Glotov, Dmitry (SS 69)	264
Goddard II, Jerome (OrgSS 16, SS 16)	73,74
Gofen, Alexander (SS 39)	171
Golden, Kenneth (SS 73)	276
Gorodetski, Anton (SS 13, SS 21)	55,92
Gottwald, Georg (SS 20)	89
Goubet, Olivier (SS 2, SS 11)	9,49
Graber, Jameson (SS 4)	19
Graef, John (OrgSS 22, SS 22)	95,96
Graff, Grzegorz (SS 51)	214
Grasselli, Maurizio (OrgSS 2, SS 4, SS 34, SS 45)	7,19,153,192
Grayshan, Katelyn (SS 48)	203
Grecksch, Wilfried (OrgSS 36, SS 36, SS 45)	160,160,192
Green, Melissa (SS 27)	121
Greenhalgh, Scott (SS 58)	236
Grigorieva, Ellina (CS 6)	322
Grooms, Ian (SS 19)	85
Gruen, Guenther (SS 2)	9
Guerra, Ignacio (SS 26, SS 29)	118,130
Guesmia, Aissa (SS 41, SS 52)	174,217
Guidotti, Patrick (SS 55)	227
Guillen, Nestor (SS 6)	30
Gumel, Abba (OrgSS 70, SS 70)	267,267
Gunesch, Roland (SS 23)	102
Guo, Daniel (SS 28)	126
Guo, Yixin (SS 25)	113
Gurski, Katharine (CS 7)	323
Gutierrez Santacreu, Juan (SS 45)	192
Guven Geredeli, Pelin (SS 4)	19
Guzman Alonso, Elisa Lavinia (SS 24)	107

## H

Haasdonk, Bernard (SS 53)	220
Habibi, Zakaria (SS 79)	295
Hagen, Thomas (SS 4, OrgSS 67)	19,254
Haller, George (SS 13, OrgSS 27, SS 27)	56,120,122
Hamdouni, Aziz (OrgSS 24)	106
Hamouda, Makram (OrgSS 12, SS 12)	52,52
Han, Daozhi (SS 12)	53
Han, Jongmin (SS 62)	242
Hanin, Leonid (SS 14)	60
Harada, Junichi (SS 15)	67
Harlim, John (OrgSS 20, SS 20, SS 79)	88,89,295
Haro, Alex (SS 13, SS 21)	56,93
Hartmann, Carsten (SS 10)	44
Hartsock, Leopold (PS)	336
Haruna, Taichi (SS 51)	214
Harvey, Emily (SS 17)	79
Hashimoto, Takahiro (SS 15)	67
Heffernan, Jane (SS 9, SS 70)	40,268
Henderson, Johnny (SS 22)	96
Henning, Patrick (SS 79)	296
Herbei, Radu (SS 20)	89
Hernandez-Duenas, Gerardo (SS 19)	85
Hessel-von Molo, Mirko (SS 10)	44
Hetzer, Georg (SS 16, SS 55, OrgSS 69)	74,227,264
Hews, Sarah (SS 14)	61
Hidalgo, Arturo (SS 69)	265
Hillen, Thomas (SS 31, OrgSS 61)	139,238
Himonas, Alex (OrgSS 48, SS 48, SS 82)	202,203,305
Hinow, Peter (SS 31, SS 67)	139,255
Hinz, Michael (SS 36)	161
Hirstoaga, Sever (SS 79)	296
Hishida, Toshiaki (SS 7)	32
Hoang, Luan (SS 62)	242
Hoell, Nicholas (SS 50, CS 9)	210,327
Hogan, John (CS 10)	332
Holliman, Curtis (SS 48)	204
Holmes, John (SS 48)	204
Hou, Xiaojie (OrgSS 18, SS 18)	81,82
Hsia, Chun-Hsiung (SS 11, SS 38)	49,166
Hsu, Sze-Bi (SS 14, SS 55)	61,227
Hu, Bei (SS 31)	139
Hu, Changbing (SS 14, OrgSS 28, SS 28)	61,125,126
Hu, Qingwen (SS 67)	256
Huang, Aimin (SS 11)	49
Huang, Hsin-Yuan (SS 13)	56
Huang, Jimmy (SS 9)	40
Huang, Rui (SS 42)	178
Huzak, Renato (SS 17)	79

Ibraguimov, Akif (SS 4, OrgSS 25)	20,111
-----------------------------------	--------

## I

Idczak, Dariusz (SS 15)	68
Ifrim, Mihaela (SS 48)	204
Ikeda, Hideo (SS 8)	35
Ikeda, Kota (SS 8)	35
Ikehata, Ryo (OrgSS 41, SS 41)	173,174
Ilhan, Onur Alp (OrgSS 52, SS 52)	217,217
Infante Acevedo, Jose Arturo (SS 53)	221
Ishida, Sachiko (SS 38, SS 61)	166,238
Ishige, Kazuhiro (SS 1)	4
Ishiwata, Michinori (SS 1, SS 15)	4,68
Israel, Haydi (SS 2)	9
Itskov, Vladimir (SS 43)	184
Izaguirre, Jesus (SS 5)	25

