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Entropy in Dynamical Systems: Complexity, Flexibility and Rigidity

Several interrelated concepts of entropy as well as closely related notions of Lyapunov characteristic exponents play a central role in the modern theory of dynamical systems.Those notions give quantitative expression of the measure of exponential complexity present in a deterministic system. After a general review of those concepts and principal relations between them I will discuss results and open problems related to two complimentary phenomena. of flexibility and rigidity. The general paradigm of flexibility can be rather vaguely formulated as follows:

> *Under properly understood general restrictions within a fixed class of smooth dynamical systems quantitative dynamical invariants take arbitrary values.*

Precise calculations are possible only in very few cases, primarily of algebraic nature such as homogeneous or affine systems. Most known constructions are perturbative and hence at best would allow to cover a small neighborhood of the values allowed by the model, or more often, not even that, since those models are often "extremal". So establishing flexibility calls for *non-perturbative or large perturbation constructions* in large families to cover possible values of invariants.
On the other hand, there is the rigidity paradigm that is better developed. It has several aspects and in the case of classical systems with discrete and continuos time one of them is related to these quantitative characteristics os exponential complexity:

> *Particular values of entropies or Lyapunov exponents or relations between those determine algebraic or similar models within a broad class of systems.*

Rigidity becomes more common and even prevalent when one passes from classical systems to systems with multi-dimensional time.