Peter D. Lax: A Life in Mathematics

There is apparently little to say about the mathematical contributions of Peter Lax and his effect on the work of so many others that has not already been said in several other places, including the Wikipedia, the web site of the Norwegian Academy's Abel Prize, and in quite a few other tributes. These lines are being written on the occasion of Peter's 90th birthday — which will be mathematically celebrated at the *11th AIMS Conference on Dynamical Systems, Differential Equations and Applications*, to be held in July 2016 in Orlando, FL — and their author merely intends to summarize some salient and relevant points of Peter's life and work for this celebration.

The Life. Peter was born on May 1st, 1926 into an upper-middle class, highly educated Jewish family in Budapest, Hungary. Fortunately for him, his family realized early on the danger to Jewish life and limb posed by the proto-fascist government of Miklós Horthy, a former admiral in the Austro-Hungarian Navy, who ruled the country from March 1920 to October 1944 with the title of Regent of the Kingdom of Hungary. The Laxes took the last boat from Lisbon to New York in November 1941.

In New York, Peter completed his high-school studies at the highly competitive Stuyvesant High School and managed three semesters of mathematics at New York University (NYU) before being drafted into the U.S. Army. His mathematical talents were already recognized by several leading Jewish and Hungarian mathematicians who had fled the fascist cloud gathering over Western and Central Europe, including Richard Courant at NYU and John von Neumann at Princeton.

Peter's wartime Army service led him, due to these talents, to the Manhattan Project and to Los Alamos, where he quickly moved from operating a calculator to more mathematical tasks. It was there that he became fascinated by the possibilities of numerically computing solutions of partial differential equations (PDEs) and by the nature of teamwork involving mathematicians, physicists and engineers.

These early experiences, along with Von Neumann's personal influence, shaped Peter's particular brand of combining pure and applied mathematics, and his style in inspiring 55 Ph.D. students and 554 "descendants," i.e., students of students to the sixth generation. He was also deeply influenced by, and influenced in turn, the very special atmosphere of NYU's Institute of Mathematics and Mechanics (IMM), now called the Courant Institute of Mathematical Sciences (CIMS). The IMM was founded by Courant, previously the Head of the Mathematics Institute at Göttingen, a mecca of mathematics both before World War I and thereafter.

The Göttingen tradition of small-town intimacy combined with incredible mathematical brilliance and distinction lived on in Greenwich Village. Brilliance is attested to by the Mathematics Department of the Institute's including 18 members of the U.S. National Academy of Sciences — more than any other mathematics department in the U.S. — and, significantly, five members of the National Academy of Engineering. The fact that several faculty members married the daughters of their teachers or colleagues reflects the small-town intimacy. Another element of tradition is the Mathematics Colloquium's still meeting on Mondays at 3:45 pm, in deference to the Central and Northern European custom of the academic quarter of an hour — in Latin "cum tempore," literally "with time," i.e., allowing for the academic quarter of an hour — as opposed to the more usual scheduling of seminars at the full or half hour, called "sine tempore" or "without time".

Peter's influence on the Courant Institute was particularly manifest when he became its fifth Director, and the last one of the line that went from the three founders — Courant (1935–1958), James J. Stoker (1958–1966) and Kurt Otto Friedrichs (1966-67), with an interlude due to Jürgen Moser (1967–1970) — to their two top students: Louis Nirenberg (1970–1972) and Peter (1972–1980); he was followed by the "outsider" Srinivasa Varadhan (1980–1984). It is during Peter's two terms of office that the Institute consolidated its special standing within the University by having its Director report directly to NYU's Provost and President. And it is Peter who was the first one, in 2005, in a sequence of four Abel Prize wins by Institute faculty — followed by Varadhan (2007), Mikhail Gromov (2009) and Nirenberg (2015) — a sequence yet to be matched by any other institution in the world.

The Work. It is hard to decide by which one of Peter's remarkable contributions to start. Having mentioned the effect of his years at Los Alamos on future endeavors, it might make sense to begin with what Gilbert Strang calls the Fundamental Theorem of Numerical Analysis, better known as the *Lax Equivalence Theorem*: Given an evolution equation and a finite-difference scheme that is consistent with it, in the sense that the truncation error of the scheme tends to zero as the differences do so, the discrete solution converges to the continuous one if and only if the scheme is stable. In other words, given consistency of a finite-difference scheme, convergence and stability are equivalent properties of the scheme.

As is often the case, one direction is easy to prove, the other one is not, and it involves some highly nontrivial functional analysis. The *Lax-Richtmyer* (1956) paper dealt with the case of linear PDEs, but the result holds, under suitable smoothness assumptions, for nonlinear ordinary differential equations (ODEs) as well. Like all of Peter's work, the proof is quite elegant and the result is incredibly useful: stability is much easier to check in practice than convergence, in any realistic problem for which no analytic solution is known.

In the general area of numerical methods for evolution PDEs, there are the first-order-in-time *Lax-Friedrichs* and the second-order-in-time *Lax-Wendroff* (1964) schemes. Both are second-order in space and the two are Peter's contributions that are known basically to any scientist or engineer who ever tried to solve a time-dependent PDE numerically.

It is the problem of capturing shocks in compressible fluids that motivated some of Von Neumann's pioneering numerical-analysis work at Los Alamos. Peter developed a highly satisfactory theory for their correct propagation under the heading of *Hyperbolic Systems of Conservation Laws*. In particular, aside from the well-known Rankine-Hugoniot condition, a shock has to satisfy the *entropy condition* that the flow to the left of the shock in the (x, t) plane, with x being the space coordinate and t being time, has to move faster than the flow to the right.