## J

Jabin, Pierre-Emmanuel (SS 76)	283
Jafari, Hossein (SS 52)	217
Jain, Harsh (SS 31)	139
Jaiswal, Dilip Kumar (CS 9)	328
Jankowski, Hanna (SS 9)	40
Javaheri, Mohammad (SS 23)	102
Jebelean, Petru (SS 32)	144
Jha, Navnit (SS 12)	53
Ji, Xia (SS 80)	300
Jia, Gao (SS 42)	178
Jiang, Huiqiang (SS 62)	242
Jiang, Yi (SS 31)	140
Jilkine, Alexandra (SS 37, CS 4)	164,318
Jimenez-Casas, Angela (SS 52)	218
Jing, Tian (SS 68)	260
Jolly, Michael (OrgSS 30, SS 30)	133,134
Joo, Soo Kyung (OrgSS 62)	241
Jordan, Pedro (SS 4)	20
Ju, Ning (OrgSS 28)	125
Juang, Jonq (CS 10)	332
Julien, Keith (SS 19)	85
Jung, Chang-Yeol (OrgSS 12, SS 12)	52,53
Junge, Oliver (OrgSS 10, SS 10)	43,44
Junxiang, Xu (SS 82)	306

---

**K**


---

Kabeya, Yoshitsugu (SS 38)	167
Kadirbayeva, Zhazira (CS 2)	314
Kahng, Byungik (SS 23, SS 67)	102,256
Kalappattil, Lakshmi (SS 16)	74
Kaltenbacher, Barbara (OrgSS 4, SS 80)	17,300
Kan-on, Yukio (SS 8)	36
Kanagawa, Shuya (SS 46, SS 68)	195,261
Kaneko, Yuki (SS 38)	167
Kang, Kyungkeun (SS 62)	242
Kang, Yun (SS 14)	61
Kano, Risei (SS 64)	251
Karapetyan, David (SS 48)	204
Karjanto, Natanael (SS 57)	232
Karp, Lavi (SS 48, SS 49)	204,207
Karper, Trygve (SS 7)	32
Karrasch, Daniel (SS 10)	45
Kaslik, Eva (SS 52)	218
Keating, Shane (SS 19, SS 20)	85,89
Keesling, James (SS 23, SS 47)	103,199
Keller, Diana (SS 36)	161
Keller, Karsten (OrgSS 51, SS 51)	213,214
Kelliher, James (SS 12, SS 28)	53,126
Kelly, Scott (CS 2)	314
Kemp, Paula (CS 7)	324
Kenmochi, Nobuyuki (OrgSS 64, SS 64)	250,251
Kennedy, Judy (SS 23)	103
Khalique, Chaudry (SS 72)	273
Khan, Adnan (SS 70)	268
Khan, Muhammad (SS 72)	273
Khanal, Netra (OrgSS 57, SS 57)	232,232
Khavinson, Dmitry (SS 49)	207
Khouider, Boualem (SS 44)	188
Kieu, Thinh (SS 25)	113
Kikuchi, Koji (SS 42)	179
Kim, Chan-Gyun (SS 33)	147
Kim, Hyejin (SS 45, CS 9)	193,328
Kim, Namhee (SS 5)	25
Kim, Sungwhan (SS 50)	210
Kim, Yangjin (OrgSS 31, SS 31)	137,140
King, Deborah (SS 23)	103
Kirane, Mokhtar (OrgSS 52)	217
Kirwan, A (SS 27)	122
Klein, Avraham (SS 49)	207
Knio, Omar (SS 20)	90
Knobloch, Juergen (SS 75)	280
Ko, Eunkyung (SS 16)	74
Ko, Wonlyul (SS 72)	273
Kobayasi, Kazuo (SS 15)	68
Kobilarov, Marin (SS 24)	108
Kocic, Vlajko (CS 1)	311
Koch, Hans (SS 38)	167
Kohler, Dominic (CS 7)	324
Kolokolnikov, Theodore (SS 3, SS 29)	13,130

Koltai, Péter (SS 10)	45
Komornik, Vilmos (SS 4)	20
Kong, Liang (SS 69)	265
Kong, Lingju (OrgSS 22, SS 22)	95,96
Koo, Namjip (SS 52)	218
Kosmatov, Nickolai (SS 22)	97
Kosovalic, Nemanja (SS 14)	62
Kostelich, Eric (SS 20)	90
Kovacic, Gregor (OrgSS 44, SS 44)	186,188
Kovats, Jay (SS 47)	199
Kowar, Richard (SS 80)	300
Kreml, Ondrej (SS 77)	288
Krueger, Helge (SS 21)	93
Krupa, Maciej (SS 75, OrgSS 78, SS 78)	280,291,292
Ku, Se-Hyun (SS 62)	242
Kuang, Yang (OrgSS 14, SS 31, SS 70)	59,140,268
Kubo, Akisato (SS 15)	68
Kubo, Hideo (SS 4, SS 41)	20,174
Kubo, Masahiro (SS 15)	68
Kumar, Atul (CS 9)	328
Kumar, Manish (SS 3)	13
Kumazaki, Kota (SS 64)	251
Kurths, Juergen (SS 51)	215
Kuto, Kousuke (SS 38)	167
Kwietniak, Dominik (SS 23)	103

