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ABSTRACTS

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Invited Plenary Lectures

John Ball
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John Ball is Sedleian Professor of Natural Philosophy in the Mathematical Institute at Oxford University and Director of the Oxford Centre for Nonlinear PDE. After obtaining his doctorate in 1972 at the University of Sussex under the supervision of David Edmunds, he joined the Department of Mathematics at Heriot-Watt University, Edinburgh, where he remained until moving to Oxford in 1996. His main research interests are in the calculus of variations and infinite-dimensional dynamical systems, together with their applications to solid mechanics, materials science and liquid crystals. He was President of the International Mathematical Union from 2002-06.

Mathematical Models of Liquid Crystals

Abstract
Recent years have seen an explosion of interest in the mathematics of liquid crystals. The lecture will survey some mathematical results and open questions related to the Oseen-Frank and Landau – de Gennes models, in particular concerning how these models describe defects.

Gang Bao
Zhejiang University, Peoples Republic of China
http://www.math.zju.edu.cn/bao/

Gang Bao is Chair Professor ("One Thousand Talent Program"), 2010-present and Dean of the School of Mathematical Sciences at Zhejiang University. After his PhD degree at Rice University in 1991, he was a Research Associate at Rice University and the IMA, University of Minnesota (1991-1994), Assistant and Tenured Associate Professor at University of Florida (1994-1999), and Full Professor at Michigan State University (1999-2014). He was the Founding Director of the Michigan Centre for Industrial and Applied Mathematics at Michigan State University (2006-2014). A recipient of the Feng Kang Prize on Scientific Computing in 2003 and the MSU University Distinguished Faculty Award in 2007, and elected SIAM Fellow in 2016, Dr. Bao's research interests include inverse problems for partial differential equations: mathematics and computation of diffractive optics, nonlinear optics, nano-optics, and electromagnetics. He has published over 150 papers in leading research journals including J. Ameri. Math. Soc., Arch. Rational Mech. Anal., J. Math. Pures Appl., Trans of AMS, Math. Comp., as well as Nature Nanotechnology. Dr. Bao is currently on the editorial boards of over 10 international research journals on mathematics including SIAM J. Appl. Math, SIAM J. Numer. Anal., J. Differential Equations, Inverse Problems, Inverse Problems and Imaging, and DCDS-B.

Recent Developments of Inverse Scattering Problems in Wave Propagation

Abstract
Inverse scattering problems arise in diverse application areas, such as nondestructive testing, seismic imaging, near-field and nano optical imaging, and medical imaging. A model scattering (direct) problem is concerned with a given wave incident on a medium enclosed by a bounded domain. The problem is to determine the scattered field or the energy distribution for the known scatterer. The inverse problem is to determine the scatterer from the boundary measurements of the fields. Although this is a classical problem in mathematical physics, mathematical issues and numerical solution of the inverse problem remain to be challenging since the problem is highly nonlinear, large and multi-scale, and most of all ill-posed! The severe ill-posedness has thus far limited in many ways the scope of inverse problem methods in practical applications. It also presents a fresh source of exciting problems in mathematical modeling, analysis, and computation. In this talk, the speaker will first introduce several inverse scattering problems of broad interest and discuss recent developments in the mathematical and computational studies of the problems. Of particular importance are inverse medium problems, inverse source problems, and inverse obstacle problems for acoustic and electromagnetic waves. Based on multi-frequency data and the uncertainty principle, effective computational and mathematical approaches will be presented for overcoming the ill-posedness of the inverse problems. Selected mathematical and computational results will be highlighted. In addition, recent stability results for inverse scattering problems in elasticity will also be presented. The talk will be concluded by remarks on related topics and open problems.

Annalisa Buffa
CNR-IMATI: PAVIA, Italy

Professor Annalisa Buffa received her degree in Computer Engineering in 1996 and her PhD in Mathematics in 2000. She obtained a research position at the CNR Institute IMATI (Italy) in 2001. She became Research Director in 2004 and led the Institute from 2013 to 2016. In September 2016, she joined the Institute of Mathematics at EPFL (Switzerland) as Full Professor in 2016. In 2008 she was granted an ERC StG, she received the ICIAM Collatz Prize in 2015 and was awarded an ERC AdG in 2016. She was invited/plenary speaker at a number of international conferences, section speaker at International Congress of Mathematicians (2014, Seoul, Korea), and plenary speaker at ICIAM (Beijing, China, 2015). She is a member of the Academia Europaea.

Numerical Methods for PDEs: Old and New Challenges

Abstract
Numerical methods for PDEs is a branch of numerical analysis which offers scientific challenges spanning from functional analysis to computer science and code design. I will present recent results for numerical methods based on the use of splines as main tool for representing both geometric models and unknowns, in the so called isogeometric analysis framework. For this class of methods, all robust and accurate techniques aiming at enhancing the flexibility of splines, while keeping their structure, are of paramount importance since the tensor product structure underlying spline constructions is far too restrictive in the context of approximation of partial differential equations (PDEs) and of construction of volumetric geometric models. I will describe various approaches, from adaptivity with regular splines, to trimming. Moreover, I will show applications and test benches involving large deformation problems in non linear mechanics.

Vincent Calvez
Institut Camille Jordan, France
http://perso.ens-lyon.fr/vincent.calvez/

I have been working on mathematical models for biology since my undergraduate studies. I did my PhD under the supervision of Benoît Perthame in Paris about models and analysis for collective motion of cells. I studied concentration waves of bacteria in a micro-channel by means of a mesoscopic model. More recently, I moved to theoretical eco-evolutionary biology, e.g. dispersal evolution, propagation of invasive species, and evolution of ageing. I am the principal investigator of the ERC starting grant project MESOPROBIO.

Mesoscopic Models for Propogation in Biology

Abstract
I will review recent results of modeling and analysis of biological invasions by means of kinetic transport equations and reaction-diffusion equations. I will focus on two case studies for which it is crucial to disentangle the structure of the wave in order to describe correctly the propagation. This leads to new and challenging mathematical problems, including wave acceleration. The first case study is about traveling waves of chemotactic bacteria. The second case study is about the evolution of dispersion during a population range expansion.

Chiun-Chuan Chen
National Taiwan University, Taiwan
http://math.cts.nthu.edu.tw/Mathematics/conf-PDE2000/CC_Chen.html

Chiun-Chuan Chen is currently a professor of mathematics at National Taiwan University. He obtained his PhD in mathematics at National Taiwan University and held academic positions at Academia Sinica and National Chung Cheng University. His research interests include elliptic equations, reaction-diffusion equations and calculus of variations.

Travelling Wave Solutions of the 3-species Lotka-Volterra Competition System with Diffusion

Abstract
A three species competition-diffusion system may display cyclic competition and complicated behavior. To study the dynamics of the system, it is important to understand the traveling waves solutions. In this lecture, we will report some of the recent progress on travelling wave solutions of the 3-species Lotka-Volterra competition system with diffusion.

Jean-Michel Coron
Université Pierre et Marie Curie, France
http://en.wikipedia.org/wiki/Jean-Michel_Coron

Jean-Michel Coron received the engineering degree from École polytechnique, Paris, France, in 1978 and from the Corps des Mines in 1981. He received the Thèse d'État in 1982. He has been a researcher at Mines ParisTech, then an associate professor at École polytechnique, and a full professor at Université Paris-Sud (Paris 11). He is currently a full professor at Université Pierre et Marie Curie (Paris 6) and a member of the French Academy of Sciences. Until the 90's, Coron worked on partial differential equations arising in differential geometry (Rellich's conjecture, Yamabe-type problems, harmonic maps) and in the physics of liquid crystals. Later, Coron moved to control theory and in particular to the stabilization of nonlinear control systems and the control of systems modeled by means of partial differential equations (Euler and Navier-Stokes equations of incompressible fluids, shallow water equations, Schrödinger equations, Korteweg-de Vries equations). Coron was selected to deliver a plenary lecture in control theory at ICIAM congress 2015 and at ICM 2010. He has received many prizes, including the W. T. and Idalia Reid Prize (2017), the Maxwell prize (2015) and the Fermat prize (1993). He was the recipient of an ERC advanced grant (2011-2016).

How the Nonlinearities can be Used to Control a System

Abstract
A control system is a dynamical system on which one can act thanks to what is called the control. For example, in a car, one can turn the steering wheel, press the accelerator pedal etc. These are the control(s). One of the main problems in control theory is the controllability problem. It is the following one. One starts from a given situation and there is a given target. The controllability problem is to see if, by using some suitable controls depending on time, the given situation and target, one can move from the given situation to the target. We study this problem with a special emphasis on the case where the nonlinearities play a crucial role. In finite dimension in this case, a key tool is the use of iterated Lie brackets as shown in particular by the Chow theorem. This key tool also gives important results for some control systems modeled by means of partial differential equations. However, we do not know how to use it for many other control systems modeled by means of partial differential equations. We present methods to avoid the use of iterated Lie brackets. We give applications of these methods to the control of various physical control systems (Euler and Navier- Stokes equations of incompressible fluids, 1-D hyperbolic systems, heat equations, shallow water equations, Korteweg-de Vries equations, Schroedinger equations...) and to the stabilization problem, another of the main problems in control theory.

Yiming Long
Nankai University, Peoples Republic of China
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Yiming Long received his Master degree from Nankai University in 1981 and Ph. D. from University of Wisconsin-Madison in 1987. After the post-doc period in FIM of ETH-Zürich, he joined the Nankai Institute of Mathematics (now Chern Institute of Mathematics) of Nankai University in late 1988 and is a professor since 1990. He is a Changjiang professor at Nankai University since 2000. Long's interest is in dynamical systems, variational methods, symplectic geometry and celestial mechanics, especially on the iteration theory of Maslov-type indices, periodic solutions of Hamiltonian systems, close geodesics, stability of periodic orbits of N-body problems. The awards obtained by him include the S.S. Chern Prize of Chinese Mathematical Society (1997), TWAS Award in Mathematics by Academy of Sciences for the Developing World (TWAS, 2002), Chinese National Natural Science Prize (2004), the Prize for Scientific and Technological Progress by Ho Leung Ho Lee Foundation (2013). He was an invited speaker of the International Congress of Mathematicians (2002). He is a member of Chinese Academy of Sciences, a fellow of TWAS, and an inaugural fellow of American Mathematical Society.

Closed Geodesics on Compact Finsler Manifolds

Abstract
The closed geodesic problem is a classical topic in dynamical systems, differential geometry and variational analysis, which can be traced back at least to H. Poincaré. A famous conjecture claims the existence of infinitely many distinct closed geodesics on every compact Riemannian manifold. But so far it has been only proved for the 2-dimentional case. On the other hand, Riemannian metrics are quadratic reversible Finsler metrics, and the existence of at least one closed geodesic on every compact Finsler manifold is well-known because of the famous work of L. Lyusternik and A. Fet in 1951. In 1973 A. Katok constructed a family of remarkable Finsler metrics on every d-dimensional sphere S^d which possesses precisely 2[(d+1)/2] distinct closed geodesics. In 2004, V. Bangert and the author proved the existence of at least 2 distinct closed geodesics for every Finsler metric on S^2, and this lower bound multiplicity estimate on S^2 is sharp by Katok's example. Since this work, many new results on the multiplicity and their stabilities of closed geodesics have been established. In this lecture, I shall give a survey on the study of closed geodesics on compact Finsler manifolds, including a brief history and results obtained in the last 10 years. I shall also introduce some recent results we obtained for the multiplicity and stability of closed geodesics on compact Finsler manifolds, sketch some ideas of their proofs, and then propose some further open problems in this field.

Hirokazu Ninomiya
Meiji University, Japan
http://www.isc.meiji.ac.jp/∼nino38/index-en.html

Hirokazu Ninomiya received his Ph.D. in mathematics in 1995 from Kyoto University under the supervision of Takaaki Nishida. He is currently Professor of School of Interdisciplinary Mathematical Sciences at Meiji University. His research interests are mainly in parabolic equations and systems including free boundary problems, especially, entire solutions, pattern formation and roles of diffusion.

Propagation Phenomena in Reaction-Diffusion Equations

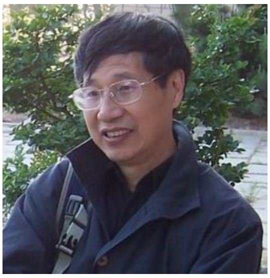
Abstract
Propagation phenomena arise in a large variety of systems in physics, chemistry and biology. In this talk we focus on propagation phenomena of the reaction-diffusion equation. Recently traveling wave solutions and entire solutions of the reaction-diffusion equation have been studied intensively. Here traveling wave solution means the solution translating with a constant speed without changing its profile and the entire solution is a solution existing for any positive and negative time. Various traveling wave solutions and entire solutions have been constructed not only in the whole space, but also in an exterior domain. I will survey the recent studies and the relation between traveling wave solution and the entire solution including the application to the propagation phenomena.

Natasa Pavlovic
The University of Texas at Austin, USA
http://www.ma.utexas.edu/users/natasa/

Natasa Pavlovic is a Professor of Mathematics at the University of Texas at Austin. She joined the Department of Mathematics at the University of Texas at Austin as an Assistant Professor in 2007, following a faculty appointment at Princeton University and postdoctoral appointments at the Institute for Advanced Study and Princeton University. Pavlovic completed her PhD at the University of Illinois at Chicago in 2002 under the supervision of Susan Friedlander and Nets Katz. The focus of Pavlovic's research is on partial differential equations, including wave and dispersive equations which are used as models for many wave phenomena from Bose-Einstein condensation to formation of freak waves in an ocean, equations of fluid motions and kinetic equations that describe dynamics of a dilute gas and are at the core of applied analysis, probability and statistical physics. She was a recipient of a Sloan Fellowship in 2008, held an Eisenbud Professorship at the Mathematical Sciences Research Institute in Berkeley in Fall 2015, and was named a Fellow of the American Mathematical Society in 2016. Also Pavlovic was an elected member at large of the Council of the American Mathematical Society and served on the Council in the period 2013 - 2016. Her teaching at the University of Texas at Austin has been recognized by John R. Durbin Teaching Excellence in Mathematics Award in 2009 and by the College of Natural Sciences Teaching Excellence Award in 2010.

Back and Forth from Quantum Many Particle Systems to Nonlinear PDE, and Applications to Kinetic Equations

Abstract
Analysis of large systems of interacting particles is a key for predicting and understanding various phenomena arising in different contexts, from physics (in understanding e.g. boson stars) to social studies (when modeling social networks). Since the number of particles is usually very large one would like to understand qualitative and quantitative properties of such systems of particles through some macroscopic, averaged characteristics. In order to identify macroscopic behavior of multi-particle systems, it is helpful to study the asymptotic behavior when the number of particles approaches infinity, with the hope that the limit will approximate properties observed in the systems with a large finite number of particles. An example of an important phenomenon that describes such macroscopic behavior of a large system of particles is the Bose-Einstein condensation. Mathematical models have been developed to understand such phenomena. Those models connect large quantum systems of interacting particles and nonlinear PDE that are derived from such systems in the limit of the number of particles going to infinity. In this talk we will focus on developments that connect a quantum many particle system of bosons and the nonlinear Schrodinger equation, and will apply some of the ideas appearing in this context to a new program of studying well-posedness of Boltzmann equation, which describes the evolution of the probability density of independent identically distributed particles modeling a rarefied gas with predominantly binary elastic interactions. This talk is based on joint works with Thomas Chen, and with Thomas Chen and Ryan Denlinger.



Shige Peng
Shandong University, Peoples Republic of China
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Shige Peng received his PhD in 1985 at University Paris-Dauphine, in the direction of mathematics and informatics, and 1986 at University de Marseille, in the direction of applied mathematics. He jointed Institute of Mathematics of Fudan University (Shanghai) as a two-year post-doc position during 1997-1999. He then come back his homeland Jinan as assistant associated professor in 1990 and full professor in 1991. His main researches are in the domains of stochastic optimal controls, backward stochastic differential equations and the corresponding partial differential equations, stochastic HJB equations. Recently he is interested in the theoretical foundation of nonlinear expectations. Some awards he has received are, the Natural Science Prize of China (1995), Su Buqing Prize of Applied Mathematics (2006), Chinese Academy of Science Tan Kah Kee Science Award (2008), Chinese Society of Mathematics Hua Loo-Keng Award (2011), and the Qiu Shi Award for Outstanding Scientists (2016). He was a plenary speaker at the ICM 2010 and ICIAM 2015.

Theoretical Study of Nonliner Expectations and Applications to Data Sequences with Essential Uncertainty of Probability Distributions

Abstract
How to calculate the essential uncertainty of probability distributions hidden behind a real data sequence is a theoretically and practically important challenging problem. Recently some fundamentally important progress has been achieved in the domain of law of large numbers (LLN) and central limit theorem (CLT) with a much weaker assumption of independence and identical distribution (i.i.d.) under a sublinear expectation. These new LLN and CTL can be applied to significantly wide classes of data sequence to construct the corresponding optimal estimators. In particular, many distribution uncertainties hidden behind data sequences are able to be quantitatively calculated by introducing a new algorithm of phi-max-mean type. In this talk, I take some typical examples to provide a more concrete explanation of the above mentioned LLN and CLT, the key idea of their proofs, as well as the new phi-max-mean estimators.

Special Session 1: Mathematical Models and Methods in Materials Science

Pierluigi Cesana, Kyushu University, Japan
John M. Ball, University of Oxford, England
Marco Cicalese, Technische Universität München, Germany

These last few years have seen the appearance of novel materials for technological applications with unusual features and properties. Synthesis of extremely low-hysteresis shape-memory alloys, soft thin membranes and programmables structures where one has the critical interaction of topological defects and geometric constraints have suggested and motivated the development of new mathematical models and methods possibly based on the variational principle as well as stochastic processes. In this proposed special session we leave the material and the application at the centre stage. Topics of interest are formation of interfaces and discontinuities in phase-transforming materials, avalanches, nucleation and evolution of complex structures, effects of topology, geometry and randomness on the mechanical properties of (smart) materials. We intend to gather mathematicians, materials scientists and experimentalists working on materials and physical systems from their own peculiar perspective with the specific purpose of stimulating constructive discussion and interaction.

Generalized Hadamard Jump Conditions and Polycrystal Microstructure

John Ball
University of Oxford, England
Carsten Carstensen

The talk will describe various generalizations of the Hadamard jump condition, and how they can lead to information about polycrystal microstructure arising from martensitic phase transformations.

Interfacial Energies on Dense Graph Sequences

Andrea Braides
University of Rome Tor Vergata, Italy
Paolo Cermelli, Simone Dovetta

Non-convex (short-range) interactions on lattices give rise, in a passage discrete-to-continuum, to interfacial energies such as those found in variational theories of Fracture. The details of the interfacial energy functions are usually captured by looking at the corresponding behaviour of Ising Systems. When long-range interactions are present, the topology of the interactions may give rise to non-local effects and diffuse interfaces. We consider the case when each node interacts with a substantial portion of the nodes of the lattice (dense graph), and give a description of the limit in terms of an energy defined in terms of the general notion of a limit graphon.

Self-Organization and Criticality in Martensite

Pierluigi Cesana
Kyushu University, Japan
John M. Ball, Ben Hambly

A martensitic phase-transformation is a first-order diffusionless transition occurring in elastic crystals and characterized by an abrupt change of shape of the underlying crystal lattice. It is the basic activation mechanism for the Shape-Memory effect. In this talk we present a probabilistic model for the description of martensitic microstructure as an avalanche process. Our approach to the analysis of the model is based on an associated general branching random walk process. Comparisons are reported for numerical and analytical solutions and experimental observations.

Design of Low-Hystereis Phase Transforming Materials by the Conditions of Compatibility

Xian Chen
Hong Kong University of Science and Technology, Hong Kong

The *Cofactor Conditions*, proposed and studied in X. Chen et al, JMPS 2013, underlie a comprehensive set of all geometric nonlinear conditions of compatibility between crystal structures of different symmetries. In this talk, we will present the mathematical formulation of the Cofactor Conditions and their insights to the design of low-hysteresis alloys undergoing reversible phase transformation. We will also discuss the algorithm of finding the lattice correspondence and transformation strains of the initial and final phases, which sometimes leads to the discovery of unconventional transformation pathways.

Quantization Error Dependent Atomistic-To-Continuum Theories for the Classical Xy Model

Marco Cicalese
TU Munich, Germany
Gianluca Orlando, Matthias Ruf

We consider the classical two-dimensional xy model for ferromagnetic materials. Its atomistic-to-continuum limit has been the object of intense studies in the last years since it can be considered as a simple representative of a wide class of energetic models leading to concentration of energy on points. In particular it is known that at certain scalings the xy model is equivalent to the Ginzburg-Landau model in superconductivity and to the screw dislocation model in plasticity. In the numerical approximation of such energies one usually has to carefully approximate the continuum range of the order parameter and quantization errors appear. In this talk, we consider the continuum approximation of the xy model for small lattice spacings and quantization errors, and we show that different theories may arise depending on the asymptotic behaviour of their quotient.

A Rigorous Approach to Describing the Mobility of Screw Dislocations

Thomas Hudson
University of Warwick, England

Discrete Dislocation Dynamics (DDD) is a phenomenological modelling and simulation technique used to study plasticity in crystalline solids on length- and timescales inaccessible with molecular dynamics. In this talk, I will present some mathematical results demonstrating that in a particular case, DDD for screw dislocations can be *derived* from a microscopic stochastic model.

Programming of Shape in Narrow Strips of Liquid Crystal Elastomers

Konstantinos Koumatos
University of Sussex, England
Virginia Agostiniani, Antonio DeSimone

Using the theory of Gamma-convergence, we derive from three-dimensional elasticity new one- dimensional models for ribbons exhibiting spontaneous curvature and twist. We apply the models to shape-selection problems for thin films of nematic elastomers with the twist and splay-bend geometries for the nematic director. For the former, we discuss the possibility of helicoid-like shapes as an alternative to spiral ribbons.

