Bernold Fiedler
Freie Universität Berlin, Germany
fiedler@math.fu-berlin.de
http://dynamics.mi.fu-berlin.de/persons/fiedler.php

Bernold Fiedler obtained his Doctorate and Habilitation in Heidelberg, and is currently Professor at the Institute of Mathematics (Free University Berlin). His research interests are: Differential Equations, Dynamical Systems, global attractors of certain parabolic PDEs, pattern formation, bifurcation theory, and a broad range of applications.

Reaction, diffusion, and advection in one space dimension – an invitation

Abstract
We give an introduction to the rich theory and phenomenology of scalar reaction-advection-diffusion PDEs

$$u_t = u_{xx} + f(x, u, u_x),$$

specifically, on the unit interval under Neumann boundary conditions. Besides growth conditions on $f \in C^2$, to guarantee existence of the (Sturm) global attractor, we only assume hyperbolicity of all equilibrium solutions $u_t = 0$. Rather than addressing superficial issues like existence and finite dimension, however, we ask: what do those Sturm global attractors actually look like?
Easy as this may sound, please recall that already the complications of the chaotic equilibrium equation $u_t = 0$, a forced pendulum, are legendary. The ODE boundary value problem exhibits intriguing links to topics like braids, billiards, and Temperley-Lieb algebras.
We give a combinatorial characterization of the PDE heteroclinic orbits between the ODE equilibria. On the more geometric and topological side, we observe how the unstable manifolds form a regular CW-complex, due to a Schoenflies Theorem on their boundaries. For illustration we show how any single closed 3-ball arises as a Sturm global attractor, but not any pair of 3-balls.
The results are joint work with Carlos Rocha (IST Lisboa) and others. They are all based on one-dimensional nodal properties, which amount to a nonlinear version of Sturm oscillation theory. See also http://dynamics.mi.fu-berlin.de/