---

**L**


---

Lam, King-Yeung (SS 6, SS 69)	30,265
Langa, Jose A (SS 10)	44
Larios, Adam (SS 30)	134
Lasiecka, Irena (SS 2, OrgSS 4, SS 7)	10,17,32
Latifizadeh, Habibolla (SS 72)	273
Laurent, Thomas (SS 3)	13
Lauterbach, Reiner (OrgSS 75)	279
Lazzo, Monica (SS 26)	118
Ledzewicz, Urszula (SS 14, SS 31)	62,141
Lee, Eun Kyoung (SS 16)	75
Lee, Ho (SS 62)	243
Lee, Jihoon (SS 62)	242
Lee, Jyh-Hao (SS 57)	233
Lee, Keonhee (SS 23)	103
Lee, Manseob (SS 23)	103
Lee, Seongwon (SS 31)	141
Legoll, Frederic (SS 79, SS 80)	296,301
Lehotzky, David (SS 67)	256
Lei, Jinzhi (SS 25, SS 37)	113,164
Lelievre, Tony (SS 5, OrgSS 53, SS 80)	25,220,301
LeMesurier, Brenton (SS 67)	256
Leonard, Christian (SS 24)	108
Lerman, Kristina (SS 3)	13

Levaggi, Laura (SS 15)	68	Ma, To Fu (SS 41)	174
Levy, Doron (SS 37)	164	Ma, Wanbiao (SS 14)	63
Li, Bingtuan (OrgSS 14, SS 14)	59,63	Maghami, Mahsa (SS 54)	224
Li, Cuiping (CS 4)	318	Mahmoudiandehkordi, Siamak (PS)	336
Li, Jiayu (OrgSS 14, SS 14, SS 68)	59,62,261	Majda, Andrew (PL, OrgSS 20)	2,88
Li, Junfang (SS 26)	118	Majewski, Marek (SS 15)	69
Li, Shuwang (SS 34)	153	Malecha, Ziemowit (SS 19)	86
Li, Wan-Tong (SS 14)	62	Malik, Tufail (OrgSS 70, SS 70)	267,269
Li, Xiaosheng (SS 28, SS 50)	126,211	Mamillapalle, Karthikeya (CS 1)	311
Li, Y. Charles (SS 28, SS 68)	126,261	Manasevich, Raul (OrgSS 26, SS 26)	117,118
Li, Yi (OrgSS 18, SS 18)	81,82	Marahrens, Daniel (SS 80)	301
Li, Yinghua (SS 42)	179	Marano, Salvatore (OrgSS 32, SS 32)	142,144
Liang, Zhian (CS 9)	328	Marchand, Richard (SS 15)	69
Liao, Jie (SS 81)	302	Marco, Jean Pierre (SS 13)	57
Lin, Chi-Tien (SS 42)	179	Maroncelli, Daniel (SS 22)	97
Lin, Kevin (SS 44)	188	Marrero, Juan Carlos (SS 24)	108
Lin, Runchang (SS 16, CS 7)	75,324	Martcheva, Maia (SS 70)	269
Lin, Zhiwu (SS 28)	126	Martin de Diego, David (SS 24)	108
Lindsay, Alan (SS 6, SS 29)	30,130	Martinez, Christina (SS 72)	273
Liu, Hao (SS 70)	268	Martinez, Salome (SS 6, SS 55)	31,228
Liu, Honghu (SS 11)	49	Martinez, Vincent (SS 30)	135
Liu, Liping (SS 81)	302	Martinez Carballo, Maria Angeles (PS)	336
Liu, Ping (SS 33)	148	Marvel, Seth (SS 3)	14
Liu, Shibo (SS 32)	144	Marx, Christoph (SS 21, SS 84)	93,308
Liu, Shitao (SS 4)	20	Matilla-Garcia, Mariano (SS 51)	215
Liu, Xinfeng (SS 37)	164	Matsuura, Kei (SS 15)	69
Liu, Yonggang (SS 73)	276	Matsuzawa, Hiroshi (SS 38)	167
Liu, Yue (OrgSS 82)	305	Matthews, Charles (SS 5)	25
Livrea, Roberto (SS 32, SS 35)	144,158	Mavi, Rajinder (SS 84)	308
Lo, Wing Cheong (SS 37)	165	Mavinga, Nsoki (SS 16)	75
Lohse, Alexander (SS 75)	281	Mayer, John (SS 23)	104
Lopes Filho, Milton (SS 34)	153	Maze, Grace (SS 73)	276
Lopez-Gomez, Julian (SS 29, OrgSS 55, SS 55)	130,226,227	Mazzucato, Anna (SS 12, SS 28)	53,127
Lorenzi, Tommaso (SS 76)	283	McCalla, Scott (SS 3)	14
Lorz, Alexander (SS 61, OrgSS 76)	238,283	Medvedev, Georgi (OrgSS 43, SS 43, SS 78)	183,184,292
Losada, Juan Carlos (SS 54)	224	Meerson, Baruch (SS 49)	208
Lou, Bendong (SS 8)	36	Mei, Ming (SS 41, OrgSS 42, SS 42)	174,177,180
Lou, Yuan (PL, OrgSS 6, SS 55)	2,28,227	Meiss, James (SS 27)	122
Lu, Qishao (OrgSS 25, SS 25, SS 68)	111,113,261	Melara, Luis (CS 6)	322
Lu, Zhuosheng (SS 68)	262	Melgaard, Michael (SS 32, SS 64)	144,252
Lucas, Stephen (SS 10, OrgSS 39, SS 39)	45,170,171	Mellet, Antoine (SS 34)	153
Lukacova, Maria (SS 7)	32	Meng, Deyu (SS 9)	40
Lukic, Milivoje (SS 21)	93	Mestdag, Tom (OrgSS 24, SS 24)	106,108
Lunasin, Evelyn (SS 30)	134	Mezic, Igor (SS 10)	45
Lundberg, Erik (SS 49)	208	Michta, Mariusz (SS 15)	69
Luo, Chong (SS 62)	243	Mickens, Ronald (SS 70)	269
Luque, Alejandro (SS 13)	56	Micu, Sorin (SS 4)	21
Luu, Quoc (SS 14)	63	Migorski, Stanislaw (OrgSS 15, SS 15)	66,69
Lyons, Jeffrey (SS 22)	97	Mikelic, Andro (SS 7)	33
		Milasi, Monica (SS 58)	236
		Milewski, Paul (SS 19)	86
		Ming, Pingbing (SS 79, SS 81)	297,302
		Minhos, Feliz (SS 22)	97
		Miranville, Alain (OrgSS 2, SS 11, SS 34, SS 45)	7,50,153,193
		Miyagaki, Olimpio (SS 32)	145