Quantitative Homogenization in Nonlinear Elasticity

Stefan Neukamm
TU Dresden, Germany
Mathias Schaeffner

We consider a nonlinear elastic composite with a periodic microstructure described by the nonconvex integral functional

$$\mathcal{E}_\varepsilon(u) := \int_\Omega W(\frac{x}{\varepsilon}, \nabla u(x)) - f(x) \cdot u(x)\, dx$$

As it is well-known, under suitable growth conditions, $\mathcal{E}_\varepsilon$ Γ-converges to a functional with a homogenized energy density W_{hom}. One of the main problems in homogenization of nonlinear elasticity is that long wavelength buckling prevents the possibility of homogenization by averaging over a single period cell, and thus W_{hom} is in general given by an infinite-cell formula. Under appropriate assumptions on W (e.g. frame indifference, minimality at identity, non-degeneracy) and on the microstructure (smooth but possibly touching inclusions), we show that in a neighbourhood of rotations W_{hom} is characterized by a single-cell homogenization formula. For this, we combine the construction of a matching convex lower bound and Lipschitz-estimates for sufficiently small solutions of nonlinear elliptic systems. Moreover, for small loads, we derive a quantitative two-scale expansion and establish existence and uniform Lipschitz estimates for minimizers of $\mathcal{E}_\varepsilon$.

Free Energies on Stochastic Lattices

Matthias Ruf
University of Brussels, Belgium
Marco Cicalese, Antoine Gloria

We study the asymptotic behavior of large volume Gibbs measures associated with discrete Hamiltonians that are defined on deformations of a stationary stochastic lattice. Assuming polynomial growth and finite range interactions for the discrete Hamiltonian, we prove a large deviation principle with a continuum elasticity-type rate functional. We then investigate this functional in the small temperature regime. Under suitable continuity assumptions on the microscopic Hamiltonian, we show that it can be well approximated by the Γ-limit of the rescaled discrete Hamiltonians.

Asymptotic Properties of Step Bunching in Epitaxial Growth with Elasticity Effects

Aaron Yip
Purdue University, USA
Tao Luo, Yang Xiang

In epitaxial thin film growth, elasticity effects often lead to self-organizing pattern formation which can be important in the fabrication of nano-structures. We discuss an elasticity model that takes into account of the lattice misfit between the substrate and the film, and the broken-bond effect due to surface steps. The former is an attractive while the latter is a repulsive interaction. It is found that uniform step train is unstable and will evolve into structures consisting of macroscopic step bunches. For the case of vicinal surface which consists of a sequence of monotonically decreasing steps, using a variational formulation, we analyze the properties of these bunches, notably their energy scaling and bunch width.
We emphasize on a discrete model but continuum description will also be discussed.

Stochastic Homogenisation of Free-Discontinuity Problems

Caterina Zeppieri
University of Muenster, Germany
Filippo Cagnetti, Gianni Dal Maso, Lucia Scardia

In this talk I present some recent results concerning the stochastic homogenization of a class of free-discontinuity functionals depending on vector-valued functions u which can be discontinuous across hypersurfaces depending on u. We show that, under the usual assumptions of stationarity and ergodicity, the homogenization procedure gives rise to a (homogeneous) deterministic free-discontinuity functional belonging to the same class.

Special Session 2: Control of Partial Differential Equations

Jean-Michel Coron, Université Pierre et Marie Curie, France
Zhiqiang Wang, Fudan University, Peoples Rep of China
Xu Zhang, Sichuan University, Peoples Rep of China

Many systems in sciences and industry are described by means of Partial Differential Equations (PDEs). For these systems, an important issue is to steer the system to some desired targets by using suitable controls, taking into account the control costs and constraints. Another important issue is to stabilize unstable equilibrium (or to improve the stability) by means of suitable feedback laws. Such problems become more and more important in various fields of applications. The mathematical problems are very challenging in both theoretical and numerical aspects.

On the Cost of Controllability of the 1-Dimensional Heat Equation

Sylvain Ervedoza
Université de Toulouse and CNRS, France
Jérémi Dardé

The goal of this talk is to present some recent results on control issues for the 1-dimensional heat equation. In particular, we shall present a new proof of the controllability of the heat equation in 1-d yielding new results on the cost of controllability of the heat equation in 1d. Our strategy is based on a Carleman type estimate inspired by the Gaussian kernel and arguments from holomorphic function theory, namely the Phragmen Lindelof principle.

Control of a Solid Immersed in a Perfect Incompressible Fluid

Olivier Glass
Université Paris-Dauphine, France
Jozsef Kolumban, Franck Sueur

We consider the dynamics of a solid immersed in a perfect incompressible fluid. The fluid is driven by the Euler equation for inviscid incompressible fluids with impermeable boundaries, and the solid is driven by the Newton equations and evolves under the influence of the fluid pressure. We study the possibility of controlling the motion of the body by using suitably chosen boundary conditions on some open part of the boundary.

Boundary Stabilization of 1-D Hyperbolic Balance Laws

Long Hu
Shandong University, Peoples Rep of China

In this talk, we are concerned with the recent development of boundary stabilization problem of 1-D hyperbolic balance laws by using backstepping methods. We will show how to design boundary feedback controllers to rapidly stabilize the general 1-D coupled hyperbolic PDEs (linear or quasilinear, autonomous or non-autonomous (especially the case with the coefficients depending on time and space simultaneously) systems). In particular, the linear systems can be stabilized in optimal finite time.

Insensitizing Control for Linear and Semi-Linear Heat Equations with Partially Unknown Domain

Pierre Lissy
Université Paris-Dauphine, France
Yannick Privat, Yacouba Simpore

In this talk, we are interested in an insensitizing control problem on a semi-linear heat equation with Dirichlet boundary conditions and globally Lipschitz nonlinearity, which consists in finding a distributed control such that some functional of the state is insensitive at the first order to the perturbations of the domain. Our first result consists of an approximate insensitization property on the semi-linear heat equation. It rests upon a linearization procedure together with the use of an appropriate fixed point theorem. Our second result is specific to the linear case. We show a property of exact insensitization for some families of deformation given by one or two parameters. Our proof relies on a geometrical approach and direct computations.

Output Feedback Sampled-Data Stabilization for Heat Equations

Hanbing Liu
China University of Geosciences (Wuhan), Peoples Rep of China
Ping Lin, Gengsheng Wang

By building up a type of observability inequality related to partial L^2-null approximate controllability of heat equation with time-invariant control, this work constructs a type of output feedback law of sampled-data form for rapid stabilization of heat equation with potential. Moreover, we get both the lower and upper bounds of the norm of the feedback operator with respect to the sampling period. We find that the norm of the feedback will go to infinity when the sampling period goes to infinity or zero. Furthermore, we show that the norm of the feedback continuously depends on the sampling period, and there exists an optimal sampling period in the sense that the norm of the corresponding feedback is minimal.

Some Problems of Localised Controllability

Jean-Pierre Puel
LMV, University of Versailles, France

We will present several problems of localised controllability, some of them being open. For example, we will consider the bizone controllability for the heat equation and the instantaneous localised energy problems for the Schrödinger equation and the wave equation.

Exact Controllability Results for Some PDEs of Parabolic Type

Lionel Rosier
MINES ParisTech, France
Camille Laurent, Philippe Martin, Ivonne Rivas, Pierre Rouchon

Recently, the issue of the determination of the reachable states for the boundary control of the one-dimensional heat equation has been revisited by several approaches. After a brief review of these results, we will discuss some recent extensions: (i) the derivation of the exact controllability of some nonlinear parabolic equations, including the viscous Burgers equation and the Allen-Cahn equation, (ii) the investigation of the reachable spaces of the Korteweg-de Vries equation with a control from the left endpoint. The above results are obtained in some spaces of analytic functions, and the analysis involves some ill-posed problems investigated in Gevrey classes.

Exponential Boundary Feedback Stabilization of a Shock Steady State for the Inviscid Burgers Equation

Peipei Shang
Tongji University, Peoples Rep of China
Georges Bastin, Jean-Michel Coron, Amaury Hayat

We study the exponential stabilization of a shock steady state for the inviscid Burgers equation on a bounded interval. Our analysis relies on the construction of an explicit strict control Lyapunov function. We prove that by appropriately choosing the feedback boundary conditions, we can stabilize the state as well as the shock location to the desired steady state in H^2-norm, with an arbitrary decay rate.

Observable Set, Observability, Interpolation Inequality and Spectral Inequality for the heat equation in $\mathbb{R}^n$

Ming Wang
China University of Geosciences, Peoples Rep of China
Gengsheng Wang, Can Zhang, Yubiao Zhang

This talk gives connections among observable sets, the observability inequality, the Hölder-type interpolation inequality and the spectral inequality for the heat equation in $\mathbb{R}^n$. We present the characteristic of observable sets for the heat equation. In more detail, we show that a measurable set in $\mathbb{R}^n$ satisfies the observability inequality if and only if it is γ-thick at scale L for some $\gamma > 0$ and $L > 0$. We also build up the equivalence among the above-mentioned three inequalities. More precisely, we obtain that if a measurable set in $\mathbb{R}^n$ satisfies one of these inequalities, then it satisfies others. Finally, we get some weak observability inequalities and weak interpolation inequalities where observations are made over a ball.

Optimization of the Principal Eigenvalue for Elliptic Operators

Jiongmin Yong
University of Central Florida, USA
Hongwei Lou

Optimization problems of the principle eigenvalue for elliptic operators of divergence form are considered. The eigen map of elliptic operator is introduced and the continuity as well as the differentiability of such a map is established. For maximization problem, the admissible control set is convexified to get the existence of optimal solutions. Whereas, for minimization problem, the relaxation of the problem under H-convergence is used to get a relaxed optimal solution. Some necessary conditions are presented for both problems and illustrative examples are presented as well.

Special Session 3: Recent Trends in Mathematical Finance

Shige Peng, Shandong University, Peoples Rep of China
Zengjing Chen, Shandong University, Peoples Rep of China
Yufeng Shi, Shandong University, Peoples Rep of China

This session will focus on some recent developments in the theory of mathematical finance, including pure analysis and applications. Questions of mathematical finance, nonlinear expectation and backward stochastic differential equations will be central topics of discussion. The goal of this minisymposium is to provide a forum to discuss the recent progress and promising directions in mathematical finance and their applications to the financial market and economical problems.

Stochastic Global Maximum Principle for Optimization with Recursive Utilities

Mingshang Hu
Shandong University, Peoples Rep of China

We study the recursive stochastic optimal control problems. The control domain does not need to be convex, and the generator of the backward stochastic differential equation can contain z. We obtain the variational equations for backward stochastic differential equations, and then obtain the maximum principle which solves completely Peng's open problem.

Representation of Limit Values for Nonexpansive Stochastic Differential Games

Juan Li
Shandong University, Peoples Rep of China
Nana Zhao

A classical problem in ergodic control theory consists in the study of the limit behaviour of $\lambda V_\lambda(\cdot)$ as $\lambda \searrow 0$, when V_λ is the value function of a deterministic or stochastic control problem with discounted cost functional with infinite time horizon and discount factor λ. We study this problem for the lower value function V_λ of a stochastic differential game with recursive cost, i.e., the cost functional is defined through a backward stochastic differential equation with infinite time horizon. But unlike the ergodic control approach, we are interested in the case where the limit can be a function depending on the initial condition. For this we extend the so-called non-expansivity assumption from the case of control problems to that of stochastic differential games.
Based on a joint work with Rainer Buckdahn (Brest, France), Nana Zhao (Weihai, China).

An Ergodic BSDE Approach to the Construction of Forward Preferences

Gechun Liang
University of Warwick, England
Ying Hu, Shanjian Tang

We present some recent progress about the construction of forward preferences using the tools from ergodic and infinite horizon backward stochastic differential equations. We derive a new type of quadratic BSDE system in infinite horizon for regime switching forward preferences, and solve the system using the multidimensional comparison theorem for BSDE. Based on a joint work with Ying Hu and Shanjian Tang.

EM Algorithm and Stochastic Control

Xianhua Peng
HSBC Business School, Peking University, Peoples Rep of China
Steven Kou, Xingbo Xu

Generalising the idea of the classical EM algorithm that is widely used for computing maximum likelihood estimates, we propose an EM-Control (EM-C) algorithm for solving multi-period finite time horizon stochastic control problems. The new algorithm sequentially updates the control policies in each time period using Monte Carlo simulation in a forward-backward manner; in other words, the algorithm goes forward in simulation and backward in optimization in each iteration. Similar to the EM algorithm, the EM-C algorithm has the monotonicity of performance improvement in each iteration, leading to good convergence properties. We demonstrate the effectiveness of the algorithm by solving stochastic control problems in revenue management for airlines and in the study of real business cycle.

On the Uniqueness of Adapted Solutions to BSDEs

Yufeng Shi
Shandong University, Peoples Rep of China
Min Li

In the theory of BSDEs, in order to guarantee the existence and uniqueness of adapted solutions to BSDEs, one usually supposes that the generator g is Lipschitz with respect to y and z or other similar hypothetical conditions in which the variable z satisfies the similar assumptions as the variable y. In this talk, we show that BSDEs also has a unique adapted solution if the generator g is Lipchitz with respect to y and is linear growth and continuous with respect to z. This implies that the variable z is determined by y. And as an application, we obtain that the corresponding partial differential equations have a unique viscosity solution.

Donsker-Type Theorem for Log-Likelihood Ratio Processes

Hanchao Wang
Institute for Financial Studies, Shandong University, Peoples Rep of China
Zhonggen Su

Let $(\Omega, \mathcal{F}, (\mathcal{F})_{t\geq 0}, P)$ be a complete stochastic basis, X a semimartinagle with predictable compensator (B, C, ν). Consider a family of probability measures $\mathbf{P} = (P^{n,\psi}, \psi \in \Psi)$, $n \geq 1$, where $P^{n,\psi} \overset{loc}{\ll} P$, and denote the likelihood ratio process by $Z_t^{n,\psi} = \frac{d\, P^{n,\psi}|_{\mathcal{F}_t}}{d\, P|_{\mathcal{F}_t}}$. We are mainly interested in the logarithmic ratio process $\log Z_t^{n,\psi}$. Under some regularity conditions in terms of logarithm entropy and Hellinger processes, we prove that $\log Z_t^n$ converges weakly to a Gaussian process in $\ell^\infty(\Psi)$ as $n \to \infty$. At last, an application of our main result is given.

Dual-Curve Term Structure Models for Post-Crisis Interest Rate Derivatives Markets

Lixin Wu
HKUST, Hong Kong
Cui Shidong

Before the 2007-08 financial crisis, the term structure of forward rates of different tenors are linked by the discount curve. As a result, interest-rate modeling could be carried out with the forward-rate curve of a particular tenor, say, the three-month tenor. Such a linkage, however, broke down during the financial crisis. Nowadays, for pricing purposes, the term structure of forward rates of different tenors are modeled separately, which is coined the multi-curve modeling and has become the new norm of LIBOR derivatives modeling. The majority of multi-curve modeling approaches, however, are at odds with the stylized pattern of basis swap curves: smooth and monotonically decreasing in terms (or maturities), which cannot be retained if forward rates of different tenors were driven by different random factors in any usual way. The multi-curve modeling has served to legitimize, undesirably, sector segregation in pricing and hedging. In this article, we decompose a LIBOR rate into an OIS forward rate and a discrete loss rate, which represent the risk-free component and the default-risk component, respectively, and model them simultaneously using some popular dynamics for interest rates. In particular, we adopt the lognormal and CEV dynamics with stochastic volatility and establish the dual-curve versions of the LIBOR market model and the SABR model, respectively. Closed-form pricing formulae are developed for caplets and swaptions under the dual-curve SABR model, along the approach of heat kernel expansion.

A Second-Order Stochastic Maximum Principle for Generalized Mean-Field Singular Control Problem

Jie Xiong
Southern University of Science and Technology, Peoples Rep of China

We study the generalized mean-field stochastic control problem when the usual stochastic maximum principle (SMP) is not applicable due to the singularity of the Hamiltonian function. In this case, we derive a second order SMP. We introduce the adjoint process by the generalized mean-field backward stochastic differential equation. The keys in the proofs are the expansion of the cost functional in terms of a perturbation parameter, and the use of the range theorem for vector-valued measures. This talk is based on a paper joint with Hancheng Guo.

Quantile Optimization Under Derivative Constraint

Zuoquan Xu
The Hong Kong Polytechnic University, Hong Kong

This talk will focus on a new type of quantile optimization problems arising from insurance contract design models. This type of optimization problems is characterized by a constraint that the derivatives of the decision quantile functions are bounded. Such a constraint essentially comes from the “incentive compatibility” constraint for any optimal insurance contract to avoid the potential severe problem of moral hazard in insurance contract design models. By a further development of the relaxation method, we provide a systemic approach to solving this new type of quantile optimization problems. The optimal quantile is expressed via the solution of a free boundary problem for a second-order nonlinear ordinary differential equation (ODE), which is similar to the Black-Scholes ODE for perpetual American options.

Approximation of Stable Law by Stein's Method

Lihu Xu
University of Macau, Peoples Rep of China

Using Stein's method, we shall prove a general inequality which is comparable with the classical Esseen's inequality, from which one can derive the convergence rate of stable type central limit theorem. We also give 4 examples as applications.

Time-Inconsistent Recursive Stochastic Optimal Control Problems

Zhiyong Yu
Shandong University, Peoples Rep of China
Qingmeng Wei, Jiongmin Yong

In this talk, a time-inconsistent stochastic optimal control problem with a recursive cost functional is studied. Equilibrium strategy is introduced, which is time-consistent and locally approximately optimal. By means of multi-person hierarchical differential games associated with partitions of the time interval, a family of approximate equilibrium strategy is constructed and by sending the mesh size of the time interval partition to zero, an equilibrium Hamilton-Jacobi-Bellman (HJB, for short) equation is derived through which the equilibrium value function can be identified and the equilibrium strategy can be obtained. Moreover, a well-posedness result of the equilibrium HJB equation is established under certain conditions, and a verification theorem is proved. This talk is based on a joint work with Dr. Qingmeng Wei and Prof. Jiongmin Yong.

The Dynamics of Stochastic Incentive Effect for "U" Shape Theory for SMEs Under Bigdata Framework

Xianzhi Yuan
Peoples Rep of China

In this paper, based on Higgs which is our Hologram engine under the bigdata, we establish the Dynamics of Stochastic Incentive Effect for "U" Shape Theory owned by SMEs (Small and medium-sized enterprises). The aim of our talk is to discuss a new quantitative method and the associated prototype system to address the issue how the venture capital incents partners especially associated with partnership success, what roles the internal/external risks play respectively, and by how to avoid risk resonance and create portfolio strategies of introducing venture capital and optimizing the portfolio risk in the practice. In another way, if taking the enterprise as the target (or say the partners), we like to describe the mechanics for venture capital finance in an environment by combining investment associated external and internal risk with consideration of capital-product switching mechanics - the "back and forth conversion of two states" (which are due to partnerships between multiple sides that share goals and strive for mutual benefit are ubiquitous both between and within the enterprises, and competition and cooperation are the fundamental characterize in partnership systems). In order to do so, we use "asymmetric bistable Cobb-Douglas utility" as the tool to describe the two states (actually, we can also use some other kinds of utility function too), then we build the new model called "nonlinear stochastic differential equation" to describe the dynamical behaviors of bilateral partnership system in the presence of periodic capital-product switches and stochastic fluctuations (called "an over-damped non-linear Langevin equation") to study when the "back and forth conversion of two state" could reach the "best" in terms of Stochastic Resonance (SR) by introducing three new concepts below for the measurements of the system (for enterprise): 1): "output signal-to-noise ratio (SNR)", 2): "stationary unit risk-return (URR)", and 3): "incentive risk". These three new concepts can be classified as two categories: systematic risk, and bilateral risk. In this way, we are able to establish the general framework for the mechanics of enterprises, in particular, to successfully explain the so-called "U" phenomenon for SMEs, the key business behavior of SMEs (from the practice in China) which mean that more external cooperators many not be better, this is against the intuition and traditional understanding (this might be one of the biggest discoveries for the SME's study under the framework of Fintech by using bigdata method), which is also called "U" Shape phenomenon for SMEs first time in this area.

Non-Zero Sum Differential Game of Backward Doubly Stochastic System with Delay

Qingfeng Zhu
Shandong University of Finance and Economics, Peoples Rep of China
Yufeng Shi

This paper is concerned with a kind of nonzero sum differential game of backward doubly stochastic system with delay, in which the state dynamics follows a backward doubly stochastic differential equation with delay. We establish a necessary condition in the form of maximum principle with Pontryagins type for open-loop Nash equilibrium point of this type of game, and then give a verification theorem which is a sufficient condition for Nash equilibrium point. The theoretical results are applied to study a nonzero sum differential game of linear-quadratic backward doubly stochastic system with delay.

Special Session 4: Dynamical Systems and Variational Methods

Yiming Long, Nankai University, Peoples Rep of China
Chao-Nien Chen, National Tsinghua University, Taiwan
Huagui Duan, Nankai University, Peoples Rep of China

In recent years, the variational methods have played an important role in the study of finite and infinite dimensional dynamical systems. Existence, multiplicity and stability problems on periodic, homoclinic, heteroclinic as well as other type solutions have been deeply studied via variational and other methods. Such methods have also been applied to the study of partial differential equations. The aim of this session is to bring together active researchers from different fields in dynamical systems and variational method, to report most recent research results, and to stimulate communications and cooperation among them. The following is a non-exhaustive list of topics to be discussed in this session.

- N-body problems in celestial mechanics
- Geodesic flows and closed geodesics on compact manifolds
- Periodic orbits in Hamiltonian systems
- Homoclinic and heteroclinic orbits in Hamiltonian systems
- Reaction-diffusion systems
- Patterns and waves in gradient or skew-gradient systems

Variational Nature of Keplerian Orbits

Kuo-Chang Chen
National Tsing Hua University, Taiwan

Keplerian orbits can be characterized as minimizers of some action functional on function spaces with natural topological or boundary constraints. This fact is useful in variational construction of periodic orbits for the n-body and n-center problems. The elliptic case, settled by W. Gordon in 1977, is considerably well-known. Parabolic case is less well-known, and hyperbolic case is virtually unknown. In this talk I will briefly outline those known facts, and describe my proof for the minimizing property of hyperbolic orbits.