While these contributions dealt with problems that involved at least some numerical dissipation, Peter made striking contributions to the theory of infinite-dimensional Hamiltonian systems. His work on the Korteweg–de Vries (KdV) equation involved the transformation of the original, nonlinear PDE for solitary waves in one space dimension (1-D) into another PDE that involved the commutator of two operators, L and A, with L a linear, Sturm-Liouville operator. The transformed equation is called the *Lax equation*, and the operators L and A form a *Lax pair*.

The eigenvalues of L are the invariants of the KdV equation and this identification explains the rather surprising existence of an infinite number of such invariants. Furthermore — and to the joy of so many more mathematicians, physicists and engineers — any nonlinear PDE that allows its reformulation as a Lax equation will have an infinite number of invariants and, therewith, solitary-wave solutions. The best-known examples include the sine-Gordon equation for surfaces of constant negative curvature, the nonlinear Schrödinger equation, and the Toda lattice model of a 1-D crystal.

The striking results of Peter, Martin Kruskal and others on 1-D solitary waves also stimulated interest in more general localized coherent structures in two and three dimensions (2-D and 3-D). An important class of such physically observed and mathematically modeled structures is that of 2–D modons in planetary-scale, rotating fluids and of their further 3-D generalizations. These structures do not possess all the remarkable features of 1-D solitons but they can still play an important role in the geophysical and other flows in which they appear.

The Abel Prize extended citation states that "Peter D. Lax has been described as the most versatile mathematician of his generation." Rather than continue with scattering theory, the Lax-Milgram theorem and its applications to the method of finite elements, and quite a few additional topics, I'd like to mention the 2005 Abel Prize citation itself: "for his groundbreaking work on the theory and application of partial differential equations and to the computation of their solutions." Long before 2005, Peter was affectionately known to his students and many others as "PDE Lax," a name that will live in history.

The Honors. Peter is a member or foreign member of seven national academies, including the U.S. National Academy of Sciences, the (former Soviet, now) Russian Academy of Sciences, the Chinese and, of course, the Hungarian Academy of Sciences. He holds nine honorary doctorates from universities on three continents.

And besides the Abel Prize, he was awarded the highest distinctions of all three U.S. mathematical societies (AMS, MAA and SIAM), as well as the Wolf Prize in Mathematics, and several other high honors. Of these, the Von Neumann Lecture of the SIAM must have been a particular pleasure for Peter to deliver.

The Man. Of course, Peter D. Lax would still be the great mathematician that he is had he been a very different person. But in the actual world, it is hard to separate the work from the man. Peter's aura of human warmth, approachability and fun radiate along with and beyond the immediate influence of his work. As a convener of the special session "SS01. Nonlinearity in Climate and the Geosciences, A Special Session Honoring Peter D. Lax" at the *11th AIMS Conference on Dynamical Systems* mentioned at the beginning, I had the opportunity to appreciate the impression he made on people who had met him only once, even fleetingly, not just on his many students and other close associates.

Peter's personal style owes a lot to the city and country of his birth, as well as to the very special *Mitteleuropa* culture of the intellectual milieu that gave birth to surprisingly many mathematicians, physicists, writers and artists of Courant's and Von Neumann's generation. These scientists and humanists have enriched the United States and the world with their professional and human contributions, and Peter is a more than worthy representative of this group in his own generation. On this account, too, much has been written already, and it won't be repeated here. Let me just conclude with two anecdotes.

It appears that I'm not the only student of Peter's that has treasured his or her "true" Ph. D. diploma: the bottle of champagne happily imbibed after the successful defense and bearing on its label the signatures of Peter and of the other Ph. D. committee members. This tradition of Peter's clearly shows the connections he always made between fine work and the civilized life. My treasured signed bottle was unfortunately lost when the truck hired for the move from NYU to UCLA in 1985 was broken into, with this "diploma" being the only irreplaceable loss. I have, of course, tried to keep the tradition and to pass it on to my own students, along with many of Peter's instructive and entertaining sayings.

Shortly after the above-mentioned defense, I was walking one day from the West 4th Street subway station to the Institute, when I saw Peter and several colleagues walking in the opposite direction toward the Minetta Tavern, a traditional lunch hangout of the Courant Institute faculty. From across the street, Peter called out to me, "Dr. Ghil …" and the rest was lost in traffic. In any case, that and the "diploma" sure made me feel like I was now part of the club, and ready to make one or two contributions of my own.

Michael Ghil
Ecole Normale Supérieure, Paris, and
University of California, Los Angeles

Additional sources

Norwegian Academy of Sciences and Letters, 2005. Abel Prize citation, including CV, with list of honors and distinguished government service, http://www.abelprize.no/c53864/binfil/download.php?tid=53922 .

R. Hersh, 2015. *Peter Lax, Mathematician, an Illustrated Memoir*, American Mathematical Society, Providence, RI, 253 pp.

T. Keve, 2000. *Triad: The Physicists, the Analysts, the Kabbalists*, Rosenberger & Krausz, London, 362 pp.

K. Marton, 2006. *The Great Escape: Nine Jews Who Fled Hitler and Changed the World*, Simon & Schuster, New York, 271 pp.

Mathematics Genealogy Project, consulted on May 6, 2016, https://www.genealogy.math.ndsu.nodak.edu/id.php?id=13415 .

C. Reid, 1996. *Courant in Göttingen and New York*, *The story of an Improbable Mathematician.* Springer-Verlag, New York–Heidelberg, 1976, reprinted by Copernicus, New York, 318 pp.

G. Strang, 2005. Peter Lax to Receive 2005 Abel Prize, *SIAM News*, **38**(3), https://www.siam.org/pdf/news/6.pdf .