Miyaji, Tomoyuki (SS 33)	148	Naz, Rehana (CS 9)	329
Miyamoto, Yasuhito (SS 33)	148	Necasova, Sarka (OrgSS 7, OrgSS 77, SS 77)	32,286,288
Mobley, David (SS 5)	26	Neidinger, Richard (SS 39)	171
Moghadas, Seyed (SS 9)	41	Nekorkin, Vladimir (SS 75)	281
Mohler, George (SS 3)	14	Nelson, Youri (CS 4)	318
Molica Bisci, Giovanni (SS 32, SS 35)	145,158	Nepomnyashchy, Alexander (OrgSS 49, SS 49)	206,208
Monetti, Roberto (SS 51)	215	Neuenkirch, Andreas (SS 45)	193
Montero, Alberto (SS 26)	118	Neugebauer, Jeffrey (SS 22)	97
Moon, Byungsoo (SS 82)	306	Neustupa, Jiri (SS 7)	33
Moore, Brian (CS 7, PS)	324,336	Nevai, Andrew (SS 6, OrgSS 14, SS 14)	31,59,64
Moradi, Sara (SS 52)	218	Newhall, Katherine (SS 43, SS 44)	184,189
Moradifam, Amir (SS 50)	211	Ngonghala, Calistus (CS 4)	319
Morales-Rodrigo, Cristian (SS 61)	238	Nguyen, Loc (SS 32, SS 50)	145,211
Morandotti, Marco (SS 76)	283	Nguyen, Truyen (SS 62)	243
Morita, Yoshihisa (SS 8, OrgSS 33, SS 33)	36,147,148	Ni, Wei-Ming (OrgSS 18)	81
Moskow, Shari (SS 50)	211	Nicholas, Sabu (SS 58)	236
Motreanu, D. (OrgSS 32)	142	Nie, Qing (OrgSS 37)	164
Motsch, Sebastien (SS 44, SS 76)	188,283	Niederman, Laurent (SS 13)	57
Motyl, Jerzy (SS 15)	70	Ninomiya, Hirokazu (OrgSS 8, SS 33)	35,148
Mouronte, Mary Luz (SS 54)	224	Nishikawa, Takashi (SS 43)	184
Mousa, Abdelrahim (SS 47)	199	Nkashama, M. (SS 26)	119
Moyles, Iain (SS 34)	153	Noris, Benedetta (SS 29)	131
Mu, Yang (SS 9)	41	Novick-Cohen, Amy (SS 2, OrgSS 49, SS 49)	10,206,208
Mubayi, Anuj (SS 70)	269	Novruzi, Arian (SS 15)	70
Mugler, Antje (SS 36)	161	Nowicki, Tomasz (SS 23)	104
Mukandavire, Zindoga (SS 70)	270	Nwachukwu, Eucharia (CS 7)	324
Mukherjee, Debasis (PS)	337	Nykamp, Duane (SS 43)	185
Mulone, Giuseppe (SS 70)	270		
Muntean, Adrian (OrgSS 64)	250		
Munther, Daniel (SS 6)	31		
Murai, Minoru (SS 38)	167		
Murakawa, Hideki (SS 29)	130		
Muraki, David (SS 19)	86		
Murase, Yusuke (SS 64)	252		
Mustafa, Muhammad (SS 52)	219		