Geometric Variational Problems Arising from Singular Limit of the FitzHugh-Nagumo Equations

Yung Choi
University of Connecticut, USA

When parameters in the FitzHugh-Nagumo equations change in a certain coordinate fashion, solutions of this system develop sharp jumps as a singular limit. Through Γ-convergence, the original variational problems become some geometric variational problems. We focus on the latter problems in this talk. We report our recent work on existence, multiplicity and stability of disc-shaped solutions in $\mathbf{R}^N$. In addition, the possibility of traveling wave for the geometric variational problems is discussed.

Closed Geodesics on Compact Simply-Connected Finsler Manifolds

Huagui Duan
Nankai University, Peoples Rep of China

Closed geodesics can be regarded as critical points of the energy functional. In this talk, we will give a recent survey about the multiplicity and stability of closed geodesics on compact simply-connected Finsler manifolds. This is based on some joint works with Prof. Yiming Long and Prof. Wei Wang.

Existence of Nodal Solutions for Problems with Robin Conditions

Michael Filippakis
University of Piraeus, Greece

We consider a semilinear Robin problem driven by the negative Laplacian plus an indefinite, unbounded potential. The reaction term is a Caratheodory function of arbitrary structure outside an interval $[-c, c]$ ($c > 0$), odd on $[-c, c]$ and concave near zero. Using a variant of the symmetric mountain pass theorem, together with truncation, perturbation and comparison techniques, we show that the problem has a whole sequence $\{u_n\}_{n\geq 1}$ of distinct nodal solutions converging to zero in $C^1(\overline{\Omega})$.
The publication of this paper has been partly supported by the University of Piraeus Research Center.

Resonance Identities of Closed Orbits in Hamiltonian Dynamics and Applications

Hui Liu
Wuhan University, Peoples Rep of China

Resonance identities of closed orbits in Hamiltonian dynamics are powerful tools to study the multiplicity and stability problems of closed orbits. In recent years, we have established some new resonance identities of closed characteristics on compact star-shaped hypersurfaces and non-contractible closed geodesics on Finsler compact space forms, which are successfully used to obtain new results on multiplicity and stability of closed characteristics and closed geodesics. In this talk, I will give a survey of our results.

Dynamical Problems Inspired by Semi-Classical Approximation

Shanzhong Sun
Capital Normal University, Peoples Rep of China

I will talk about some problems related to Hamiltonian dynamical systems encountered when we try to understand the semiclassical approximation in quantum mechanics.

Closed Orbits in Nonlinear Hamiltonian Systems

Wei Wang
Peking University, Peoples Rep of China

In this talk, we will describe the multiplicity and stability results on closed characteristics on compact convex hypersurfaces in R^{2n} obtained in these years. These are joint work with Professor Y. Long, X. Hu, H. Duan and H. Liu.

I_{L_0} Index Theory and Brake Orbits on Tori

Duanzhi Zhang
Nankai University, Peoples Rep of China
Hui Liu, Fanjing Wang

We will talk about the Maslov-type index i_{L_0} on the brake orbit boundary for symplectic paths starting from identity I_{2n} for any positive integer n. We prove that $-n \leq i_{L_0}(\gamma^m) - m\hat{i}_{L_0}(\gamma) \leq 0$ for any positive integer m and any symplectic path γ starting from identity, where $\hat{i}_{L_0}$ is the mean index of i_{L_0}, γ^m is the m-th iteration of γ in the sense of brake orbit boundary. As application, we study the multiplicity of brake orbits of Hamiltoninan equation on tori. This is a joint work with Hui Liu and Fanjing Wang.

Special Session 5: Recent Advances in Inverse Problems

Gang Bao, School of Mathematical Sciences, Zhejiang University, Peoples Rep of China
Jun Lai, School of Mathematical Sciences, Zhejiang University, Peoples Rep of China
Shuai Lu, School of Mathematical Sciences, Fudan University, Peoples Rep of China

Inverse problems are frequently arising in science and engineering, which are concerned with determining desired properties with observed data. The reason that makes inverse problems mathematically challenging is due to their "ill-posedness", which means that a solution to an inverse problem might neither exist nor be unique, or even if a "weak" type solution is introduced, the solution does not depend on the data continuously. These difficulties render traditional numerical methods not applicable or inherently unstable.
The special session aims to discuss recent advances in inverse problems, mainly focusing on the theory of partial differential equations with unknown sources or coefficients, numerical fast algorithms and applications. The purpose is to promote international collaboration among researchers who are working in this exciting field.

Inverse Medium Scattering with a Single Incoming Wave

Guanghui Hu
Beijing Computational Science Research Center, Peoples Rep of China

Assume an electromagnetic plane wave is incident onto an infinitely long cylinder with constant refractive index. In the TE and TM polarization cases, we prove that a single far-field pattern uniquely determines the cross section of the cylinder. In particular, this implies the absence of non-scattering wave numbers for the two-dimensional Maxwell equation in a right corner domain.

A Fast Algorithm for Electromagnetic Scattering of Three Dimensional Penetrable Axis-Symmetric Objects

Jun Lai
Zhejiang University, Peoples Rep of China

Fast algorithm for the electromagnetic scattering of dielectric objects is of great importance in optics, biomedical imaging and inverse scattering. In this talk, we apply a second kind integral formulation to the three dimensional scattering problem of axis-symmetric objects. The resulted surface integral equation is reduced to a sequence of line integral equations by Fourier transform along the azimuthal direction. These equations are decoupled from each other and a high order kernel splitting technique is applied to the evaluation of singular integrals. The algorithm is efficient and high order even for the scattering of non-smooth objects by using generalized Gaussian quadrature. Numerical experiments are presented to demonstrate the efficiency of the algorithm. Application to inverse scattering will be discussed.

Inverse Elastic Surface Scattering with Far-Field Data

Peijun Li
Purdue University, USA

A rigorous mathematical model and an efficient computational method are proposed to solving the inverse elastic surface scattering problem which arises from the near-field imaging of periodic structures. We demonstrate how an enhanced resolution can be achieved by using more easily measurable far-field data. The surface is assumed to be a small and smooth perturbation of an elastically rigid plane. By placing a rectangular slab of a homogeneous and isotropic elastic medium with larger mass density above the surface, more propagating wave modes can be utilized from the far-field data which contributes to the reconstruction resolution. Requiring only a single illumination, the method begins with the far-to-near (FtN) field data conversion and utilizes the transformed field expansion to derive an analytic solution for the direct problem, which leads to an explicit inversion formula for the inverse problem. Moreover, a nonlinear correction scheme is developed to improve the accuracy of the reconstruction. Results show that the proposed method is capable of stably reconstructing surfaces with resolution controlled by the slab's density.

Increasing Stability in the Inverse Source Problem with Attenuation and Many Frequencies

Shuai Lu
School of Mathematical Sciences, Fudan University, Peoples Rep of China
Victor Isakov

We study the interior inverse source problem for the Helmholtz equation from boundary Cauchy data of multiple wave numbers. The main goal of this paper is to understand the dependence of increasing stability on the attenuation, both analytically and numerically. To implement it we use the Fourier transform with respect to the wave numbers, explicit bounds for analytic continuation, and observability bounds for the wave equation. In particular, by using Carleman estimates for the wave equation we trace the depen-

dence of exact observability bounds on the constant damping. Numerical examples in 3 spatial dimension support the theoretical results. It is a joint work with Prof. Victor Isakov (Wichita State University).

Extended Sampling Method in Inverse Scattering

Jiguang Sun
Michigan Technological University, USA
Juan Liu

We propose a novel sampling method as an extension of the linear sampling method for inverse scattering problems. It can determine the location and approximate the support with little a priori information on the physical properties of the unknown target. A key feature is that the input data is the far-field measurement of one incident wave. The behavior of the ill-posed linear integral equations is studied. Furthermore, a multilevel technique is employed to better approximate the scatterer. Numerical experiments show that this method is stable and effective.

Determination of an Inclusion Using the Multifrequency Measurements

Chun-Hsiang Tsou
Grenoble-Alpes University, France
Habib Ammari, Faouzi Triki, Eric Bonnetier

We are interested in the inverse inclusion problem with multi-frequency measurements. Assuming an inclusion D has a conductivity that depends on the frequency ω, we recover its position and shape for boundary measurements. In this work, we assume the conductivity inside the inclusion satisfies Drude model, which is adapted in many case of metals or biological tissues. We prove that the unique solution to the conductivity equation admits a spectral decomposition $u = u_0 + u_f$ with u_0 independent to the frequency and u_f depends on the frequency. Based on this decomposition, we derive a numerical scheme to reconstruct the inclusion and the conductivity profile. The numerical method has two main steps, the first is to reconstruct the scalar part u_0 from the multifrequency measurements, and the second step is to reconstruct the inclusion D from the u_0 function obtained in the previous step.

Identification of Singular Potentials in the Plane by Boundary Measurements

Jenn-Nan Wang
National Taiwan University, Taiwan
Eemeli Blasten, Leo Tzou

In this talk, I would like to discuss the global uniqueness of determining a singular potential in the plane by the full Cauchy data on the boundary. We discuss the Schrödinger equation with potentials in L^p space, where $p > 4/3$. We will make use of an approach introduced by Bukhgeim for solving inverse boundary values in the plane. This approach consists of two key ingredients: complex geometrical optics solutions with quadratic phases and the method of stationary phase. The talk is based on a joint work with E. Blasten and L. Tzou.

Inverse Problems on Piezoelectric Equation

Xiang Xu
Zhejiang University, Peoples Rep of China

In this talk, we will discuss about the recent progress on an inverse source and an inverse coefficient prolem on piezoelectric equation, respectively. Based upon Carleman estimate, we can prove local Holder stability estimates both for these two inverse problems provided that additional measurement data are given on an interior sub-domain. Furthermore, two numerical algorithms are also proposed for solving the inverse problems. Numerical examples are presented to demonstrate the effectiveness of the proposed schemes.

Mathematical Studies of Extraordinary Field Enhancement in Subwavelength Slit Structures

Hai Zhang
HKUST, Hong Kong
Junshan Lin

Since the discovery of the extraordinary optical transmission through nanohole arrays in metallic films by Ebbesen, a wealth of research has been sparked in the experimental and theoretical investigation of localized electromagnetic field enhancement in subwavelength nanostructures. This remarkable phenomenon can lead to potentially significant applications in near-field imaging, bio-sensing, etc. However, there has been a long debate on the interpretation of the enhancement effect since Ebbesen's work. In addition, a quantitative analysis of the field enhancement in subwavelength structures is still widely open. In this talk, using two-dimensional slits as a prototype, I will present mathematical studies of the field enhancement in the subwavlength structures. Based upon the layer potential technique, asymptotic analysis and homogenization theory, the enhancement mechanisms for both the single slit and an array of slits are studied quantitatively.

Special Session 6: Ergodic-Theoretical Techniques in Partial Differential Equations

Sinisa Slijepcevic, University of Zagreb, Croatia

The session would focus on deterministic (rather than stochastic) PDE, and would explore recent progress in topics such as SRB measures for PDE, non-uniform hyperbolicity in general for PDE, uniqueness of invariant measures for Burger's-like equations, a.e.-existence of solutions, and other related applications of ergodic-theoretical tools to partial differential equations.

SRB Measures for Infinite-Dimensional Dynamical Systems with Potential Applications to PDE

Alex Blumenthal
University of Maryland, USA
Lai-Sang Young

I will talk about the extension to the setting of Banach space mappings a concept which has proven highly useful in the study of finite-dimensional dynamical systems exhibiting chaotic behavior, that of SRB measures. This extended notion of SRB measure and our results potentially apply to a large class of dissipative PDE, including dissipative parabolic and dispersive wave equations.
We generalize two results known in the finite-dimensional setting. The first is a geometric result, absolute continuity of the stable foliation, which in particular implies that an SRB measure with no zero exponents is visible, in the sense of time averages converging to spatial averages, with respect to a large subset of phase space. The second is the characterization of the SRB property in terms of the relationship between a priori different quantifications of chaotic behavior, Lyapunov exponents and metric entropy.
Complications of our infinite-dimensional environment include: (1) the absence of Lebesgue measure as a reference measure, not even k-dimensional volume elements (whereas the finite dimensional theory heavily involves the notion of volume growth along unstable leaves); and (2) mappings in our setting are not locally onto or differentiably invertible, possibly exhibit arbitrarily strong rates of contraction (even near attractors).
This work is joint with Lai-Sang Young.

Topological Horseshoes in Travelling Waves of Discretized Nonlinear Wave Equations

Yi-Chiuan Chen
Academia Sinica, Taiwan
S-S Chen, J-M Yuan

Applying the concept of anti-integrable limit to coupled map lattices originated from space-time discretized nonlinear wave equations, we show that there exist topological horseshoes in the phase space formed by the initial states of travelling wave solutions. In particular, the coupled map lattices display spatio-temporal chaos on the horseshoes.

Periodic Approximation of Exceptional Lyapunov Exponents for Semi-Invertible Operator Cocycles

Davor Dragicevic
University of Rijeka, Croatia
Lucas Backes

We prove that for semi-invertible and Holder continuous linear cocycles A acting on an arbitrary Banach space and defined over a base space that satisfies the Anosov Closing Property, all exceptional Lyapunov exponents of A with respect to an ergodic invariant measure for base dynamics can be approximated with Lyapunov exponents of A with respect to ergodic measures supported on periodic orbits. Our result is applicable to a wide class of infinite dimensional dynamical systems.

Birkhoff and Non-Birkhoff Solutions for Monotone Recurrence Relations

Wen-Xin Qin
Soochow University, Peoples Rep of China

For variational monotone recurrence relations we know from the Aubry-Mather theory the existence and properties of foliation or lamination consisting of Birkhoff solutions. In this talk, we discuss for the general monotone recurrence relations the existence of Birkhoff solutions and implications of non-Birkhoff solutions. In particular, we show that a solution with bounded action implies the existence of a Birkhoff solution and the rotation set contains an interval with end points being the Farey neighbours of p/q provided there is a non-Birkhoff (p, q) periodic solution.

Special Session 7: Recent Trends and Progress in Mathematical Fluid Dynamics

Eduard Feireisl, Czech Academy of Sciences, Prague, Czech Rep
Antonin Novotny, Université du Sud Toulon-Var, France
Milan Pokorny, Charles University, Prague, Czech Rep

The session will focus on recent developments in the mathematical theory of fluids in motion. The talks will cover both viscous and inviscid, compressible or incompressible fluids. Possible applications in numerical analysis will also be considered.

Steady Flows of Dense Compressible Fluids

Simon Axmann
Charles University, Czech Rep
Milan Pokorný

We study the existence of strong solutions to the model describing the steady flow of compressible heat-conducting chemically reacting mixture under the additional assumption that the fluid is sufficiently dense. The diffusion is supposed to be driven by Fick's law. We work in the L^p-setting combining the methods for the weak solutions with the method of decomposition. The result is a generalization of our previous work concerning the case of single-constituted fluid.

On Measure-Valued Solutions

Jan Brezina
Tokyo Institute of Technology, Japan

In my talk I present some recent results related to measure-valued solutions.

Counterexample of Boundary Caccioppoli's Inequality in Navier-Stokes Equations

Tongkeun Chang
Yonsei University, Korea
Kang, Kyungkeun

We study Caccioppoli's inequality of the non-stationary Stokes system and Navier-Stokes system. Our analysis is local near boundary and we prove that, in contrast to the interior case, the Caccioppoli's type inequality of the Stokes system and the Navier-Stokes system, in general, fail.

Singular Set of Critical Space Solution to Navier-Stokes Flow

Hi Jun Choe
Yonsei University, Korea
Minsuk Yang, Joeg Wolf

We study local regularity properties of a weak solution u to the Cauchy problem of the incompressible Navier-Stokes equations. We present a new regularity criterion for the weak solution u satisfying the condition $L^\infty(0,T;L^{3,w}(\mathbb{R}^3))$ without any smallness assumption on that scale, where $L^{3,w}(\mathbb{R}^3)$ denotes the standard weak Lebesgue space. As an application, we conclude that there are at most a finite number of blowup points at any singular time t. The condition that the weak Lebesgue space norm of the velocity field u is bounded in time is encompassing type I singularity and significantly weaker than the end point case of the so-called Ladyzhenskaya-Prodi-Serrin condition proved by Escauriaza-Sergin-Šveràk.

Density-Dependent Incompressible Fluids in Fast Rotation

Francesco Fanelli
Université de Lyon, France

In this talk we consider a class of singular perturbation problems for non-homogeneous flows whose dynamics is influenced by the Earth rotation. We specilize on the 2-D density-dependent incompressible Navier-Stokes equations with Coriolis force: our goal is to characterize the asymptotic dynamics of weak solutions to this model, in the limit when the rotation becomes faster and faster.
We present two kinds of results, deeply different from each other from a qualitative viewpoint. If the initial density is a small perturbation of a constant state, we prove that the limit dynamics is essentially described by a homogeneous Navier-Stokes system with an additional forcing term, which can be seen as a remainder of density variations and which is a remainder of the action of the Coriolis force. If, instead, the initial density is a small perturbation of a truly variable reference state, we show that the final equations become linear, and moreover one can identify only a mean motion, described in terms of the limit vorticity and the limit density fluctuation function; this issue can be interpreted as a sort of turbulent behaviour of the limit flow. This talk is based on a joint work with *Isabelle Gallagher*.

Geostrophic Equations As a Rigorous Limit of Compressible Rotating and Heat Conducting Fluids

Young-Sam Kwon
Dong-A University, Korea
Antonin Novorny

We consider the full Navier-Stokes-Fourier system under rotation in the singular regime of small Mach and Rossby, and large Reynolds and Péclet numbers, with ill prepared initial data. We derve the quasi-geostropic equation in the framework of weak solutions from the full Navier-Stokes-Fourier system.

Relative Entropy, Weak-Strong Uniqueness and Conditional Regularity for a Compressible Oldroyd–B Model

Yong Lu
Nanjing University, Peoples Rep of China
Zhifei Zhang

We consider the compressible Oldroyd–B model derived in the paper *Existence of large-data finite-energy global weak solutions to a compressible Oldroyd–B model*, Comm. Math. Sci. 15 (2017), 1265–1323 by J.W. Barrett, Y. Lu and E. Süli, where the existence of global-in-time finite-energy weak solutions was shown in two dimensional setting. In this paper, we first state a local well-posedness result for this compressible Oldroyd–B model. In two dimensional setting, we give a (refined) blow-up criterion involving only the upper bound of the fluid density. We then show that, if the initial fluid density and polymer number density admit a positive lower bound, the weak solution coincides with the strong one as long as the latter exists. Moreover, if the fluid density of a weak solution issued from regular initial data admits a finite upper bound, this weak solution is indeed a strong one; this can be seen as a corollary of the refined blow-up criterion and the weak-strong uniqueness.

A Drop of Water

Piotr Mucha
University of Warsaw, Poland

I will talk about the issue of existence and uniqueness of solution to the Inhomogeneous Navier-Stokes system (INS) in a two dimensional domain. The key point it that the initial density can be just a characteristic function of a set. Even for such rough solutions the result provides unique solutions. As a historical remark, it is emphasized that it solves the problem put by PL Lions in his book concerning weak solutions to INS. The talk will be based on joint results with Raphael Danchin (Paris). Preprint: https://arxiv.org/abs/1705.06061

Existence of Weak Solutions for Some Multi-Fluid Models of Compressible Fluids

Antonin Novotny
University of Toulon, France
Milan Pokorny

Existence results in large for fully non-linear compressible multi-fluid models are in the mathematical literature in a short supply (if not inexisting). In this talk, we shall recall the main ideas of the Lions proof of existence of weak solutions to the compressible (mono-fluid) Navier-Stokes equations in barotropic regime. We shall then explain how this approach can be adapted to the construction of weak solutions to some simple multi-fluid models. The main tools in the proofs are the renormalization techniques for the continuity and transport equations. They will be handled in more details.

Steady Equations Describing Flow of Chemically Reacting Heat Conducting Compressible Mixtures

Milan Pokorny
Charles University, Prague, Czech Rep
Tomasz Piasecki

We consider the following system of equations

$$\begin{aligned}
&\operatorname{div}(\varrho\mathbf{u}) = 0,\\
&\operatorname{div}(\varrho\mathbf{u}\otimes\mathbf{u}) - \operatorname{div}\mathbf{S} + \nabla\pi = \varrho\mathbf{f},\\
&\operatorname{div}(\varrho E\mathbf{u}) + \operatorname{div}(\mathbf{Q} + \pi\mathbf{u} - \mathbf{S}\mathbf{u}) = \varrho\mathbf{f}\cdot\mathbf{u},\\
&\operatorname{div}(\varrho Y_k\mathbf{u}) + \operatorname{div}\mathbf{F}_k = m_k\omega_k, \quad k = 1,\dots,L
\end{aligned}$$

which describes the steady flow of a compressible heat conducting mixture of gases whose component may chemically react, where only the barycentric velocity is taken into account. We consider either the no-slip or the Navier boundary conditions for the velocity and the Newton boundary condition for the temperature. Based on papers [1]–[3] we present existence results of weak and variational entropy solutions in different situations.

References

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[3] Piasecki, T., Pokornỳ, M.: *On steady solutions to a model of chemically reacting heat conducting compressible mixture with slip boundary conditions*, to appear in Contemporary Mathematics (2018).

Well-Posedness of the Plasma-Vacuum Interface Problem for Ideal Incompressible MHD

Yongzhong Sun
Nanjing University, Peoples Rep of China
Wei Wang, Zhifei Zhang

We show the local well-posedness of plasma-vacuum interface problem for *ideal incompressible* magnetohydrodynamics under the stability condition that the magnetic field $\mathbf{h}$ and the vacuum magnetic field $\hat{\mathbf{h}}$ are non-collinear on the free interface(i.e., $|\mathbf{h} \times \hat{\mathbf{h}}| > 0$). Such a stability condition was first proposed by Y. Trakhinin for the *compressible* plasma-vacuum interface problem.

The Minkowski Dimension of Possible Singular Set in the Incompressible Navier-Stokes Equations

Minsuk Yang
Yonsei University, Korea

In this talk, we shall discuss about possible singular points of suitable weak solutions to the three dimensional incompressible Navier-Stokes equations. In particular, we investigate the Minkowski dimension of the singular set, which reflects the geometric distribution. In order to do that, we need a new regularity criterion.

On the Existence of Solutions to the Two-Fluids Systems

Ewelina Zatorska
University College London, England
Didier Bresch, Piotr Mucha

The aim of this talk is to present the recent developments in the topic of existence of solutions to the two-fluid systems. I will discuss the application of approach developed by P.-L. Lions and E. Feireisl and explain the limitations of this technique in the context of multi-component flow models. A particular example of such a model is two-fluids Stokes system with single velocity field and two densities, and with an algebraic pressure law closure. I will present the existence result that uses the compactness criterion introduced for the Navier-Stokes system by D. Bresch and P.-E. Jabin. I will also mention an innovative construction of solutions relying on the G. Crippa and C. DeLellis stability estimates for the transport equation.

Special Session 8: Propagation Phenomena in Reaction-Diffusion Systems

Hirokazu Ninomiya, Meiji University, Japan
Masaharu Taniguchi, Okayama University, Japan

This special session is concerned with mathematical analysis on propagation phenomena or pattern formation appearing in reaction-diffusion systems. Related topics are traveling waves, entire solutions and asymptotic behavior of solutions.

Entire Solutions Originating from Fronts to the Allen-Cahn Equation

Yanyu Chen
Tamkang University, Taiwan
Jong-Shenq Guo, Hirokazu Ninomiya, Chih-Hong Yao

In this paper, we study entire solutions of the Allen-Cahn equation in one-dimensional Euclidean space. This equation is a scalar reaction-diffusion equation with a bistable nonlinearity. It is well-known that this equation admits three different types of traveling fronts connecting two of its three constant states. Under certain conditions on the wave speeds, the existence of entire solutions originating from three and four fronts is shown by constructing some suitable pairs of super-sub-solutions. Moreover, we show that there are no entire solutions originating from more than four fronts.

Reduction Approach to a Reaction-Diffusion System for Collective Motions of Camphor Boats

Kota Ikeda
Meiji University, Japan
Shin-Ichiro Ei

The collective motion of camphor boats in the water channel exhibits both a homogeneous and an inhomogeneous state, depending on the number of boats. The motion of each boat is described by a traveling pulse in a reaction-diffusion model proposed in Nagayama et al. (2004), in which boats are assumed to interact with each other by the change of surface tension by diffusive molecules on the water surface. In order to study the inhomogeneous motion of camphor boats, we have to treat the linearized eigenvalue problem. However, the eigenvalue problem is too difficult to analyze. Then we would like to derive a reduced system from the original model and analyze it by applying the center manifold theorem. Several reaction-diffusion systems can generate a solution with a pulse shape. Pulse-pulse interaction is treated mathematically in Ei et al. (2002), in which a reduced system is derived from a reaction-diffusion model by applying a center manifold theorem. Since the delta functions naturally arise in our model, the theory established in L2-framework cannot be applied directly. In this talk, we modify the previous results in Ei et al. (2002) and propose a new approach of reduction to systems with the delta function.

Complex Dynamics of Bifurcating Front Solutions in a Three-Component Reaction-Diffusion System

Hideo Ikeda
University of Toyama, Japan

In the paper Butterfly catastrophe for fronts in a three-component reaction-diffusion system (J. Nonlinear Science (2015) 25, 87-129) by C.-Bruckner et al, several interesting front dynamics are studied. Based on this work, we want to discuss a bifurcation from a standing front solution of the above system. That is, we reduce the PDE dynamics to a finite-dimensional ODE system explicitly on a center manifold near a drift bifurcation point and analyze the dynamics of the reduced ODE system for several parameters. This indicates that the three-component system show a complex dynamics compared to the corresponding two-component system. Finally, we consider the type of criticality of the triple zero eigenvalue of the linearized eigenvalue problem.

On Limit Systems and Their Solution Structure for the Shigesada-Kawasaki-Teramoto Model with Large Cross-Diffusion Rate

Yukio Kan-On
Ehime University, Japan

In this talk, we treat the competition-diffusion system with nonlinear diffusion term, which was proposed by Shigesada, Kawasaki and Teramoto, and discuss the bifurcation structure of positive stationary solution for the system when the cross-diffusion rate is sufficiently large. To do this, we shall derive a limit system as the cross-diffusion rate tends to infinity, and study the bifurcation structure of positive solution for the limit system by employing the comparison principle and the bifurcation theory.

Global Stability of Critical Traveling Waves with Oscillations for Time-Delayed Reaction-Diffusion Equations

Ming Mei
Champlain College & McGill University, Canada

For a class of non-monotone reaction-diffusion equations with time-delay, the large time-delay usually causes the traveling waves to be oscillatory. In this paper, we are interested in the global stability of

these oscillatory traveling waves, in particular, the challenging case of the critical traveling waves with oscillations. We prove that, the critical oscillatory traveling waves are globally stable with the algebraic convergence rate $t^{-1/2}$, and the non-critical traveling waves are globally stable with the exponential convergence rate $t^{-1/2}e^{-\mu t}$ for some positive constant μ, where the initial perturbations around the oscillatory traveling wave in a weighted Sobolev can be arbitrarily large. The approach adopted is the technical weighted energy method with some new development in establishing the boundedness estimate of the oscillating solutions, which, with the help of optimal decay estimates by deriving the fundamental solutions for the linearized equations, can allow us to prove the global stability and to obtain the optimal convergence rates. This is the first framework to show the global stability for oscillatory traveling waves with optimal convergence rates. This is a joint work with Kaijun Zhang and Qifeng Zhang.

Compact Traveling Wave Solutions to Mean Curvature Flow with a Driving Force in Higher Dimensional Space

Harunori Monobe
Okayama University, Japan
Hirokazu Ninomiya

Traveling waves appear in various phenomena and there are a lot of results related to traveling waves in PDEs describing such phenomena. Here we pay attention to traveling wave solutions for mean curvature flow with a driving force. As is well known, Grim reaper and V-shaped traveling front are typical examples of traveling wave solutions for mean curvature flow and eikonal-curvature flow, respectively.
However they are defined in whole space and not compact. In recent years, the authors have studied about the existence and uniqueness of traveling waves composed of Jordan curve in two-dimensinoal space, called “compact traveling wave“, under a general driving force.
In this talk, we consider the same problem in more than three-dimensional space. In particular, we show the existence and uniqueness of axisymmetric compact traveling waves, which is symmetric with respect to traveling direction, for the mean curvature flow equation with driving force. This is a joint work with Hirokazu Ninomiya.

Minimization Problems for the Energy Functional with a Generalized Nonlocal Term

Yoshihito Oshita
Okayama University, Japan

We talk about the minimization problems for the energy functional with a generalized nonlocal term, and consider the asymptotic behavior of the minimum energy and the rapid oscillation of minimizers and its average wave length.

Small Data Blow-Up and Ecological Damping in Certain Three Species Food-Chain Models

Rana Parshad
Iowa State University, USA
Matthew Beauregard, Said Kouachi, Kelly Black, Emmanuel Quansah

In this talk I will present some recent finite time blow-up results in three species population models, as a lead into studying certain explosive invasive populations, such as the Burmese python in South Florida in the US. The blow-up is seen to occur for both small and large initial conditions. I will next introduce the idea of ecological damping as a means of controlling blow-up in such invasive populations, whilst avoiding non-target effects, due to classical chemical and biological control.

Nonlocal PDEs for the Dynamics of Fitness Distributions in Asexual Populations

Lionel Roques
INRA - BioSP, France
Marie-Eve Gil, François Hamel, Guillaume Martin

Understanding the complex interplay between mutation and selection in asexuals is a central issue of evolutionary biology. To study the adaptation of a population under these two forces, in the presence of a phenotype optimum, we propose two frameworks: (1) an integro-differential approach with context-dependent mutation kernels $\partial_t p(t,m) = U\,(J_y \star p - p)(t,m) + p(t,m)(m - \overline{m}(t))$, with $(J_y \star p)(t,m) = \int_{\mathbb{R}} J_y(m-y)p(t,y)\,dy$ and $\overline{m}(t) = \int_{\mathbb{R}} y\,p(t,y)\,dy$. In this case, we follow the dynamics of the fitness distribution $p(t,m)$; (2) a nonlocal nonlinear transport equation satisfied by a moment generating function of the fitness distribution; the derivation of this equation is based on microscopic arguments. We show that these two equations are connected and we derive several properties of their solutions. These properties have implications in evolutionary biology, regarding the effect of the parameters (e.g., mutation rate, dimension of the phenotypic space) on the trajectories of adaptation and on the stationary states. In particular, we give simple sufficient conditions on the parameters for the existence and non-existence of a concentration phenomenon at the optimal fitness value $m = 0$. We compare our results with empirical results given by stochastic individual-based simulations of Wright-Fisher type models. This is a joint work with Marie-Eve Gil (BioSP, I2M), François Hamel (I2M) and Guillaume Martin (ISEM).

Asymptotic Behavior of Solutions to the Logarithmic Diffusion Equation

Masahiko Shimojo
Okayama University of Science, Japan
Eiji Yanagida, Peter Takac

We investigate the behavior of positive solutions to the Cauchy problem of logarithmic diffusion equation with non-symmetric flux boundary condition at space infinity. We show that extinction of the solution occurs in a finite time and a re-scaled solution converges to the traveling wave. Our results also include some log-concavity properties of solutions.

Bifurcation Structure of Steady States for the One-Dimensional Nonlocal Allen-Cahn Equation

Tohru Tsujikawa
University of Miyazaki, Japan

We consider the Newmann problem of a 1D stationary Allen-Cahn equation with nonlocal term. This equation is a typical model of the limiting system of reaction-diffusion systems. For example, a diffusion coefficient of the system tends to infinity, which is called a shadow system.
It is shown that the global behavior of the branch of asymmetric solutions which bifurcate from a point on the branch of odd-symmetric solutions. This solutions also bifurcate from a trivial solution. It is difficult to show the appearance of the secondary bifurcation point due to be apart from the first bifurcation point. The method using the asymptotic analysis and the complete elliptic integrals proves the existence and uniqueness of secondary bifurcation point. The proof of the global behavior of the bifurcation branch is based on a levelset analysis for an integral map associated with the nonlocal term. Moreover, we explain that the stability of the symmetric solutions loses at the secondary bifurcation point. The global structure of bifurcation branches is shown by some numerical simulations. This talk is based on joint work with Prof. K. Kuto (Univ. Electro Commu.), Prof. Y. Miyamoto (Univ. of Tokyo), Prof. T. Mori (Osaka Univ.), Prof. S. Yotsutani (Ryukoku Univ.).

Spatial Decay and Stability of Multidimensional Cylinder Fronts for Degenerate Fisher Type Equation

Yaping Wu
Capital Normal University, Peoples Rep of China
Junfeng He

Consider the degenerate Fisher equation in a cylinder $R \times \Omega$, with bounded $\Omega \subset R^n$,

$$\begin{cases} u_t = \triangle u + \beta(y)u_x + g(u), & t > 0,\ (x,y) \in R \times \Omega, \\ \frac{\partial u}{\partial n} = 0, & t > 0,\ (x,y) \in R \times \partial\Omega, \end{cases} \tag{1}$$

with $g(0) = g(1) = g'(0) = 0$ and $g(u) > 0$ for $u \in (0,1)$.
In this talk we shall talk about our recent work on the spatial decay and stability of multidimensional cylinder fronts $\phi_c(x-ct, y)$ for the degenerate Fisher type equation. By applying the generalized center manifold theorem and asymptotic expansions we obtain the spatial decay of traveling fronts with all speeds, especially for the p-degree Fisher type equation we get the precise algebraic decaying rates and the higher order expansion of traveling fronts with non-critical speeds. By applying spectral analysis and sub-super solution method we prove the exponential stability of all traveling fronts in some exponentially weighted spaces and the Lyapunov stability of traveling fronts with noncritical speeds in some polynomially weighted spaces. We also investigate the asymptotic behavior and spreading speed of the solution with more general initial data, which are proved to be solely determined by the spatial decay of the initial data at one end.

Special Session 9: Nonlinear Evolution PDEs, Interfaces and Applications

Alain Miranville, University of Poitiers, France
Gunduz Caginalp, University of Pittsburgh, USA
Maurizio Grasselli, Politecnico di Milano, Italy

Many issues in applied science can be formulated as interface problems which can be regarded as limiting cases of evolution equations exhibiting transition layers. The study of phase field or diffusive interface problems, Allen-Cahn and Cahn-Hilliard equations have been an active area for the past few decades. This has also been an additional motivation for studying general nonlinear evolution equations. This session will focus on the mathematical properties (well-posedness, regularity, stability, asymptotic behavior of solutions, ...) and applications of these equations.

Sharp Interface Limit for the Allen-Cahn Equation with a Contact Angle

Helmut Abels
University of Regensburg, Germany
Maximilian Moser

We consider the sharp interface limit of the Allen-Cahn equation, when a parameter $\varepsilon > 0$ that is proportional to the thickness of the diffuse interface tends to zero, in a two dimensional bounded domain with Neumann boundary conditions. We prove convergence of the solutions of the Allen-Cahn equation to solutions of the sharp interface limit, which is the mean curvature flow with a 90 degree contact angle, provided the limit problem possesses a smooth solution on a certain time interval. To this end we construct a suitable approximation of the Allen-Cahn equation, using three levels of the terms in the formally matched asymptotic calculations, and estimate the difference with the aid of a suitable spectral estimates of the linearized Allen-Cahn operator. Moreover, we will discuss recent extensions of this results e.g. to the Navier-Stokes/Allen-Cahn system or the Stokes/Cahn-Hilliard system. This is a joint-work with Maximilian Moser.

Convergence in the Hölder Space of the Solution of the Two Phase Stefan Problem for the Parabolic Equations with Two Small Parameters

Galina Bizhanova
Institute of Mathematics & Mathematical Modeling, Kazakhstan

We study two–phase multidimensional Stefan problem with two small parameters $\varepsilon > 0$, $\kappa > 0$ at the highest derivatives in the condition on the free boundary. This problem is considered in the Hölder space.
The small parameters in the problems have very deep physical cense. They characterize the properties of the medium, in which the processes go. And it is very important to know the behavior of the solutions of the problems, when the properties of the medium are changes, i.e. small parameters tend to zero.
The aim of the work is to obtain from the solution of the fully perturbed nonlinear problem (problem A) with $\varepsilon > 0$, $\kappa > 0$ the solutions of the partially and fully unperturbed B, C, D problems with $\varepsilon > 0$, $\kappa = 0$; $\kappa > 0$, $\varepsilon = 0$; $\kappa = 0$, $\varepsilon = 0$ respectively do not solve them.
It is shown that the solution of the problem A converges as $\kappa \to 0$, $\varepsilon > 0$; $\kappa > 0$, $\varepsilon \to 0$; $\kappa \to 0$, $\varepsilon = 0$ to the unique solution of the corresponding partially and fully unperturbed problem. We prove that the limit solutions have the maximal regularity, that is the solutions of the problems B, C, D are obtained without loss of the smoothness of the given functions. The coercive estimates of all problems are established in the Hölder space.
From here it follows that the boundary layers do not appear, although the small parameters are at the principal terms in the condition on a free boundary.

A Nonlinear Model for Marble Sulphation Including Surface Rugosity: Theoretical and Numerical Results

Cecilia Cavaterra
Università degli Studi di Milano, Italy
Elena Bonetti, Francesco Freddi, Maurizio Grasselli, Roberto Natalini

We consider an evolution system describing the phenomenon of marble sulphation of a monument, accounting of the surface rugosity. We first prove a local in time well posedness result. Then, stronger assumptions on the data allow us to establish the existence of a global in time solution. Finally, we perform some numerical simulations that illustrate the main feature of the proposed model.

A Convergent Convex Splitting Scheme for a Nonlocal Cahn-Hilliard-Oono Type Equation with a Transport Term

Laurence Cherfils
University of La Rochelle, France
H. Fakih, M. Grasselli, A. Miranville

In this talk, we will introduce a first-order in time convex splitting scheme for a nonlocal Cahn-Hilliard-Oono type equation with a transport term and subject to Neumann boundary conditions. The presence of the transport term is not a minor modification, since, for instance, we lose the unconditional stability. However, we will prove the stability of our scheme when the time step is sufficiently small. Furthermore, we will prove the consistency of the scheme and the convergence to the exact solution. Finally we will give numerical simulations which illustrate our theoretical results and demonstrate the performance of our scheme for phase separation as well as for crystal nucleation, with different choices of the interaction kernel.

Estimation of the Mobility in Front Propagation

Julien Dambrine
University of Poitiers, France
Romain Chassagne

Front propagation is a generic term for phenomena occurring in scientific domains ranging from cancer modelling (tumor growth) to petroleum engineering (water flooding). These phenomena all involve the monotonic growth of a domain, with a non-uniform and possibly non-isotropic mobility. Our goal is to estimate this mobility given several snapshots of the front. In the spirit of shooting methods, we introduce a system of coupled Hamilton-Jacobi equations in which the mobility is carried by the interface.

A Quantitative Riemann-Lebesgue Lemma with Application to Equations with Memory

Filippo Dell Oro
Politecnico di Milano, Italy
Enrico Laeng, Vittorino Pata

An elementary proof of a quantitative version of the Riemann-Lebesgue lemma for functions supported on the half line is given. Applications to differential models with memory are discussed.

Stability Properties of Non-Radial Steady Ferro Fluidic Patterns

Joachim Escher
Leibniz University Hannover, Germany
Bogdan V. Matioc

Of concern is the dynamic of a fixed volume of ferrofluid in a Hele-Shaw cell under the influence of centrifugal and magnetic forces. The steady-state solutions of the associated moving boundary problem are the periodic solutions of a generalized Laplace-Young equation. We use bifurcation theory to find analytic curves consisting of non-radial steady-state solutions of the problem. The stability of these solutions is discussed by using the exchange of stability theorem.

A Diffuse Interface Model of a Two-Phase Flow with Thermal Fluctuations

Eduard Feireisl
Czech Academy of Sciences, Czech Rep
Madalina Petcu

We consider a model of a two phase flow proposed by Anderson, McFadden and Wheeler taking into account possible thermal fluctuations. The mathematical model consists of the compressible Navier–Stokes system coupled with the Cahn–Hilliard equation, where the latter is driven by a multiplicative temporal white noise accounting for thermal fluctuations. We show that existence of emphdissipative martingale solutions satisfying the associated total energy balance.

Some Regularity Results for Nonlocal Cahn-Hilliard/Navier-Stokes Systems

Sergio Frigeri
Università Cattolica del Sacro Cuore, Brescia, Italy

The talk will be focused on the presentation of some recent results concerning the regularity of solutions to Cahn-Hilliard/Navier-Stokes systems which describe flow and phase separation of mixtures of incompressible viscous binary fluids. Attention will be paid on the different situations in which these results have been established, regarding the mobility, the double-well potential and the viscosity. Some attention will also be given to the techniques employed, and, in particular, a recent approach based on maximal parabolic regularity theory will be mentioned. Furthermore, if time allows, some relevant applications will be discussed.

A Nonlinear Fourth-Order Approximation of Forward-Backward Parabolic Equations

Lorenzo Giacomelli
Sapienza University of Rome, Italy
Michiel Bertsch, Alberto Tesei

Forward-backward parabolic equations are well-known to be ill posed. Various types of regularisation have therefore been proposed, such as a pseudo-parabolic (PP) and a linear fourth-order (L-IV) one. Due to the intrinsically unstable character of backward parabolic equations, one may reasonably expect that the qualitative behaviour of approximating solutions strongly depends on the type of regularisation. In one space dimension, I will introduce a *nonlinear* fourth-order regularization and I will present some first results of a joint program with M. Bertsch and A. Tesei. In particular, I will show that this regularization differs from the above-mentioned ones in two aspects: on one hand, unlike (L-IV), it admits solutions with singularities; on the other hand, unlike

(PP), singularities can both appear *and* disappear. We hope that these features will permit a better description of some qualitative properties of the limiting solutions, e.g. in terms of time scales.

The Three-Dimensional Cahn-Hilliard-Brinkman System with Unmatched Viscosities

Andrea Giorgini
Politecnico di Milano, Italy

Diffuse interface models play nowadays an important role in Fluid dynamics as analytical and numerical methods to describe the behavior of multicomponent or multi-phase fluids flows. The Cahn-Hilliard-Brinkman (CHB) system belongs to a class of diffuse interface models for the description of the interaction between incompressible and viscuos binary fluids. In particular, it is employed for phase separation phenomena in porous media and constitutes a relaxation of the well-known Cahn-Hilliard-Navier-Stokes and Cahn-Hilliard-Hele-Shaw systems. The CHB model couples a modified Darcy's law which rules the volume-averaged fluid velocity with a convective Cahn-Hilliard equation for the difference of the fluid concentrations. I will present in this talk a fairly complete mathematical theory for the CHB model with unmatched viscosities and logarithmic potential in three dimensions. More precisely, I will discuss about uniqueness of weak solutions, global well-posedness of strong solutions and validity of the separation property. This analysis validates the CHB system as a robust diffuse interface model for the description of three dimensional two-component flows.

Determining Nodes for Damped Forced Periodic Korteweg-de Vries Equation

Olivier Goubet
Université de Picardie Jules Verne, France

We show that solutions of periodic damped forced KdV equations are asymptotically determined by their values at three points, that is if the difference of two solutions at three points vanishes when time diverge to the infinity, then the difference of these two solutions converges towards 0 in the energy space (and then everywhere).

On the Field-Induced Transport of Magnetic Nanoparticles in Incompressible Flow

Günther Grün
University of Erlangen-Nuremberg, Germany
Patrick Weiss

We propose a thermodynamically consistent model for the motion of magnetized nanoparticles – suspended in an incompressible liquid and under the influence of magnetic fields. Combining Onsager's variational principle with the equations of magnetostatics, we derive a coupled elliptic-parabolic system for the evolution of fluid velocity, pressure, magnetization, particle density, and magnetic field. Based on a stable discretization with mixed finite elements, numerical simulations in 2D are presented. Moreover, existence of weak solutions is addressed, too.