---

## N

---

Nadal, Jean-Pierre (SS 3)	14
Naeem, Imran (CS 9)	328
Nagamachi, Shigeaki (SS 46)	196
Nagaraj, B.R. (SS 62)	243
Nagayama, Masaharu (SS 38)	168
Nagy, John (SS 14)	63
Nakaguchi, Etsushi (SS 61)	239
Nakajima, Kohei (SS 51)	215
Nakamura, Ken-Ichi (SS 1, SS 8)	5,36
Nakazawa, Hideo (SS 41)	175
Nara, Mitsunori (SS 8)	36
Nascimento, Arnaldo (SS 32)	145
Natali, Fábio (SS 41)	175
Naudot, Vincent (SS 17)	79

---

## O

---

Obabiyi, Olawale (CS 4)	319
Ober-Bloebaum, Sina (SS 10)	45
Oeda, Kazuhiro (SS 38)	168
Oh, Seungly (SS 48)	204
Ohta, Masahito (SS 15)	70
Okabe, Shinya (SS 33)	149
Olascoaga, Maria (SS 27, SS 73)	122,276
Oliveira, Bruno (SS 47)	200
Oliveira, Tania (SS 47)	199
Olson, Eric (SS 63)	247
Omari, Pierpaolo (SS 35)	158
Omata, Seiro (SS 42)	180
Ong, Darren (SS 21)	93
Onodera, Michiaki (SS 49)	209
Onofrei, Daniel (SS 64)	252
Oprocha, Piotr (OrgSS 23, SS 23)	100,104
Oshita, Yoshihito (SS 33)	149
Otani, Mitsuharu (OrgSS 15)	66
Ott, William (SS 23)	104
Ozawa, Tohru (OrgSS 15)	66

## P

Padberg-Gehle, Kathrin (SS 10, SS 27)	46,123
Padial, Juan Francisco (SS 69)	265
Padron, Edith (SS 24)	109
Painter, Kevin (SS 76)	284
Palacios, Antonio (CS 10)	333
Palaniappan, Devanayagam (SS 72)	274
Palmer, Ken (SS 10)	46
Pandit, Purnima (CS 7)	325
Pandit, Sudhakar (SS 26)	119
Panigrahi, Saroj (SS 22, SS 35)	98,158
Pantazi, Chara (SS 39)	172
Papageorgiou, Nikolaos (SS 32)	145
Papatzacos, Paul (SS 34)	154
Park, Jinhae (Org SS 62)	241
Parker, Philip (SS 39)	171
Parlitz, Ulrich (SS 51, CS 4)	216,319
Parshad, Rana (SS 63)	247
Patel, Jigarkumar (SS 67)	257
Patidar, Kailash (CS 2)	315
Paullet, Joseph (SS 22)	98
Pazoto, Ademir (SS 4)	21
Pereira, Marcone (CS 9)	329
Perera, Kanishka (SS 32)	146
Perkins, Steven (SS 47)	200
Pestov, Leonid (SS 50)	212
Petcu, Madalina (OrgSS 11, SS 11, SS 28)	48,50,127
Petronilho, Gerson (OrgSS 48, SS 48)	202,205
Phan, Tuoc (SS 12, OrgSS 62)	53,241
Phillips, Dan (SS 62)	244
Piasecki, Tomasz (SS 77)	288
Pinski, Frank (SS 5)	26
Pinto, Alberto (OrgSS 47, SS 47)	198,200
Piro Vernier, Stella (SS 55)	228
Plechac, Petr (SS 49)	209
Podio-Guidugli, Paolo (SS 2)	10
Podvigina, Olga (SS 75)	281
Polacik, Peter (OrgSS 1, SS 1, SS 29)	4,5,131
Pompe, Bernd (SS 51)	216
Ponce, Enrique (SS 78, CS 2)	292,315
Popescu, Marius (SS 15)	70
Porru, Giovanni (SS 55)	228
Porter, Mason (SS 3)	15
Porzio, Maria Michaela (SS 42)	180
Postlethwaite, Claire (SS 75)	281
Poulou, Marilena (CS 9)	329
Pradhan, Saroj (SS 67)	257
Prazak, Dalibor (SS 2)	10
Prokopczyk Arita, Andrea Cristina (CS 1)	311
Prosper, Olivia (SS 70)	270
Ptashnyk, Mariya (SS 64)	252
Pujo-Menjouet, Laurent (CS 4)	319