The Existence of Solutions for the Non-Linear Hardening Problem

Risei Kano
Kochi University, Japan

In this talk, we discuss the parabolic problem form the hardening phenomena. The unknown functions u and sigma describe the displacement and stress, respectively in the one-dimensional interval $(0, L)$. Our problem means the hardening problem that the materials are harden by plasticity. That is derived from the hardening model by Visintin (2006), and the perfect plasticity model by Duvaut-Lions (1976). In the perfect plasticity model, the function that is threshold value in the plastic deformation, is a constant.
In this talk, we discuss the solvability for the above model with the threshold function depending upon time or unknown function, based on the idea of Duvaut-Lions (1976).
The problem equipped with the constraint set depend on the unknown function, is called quasi-variational inequality. The solvabilities of quasi-variational inequality have been dealt with in some papers.

On Qualitative Properties of Solutions to Microelectromechanical Systems with General Permittivity

Christina Lienstromberg
Leibniz University Hannover, Germany
Joachim Escher

Qualitative properties of solutions to the evolution problem modelling microelectromechanical systems (MEMS) with general permittivity profile are investigated. The system couples a parabolic evolution problem for the displacement of a membrane with an elliptic free boundary value problem for the electric potential in the region between the membrane and a rigid ground plate. Conditions are specified which ensure the non-positivity of the membrane's displacement. Moreover, assuming to have a non-positive displacement, it is shown that the solution develops a singularity after a finite time of existence.

A Class of Quasilinear Type Kobayashi–Warren–Carter Systems Including Dynamic Boundary Conditions

Ryota Nakayashiki
Chiba University, Japan

In this talk, we consider a class of parabolic systems, which can be called *quasilinear type Kobayashi–Warren–Carter systems*. Each constituent system is based on the mathematical model of grain boundary motion, proposed by [Kobayashi-et. al, Physica D., 140 (2000), 141–150], and the principal part of the system consists of a quasilinear diffusion equation of singular type, subject to the dynamic boundary condition. The objective of this study is to obtain a uniform mathematical method for the quasilinear type Kobayashi–Warren–Carter systems including dynamic boundary conditions. On this basis, we here address three issues, concerned with the qualitative properties of the systems. The first is to show the existence of solutions to the systems, including the rigorous expressions of solutions. The second is to show the continuous associations among the different systems. The final is to show the large time behavior of solutions. The three issues will be demonstrated in forms of the Main Theorems of this talk.

The Deep Quench Obstacle Problem

Amy Novick-Cohen
Technion, Israel

Results are presented with regard to evolution and steady states for the deep quench obstacle problem.

Higher-Order Anisotropic Phase Separation Models: Theoretical Analysis and Numerical Simulations

Shuiran Peng
University of Poitiers, France
Laurence Cherfils, Alain Miranville

The aim of this talk is to present theoretical analysis and numerical simulations on higher-order (in space) anisotropic phase separation models, corresponding to a modified free energy in which the temperature is omitted. In particular, we will discuss a priori estimates and the well-posedness results, also the dissipativity of semigroup, as well as the existence of the global attractor. Moreover, for the Allen-Cahn models with periodic boundary conditions, numerical simulations which illustrate the effects of the higher-order terms and the anisotropy, are displayed. We will further discuss the Cahn-Hilliard cases and show the numerical simulations.

Energy Stable Numerical Scheme for the Viscous Cahn-Hilliard-Navier-Stokes Equations with Moving Contact Lines

Madalina Petcu
University of Poitiers, France
Laurence Cherfils

In this work we consider the coupling between the incompressible Navier-Stokes equations with the viscous Cahn-Hilliard equations, coupling that models the motion of isothermal mixture of two confined immiscible and incompressible fluids with comparable densities and viscosities. The dynamic boundary conditions that we endow the model with, take into account the existence of a moving contact line defined at the intersection of the fluid-fluid interface with the solid wall of the physical domain. For this model we propose an energy stable temporal scheme and we prove the unconditional solvability and the stability of the discretization proposed. We also propose a fully discrete scheme for which we prove the stability and we present some numerical simulations for this problem.

Convergence of Exponential Attractors for a Finite Element Approximation of the Allen-Cahn Equation

Morgan Pierre
Université de Poitiers, France

We consider a space semidiscretization of the Allen-Cahn equation by conforming Lagrange finite elements. We build a family of exponential attractors associated to the discretized equations which is robust as the mesh parameter h tends to 0. As a consequence, we obtain an upper bound on the fractal dimension of the global attractor which is independent of h. Our proof is adapted from the result of Efendiev, Miranville and Zelik concerning the continuity of exponential attractors under perturbation of the underlying semigroup. Here, for the first time, the perturbation is a space discretization. We will also discuss the case of a time discretization and some perspectives.

On a Multi-Component Model for Tumor Growth

Giulio Schimperna
University of Pavia, Italy
Sergio Frigeri, Kei-Fong Lam, Elisabetta Rocca

We consider a model describing the evolution of a tumor inside of a host tissue. The process is described by the phase parameters representing the concentrations of proliferating and dead cells (satisfying a multi-component variant of the Cahn-Hilliard system with singular potential), by the macroscopic flow velocity (obeying to Darcy's law), and by the nutrient concentration (linked to the phase variables by an elliptic relation). Our main result is related to existence of weak solutions to the resulting evolutionary PDE system in a proper variational setting.

Damped Wave Equations with Quintic Nonlinearities in Bounded Domains: Asymptotic Regularity and Attractors

Sergey Zelik
University of Surrey, England

We discuss the recent achievements in the attractors theory for damped wave equations in bounded domains which are related with Strichartz type estimates. In particular, we present the results related with the well-posedness and asymptotic smoothness of the solution semigroup in the case of critical quintic nonlinearity. The non-autonomous case will be also considered

Special Session 10: Nonlocal Nonlinear Partial Differential Equations and Applications

Bruno Volzone, Università degli Studi di Napoli “Parthenope”, Italy
José Antonio Carrillo, Imperial College London, England
Jinhuan Wang, Liaoning University, Peoples Rep of China

This special session aims to bring together experts in the research area of nonlocal equations, in order to stimulate fruitful discussions concerning the following primary topics:

- advances in nonlinear diffusion involving nonlocal operators: regularity problems, long time asymptotic behaviour and entropy methods, existence or no existence of free boundaries, discussion of models involving general non-local operators;
- nonlocal aggregations: state of art concerning many open problems such as typical profiles of blow-up, qualitative properties of the minimizers or steady states and conditions for their existence;
- balance between nonlinear diffusions and nonlocal attraction for Keller-Segel type related models: new perspectives in the study of a general class of equations in which repulsion is modelled by nonlinear diffusion (fractional or porous medium like) and attraction by nonlocal operators.
- elliptic theory with nonlocal operators and connections with geometric and probabilistic approaches

Global Existence and Uniqueness Analysis of Reaction-Cross-Diffusion Systems

Xiuqing Chen
Beijing University of Posts and Telecommunications, Peoples Rep of China

The global-in-time existence of weak and renormalized solutions to reaction-cross-diffusion systems for an arbitrary number of variables inbounded domains with no-flux boundary conditions are proved. The cross-diffusion part describes the segregation of population species and is a generalization of the SKT model. The diffusion matrix is not diagonal and generally neither symmetric nor positive semi-definite, but the system possesses a formal gradient-flow or entropy structure. The reaction part is of Lotka-Volterra type for weak solutions or includes reversible reactions of mass-action kinetics and does not obey any growth condition for renormalized solutions. Furthermore, we prove the uniqueness of bounded weak solutions to a special class of cross-diffusion systems, and the weak-strong uniqueness of renormalized solutions to the general reaction-cross-diffusion cases.

Minimisers and Critical Points of Aggregation Diffusion Equations

Matias Delgadino
Imperial College, Argentina
Carrillo, Patacchini

We analyse free energy functionals for macroscopic models of multi-agent systems interacting via pairwise attractive forces and localized repulsion. We derive conditions for the energy to allow for critical points or global minimisers. In the fast diffusion regime, we prove the reversed Hardy-Littlewood-Sobolev inequality and we discuss the condensation phenomena for minimisers.

Singular Limits of the Isentropic Compressible Viscous Magnetohydrodynamic Equations

Fucai Li
Nanjing University, Peoples Rep of China

In this talk we shall discuss two kinds of singular limits to the isentropic compressible viscous magnetohydrodynamic equations in a bounded domain $\Omega \subset \mathbb{R}^3$. One is the incompressible limit, and the other is the inviscid limit. In the first case, the initial data are assumed to be “ill-prepared”. We show that the weak solutions (velocity and magnetic field) of the compressible magnetohydrodynamic equations converge weakly in $L^2(0,\infty;H^1(\Omega))$ to that of the incompressible viscous magnetohydrodynamic equations. In the other case, the initial data are assumed to be “well-prepared”. It is shown that the weak solutions of the compressible magnetohydrodynamic equations converge strongly in $L^2(0,T;L^2(\Omega))$ to the local strong solution of ideal isentropic compressible magnetohydrodynamic equations. Furthermore, the convergence rates are also obtained. Some related results are also reviewed.

Global Weak Solutions for the Three-Dimensional Chemotaxis-Navier-Stokes System with Slow P-Laplacian Diffusion

Yuxiang Li
Southeast University, Peoples Rep of China
Tao Weirun

In this talk we present an incompressible chemotaxis-Navier-Stokes system with slow p-Laplacian diffusion

$$\begin{cases} n_t + u\cdot\nabla n = \nabla\cdot(|\nabla n|^{p-2}\nabla n) - \nabla\cdot(n\chi(c)\nabla c), & x\in\Omega,\ t>0,\\ c_t + u\cdot\nabla c = \Delta c - nf(c), & x\in\Omega,\ t>0,\\ u_t + \kappa(u\cdot\nabla)u = \Delta u + \nabla P + n\nabla\Phi, & x\in\Omega,\ t>0,\\ \nabla\cdot u = 0, & x\in\Omega,\ t>0 \end{cases}$$

under homogeneous boundary conditions of Neumann type for n and c, and of Dirichlet type for u in a bounded convex domain $\Omega \subset \mathbb{R}^3$ with smooth boundary. Here, $\Phi \in W^{1,\infty}(\Omega)$, $0\frac{32}{15}$ and under appropriate structural assumptions on f and χ, for all sufficiently smooth initial data (n_0, c_0, u_0) the model possesses at least one global weak solution.

Asymptotic Estimates for Nonlocal Evolution Equations

Edoardo Mainini
University of Genoa, Italy

We consider nonlocal evolution models such as the Keller-Segel system or fractional porous media equations. We provide asymptotic rates of growth/decay for the L^p norms of solutions, within the minimizing movements scheme.

Fractional Porous Medium Equations: Well-Posedness and Asymptotics

Matteo Muratori
Politecnico di Milano, Italy
G. Grillo, F. Punzo

The porous medium equation (PME) is a well studied nonlinear version of the heat equation, which reads $u_t = \Delta u^m$ for $m > 1$. In particular, the diffusion becomes degenerate where the solution tends to zero, while it becomes singular where the solution tends to infinity, and compactly supported data stay with compact support at all times, even though the support eventually becomes the whole space. There is a special family of solutions, known as Barenblatts, which are explicit and, at least in Euclidean space, drive asymptotics of a quite general class of solutions. Fractional versions of the PME have also been investigated recently by J.L. Vazquez and coauthors, especially with regards to well-posedness, regularity and long-time asymptotics; one of the main issues is the fact that Barenblatts are no more explicit, and even their existence and uniqueness are nontrivial. Here I will describe some results dealing with well-posedness when finite measures are taken as initial data and asymptotics in the presence of weights. Then I will also discuss some open problems related to growing initial data and very weak solutions.

On the Structure of Solutions of Keller-Segel Systems with Sinks of Fluid

Yoshie Sugiyama
Osaka University, Japan
Eduardo Espejo, Hideo Kozono, Masanari Miura

We deal with the chemotaxis model under effect of Navier-Stokes fluids, *i.e.*, incompressible viscous fluids. We show the existence of a local *mild solution* for large initial data and a global *mild solution* for small initial data in the scale invariant class demonstrating that $n_0 \in L^1(\mathbb{R}^2)$ and $u_0 \in L^2_\sigma(\mathbb{R}^2)$. Our method is based on a perturbation of the linearization together with the $L^p - L^q$-estimates of the heat semigroup. As a by-product of our method, we prove the smoothing effect and uniqueness of our *mild solution.* In addition, we explore a type of fluid which can be forced up or down depending on the flow, under a given force. From this, we construct a solution for our model using this type of fluid, which has an arbitrary threshold number (different from 8π) for the initial mass. Here, the threshold number determines whether or not the solution diverges in some norm. In the divergent case, we give the characteristics of the time at which our solution diverges.

Long-Time Asymptotics for Nonlocal Porous Medium Equation with Absorption Or Convection

Bruno Volzone
Università degli Studi di Napoli "Parthenope", Italy
Filomena Feo, Yanghong Huang

In this talk, I will present results concerning the long-time asymptotic behaviours of nonlocal porous medium equations with absorption or convection. In the parameter regimes when the nonlocal diffusion is dominant, the entropy methods are essential for the derivation of the exponential convergence of relative entropy of solutions in similarity variables. The results are objects of a recent joint work with F. Feo and Y. Huang.

Supercritical Degenerate Parabolic-Parabolic Keller-Segel System – Existence Criterion Given by the Best Constant in Sobolev's Inequality

Jinhuan Wang
Liaoning University, Peoples Rep of China

This article presents a relationship between the sharp constant of the Sobolev inequality and the initial criterion to the global existence of degenerate parabolic-parabolic Keller-Segel system with the special diffusion exponent

Hydrodynamic Models of Collective Behavior with Damping and Nonlocal Interactions

Aneta Wròblewska-Kamiǹska
Polish Academy of Sciences / Imperial College London, Poland
Jose A. Carrillo, Ewelina Zatorska

Hydrodynamic systems for interacting particles where attraction is taken into account by nonlocal forces derived from a potential and repulsion is introduced by local pressure arise in swarming modelling. We focus on the case where there is a balance between nonlocal attraction and local pressure in presence of confinement in the whole space. Under suitable assumptions on the potentials and the pressure functions, we show the global existence of solutions for the compressible Navier-Stokes system with linear damping and nonlocal interaction force. Moreover, we show that global weak solutions converge for large times to the set of these stationary solutions in a suitable sense. In particular cases, we can identify the limiting density uniquely as the global minimizer of the free energy with the right mass and center of mass. This is a joint result with Jose A. Carrillo and Ewelina Zatorska.

Enhancement of Biological Reaction by Chemotaxis

Yao Yao
Georgia Institute of Technology, USA
Alexander Kiselev, Fedor Nazarov, Lenya Ryzhik

In this talk, we consider a system of equations arising from reproduction processes in biology, where two densities evolve under diffusion, absorbing reaction and chemotaxis. We prove that chemotaxis plays a crucial role to ensure the efficiency of reaction: Namely, the reaction between the two densities is very slow in the pure diffusion case, while the presence of a chemotaxis term greatly enhances reaction. While proving our main results we also obtain a weighted Poincare's inequality for the Fokker-Planck equation, which might be of independent interest.

Special Session 11: Dynamical System Modeling for Ecological Effects and Evolution of Dispersal in Biological Systems

Adrian Lam, The Ohio State University, USA
Robert Stephen Cantrell, University of Miami, USA
Chris Cosner, University of Miami, USA
Yuan Lou, The Ohio State University, USA

The scope of this special session covers the broad area of dynamical systems modeling arising in ecology and evolutionary biology, with emphasis on the evolution of dispersal. Researchers from various backgrounds such as modeling, analysis, and numerical analysis will be invited to stimulate communication and synthesis of approaches. We expect roughly 16-20 talks to be delivered in this session.

A PDE Model of Intraguild Predation with Cross-Diffusion

Robert Stephen Cantrell
University of Miami, USA
Xinru Cao, King-Yeung Lam, Tian Xiang

This talk concerns a quasilinear parabolic system modeling an intraguild predation community in a focal habitat in $\mathbb{R}^n$, $n \geq 2$. In this system the intraguild prey employs a fitness-based dispersal strategy whereby the intraguild prey moves away from a locale when predation risk is high enough to render the locale undesirable for resource acquisition. The system modifies an earlier model considered by Ryan and Cantrell by adding an element of mutual interference among predators to the functional response terms in the model, thereby switching from Holling two forms to Beddington-DeAngelis forms. We show that the resulting system can be realized as a semi-dynamical system with a global attractor for any $n \geq 2$. In contrast, the original model was restricted to two dimensional spatial habitats. The permanence of the intraguild prey then follows as in Ryan and Cantrell by means of the Acyclicity Theorem of Persistence Theory.

A Tridiagonal Patch Model of Bacteria Inhabiting a Nanofabricated Landscape

Brian Coomes
University of Miami, USA
Robert Stephen Cantrell, Yifan Sha

Recently we employed a discrete-diffusion modeling framework to examine a system inspired by the nanoecology experiments on the bacterium *Escherichia coli* reported upon in a 2006 paper of Keymer et al. In these experiments, the bacteria inhabit a linear array of 85 microhabitat patches (MHP's), linked by comparatively thinner corridors through which bacteria may pass between adjacent MHP's. Nutrients flow into each of the patch substrates at different rates. In the absence of bacteria dispersal, patches where the substrate nutrient flow rate is sufficiently high may be regarded as population sources, while those with insufficient substrate flow may be regarded as population sinks. We discuss the role of dispersal in determining the predictions of the model under source-sink dynamics. We will also discuss some remaining questions.

Evolutionarily Stable Dispersal in Time Periodic Environments

Chris Cosner
University of Miami, USA
Robert Stephen Cantrell

The evolution of dispersal has been the subject of a considerable amount of interest in recent years. In the case of environments that are variable in space but static in time, it has been shown in several modeling contexts, including reaction-advection-diffusion equations and their spatially discrete or nonlocal analogues, that there are dispersal strategies that are evolutionarily stable, and those are the ones which can produce an ideal free distribution of the population. This talk will describe an extension of such ideas and results to reaction-diffusion-advection models in certain situations where the environment is periodic in time, specifically when the total amount of resources in the environment is constant but the location of the resources is time dependent. One difference between the static and time varying cases is that in the static case, for reaction-advection-diffusion equations, it is possible to choose strategies leading to an ideal free distribution that use only spatially local information. In the analogous time periodic cases, it appears that nonlocal spatial information is required.

Oscillations in the Near-Field Feeding Current of a Calanoid Copepod Are Useful for Particle Sensing

Peter Hinow
University of Wisconsin-Milwaukee, USA
Carl Giuffre, Houshuo Jiang, J. Rudi Strickler

Copepods are small crustaceans that constitute a major element of aquatic ecosystems. Key to their success is their feeding apparatus consisting of sensor-studded mouth appendages that are in constant motion. These appendages generate a feeding current to enhance the encounter probability with food items. Additionally, sensing enables the organism to determine the position and quality of food particles, and to alter the near-field flow to capture and manipulate the particles for ingestion or rejection. We observe a freely swimming copepod *Leptodiaptomus sicilis* in multiple perspectives together with suspended particles that allow us to analyze the flow field created by the animal. We observe a highly periodic motion

of the mouth appendages that is mirrored in oscillations of nearby tracer particles. We propose that the phase shift between the fluid and the particle velocities is sufficient for mechanical detection of the particles entrained in the feeding current.

Spatial Population Dynamics in Meandering Rivers

Yu Jin
University of Nebraska-Lincoln, USA
Frithjof Lutscher, Pei Yuan

We present a novel model that considers the longitudinal variation as introduced by the sinuosity of a meandering river where a main channel is laterally extended to point bars in bends. These regions offer different habitat conditions for aquatic populations and therefore may enhance population persistence. Our model is a nonstandard reaction-advection-diffusion model where the domain of definition consists of the real line (representing the main channel) with periodically added intervals (representing the point bars). We give an existence and uniqueness proof for solutions of the equations. We then study population persistence as the (in-)stability of the trivial solution and population spread as the minimal wave speed of traveling periodic waves. We conduct a sensitivity analysis to highlight the importance of each parameter on the model outcome. We find that sinuosity can enhance species persistence.

On the Preservation of Cooperation in Two-Strategy Games with Nonlocal Interactions

Yun Kang
Arizona State University, USA
Ozgur Aydogmus, Wen Zhou

Nonlocal interactions such as spatial interaction are ubiquitous in nature and may alter the equilibrium in evolutionary dynamics. Models including nonlocal spatial interactions can provide a further understanding on the preservation and emergence of cooperation in evolutionary dynamics. In this paper, we consider a variety of two-strategy evolutionary spatial games with nonlocal interactions based on an integro-differential replicator equation. By defining the invasion speed and minimal traveling wave speed for the derived model, we study the effects of the payoffs, the selection pressure and the spatial parameter on the preservation of cooperation. One of our most interesting findings is that, for the Prisoners Dilemma games in which the defection is the only evolutionary stable strategy for unstructured populations, analyses on its asymptotic speed of propagation suggest that, in contrast with spatially homogeneous games, the cooperators can invade the habitat under proper conditions. Other two-strategy evolutionary spatial games are also explored. Both our theoretical and numerical studies show that the nonlocal spatial interaction favors diversity in strategies in a population and is able to preserve cooperation in a competing environment. A real data application in a virus mutation study echoes our theoretical observations. In addition, we compare the results of our model to the partial differential equation approach to demonstrate the importance of including non-local interaction component in evolutionary game models.

Invasion of an Empty Habitat by Two Competing Species

Adrian Lam
The Ohio State University, USA
Leo Girardin, King-Yeung Lam

We will discuss some spreading properties of monostable Lotka- Volterra two-species competition-diffusion systems when the initial values are null or exponentially decaying in a right half-line. Thanks to a careful construction of super-solutions and sub-solutions, we improve previously known results and settle open questions. In particular, we show that if the weaker competitor is also the faster one, then it is able to evade the stronger and slower competitor by invading first into unoccupied territories. The pair of speeds depends on the initial values. If these are null in a right half-line, then the first speed is the KPP speed of the fastest competitor and the second speed is given by an exact formula depending on the first speed and on the minimal speed of traveling waves connecting the two semi-extinct equilibria. Furthermore, the unbounded set of pairs of speeds achievable with exponentially decaying initial values is characterized, up to a negligible set.

Propagation Dynamics of Nonlocal Dispersal Equations

Wantong Li
Lanzhou University, Peoples Rep of China
Jia-Bing Wang, Li Zhang

This talk is concerned with the new types of entire solutions other than traveling wave solutions of nonlocal dispersal equations with monostable nonlinearity in space periodic habitats. We first establish the existence and properties of spatially periodic solutions connecting two steady states. Then new types of entire solutions are constructed by combining the rightward and leftward pulsating traveling fronts with different speeds and a spatially periodic solution. Finally, for a class of special heterogeneous reaction, we further establish the uniqueness of entire solutions and the continuous dependence of such an entire solution on parameters, such as wave speeds and the shifted variables. In other words, we build a five-dimensional manifold of solutions and the traveling wave solutions are on the boundary of the manifold.

A Nonlocal Reaction-Diffusion Growth Model with Periodic Delay and Competition

Yijun Lou
Hong Kong Polytechnic University, Hong Kong
Kaihui Liu, Zhi-Cheng Wang, Liang Zhang

Each species is subject to various biotic and abiotic factors during growth. This talk presents a deterministic model with the consideration of various factors regulating population growth such as age-dependent birth and death rates, spatial movements, seasonal variations, intra-specific competition and time-varying maturation simultaneously. The model takes the form of two coupled reaction-diffusion equations with time-dependent delays, which bring novel challenges to the theoretical analysis. The well-posedness of the system is established. Then the model is analyzed when competition among immatures is negligible, in which situation one equation for the adult population density is decoupled. The basic reproduction number is defined and shown to determine the global attractivity of either the zero equilibrium or a positive periodic solution by using the dynamical system approach on an appropriate phase space. When the competition is included, the model consisting of two coupled equations is neither cooperative (where the comparison principle holds) nor reducible to a single equation. In this case, the threshold dynamics about the population extinction and uniform persistence are established by using the basic reproduction number as a threshold index.

Dynamical Fisheries Models with Bioeconomy

Tri Nguyen-Huu
IRD, France

Many bioeconomy models of fisheries often do not take into account the evolution of the market price of landed stocks. However, it is a key variable in order to understand the sustainability of fishing activities, and allow to exhibit different long term behaviors for the resource evolution as well as for the fishing effort. We present a general fishing model that includes a model of price, demand and offer based on the classical theory of economy, which exhibit how the market can drive to either overexploitation or sustainability. We also present an example on how indirect policies (using Fish Agregation Devices) can allow fisheries to recover from overexploitation.

A Consumer-Resource Model in Population Dynamics

Wei-Ming Ni
ECNU, Peoples Rep of China
X. He, K.Y. Lam, Y. Lou

In this talk, I shall discuss some of the recent advances on a consumer-resource model proposed by B. Zhang et al. in Ecology Letters 20 (2017), 1118-1128.

Single Phytoplankton Species Growth with Light and Crowding Effect in a Water Column

Hua Nie
Shaanxi Normal University, Peoples Rep of China

We investigate a nonlocal reaction-diffusion-advection model which describes the growth of a single phytoplankton species in a water column with crowding effect. The longtime dynamical behavior of this model and the asymptotic profiles of its positive steady states for large sinking or buoyant rates are established. The results show that there is a critical death rate such that the phytoplankton species survives if and only if its death rate is less than the critical death rate. In contrast to the model without crowding effect, our results show that the density of the phytoplankton species will have a finite limit rather than go to infinity when the death rate disappears. Furthermore, for large sinking rate, the phytoplankton species concentrates at the bottom of the water column with a finite population density. For large buoyant rate, the phytoplankton species concentrates at the surface of the water column with a finite population density.

Two-Locus Clines Maintained by Diffusion and Recombination in a Heterogeneous Environment

Linlin Su
Southern University of Science and Technology, Peoples Rep of China
Reinhard Bürger, King-Yeung Lam

We study a system of semilinear parabolic partial differential equations which describes the evolution of gamete frequencies in a geographically structured population of migrating individuals. Fitness of individuals is determined by two recombining, diallelic genetic loci that are subject to spatially varying selection. Migration is modeled by diffusion. Of most interest are spatially non-constant stationary solutions, so-called clines. In a two-locus cline, all four gametes are present in the population. The key problem is the study of existence, uniqueness, and stability of two-locus clines and how their existence and properties depend on the interaction of diffusion, selection, and recombination. We provide conditions for existence and linear stability of a two-locus cline if recombination is either sufficiently weak or suffi-

ciently strong relative to selection and diffusion. For strong recombination, we also prove uniqueness and global asymptotic stability. For arbitrary recombination, we determine the stability properties of the monomorphic equilibria, which represent fixation of a single gamete.

Effectiveness of Control and Preventive Measures Influenced by Pathogen Trait Evolution: Example of *Escherichia coli* O157:H7

Xueying Wang
Washington State University, USA
Majid Bani-Yaghoub, Patrick Pithua, Sharif S. Aly

This work is the first step towards assessment of control and preventive measures with respect to pathogen trait evolution. Our goal is to reassess the effectiveness of environmental decontamination (ED) and antimicrobial drug administration (ADA) by taking into account pathogen trait evolution within and between the hosts. To achieve this goal, we construct a multi-strain disease transmission model with cross immunity and intermittent shedding, where transmission occurs via environmental and host reservoirs. The ecology of infection is investigated by establishing conditions for stability and existence of equilibria. Using the stability results, it is demonstrated that pathogen trait evolution neutralizes or diminishes the effectiveness of ADA and ED. Specifically, first the model is fitted to the prevalence data of *Escherichia coli* O157:H7 in a dairy farm. Then the effectiveness of ADA and ED is explored under four hypothesized cases of pathogen trait evolution. It is shown that excessive implementation of ADA or ED can result in pathogen traits capable of destabilizing the disease free equilibrium and establishing stable multi-strain or single-strain endemic equilibria.

Error Estimates for Diffusion Equations on Coated Bodies and Lifespan of Effective Boundary Conditions

Xuefeng Wang
Southern Univ. of Science and Technology, Peoples Rep of China

Imagine the scenario of a heat/diffusion equation on a body, which is surrounded by a thin layer whose diffusion property is drastically different from that on the body (suppose Dirichlet boundary condition is imposed on the outer boundary of the thin layer). To simplify the numerics and to be able to see the effects of the coating, in previous works we have found "effective boundary condition"which is imposed on the boundary of the uncoated body, resulting in a simpler "effective model"that approximates the original boundary value problem (the full model). I will talk about error estimate results from which we see how well the effective model approximates the full model. It turns out that in some cases this approximation is not possible to be a good one for the entire time interval $[0, \infty)$; in these cases we give estimates for the time duration when the approximation is good–that time duration is what we call "lifespan of effective boundary condition".

Spatial Spread of Epidemic Diseases in Geographical Settings: Seasonal Influenza Epidemics in Puerto Rico

Yixiang Wu
Vanderbilt University, USA
Pierre Magal, Glenn F. Webb

Deterministic models are developed for the spatial spread of epidemic diseases in geographical settings. The models are focused on outbreaks that arise from a small number of infected hosts imported into subregions of the geographical settings. The goal is to understand how spatial heterogeneity influences the transmission dynamics of the susceptible and infected populations. The models consist of systems of partial differential equations with diffusion terms describing the spatial spread of the underlying microbial infectious agents. The model is compared with real data from seasonal influenza epidemics in Puerto Rico.

Dynamics and Asymptotic Profiles of Endemic Equilibrium for Two Frequency-Dependent SIS Epidemic Models with Cross-Diffusion

Tian Xiang
Renmin University of China, Peoples Rep of China

We consider two SIS epidemic reaction-diffusion models in heterogeneous environment, with a cross-diffusion term modeling the effect that susceptible individuals tend to move away from higher concentration of infected individuals. It is first shown that the corresponding Neumann initial-boundary value problem in an n-dimensional bounded smooth domain possesses a unique global classical solution which is uniformly-in-time bounded regardless of the strength of the cross-diffusion and the spatial dimension n. It is further shown that, even in the presence of cross-diffusion, the models still admit threshold-type dynamics in terms of the basic reproduction number $\mathcal{R}_0$; that is, the unique disease free equilibrium is globally stable if $\mathcal{R}_0 < 1$, while if $\mathcal{R}_0 > 1$, the disease is uniformly persistent and there is an endemic equilibrium, which is globally stable in some special cases Our results on the asymptotic profiles of endemic equilibrium illustrate that restricting the motility of susceptible population may eliminate the infectious disease entirely for the first model with constant total population but fails for the second model with varying total population. In particular, this implies that such cross-diffusion does not contribute to the elimination of the infectious disease modelled by the second one. This is a joint work with H. Li and R. Peng.

In 2-D, it is shown that, for any regular initial data, the system has a unique global -in-time smooth solution for arbitrary size of chi, and it is uniformly bounded in time in the case that the net growth rate of prey are non-positive. In the latter case, we further study its long time dynamics, implying the cross-diffusion and the instability of certain semi-trivial constant equilibria are still unable to induce pattern formations. In particular, we find that the long time behaviors of the PDE may not always be determined by its corresponding ODE system. This seems to be a new phenomenon compared to existing long time dynamics. The technical condition of the convexity on the domain and the smallness condition on the prey-taxis sensitivity are removed. The (boundedness and) convergence are proved in n-D by the simple and transparent energy method rather than the dissipative dynamical system techniques and Lyapunov function techniques used in early related works.

Modeling the Spatial Spread of Mosquitoes with Serratia Infection

Yanyu Xiao
University of Cincinnati, USA

Engineered symbiotic bacteria have been shown to equip mosquitoes with resistant to Malaria parasites. But the introduction of the lab-borned mosquitoes to wild mosquito population is a big challenge. In this work, we use a partial differential equation model to examine the spatial spread of these genetically modified mosquitoes through mating and birth.

Invasion and Coexistence of Competition-Diffusion-Advection System with Heterogeneous Vs Homogeneous Resources

Benlong Xu
Shanghai Normal University, Peoples Rep of China
Hongyang Jiang

In this talk, we shall present some recent results of the dynamics of a Lotka-Volterra reaction-diffusion-advection model for two competing species which disperse by both random diffusion and advection along environmental gradient. In this model, the species are assumed to be identical except spatial resource distribution: heterogeneity vs homogeneity. It is shown that the species with heterogeneous resources distribution is always in a better position, that is, it can always invade when rare. The ratio of advection strength and diffusion rate of the species with heterogeneous distribution plays a crucial role in the dynamics behavior of the system. Some conditions of invasion, driving extinction, and coexistence are given in terms of this ratio and the diffusion rate of its competitor. Some open problems will be presented.

Long Time Behaviors of the Fisher-KPP Equation in the River Network

Maolin Zhou
University of New England, Australia
Yihong Du, Bendong Lou, Rui Peng

In this work, we consider the propagation phenomena in the river with two or three branches and the effect of water speed on it. And we give out a classification of the asymptotic behaviors of solutions for different cases.

Special Session 12: Numerical Methods for Phase Field Models

Zhonghua Qiao, The Hong Kong Polytechnic University, Hong Kong
Jie Shen, Purdue University, USA
Xiaoping Wang, Hong Kong University of Science and Technology, Hong Kong

Phase field modeling is a popular technique for simulating microstructure evolution at the mesoscale in the study of many multi-phase physical, chemical or biological processes. Numerical simulations have played an important role in the study of phase field models. Although many numerical methods have been developed, new challenges appear in designing efficient numerical methods for various phase field models that are of importance in applications. The session is devoted to discussing the recent progress and promising directions in numerical methods relevant to phase field models.

Maximum Principle-Preserving Exponential Time Differencing Schemes for Nonlocal Allen-Cahn Equations

Lili Ju
University of South Carolina, USA
Qiang Du, Xiao Li, Zhonghua Qiao

In this work we design and analyze first and second order exponential time differencing (ETD) schemes for solving the nonlocal Allen-Cahn (NAC) equation, a generalization of the classic Allen-Cahn equation by replacing the Laplacian with a parameterized nonlocal diffusion operator. The solution of the NAC equation satisfies the maximum principle, thus it is highly desired that the approximated solutions also preserve such a physical property in the fully discrete sense. Our numerical schemes are obtained by using the quadrature-based finite difference method for spatial discretization and applying the stabilized ETD-based approximations for time integration. We prove the discrete maximum principle (DMP) and energy stability of the proposed schemes; in particular, the DMP are unconditionally held for both schemes with respect to any time step size and so does the energy dissipation law for the first-order scheme. We then derive their respective optimal maximum-norm error estimates and further show that the schemes are asymptotic compatible, i.e., the approximated solutions converge to the classic AC solution when the horizon, the spatial mesh size and the time step size all goes to zero. Various experiments are performed to verify these theoretical results, and to numerically investigate the relationship between the discontinuities and the nonlocal parameters.

Exponential Time Differencing Schemes for the Epitaxial Growth Model Without Slope Selection

Xiao Li
Beijing Computational Science Research Center, Peoples Rep of China

In this talk, we present a class of exponential time differencing (ETD) schemes for solving the epitaxial growth model without slope selection. A linear convex splitting is first applied to the energy functional of the model, and then Fourier collocation and ETD-based multistep approximations are used respectively for spatial discretization and time integration of the corresponding gradient flow equation. Energy stabilities and error estimates of the first and second order ETD schemes are rigorously established in the fully discrete sense. We also numerically demonstrate the accuracy of the proposed schemes and simulate the coarsening dynamics with small diffusion coefficients. The results show the logarithm law for the energy decay and the power laws for growth of the surface roughness and the mound width, which are consistent with the existing theories in the literature.

Reciprocal Theorem: from Local Equations to Symmetry Over the Whole System

Tiezheng Qian
Hong Kong University of Science and Technology, Hong Kong

Onsager's reciprocal symmetry is typically used to derive local constitutive equations through a variational approach. We consider a non-equilibrium system close to the global equilibrium state. We show that the reciprocal symmetry for local constitutive equations can be extended to a new symmetry over the whole system. This symmetry is manifested in the kinetic coefficients connecting the forces and fluxes defined at the system boundary. Our results generalize the Lorentz theorem for Stokes flows. A phase field model will be used to demonstrate the generalized theorem.

A Third Order Exponential Time Differencing Numerical Scheme for No-Slope-Selection Epitaxial Thin Film Model

Zhonghua Qiao
The Hong Kong Polytechnic University, Hong Kong
Cheng Wang

In this paper we study and analyze a (temporally) third order accurate exponential time differencing (ETD) numerical scheme for the no-slope-selection (NSS) equation of the epitaxial thin film growth model, with Fourier pseudo-spectral discretization in space. A linear splitting is applied to the physical model, for the sake of energy stability analysis, and an ETD-based multistep approximation is used for time integration of the corresponding equation. In turn, the energy stability is established in a modified

version. And also, the optimal rate convergence analysis and error estimate are derived in details, in the $\ell^\infty(0,T;H_h^1)\cap\ell^2(0,T;H_h^3)$ norm, with the help of a careful eigenvalue bound estimate, combined with the nonlinear analysis for the NSS model. This convergence estimate is the first such result for a third order accurate scheme for a gradient flow. Some numerical simulation results are presented to demonstrate the efficiency of the numerical scheme and the third order convergence. In particular, the long time simulation results for $\varepsilon = 0.02$ (up to $T = 3\times10^5$) have indicated a logarithm law for the energy decay, as well as the power laws for growth of the surface roughness and the mound width. In particular, the power index for the surface roughness and the mound width growth, created by the third order numerical scheme, is more accurate than those produced by certain second order energy stable schemes in the existing literature.

Multiple Scalar Auxiliary Variable (MSAV) Approach and Its Application to the Phase Field Vesicle Membrane Model

Jie Shen
Purdue University and Xiamen University, USA

We consider in this paper gradient flows with disparate terms in the free energy that can not be efficiently handled with the scalar auxiliary variable (SAV) approach, and develop the multiple scalar auxiliary variable (MSAV) approach to deal with these cases. We apply the MSAV approach to the phase-field vesicle membrane (PF-VMEM) model which, in addition to some usual nonlinear terms in the free energy, has two additional penalty terms to enforce the volume and surface area. The MSAV approach enjoys the same computationally advantages as the SAV approach, but can handle free energies with multiple disparate terms such as the volume and surface area constraints in the PF-VMEM model. The MSAV schemes are unconditional energy stable, second-order accurate in time, and can be decoupled, at each time step, into three linear fourth-order equations with constant coefficients, each can be further reduced to two Poisson type equations. Hence, these schemes are easy to implement and extremely efficient when coupled with an adaptive time stepping. Ample numerical results are presented to validate the stability and accuracy of the MSAV schemes.

Positivity Preserving Numerical Schemes for Phase-Field Models

Xiaoming Wang
Fudan University and Florida State University, Peoples Rep of China
Wenbin Chen, Cheng Wang

We consider numerical schemes for phase-field models with logarithmic potential (Flory-Huggins type potential). We present energy stable numerical algorithms for phase-field models that are able to preserve the positivity of certain physical quantities.

2nd Law of Thermodynamics, Onsager Principle and Numerical Methods for Multiphase Systems

Qi Wang
CSRC/Univ. of South Carolina, Peoples Rep of China
Jia Zhao, Qi Wang

I will present a paradigm to numerically discretize continuum models derived from the Onsager Principle, which therefore satisfy the 2nd law of thermodynamics. This general strategy gives one a unified treatment of thermodynamically consistent models for a host of physical systems. It demonstrates the benefits of developing thermodynamical consistent models and the use of the models in numerical simulations.

The So-Called Invariant Energy Quadratization Approach for Developing Unconditionally Energy Stable Schemes of Phase Field Models

Xiaofeng Yang
University of South Carolina, USA

The free energies of gradient flow systems usually consist of various nonlinear potentials formulated in diverse complex formats which present a major challenge in the construction of efficient and accurate time discretization schemes. We overcome this challenge by developing a flexible and robust IEQ approach which enables us to develop time discretization schemes for a large class of gradient flow systems. More precisely, the developed schemes (i) are accurate (up to second order in time); (ii) are stable (unconditional energy dissipation law holds); and (iii) are efficient and easy to implement (only need to solve some positive definite linear system at each time step).

Thermodynamic-Consistent Multiple-Relaxation-Time Lattice Boltzmann Equation Model for Nonideal Fluids with Peng-Robinson Equation of State

Zhang Yuze
The Hong Kong Polytechnic University, Hong Kong
Zhonghua Qiao, Xuguang Yang

In this work, a multiple-relaxation-time (MRT) lattice Boltzmann (LB) equation model with Beam-Warming (B-W) scheme is proposed to simulate multi-phase fluid system with Peng-Robinson (P-R) equation of state (EOS). The mathematical model of the multi-phase fluid flow is derived based on the NVT-based framework, where the Helmholtz free energy of P-R fluid is introduced. The nonideal force in multi-phase flow is directly computed from the free energy so that a more compact formulation of hydrodynamic equations, which is termed as potential form, can be obtained. The MRT-LB model is developed based on the potential form of hydrodynamic equations, which can eliminates the parasitic currents effectively. In addition, to capture the tiny nonconvex perturbation from the linear trend of P-R model precisely, the B-W scheme is utilized in the present MRT-LB model, which leads to an adjustable Courant-Friedrichs-Lewy (CFL) number and the second order accuracy can be naturally achieved by this scheme without any other requirement and numerical boundary conditions. In the numerical experiments, a realistic hydrocarbon component, such as isobutane, in three dimensional space is simulated by the proposed MRT-LB model. Numerical results show that the magnitude of parasitic currents can be significantly reduced by the present MRT-LB model.

Phase Field Modeling of Cell Polarity and Cell Delamination

Lei Zhang
Peking University, Peoples Rep of China

Control of cellular behaviors plays a critical role in pattern formation, growth regulation and regeneration. Numerous developmental processes have been extensively studied from a mechanistic perspective, but only recently have serious efforts been directed toward systems biology approach. In this talk, I will present two biological systems to study pattern formation by using phase field model. First, we present a mathematical model that incorporates the interplays between Rac, filamentous actin (F-actin), and membrane tension for the formation of cell polarity. Second, I present a phase field approach to study the neuroblast delamination in Drosophila. Dynamics of cell ingression and role of actin-myosin network in apical constriction reveal that the myosin signaling drives neuroblast delaminiation in such rare event. The joint work with Feng Liu (PKU), Yan Yan (HKUST).

Bubble Assemblies in Ternary Systems with Long Range Interaction

Yanxiang Zhao
George Washington University, USA
Chong Wang, Xiaofeng Ren

A nonlocal diffuse interface model, based on the Nakazawa-Ohta density functional theory for triblock copolymers, is used to study bubble assemblies in ternary systems. The model has three parameters weighing three types of long range interaction and two parameters that fix the total area of each constituent. As the parameters vary, a large number of morphological phases appear as stable stationary states. One open question related to the polarity direction of double bubble assemblies is answered numerically. Moreover, it is shown that the average size of bubbles in a single bubble assembly depends on the sum of the minority constituent areas and the long range interaction coefficients. One further identifies the ranges for area fractions and the long range interaction coefficients for double bubble assemblies.

Special Session 13: Measurable and Topological Dynamics

Yonatan Gutman, Institute of Mathematics of the Polish Academy of Sciences, Poland
Hitoshi Nakada, Keio University, Japan
Kyewon Koh Park, Korea Institute for Advanced Study, Korea
Xiangdong Ye, University of Science and Technology of China, Peoples Rep of China

Ergodic theory (or measurable dynamics) and topological dynamics are two sister branches of the theory of dynamical systems whose origins can be traced back to Poincaré's work on celestial mechanics. The relation between the two disciplines is reflected by a common vocabulary which describes different but parallel notions as well as by deep and sometimes unexpected connections. A significant impetus for the development of the two disciplines was given by Furstenberg's celebrated proof of Szemeredi's theorem. Nowadays the circle of ideas behind this proof plays as an important role as it did 40 years ago.
The aim of the session is to bring together researchers working in both fields, in order to report the newest progress, exchange ideas, forge new cooperation and to discuss both classical and merging open problems.

A Note on Holomorphic Shadowing for Henon Maps

Yi-Chiuan Chen
Academia Sinica, Taiwan

In studying the complex Henon maps, Mummert defined an operator the fixed points of which give rise to bounded orbits. This enabled him to obtain an estimate of the solenoid locus. Instead of the contraction mapping theorem, in the talk, I shall present an implicit function theorem version of his result, with some generalisation.

Rigidity of Group Actions

Nhan Phu Chung
Sungkyunkwan University, Korea
Yongle Jiang

In this talk, we will present Livsic theory of continuous (Holder) cocycles for group actions. In contrast to the map case, when the acting groups are higher rank, for certain systems we always have cocycle superrigidity without any assumptions of cocycles on periodic data.