## Q

Qi, Yuanwei (OrgSS 18, SS 18)	81,82
Qu, Changzheng (OrgSS 82, SS 82)	305,306
Quaas, Alexander (OrgSS 6)	28

## R

Radu, Petronela (OrgSS 4, SS 4, SS 41)	17,21,175
Rafie-Rad, Mehdi (SS 24)	109
Raheem, Abdur (SS 16)	75
Rahimi, Hamidreza (CS 9)	330
Rammaha, Mohammad (SS 4)	21
Rautmann, Reimund (OrgSS 77, SS 77)	286,288
Razafindralandy, Dina (OrgSS 24, SS 24)	106,109
Regmi, Dipendra (SS 57)	233
Reichel, Wolfgang (SS 1)	5
Reisi Gahrooei, Mostafa (PS)	337
Reitmann, Volker (SS 15, SS 63)	70,247
Rejniak, Katarzyna (SS 31)	141
Ren, Kui (SS 50)	212
Ren, Weiqing (SS 81)	303
Ren, Xiaofeng (SS 29)	131
Renault, Jerome (SS 47)	200
Restrepo, Juan (SS 20)	90
Restrepo, Juan (SS 43)	185
Rieger, Christian (SS 53)	221
Riess, Thorsten (SS 75)	282
Rifford, Ludovic (SS 13)	57
Ritchie, Hannah (PS)	337
Ritter, Lake (SS 25)	114
Rivas, David (OrgSS 73, SS 73)	275,277
Rivera, Joaquin (SS 18)	82
Rizwan, Syed Mohd (CS 6)	322
Robinson, James (SS 10, SS 11, OrgSS 63)	46,50,246
Robinson, Stephen (SS 58)	237
Rocca, Elisabetta (SS 2, SS 7)	10,33
Rocha, Carlos (SS 33)	149
Rodrigues, Alexandre (SS 75)	282
Rodrigues, Tatiana (SS 21)	93
Rodriguez, Jesus (SS 22)	98
Rodriguez, Nancy (SS 3, SS 76)	15,284
Roeckner, Michael (SS 36, SS 45)	161,193
Roeger, Lih-Ing (SS 14)	64
Rogers, Lisa (SS 44)	189
Rom-Kedar, Vered (SS 13)	57
Romance, Miguel (OrgSS 54, SS 54)	222,224
Ros, Javier (CS 10)	333
Rosado Linares, Jesus (SS 76)	284
Rosu, Daniela (SS 15)	71



Roth, Christian (SS 36)	162	Sherwood, William (SS 78)	292
Rousseau, Antoine (SS 11, SS 79)	297	Shi, Junping (SS 16, OrgSS 33, SS 33)	76,147,150
Ruan, Weihua (SS 42)	180	Shieh, Tien-Tsan (SS 38)	168
Rudmin, Joseph (SS 39)	171	Shino, Masatoshi (SS 25)	114
Ruggiero Rodriguez, Rafael (SS 13)	57	Shilnikov, Andrey (SS 17, SS 78)	80,292
Ruiz-Herrera, Alfonso (SS 35)	158	Shimojo, Masahiko (SS 8)	37
Rumpf, Benno (SS 30)	135	Shinde, Kishor (CS 2)	315
Rupp, Florian (SS 67)	257	Shiue, Ming-Cheng (SS 11, SS 57)	50,233
Ryan, Daniel (SS 6)	31	Shivaji, Ratnasingham (OrgSS 16, SS 16)	73,76
Ryham, Rolf (SS 62)	244	Shlizerman, Eli (SS 13, SS 43)	58,185
Ryu, Kimun (SS 55, CS 9)	229,330	Shmarev, Sergey (SS 42, SS 69)	180,265

---

## S

---

Saal, Juergen (SS 77)	288	Siegel, Michael (SS 34)	154
Sadel, Christian (SS 21)	94	Siegmund, Stefan (OrgSS 10)	43
Sabuwala, Adnan (CS 9)	330	Sierzega, Mikolaj (SS 63)	248
Sadhu, Susmita (SS 16, CS 2)	75,315	Silva, Ricardo (CS 9)	330
Sadowski, Witold (SS 63)	248	Silvestre, Ana (SS 77)	289
Saez, Mariel (SS 26)	119	Sim, Inbo (SS 16)	77
Sahai, Tuhin (SS 10)	46	Simonnet, Eric (SS 11)	50
Said-Houari, Belkacem (SS 4, SS 41)	21,175	Simpson, David (SS 45)	194
Sakamoto, Kunimochi (SS 8, SS 33)	36,149	Singh, Anuraj (SS 25)	114
Salceanu, Paul (SS 70)	270	Sinn, Mathieu (SS 51)	216
Salgado, Modesto (SS 24)	109	Skeel, Robert (SS 5)	26
Salvatore, Addolorata (SS 32)	146	Smital, Jaroslav (SS 23)	104
Sammartino, Marco (SS 12, SS 63)	54,248	Smith, Andrew (CS 2)	316
Sanz-Serna, J (PL, OrgSS 5, SS 5)	3,24,26	Smith, Leslie (OrgSS 19, SS 19, SS 30)	84,87,135
Sapsis, Themistoklis (SS 20, SS 27)	90,123	So, Paul (SS 43)	185
Sasi, Sarath (SS 16)	76	Soave, Nicola (SS 35)	159
Sauer, Martin (SS 36)	162	Sochacki, James (OrgSS 39, SS 39, SS 72)	170,171,274
Savin, Tatiana (OrgSS 49, SS 49)	206,209	Sonck, Charlotte (CS 4)	320
Scheel, Arnd (SS 8, SS 33)	37,149	Song, Changming (SS 42)	181
Schmidt, Paul (SS 16, SS 26)	76,119	Song, Pai (CS 1)	311
Schneider, Reinhold (SS 53)	221	Song, Xinyu (SS 14)	64
Schober, Constance (SS 67)	257	Soto, Herme (SS 52)	219
Schonbek, Maria (SS 7, SS 77)	33,289	Specovius-Neugebauer, Maria (SS 77)	289
Schonbek, Tomas (SS 77)	289	Spradlin, Greg (SS 35, SS 50)	159,212
Schugart, Richard (SS 25, SS 37)	114,165	Srivastava, Shilpee (SS 16)	77
Schurz, Henri (SS 67)	257	Staicu, Vasile (SS 15)	71
Scott, Sherry (SS 27, OrgSS 73, SS 73)	123,275,277	Starkloff, Hans-Joerg (SS 36)	162
Scrimali, Laura (SS 58)	237	Stathopoulos, Vassilios (SS 5)	26
Seara, Tere (OrgSS 13)	55	Stebel, Jan (SS 77)	289
SenGupta, Indranil (SS 68)	262	Stechmann, Samuel (OrgSS 19, SS 19, SS 20)	84,87,91
Sepulveda, Alex (SS 52)	219	Stefankova, Marta (SS 23)	105
Sequeira, Adelia (SS 77)	289	Stinchcombe, Adam (CS 4)	320
Servadei, Raffaella (SS 32)	146	Stinner, Christian (SS 42, SS 61)	181,239
Shadden, Shawn (SS 27)	123	Stoiciu, Mihai (SS 21)	94
Shahin, Sami (SS 72)	274	Stremler, Mark (SS 27)	124
Shamis, Mira (SS 21, SS 84)	94,308	Strickland, Christopher (CS 4)	321
Sharples, Nick (SS 63)	248	Strunin, Dmitry (CS 9)	330
Shen, Wenxian (OrgSS 69)	264	Su, Jianzhong (OrgSS 25, SS 25)	111,114
		Su, Linlin (SS 62)	244



Vitagliano, Luca (SS 24)	110	Wrzosek, Dariusz (SS 61)	239
Vitanza, Carmela (SS 58)	237	Wu, Donglun (SS 35)	159
Vitolo, Antonio (SS 68)	262	Wu, Jianhua (SS 14)	65
Vivas-Barber, Ana (SS 14)	65	Wu, Jianhong (PL, OrgSS 9)	3,39
Vladimirova, Natalia (SS 30)	135	Wu, Xiaotian (SS 70)	271
von Renesse, Max (SS 36)	163	Wu, Yaping (SS 8, OrgSS 18)	38,81
Vrabie, Ioan (OrgSS 15, SS 15)	66,72	Wu, Ying (SS 25)	115
Vucelja, Marija (SS 30, SS 44)	136,190	Wusterhausen, Frank (SS 36)	163