Non-Singular Actions of Amenable Groups

Anthony Dooley
University of Technology Sydney, Australia

For a non-singular action of the integers, the author and collaborators have introduced the critical dimension, the index α for which $\Sigma_{k=0}^{n} \frac{d\mu \circ T^k}{d\mu}$ grows as $O(n^\alpha)$. It's an interesting invariant of metric equivalence which is related to some kind of non-singular entropy.
Recently, with my student Kieran Jarrett, we have been investigating how this works for other amenable group actions including $\mathbb{Z}^d$, the Heisenberg group and the lamplighter group.

Metric Mean Dimension and Almost Lossless Analog Compression

Yonatan Gutman
Institute of Mathematics, Polish Academy of Sciences, Poland
Adam Ŝpiewak

Wu and Verdù developed a theory for almost lossless analog compression where one imposes various regularity conditions on the compressor and the decompressor and the input signal is modeled by a (typically infinite-entropy) Bernoulli process. In this work we consider the broader class of signals modeled by time-invariant probability measures and find uniform lower and upper bounds in terms of *metric mean dimension*, *mean box dimension* and *mean Rènyi information dimension*. An essential tool is the recent Lindenstrauss-Tsukamoto variational principal expressing metric mean dimension in terms of certain rate-distortion functions.

Quantitative Recurrence for Chacón Transformation

Michihiro Hirayama
University of Tsukuba, Japan

We consider a quantitative aspect of multiple recurrence for probability measure preserving systems. For the Chacòn transformation, we derive a Kac inequality for the simultaneous first return time.

On the Number of Ergodic Measures Over an Ergodic Measures for Factor Maps Between Shifts of Finite Type

Uijin Jung
Ajou University, Korea
Jisang Yoo

Given an entropy preserving factor map between two irreducible shifts of finite type X and Y, there is a number d, called the degree of f, such that almost all points in Y have d preimages in X. The factor map naturally induces a factor map between the sets of invariant measures of X and Y, and it is folklore

that each fully supported ergodic measure on Y has at most d ergodic preimage measures on X. This number may vary: For example, if a given measure is Markov, then it lifts to a unique measure which is also Markov (Boyle and Tuncel). Recently, Yoo defined the notion of the multiplicity of each lifted measures and showed that the degree is the sum of the multiplicity of all ergodic measures over it. In this talk after explaining the history and known results regarding the multiplicity structure of invariant measures, we show that there is a residual set of ergodic invariant measures on Y number of whose ergodic invariant measures is exactly d.

Ergodic Theorems for Nonconventional Arrays and an Extension of the Szemeredi Theorem

Yuri Kifer
Hebrew University of Jerusalem, Israel

The study of nonconventional sums

$$S_N = \sum_{n=1}^{N} F(X(n), X(2n), ..., X(\ell n)),$$

where $X(n) = g \circ T^n$ for a measure preserving transformation T, has a 40 years history after Furstenberg showed that they are related to the ergodic theory proof of Szemerédi's theorem about arithmetic progressions in sets of integers of positive density. Recently, it turned out that various limit theorems of probability theory can be successfully studied for sums S_N when $X(n)$, $n = 1, 2, ...$ are weakly dependent random variables. I will talk about a more general situation of nonconventional arrays of the form $S_N = \sum_{n=1}^{N} F(X(p_1 n + q_1 N), X(p_2 n + q_2 N), ..., X(p_\ell n + q_\ell N))$ and how this is related to an extended version of Szemerédi's theorem. I'll discuss also ergodic and limit theorems for such and more general nonconventional arrays.

Families of Directed Graphs and Topological Conjugacy of the Associated Markov-Dyck Shifts

Wolfgang Krieger
University of Heidelberg, Germany
Toshihiro Hamachi

Markov-Dyck shifts are symbolic dynamical systems, that are constructed from The inverse semigroup of the graph appears as an intermediate step in this construction. We consider Markov-Dyck shifts, that are constructed from strongly connected finite directed graphs, such that by removing all edges from the graph, whose target vertex has more than one incoming edge, one obtains a tree. This tree we call the contracting subtree of the graph. We consider certain families of graphs with a contracting subtree. We characterize the Markov-Dyck shifts of the graphs, that belong to these families, among Markov-Dyck shifts, by invariants of topological conjugacy. We also show for the graphs in these families, that the topological conjugacy of their Markov-Dyck shifts implies the isomorphism of the graphs. Among the families, that we consider, is the family of graphs, whose contracting subtree is spherically homogeneous of depth two. We also consider families, that contain graphs with four edges in the complement of their contracting subtrees. In the proofs we use semigroup invariants and periodic orbit counts.

Effective Uniqueness of Entropy-Maximizing Measure of Geodesic Flows on Graphs

Sanghoon Kwon
Catholic Kwandong University, Korea
Rene Ruhr

We discuss the effective uniqueness of entropy-maximizing measure of discrete time geodesic flows on graphs. Namely, if ϕ is a discrete time geodesic flow on a graph and μ is a ϕ-invariant probability measure with measure-theoretical entropy close to the topological entropy of ϕ, then μ is close to the unique entropy-maximizing measure of ϕ. Analogous results has been obtained for toral automorphisms by Polo, for shift maps on symbolic systems by Kadyrov, and for Cartan actions on p-adic groups by Ruhr.

On Substitution Tilings with Infinite Local Complexity

Jeong-Uup Lee
Catholic Kwandong University, Korea
Boris Solomyak

There has been a lot of study on substitution tilings with finite local complexity in terms of their dynamical, spectral, and geometric properties. However very little is known for substitution tilings with infinite local complexity and it is getting more attention. We develop a sufficient condition for the substitution dynamical system to be uniquely ergodic. The unique ergodicity on substitution dynamical system has been developed already in [Frank-Sadun '14] and [Frettloh-Richard '14]. But we find a concrete measure of cylinder sets and it is used to make a connection to dynamical and spectral properties of substitution tilings. As a result, we find four equivalent properties for the substitution dynamical systems to be not weakly mixing.

Mean Equicontinuity, Bounded Complexity and Discrete Spectrum

Jian Li
Shantou University, Peoples Rep of China

On the Construction of Translation Surfaces from Piecewise Rotation Maps of the Circle

Hitoshi Nakada
Keio University, Japan
Kae Inoue

The notion of the critical iterate of piecewise rotation maps of the circle was first introduced by Cruz and da Rocha (2005). We will see that for any given singularity order vector and any marked singularity, there exists a piecewise rotation of the circle such that one of discontinuous points and one of its associated critical iterate generate a translation surface which has the given singularity orders with the given marked singularity.

On the Construction of the Natural Extensions of the Nearest Integer Complex Continued Fraction Maps

Rie Natsui
Japan Women's University, Japan

We consider the nearest integer complex continued fraction map associated to the Euclidean fields $\mathbb{Q}(\sqrt{-d})$, $d = 1, 2, 3, 7, 11$.
For each map, we see that there is an absolutely continuous ergodic invariant probability measure.
We will explain how to construct the natural extension of each map on a subset of $\mathbb{C} \times \mathbb{C}$. Then the invariant measure for this extension is derived from the hyperbolic measure on $\mathbb{H}^3$ and the density function of the absolutely continuous invariant measure is given as its marginal.

Universality of Toral Automorphisms

Anthony Quas
University of Victoria, Canada
Terry Soo

We prove that an ergodic automorphism T of the torus, $\mathbb{T}^d$ is universal. That is: for any ergodic measure-preserving transformation, S, of a space (X, μ) such that $h_\mu(S)$ is smaller than the topological entropy of T, there exists an injective factor map $\pi \colon X \to \mathbb{T}^d$ (informally, the dynamical system T contains a copy of (X, μ, S)).
Benjy Weiss has shown how to use this to obtain as a corollary that an invariant proposed by Halmos contains less information than the rational spectrum of a transformation.

Topological Transitivity and Wandering Intervals for Group Actions on the Line $\mathbb{R}$

Enhui Shi
Soochow University, Peoples Rep of China
Lizhen Zhou

For every group G, we show that either G has a topologically transitive action on the line $\mathbb{R}$ by orientation-preserving homeomorphisms, or every orientation-preserving action of G on $\mathbb{R}$ has a wandering interval. According to this result, all groups are divided into two types: transitive type and wandering type, and the types of several groups are determined. We also show that every finitely generated orderable group of wandering type is indicable. As a corollary, we show that if a higher rank lattice Γ is orderable, then Γ is of transitive type.

A Dense Subset of Continuous Functions with Uncountably Many Ergodic Maximizine Measures

Mao Shinoda
Keio University, Japan

The main purpose of the ergodic optimization is to describe invariant measures which maximize the space average of a continuous function on a dynamical system. The generic uniqueness and the low-complexity of maximizing measures are proved by several authors. We prove, on the other hand, there exits a dense subset of continuous functions which have uncountably many ergodic maximizing measures. Moreover in the case of subshift of finite type we can show that the uncountably many measures have positive entropy. The main idea of our proof is the application of the Bishop Phelps theorem to the context of maximizing measures.

Limit Sets in Topologically Transitive Cylinder Transformations

Artur Siemaszko
University of Warmia and Mazury in Olsztyn, Poland
Jan Kwiatkowski

Let X be a compact metric monothetic group and $T : X \longrightarrow X$ be a homeomorphism of X. Let $f : X \longrightarrow \mathbb{R}$ be a continuous function (called *a cocycle*). By a *cylinder transformation* we mean a homeomorphism. $T_f : X \times \mathbb{R} \longrightarrow X \times \mathbb{R}$ (or rather a $\mathbb{Z}$–action generated by it) given by the formula

$$T_f(x, r) = (Tx, f(x) + r).$$

In [1] H. Poincaré addressed the problem of what types of orbits may coexist in such a system. We give a very brief history of the problem and describe some recent results.

References

[1] H. Poincaré, *Mèmoire sur les courbes dèfinies par une èquation diffèrentielle*, 1882.

Enveloping Semigroup of the Induced Systems

Manpreet Singh
Indian Institute of Technology, Delhi, India
Anima Nagar

Let (X, d) be a compact metric space and f be a continuous surjection on X. The dynamical system (X, f) induces the system $(2^X, f_*)$ where 2^X is the set of all non-empty closed subsets of X with the Hausdorff metric.
The enveloping semigroup of (X, f) is the closure of $\{f^n : n \in \mathbb{N}\}$ in X^X with the topology of pointwise convergence. We study some comparison between the enveloping semigroups $E(X)$ and $E(2^X)$.

Birkhoff Sum Fluctuations in Substitution Dynamical Systems

Younghwan Son
POSTECH, Korea
E. Paquette

In this talk we consider Birkhoff sums of eigenfunctions for substitution dynamical systems with incidence matrix having eigenvalues of modulus 1. We will show a central limit theorem for non-coboundary eigenfunctions of the incidence matrix with eigenvalue modulus 1.

Factor Maps and Equilibrium States

Jisang Yoo
Sungkyunkwan University, Korea

We consider an arbitrary finite-to-one extension X of a topological dynamical system Y and count the number of ways to lift an invariant measure on the base system Y, up to multiplicity. In addition, we analyze the structure of arbitrary (possibly infinite-to-one) factor maps between symbolic dynamical systems and consider the problem of counting the number of ways to lift an invariant measure in an entropy maximizing way. This will involve the use of tools like class degree, relative equilibrium states, and Poulsen simplex.

Topology and Topological Sequence Entropy

Ruifeng Zhang
Hefei University of Technology, Peoples Rep of China

Let X be a compact metric space and $T : X \longrightarrow X$ be continuous. Let $h^*(T)$ be the supremum of topological sequence entropies of T over all subsequences of $\mathbb{Z}_+$ and $S(X)$ be the set of the values $h^*(T)$ for all continuous maps T on X. It is known that $\{0\} \subseteq S(X) \subseteq \{0, \log 2, \log 3, \ldots\} \cup \{\infty\}$. Only three possibilities for $S(X)$ have been observed so far, namely $S(X) = \{0\}$, $S(X) = \{0, \log 2, \infty\}$ and $S(X) = \{0, \log 2, \log 3, \ldots\} \cup \{\infty\}$. In this paper we completely solve the problem of finding all possibilities for $S(X)$ by showing that in fact for every set $\{0\} \subseteq A \subseteq \{0, \log 2, \log 3, \ldots\} \cup \{\infty\}$ there exists a one-dimensional continuum X_A with $S(X_A) = A$. In the construction of X_A we use Cook continua.

Special Session 14: Topological Nonlinear Analysis and Applications

Zalman Balanov, University of Texas at Dallas, USA
Jianshe Yu, Guangzhou University, Peoples Rep of China
Slawomir Rybicki, Nicolaus Copernicus University, Poland
Meymanat Farzamirad, University of Alberta, Canada

This session aims at an overview of new topological and variational methods developed for studying nonlinear differential equations and dynamical systems. It will cover topics ranging from Conley index theory, topological degree, bifurcation theory, mini-max variational methods to n-body problems, Hamiltonian systems, Schrödinger equations and other applied problems.

Hopf Bifurcation of Relative Periodic Solutions in Symmetric Delay Differential Systems: Equivariant Degree Approach

Zalman Balanov
University of Texas at Dallas, USA
P. Kravetc, W. Krawcewicz, D. Rachinskii, H. Wu

The analysis of Hopf bifurcation of periodic solutions from an equilibrium state (both in non-equivariant and equivariant setting) has been done by many authors using different techniques. A natural counterpart of an equilibrium state (resp. periodic solution) in dynamical systems with continuous symmetries is a relative equilibrium, i.e. an equilibrium modulo the group action (resp. relative periodic solution). M. Krupa proposed a method for analysis of the bifurcation of relative periodic solutions from a relative equilibrium for generic systems of smooth ODEs based on normal slice and center manifold reductions. Although, in principal, Krupa's method can be applied to generic systems of smooth equivariant FDEs, we are not aware of any published work in this direction. On the other hand, if a system of ODEs/FDEs exhibits a lack of smoothness (systems with hysteresis), or lack of genericity (multiple/resonant eigenvalues crossing the imaginary axis non-transversally), absence of a flow (mixed FDEs), then the center manifold reduction is either impossible or meets serious difficulties. In my talk, I will show how to adapt the equivariant degree method for analysis of Hopf bifurcation of relative periodic solutions to FDEs with a lack of smoothness/genericity. As an application, a delay differential model of coupled identical passively mode-locked semiconductor lasers will be considered.

On Three Critical Points Theorems with and Without Local Minimums

Marlene Frigon
University of Montreal, Canada
Marlene Frigon

We present three different types of theorems establishing the existence of at least three critical points for suitable functionals. In the first type, there might be no local minimum among the three critical points. In the second type, at least one of them is a local minimum. Finally, in the third type, there are two local minimums among the three critical points. The conditions imposed on the functionals ensure that the existence of each of these three critical points relies on a suitable linking pairs geometry. Many particular cases of these results are presented.

Variational Approaches to Periodic Solutions to Delay Differential Equations

Zhiming Guo
Guangzhou University, Peoples Rep of China
Zhiming Guo, Jianshe Yu

The study of periodic solution problem to delay differential equations, which has extensively received a lot of attention, has had a long history. Many excellent results on existence of their periodic solutions were obtained by using some fixed point theorems, Hopf bifurcation theorems, coincidence degree theory and Poincaré-Bendixson theorem, etc. Since 1997, variational methods were applied to study the existence and multiplicity of periodic solutions to delay differential equations. A series of interesting and significant results are obtained via variational approaches in the literature. In this talk, we present a detailed survey of basic results and recent development on this research area. Some interesting open problems are also addressed.

Distribution Profiles in Gene Transcription Activated by the Cross-Talking Pathway

Feng Jiao
Guangzhou University, Peoples Rep of China
Jianshe Yu

Gene transcription is a stochastic process, manifested by the heterogeneous mRNA distribution in an isogenic cell population. Bimodal distribution has been observed in the transcription of stress responsive genes which have evolved to be easily turned on and easily turned off. This is against the conclusion in the classical two-state model that bimodality occurs only when the gene is hardly turned on and hardly turned off. In this paper, we extend the gene activation process in the two-state model by introducing the cross-talking pathway that involves the random selection between a spontaneous weak basal pathway and a stress-induced strong signaling pathway. By deriving exact forms of mRNA distribution at steady-state, we find that the cross-talking pathway is much more likely to trigger the bimodal distribution. Our fur-

ther analysis reveals an observed transition among the decaying, bimodal and unimodal mRNA distribution for stress gene upon enhanced stimulations. Especially, the bimodality occurs when the stress-induced signalling pathway is more frequently selected, reinforcing the assertion that bimodal transcription is a general feature of stress genes in response to environmental change.

Application of the Equivariant Degree Method in Nonlinear Reversible Differential Equation

Wieslaw Krawcewicz
University of Texas at Dallas, USA

The existence of periodic solutions in Γ-symmetric reversible second order systems of type $\ddot{x} = F(x)$ can be effectively studied by means of the $\Gamma \times O(2)$-equivariant degree with values in the Bernside ring $A((\Gamma \times O(2))$. In this talk we will illustrate the techniques and methods based on the equivariant degree to study the existence of periodic solutions, topological classification of their spatio-temporal symmetries, and the related bifurcation problems including global continuation. We will present several examples of possible applications in various applied areas.

Ize Conjecture and Low Dimensional Equivariant Dynamics

Reiner Lauterbabach
University of Hamburg, Germany

In this talk we present recent progress on Ize's conjecture, i.e. we give an overview of known counter examples to the Ize conjecture. The main issue is the question of bifurcation in those situations where all fixed point spaces are even dimensional. Besides presenting such bifurcation results, we start a discussion of dynamics beyond steady state bifurcation. A general theory of such bifurcations is still open.

Uniqueness of Positive Solutions of the Brezis-Nirenberg Problems in Thin Spherical Annular Domains and Its Application

Naoki Shioji
Yokohama National University, Japan
Kohtaro Watanabe

We study the uniqueness of a positive solution of the Brezis-Nirenberg problems in thin spherical annular domains by using the Pohozaev identities. As an application, we consider symmetry breaking bifurcations of the problems.

Ambrosetti-Prodi Type Result Under Local Coercivity Conditions

Elisa Sovrano
University of Udine, Italy
Fabio Zanolin (University of Udine, Italy)

Starting from the pioneering work by Ambrosetti and Prodi [Ann. Mat. Pura Appl., 1972], by means of the expression "Ambrosetti-Prodi type problems" one refers to nonlinear parameter-depending boundary value problems (BVPs) of the form $Lu+g(u) = \mu$ where the derivative of the nonlinear term g crosses some eigenvalues of the linear differential operator L and there is a change in the number of the solutions, as the real parameter μ varies. Motivated by a problem proposed by Ambrosetti [Atti Accad. Naz. Lincei, 2011], which regards the periodic case of these problems, we consider periodic BVPs associated with first order or second order ODEs of the form $x' + g(t, x) = \mu$ or $x'' + f(x)x' + g(t, x) = \mu$, respectively. We present results of existence and multiplicity of Ambrosetti-Prodi type assuming local coercivity conditions on g, in order to relax assumptions of uniform type previously considered in the literature. The proofs are carried out by both the use of a classical topological argument, that is the Mawhin coincidence degree, and also by means of new tools which permit to deal with the more general case of a local coercive nonlinearity. The results are based on joint works with F. Zanolin (University of Udine, Italy).

Bifurcation of Space Periodic Solutions in Symmetric Reversible FDEs

Haopin Wu
The University of Texas at Dallas, Taiwan
Z. Balanov

Second order differential equation $v'' + g(v) = 0$ satisfies the property: if $v(t)$ is a solution, then so is $v(-t)$. In the context of first order (time evolutionary) ODEs, this property, which is called *time-reversibility*, means that the system respects the involutionary symmetry connecting time-reversed evolution and forward-time evolution. Local bifurcations of periodic solutions to parametrized reversible systems of ODEs have been studied intensively by Buzzi, Lamb, Vanderbauwhede et al.
In FDE systems, *time-reversal symmetry* involves "information traveling back in time", which is paradoxical in common sense. Therefore, instead of time-reversible FDEs, one may consider (networks of) *space-reversible* FDEs. Motivating physical examples are related to stationary solutions to PDEs with nonlocal interaction: *mixed delay differential equations* (MDDEs) and *integro-differential equations* (IDEs).
In this talk, a $(\Gamma \times O(2))$-equivariant degree based method is proposed to study bifurcation of 2π-periodic solutions in symmetric networks of reversible FDEs (the finite group Γ reflects symmetries of coupling while $O(2)$ is associated with periodicity and reversibility). Such a bifurcation occurs when eigen-

values of linearization move along the imaginary axis (without change of stability of the trivial solution and possibly without $1 : k$ resonance). In the case of S_4-symmetric networks of MDDEs and IDEs, we present exact computations of full equivariant bifurcation invariants.

Mathematical Models on Killing Mosquitoes with More Mosquitoes

Bo Zheng
Guangzhou University, Peoples Rep of China
Moxun Tang, Jianshe Yu, Zhiyong Xi, Linchao Hu, Megen Huang

Mosquitoes are primary vectors of life-threatening diseases such as dengue, malaria, and Zika. A new control method involves releasing mosquitoes carrying bacterium Wolbachia into the natural areas to infect or eradicate wild mosquitoes and block disease transmission. In this talk, some mathematical models, including models of ordinary differential equations, delay differential equations, stochastic equations, and reaction-diffusion equations, on the dynamics of Wolbachia spread in mosquitoes will be presented. Instead of elaborating the mathematical proof, I will focus more on what our results imply biologically that may be instructive to the design of release strategies. This is a joint work with Prof. Moxun Tang, a microbiologist Prof. Zhiyong Xi from Michigan State University, Prof. Jianshe Yu and Dr. Mugen Huang from Guangzhou University, and Dr. Linchao Hu from Sun Yat-Sen University.

Homoclinic Solutions for Some Discrete Systems

Zhan Zhou
Guangzhou University, Peoples Rep of China

In this talk, I will show some new sufficient conditions on the existence of homoclinic solutions for some discrete systems, especially the existence of homoclinic solutions with both components are not zero are obtained for coupled discrete system.