---

## W

---

Wadade, Hidemitsu (SS 38)	168
Wakasa, Kyouhei (CS 9)	331
Wakasa, Tohru (SS 8, SS 38)	37,169
Wakolbinger, Tina (SS 58)	237
Wang, Bixiang (SS 45)	194
Wang, Guangwa (SS 10, SS 22)	47,99
Wang, Jin (SS 67)	258
Wang, Min (SS 22)	99
Wang, Peiyong (SS 81)	303
Wang, Shin-Hwa (SS 33)	150
Wang, Shouhong (OrgSS 11, SS 11, SS 62)	48,51,244
Wang, Shu (SS 77)	290
Wang, Wei (SS 28)	127
Wang, Xiaohui (SS 68)	263
Wang, Xiaoming (SS 11, OrgSS 34)	51,152
Wang, Xiaoping (OrgSS 81, SS 81)	302,303
Wang, Xiaoqiang (SS 34)	155
Wang, Yan (SS 73)	277
Wang, Zhi-Cheng (SS 8)	37
Wang, Zhian (SS 61)	239
Wang, Zhongming (SS 28)	127
Ward, James (OrgSS 26)	117
Warne, Paul (OrgSS 39)	170
Watanabe, Hiroshi (SS 64)	253
Watanabe, Tatsuya (SS 15)	72
Weare, Jonathan (SS 20)	91
Wechselberger, Martin (OrgSS 17)	78
Wei, Juncheng (SS 1, OrgSS 29, SS 29)	5,129,132
Wei, Lei (SS 55, SS 68)	230,263
Weinberger, Hans (SS 8)	38
West, Matthew (SS 10)	47
Wijayasinghe, Indika (SS 25)	115
Willmott, Andrew (SS 73)	277
Winkler, Michael (SS 1, SS 42, OrgSS 61)	5,181,238
Winter, Matthias (SS 8, SS 29)	38,132
Wirosoetisno, Djoko (SS 11)	51
Wise, Steven (SS 2, OrgSS 34)	11,152
Wolfram, Marie-Therese (SS 76)	284
Wong, James (SS 22)	96
Woodruff, Celestine (SS 44)	190

---

## X

---

Xiang, Yang (OrgSS 81, SS 81)	302,304
Xiao, Yanyu (SS 9, SS 14)	65
Xie, Xiaoxia (SS 69)	266
Xie, Zhifu (SS 18, SS 55)	83,230
Xu, Feng (SS 9)	42
Xu, Jian (SS 25)	116
Xu, Xianmin (SS 81)	304
Xu, Zibo (SS 47)	201
Xu, Zongben (OrgSS 9, SS 9)	39,41
Xue, Chuan (SS 25)	116

---

## Y

---

Yagdjian, Karen (SS 48)	205
Yakubu, Abdul-Aziz (SS 14)	65
Yamada, Yoshio (OrgSS 38)	166
Yamauchi, Yusuke (SS 15)	72
Yamazaki, Kazuo (SS 57)	233
Yamazaki, Noriaki (SS 64)	253
Yan, Xiaodong (SS 62, SS 81)	245,304
Yanagida, Eiji (OrgSS 1, SS 1)	4,6
Yang, Bo (OrgSS 22)	95
Yang, Xiongfeng (SS 42)	181
Yang, Zhuoqin (SS 25)	116
Yanling, Shi (SS 63)	249
Yessen, William (SS 21)	94
Yip, Nung Kwan Aaron (SS 62)	245
Yokota, Tomomi (SS 61)	240
Yokoyama, Keita (SS 46)	196
Yomba, Emmanuel (SS 82)	306
Yoon, Jeong-Rock (SS 50)	212
Yoshikawa, Shuji (SS 41)	176
Yotsutani, Shoji (OrgSS 38, SS 38)	166,169
You, Yuncheng (SS 25, OrgSS 63)	116,246
Yu, Xiang (SS 24)	110
Yu, Xinwei (SS 28)	128
Yu, Zhixian (SS 42)	182
Yuan, Juan-Ming (OrgSS 57, SS 57)	232,234
Yuan, Pengfei (SS 22)	99

---

## Z

---

Zaman, Muhammad (SS 37)	165
Zambrini, Jean-Claude (SS 24)	110
Zanin, Massimiliano (SS 54)	225
Zanolin, Fabio (OrgSS 35, SS 35, SS 55)	156,159,231
Zatorska, Ewelina (SS 77)	290
Zenkov, Dmitry (SS 24)	110
Zhang, Aijun (SS 69)	266
Zhang, Dajun (SS 82)	307
Zhang, Fubao (SS 82)	307
Zhang, Hong-kun (CS 10)	333
Zhang, Jing (SS 4)	22
Zhang, Lei (SS 29)	132
Zhang, Lei (SS 79)	297
Zhang, Lijun (SS 68)	263
Zhang, Xuguo (SS 9)	42
Zhang, Zhenghe (SS 21)	94
Zhang, Zhou (SS 9)	42
Zhao, Chunshan (SS 18)	83
Zhao, Guangyu (SS 18)	83
Zhao, Kun (SS 2, SS 18)	11,83
Zheng, Shenzhou (SS 68)	263
Zhijian, Yang (SS 42)	182
Zhou, Jun (SS 33)	151
Zhou, Ruhai (SS 62)	245
Zhu, Zuo-nong (SS 82)	307
Zilio, Alessandro (SS 29)	132
Zinchenko, Maxim (SS 84)	309
Zubelli, Jorge (SS 47, SS 76)	201,285
Zuloeta Jimenez, Manuel (PS)	337
Zumbrum, Matt (PS)	338