Special Session 15: Analysis of Evolutionary Systems of Partial Differential Equations for Complex Materials

Anja Schlömerkemper, University of Wüzburg, Germany
Sarka Necasova, Institute of Mathematics, Academy of Sciences, Czech Rep
Arghir Zarnescu, Basque Center for Applied Mathematics, Bilbao, Spain
Giulio Schimperna, University of Pavia, Italy

This session is devoted to the analytical investigation of partial differential equations for systems that show underlying microstructures as for instance in complex fluids or in magnetic, elastic or magnetoelastic materials. We will bring together international experts who will exchange ideas on how to deal with the various microstructures that show up in the systems mentioned. We will learn about new mathematical methods from the different but related research directions, which may foster collaborations and lead to new insights. This will help to better understand the effective behaviour caused by the microstructures in the system.

A Coupled Bulk-Surface Model for Lipid Raft Formation in Cell Membranes

Helmut Abels
University of Regensburg, Germany
Johannes Kampmann

We investigate a model for lipid raft formation and dynamics in biological membranes which was proposed by Garcke, Kampmann, Rätz and Röger. The model includes a cholesterol exchange between cytosol and cell membrane and takes lipid-cholesterol interaction energy into account. This leads to a diffusion equation in the bulk coupled to a Cahn-Hilliard type equation on the surface that describes the membrane. In particular, the evolution on the surface describes a relaxation dynamics for an energy which includes lipid-phase separation and lipid-cholesterol interaction energy. In dependence on the constitutive laws for the exchange between cell membrane and cytosol, the resulting model can include nonequilibrium processes. We discuss mathematical properties of the model and a connection to the Ohta-Kawasaki equation by showing that weak solutions to this model converge to the solutions to the Ohta-Kawasaki equation if certain parameters in the model tend to zero.

Partial Energy-Dissipation and Smoothing Effect for Constrained Allen-Cahn Equations

Goro Akagi
Tohoku University, Japan
Goro Akagi

In this talk, we shall discuss energy-dissipation phenomena and smoothing effect of solutions for an Allen-Cahn equation with nondecreasing constraint. More precisely, we shall treat the Cauchy-Dirichlet problem for the equation

$$u_t = \Big(\Delta u - W'(u)\Big)_+,$$

where $W(\cdot)$ is a double-well potential and $(\cdot)_+$ is the positive-part function. Hence solutions are constrained to be nondecreasing. Such a constraint prevents emergence of the energy-dissipation and smoothing effect, which are completely realized for classical Allen-Cahn equation. As a result, one can prove non-existence of global attractor in any L^p-spaces (and hence, in any Sobolev spaces). On the other hand, this equation still involves a gradient structure, and hence, energy-dissipation and smoothing effect emerge in an incomplete way. Main purpose of this talk is to explain how to extract such an incomplete emergence of energy-dissipation and smoothing effect for evolution equations with nondecreasing constraint from a functional analytic point of view. This talk is based on a joint work with M. Efendiev (München).

Dispersal Towards Food: the Singular Limit of an Allen-Cahn Equation

Danielle Hilhorst
CNRS/Univ. Paris-Sud, France
Yongjung Kim, Dohyun Kwon, Thanh Nam Nguyen

The effect of dispersal under heterogeneous environment is studied in terms of the singular limit of an Allen-Cahn equation. Since biological organisms often slow down their dispersal if food is abundant, a food metric diffusion is taken to include such a phenomenon. The migration effect of the problem is approximated by a mean curvature flow after taking the singular limit which now includes an advection term produced by the spatial heterogeneity of food distribution. It is shown that the interface moves towards a local maximum of the food distribution. In other words, the dispersal considered here is not a trivialization process anymore, but an aggregation one towards food.

Wellposedness of Weak Solutions to Viscoelasticity

Xianpeng Hu
City University of Hong Kong, Hong Kong

We will discuss some recent progress in the mathematical analysis for the viscoelastic fluid flow. Global existences of weak solutions will be the main subject.

The oscillation and concentration of approximating solutions are two main obstacles. A variant of “effective” viscous flux turns again to be a key tool to deal with the weak stability.

Analysis of a Model for a Magneto-Viscoelastic Material

Martin Kalousek
University of Würzburg, Germany

The talk is concerned with a mathematical model for a class of materials which possess the special property that they respond mechanically to applied magnetic fields and they change their magnetic properties in response to mechanical stresses. The model consists of a system of equations for the balance of momentum that is coupled with systems of equations describing the evolution of quantities related to elastic and magnetic properties of the material. The issue of existence as well as uniqueness of a solution to the system of partial differential equations under consideration will be discussed.

On the Passage from Nonlinear to Linearized Viscoelasticity

Martin Kruzik
Czech Academy of Sciences, Czech Rep
Manuel Friedrich

We formulate a quasistatic nonlinear model for non-simple viscoelastic materials at a finite-strain setting in the Kelvin's-Voigt's rheology where the viscosity stress tensor complies with the principle of time-continuous frame-indifference. We identify weak solutions in the nonlinear framework as limits of time-incremental problems for vanishing time increment. Moreover, we show that linearization around the identity leads to the standard system for linearized viscoelasticity and that solutions of the nonlinear system converge in a suitable sense to solutions of the linear one. The same property holds for time-discrete approximations and we provide a corresponding commutativity result.

Tumour Dynamics with the Cahn-Hilliard Equation: Modelling, Analysis and Estimation

Kei Fong Lam
The Chinese University of Hong Kong, Hong Kong

Tumours are highly complex entities coupling many chemical and physical reactions across multiple spatial and temporal scales. To obtain a tractable description of the main dynamical features, such as proliferation, apoptosis, necrosis and chemotaxis, we propose to use a class of diffuse interface models consisting of the Cahn-Hilliard equation. We discuss a collection of analytical results such as well-posedness and asymptotic limits, as well as some recent results concerning parameter identification with optimal control and Bayesian inversion, so that practitioners can match model predictions with experimental data.

Approximating the Fluctuations in Random Heterogeneous Problems

Frederic Legoll
Ecole des Ponts, France

Computing the homogenized properties of random materials is often very expensive. A standard approach consists in considering a large domain, and solving the so-called corrector problem on that domain, submitted to e.g. periodic boundary conditions. Because the computational domain is finite, the approximate homogenized properties are random, and fluctuate from one realization of the microstructure to another. We have recently introduced several efficient numerical approaches to reduce the statistical noise. These approaches allow to compute the expectation of the homogenized coefficients in a more efficient manner than brute force Monte Carlo methods.

Beside the (averaged) behavior of the material response on large space scales (which is given by its homogenized limit), another question of interest is to understand how much this response fluctuates around its coarse approximation, before the homogenized regime is attained. More generally, we aim at understanding which parameters of the distribution of the material coefficients affect the distribution of the response, and whether it is possible to compute that latter distribution without resorting to a brute force Monte Carlo approach.

This talk, based on joint works with P.-L. Rothe, will review the recent progresses made on these questions, both from the theoretical and numerical viewpoints.

On Positivity of Solutions to Microelectromechanical Systems with General Permittivity Profile

Christina Lienstromberg
Leibniz University Hannover, Germany
Joachim Escher, Pierre Gosselet

Qualitative properties of solutions to the evolution problem modelling microelectromechanical systems (MEMS) with general permittivity profile are investigated. The system couples a parabolic evolution problem for the displacement of a membrane with an elliptic moving boundary problem for the electric potential in the region between the membrane and a rigid ground plate. Conditions are specified which ensure non-positivity of the membrane's displacement. Moreover, numerical evidence is provided for the existence of non-constant permittivity profiles that induce positive membrane displacements while the corresponding small-aspect ratio model possesses only non-positive displacements.

A New Class of Approximate Lennard-Jones Potentials

Tai-Chia Lin
National Taiwan University, Taiwan

The Lennard-Jones (LJ) potential, a well-known mathematical model for the interaction between a pair of ions, has important applications in many fields of biology, chemistry and physics. Using band-limited functions, we obtained a class of approximate LJ (LJ_a) potentials which can be used to derive PNP-steric equations as a model to describe the ion transport through (biological) channels (with B. Eisenberg, 2014). However, due to the strong singularities of LJ potentials, it is difficult to calculate the Fourier transform of LJ_a potentials. In this lecture, a new class of approximate LJ (LJ_{na}) potentials with precise formulas of Fourier transform will be introduced. Using techniques of complex analysis, we may prove that the energy of ions interacting by the LJ_{na} potential can approach to the energy of ions interacting by the LJ_a potential. This may provide a new PNP type model for the ion transport through channels.

Thermal Effects for Liquid Crystal Materials

Chun Liu
Illinois Institute of Technology, USA
Francesco de Anna

In this work, I will present work on temperature dependent models for macroscopic liquid crystal materials. The model is consistent with the general thermodynamics laws, as well as the energetic variational framework for these systems in isothermal situations.

A Property in Smectic Liquid Crystals

Jinhae Park
Chungnam National University, South Korea
Jinhae Park

In this talk, we consider a simplified version of energy functional in Smectic Liquid Crystals and study a special solution with a certain symmetry. We discuss some local minimzers of the energy functional.

Asymptotic Analysis of Discrete Energies with Angular Terms

Annie Raoult
Université Paris Descartes, France
Hervé Le Dret

In the past few years, much attention was devoted to the derivation of effective continuous energies for discrete lattices. Applications encompass atomic nanostructures and, more surprisingly, living tissues. The variational framework is widely used and allows to obtain rigorous results valid for large deformations. We will give a review of several related works and we will show that because of their intricate hexagonal geometry, graphenes require specific analysis.

Topological Defect Structures in Nematic Complex Fluids

Miha Ravnik
University of Ljubljana, Slovenia

Nematic complex fluids are interesting for a variety of mechanisms, including self-assembly, activity, topology, and material flow, that can be used to create complex three-dimensional field structures. Here, we present various nematic structures, bulk fields or in combination with colloidal particles, as recently achieved by numerical modelling and in collaborations with experiments that show potential for application in complex optics and photonics. Central to the structures are complex conformations of topological defects, as they can bind, stabilise, or distort the structure. More specifically, we show knotted defects, fractal defect states and design of quasicrystalline symmetry in the structure, all unusual properties with further possible implications or use. More generally, complex soft matter is shown to offer a fascinating testbed and platform for designing and testing of topology-conditioned complex materials.

On an Evolutionary Model for Magnetoelasticity in Eulerian Description - Existence of Weak Solutions

Anja Schlömerkemper
University of Würzburg, Germany

A fundamental issue in the modeling of magnetoelastic materials is that elasticity is phrased in Lagrangian coordinates whereas magnetism is phrased in Eulerian coordinates. We discuss a model that is completely phrased in Eulerian coordinates and takes microstructures of the magnetization into account. The model presented is a system of partial differential equations that contains (1) the incompressible Navier-Stokes equations with magnetic and elastic terms in the stress tensor obtained by a variational approach, (2) a regularized transport equation for the deformation gradient and (3) the Landau-Lifshitz-Gilbert equation for the dynamics of the magnetization. We will indicate the derivation of the model and will present results on the analytical properties of the system.

The Dugdale Criterion As Generalized Energy Criterion

Maria Specovius-Neugebauer
University of Kassel, Germany
Sergej Nazarov

We consider a plane elasticity problem in a domain $\Omega \subset \mathbb{R}^2$ with an internal straight crack. Dealing with small strains is an essential assumption while using the linear elasticity theory, however, the asymptotic behavior of the solution u near the crack tips leads to unbounded strains (and stresses) which needs further physical interpretation. In the model of Dugdale it is assumed that the stress cannot overcome a certain yield stress σ_c. Therefore ahead of the crack Λ a (one dimensional) plastic zone appears, where $\sigma_{22} = -\sigma_c$ and σ_c is large in comparison with the external loading. Leonov and Panasyuk used the argument that in the crack mouth there exist strong adhesive forces which must be overcome to initiate the growth of the crack. Both models lead to the same nonlinear mathematical problem and the same fracture criterion for crack growth under Mode I loading. It can be generalized to the case of orthotropic media, like in the case of isotropic media it coincides with the classical energy criterion (and Irwin's criterion, resp.) asymptotically. With asymptotic analysis it can be shown how the geometry of the problem influences the solution, this leads to a refined Dugdale-criterion. By introducing a generalized energy functional this refined criterion can be interpreted as an energy criterion as well.

Special Session 16: Stochastic Modeling in Biology, Phase transitions and Fluid Dynamics: Theory and Approximation

Tadahisa Funaki, University of Tokyo, Japan
Danielle Hilhorst, CNRS and University Paris-Sud Paris-Saclay, France
Roger Temam, Indiana University, Bloomington, USA

Stochastic models play a significant role in science, with numerous applications such as biology, phase transition, climate science, geophysics and engineering. Moreover, sitting at the interface between probability theory, mathematical analysis and the theory of parabolic and hyperbolic partial differential equations, these problems provide interesting and challenging mathematical complications. Motivated by the need from both the outside and inside of the mathematical community, our session will focus on the recent advances in various application fields. We aim to bring together researchers to discuss such models from both the theoretical and applied points of view, with topics including the regularity behavior of solutions, the stochastic dynamics and the stochastic numerical analysis and implementations.

Stochastic Motion of Droplets in the Cahn-Hilliard Equation

Dirk Blömker
Universität Augsburg, Germany
Alexander Schindler (Augsburg)

We study the stochastic Cahn-Hilliard equation, which is a model describing the phase separation and subsequent coarsening of binary alloys. In the nucleation regime almost spherical droplets appear, and we approximate the infinite dimensional stochastic dynamics of these droplets by the motion along a finite dimensional slow manifold. The main results are effective equations (given by stochastic ordinary differential equations) on the slow manifold and the stochastic stability of the manifold.

Some Results on Leslie-Gower Competition Models

Yunshyong Chow
Academia Sinica, Taiwan

Leslie-Gower competitive systems in discrete time for d species can be described as follows: For time $t \geq 0$ and $i = 1, \ldots, d$,

$$x_i(t+1) = \frac{a_i x_i(t)}{1 + \sum_{j=1}^{n} c_{ij} x_j(t)}, \qquad (1)$$

where $x_i(t)$ means the population size of the i-th species at time t, all carrying capacities $(a_i - 1)/c_{ii} > 0$ and the inter-specific coefficients $c_{ij} > 0$.

By rescaling, we may assume wlog that all $c_{ii} = 1$. Biologists believe that only the species with highest carrying capacity will survive as time goes to ∞. It is not always so as Cushing et ce. (2004) gave a complete description of the global behavior for $d = 2$. Ackleh and Jang (2005) conjectured that such a principle holds when all $c_{ij} = 1$. Chow and Hsieh (2013) verified this conjecture. A simple proof can be found in Ackleh, Sacker and Salceanu (2014). For two species, Chow and Jang (2014) added Allee effect to Eq (1) which becomes

$$\begin{cases} x(t+1) = \frac{a_1 x(t)}{1+x(t)+c_1 y(t)} \frac{x(t)}{m_1 + x(t)}, \\ y(t+1) = \frac{a_2 y(t)}{1+c_2 x(t)+y(t)} \frac{y(t)}{m_2 + y(t)}. \end{cases} \qquad (2)$$

The global behavior of the system, which may possess four interior steady states, is clarified. Study on the special case that $d = 3$ and all $c_{ij} = c$ for $i \neq j$ will be reported.

Geometrical Methods for Stochastic Dynamics

Jinqiao Duan
Huazhong University of Science and Technology, Peoples Rep of China

Dynamical systems arising in engineering and science are often subject to random fluctuations. The noisy fluctuations may be Gaussian or non-Gaussian, which are modeled by Brownian motion or α-stable Levy motion, respectively. Non-Gaussianity of the noise manifests as nonlocality at a macroscopic level. Stochastic dynamical systems with non-Gaussian noise (modeled by α-stable Levy motion) have attracted a lot of attention recently. The non-Gaussianity index α is a significant indicator for various dynamical behaviors. The speaker will overview recent advances in geometrical methods for stochastic dynamical systems, including random invariant sets, random invariant manifolds, stochastic bifurcation, mean exit time, escape probability, tipping time, most probable orbits, and transition pathways between metastable states.

Existence and Uniqueness Results for a Stochastic Phase-Field Problem

Perla El Kettani
University of Paris-Sud, France
Danielle Hilhorst

In this talk, I will present the proof of the existence and uniqueness of the solution of a stochastic phase-field problem modelling the melting of ice. I will consider a multiplicative noise induced by a Q-Brownian motion. The starting point is to perform a Galerkin approximation and establish a priori estimates for the solution pair of the corresponding approximate system. Since in the stochastic case the solutions are not differentiable in time and since we have an additional random variable, the usual compactness method used in the theory of deterministic nonlinear partial differential equations cannot be applied; therefore we use a stochastic compactness method based upon fractional Sobolev spaces.

2D Euler Equations with Random Initial Conditions

Franco Flandoli
Scuola Normale Superiore of Pisa, Italy

S. Albeverio and coworkers, including A.B. Cruzeiro, initiated long ago the investigation of stochastic solutions of 2D Euler equations having Gaussian properties. Stationary inverse cascade turbulence has almost Gaussian statistics and the previous model could be an idealization of this particular turbulent regime. We add results to the previous investigations of this model in several directions including: i) point vortex approximation; ii) probabilistic analysis of regions of high vorticity intensity.

Motion by Mean Curvature from Glauber-Kawasaki Dynamics

Tadahisa Funaki
Waseda University, Japan
Kenkichi Tsunoda

We derive the motion by mean curvature directly from a particle system on a periodic square lattice called Glauber-Kawasaki dynamics. This problem was discussed by Katsoulakis and Souganidis, but our method is different and relies on an estimate on the relative entropy.

Existence and Uniqueness Results for a First Order Conservation Law Involving a Q-Brownian Motion

Yueyuan Gao
Tohoku University, Japan
Tadahisa Funaki, Danielle Hilhorst

We consider a first order stochastic conservation law with a multiplicative source term involving a Q-Brownian motion. We first recall a result stating that the discrete solution obtained by a finite volume method converges along a subsequence in the sense of Young measures to a measure-valued entropy solution as the maximum diameter of the volume elements and the time step tend to zero [1]. This convergence result yields the existence of a measure-valued entropy solution. We then present the Kato's inequality and as a corollary we deduce the uniqueness of the measure-valued entropy solution as well as the uniqueness of the weak entropy solution. The Kato's inequality is proved by a doubling of variables method; in order to apply this method, we prove the existence and the uniqueness of the weak solution of an associated nonlinear parabolic problem [2]. This is joint work with Tadahisa Funaki and Danielle Hilhorst.

References

[1] T. Funaki, Y. Gao and D. Hilhorst, Convergence of a finite volume scheme for a stochastic conservation law involving a Q-Brownian motion, Accepted for publication in DCDS B, AIMS, hal-01404119.

[2] T. Funaki, Y. Gao and D. Hilhorst, Uniqueness of the entropy solution of a stochastic conservation law with a Q-Brownian motion, in preparation.

Path-By-Path Regularization by Noise for Scalar Conservation Laws

Benjamin-Manuel Gess
MPI MIS Leipzig, Germany
Khalil Chouk

We prove a path-by-path regularization by noise result for scalar conservation laws. In particular, this proves regularizing properties for scalar conservation laws driven by fractional Brownian motion and generalizes the respective results obtained in [G., Souganidis; Comm. Pure Appl. Math. (2017)]. We show that rho-gamma-irregularity is a sufficient path-by-path condition implying such regularizing effects. In addition, we introduce a new path-by-path scaling property which is also shown to be sufficient to imply regularizing effects.

Weak and Strong Orders of Convergence for Approximations of the Allen-Cahn Equation Using Splitting Strategies

Ludovic Goudenege
CNRS, France
Charles-Edouard Brehier

The numerical schemes for the approximation of stochastic partial differential equations relies on classical schemes for stochastic differential equations with all the theory concerning strong and weak orders of convergence in time. In this talk the spatial discretization will not be considered, but I will present the proof for the time discretization on the example of phase transitions and fluids dynamics of Allen-Cahn type. Usually proving the existence of solution and orders of convergence depends on the global Lipschitz regularity of nonlinear terms. But when the nonlinear terms have dissipative effects, it is again possible to define a solution. Actually we can use this type of proof to study the orders of convergence of time schemes, as soon as we can explicitly treat the nonlinear term, for instance via splitting strategies. In this talk, I will explain how we can prove the weak and strong orders of convergence for approximations of Allen-Cahn equation using splitting strategies, thanks to an explicit treatment of the nonlinear terms. All these results will be validated by numerical experiments.

Stationary Solutions to Stochastic Compressible Navier-Stokes System

Martina Hofmanova
University Bielefeld, Germany

We study the long-time behavior of solutions to a stochastically driven Navier-Stokes system describing the motion of a compressible viscous fluid driven by a temporal multiplicative white noise perturbation. The existence of stationary solutions is established in the framework of Lebesgue-Sobolev spaces pertinent to the class of weak martingale solutions. The methods are based on new global-in-time estimates and a combination of deterministic and stochastic compactness arguments. In contrast with the deterministic case, where related results were obtained only under rather restrictive constitutive assumptions for the pressure, the stochastic case is tractable in the full range of constitutive relations allowed by the available existence theory. This can be seen as a kind of stabilizing effect of the noise on the global-in-time solutions.

A Relation Between Modeled Distributions and Paracontrolled Distributions

Masato Hoshino
Kyushu University, Japan

In the world of singular SPDEs, there are two big theories: the theory of regularity structures by Hairer and the paracontrolled calculus by Gubinelli, Imkeller and Perkowski. In Hairer's theory, the solution is defined as a modeled distribution, which represents a local behavior of the solution. In the GIP theory, the solution is defined as a paracontrolled distribution, which is defined by global but nonlocal operators. Our aim is to find an equivalence between these two notions.

Boundary Layer Analysis for the Stochastic Nonlinear Reaction-Diffusion Equations

Chang-Yeol Jung
UNIST, Korea
Youngjoon Hong, Roger Temam

Singularly perturbed stochastic (and deterministic) nonlinear reaction-diffusion equations are considered. We first study the governing problem posed in the channel domain with lateral periodicity and extend the results to general smooth domains.
Introducing corrector functions, which correct the boundary values discrepancies, we are able to develop the convergence analysis. For the analysis, we make use of the maximum principle to estimate the corrector functions. The stochastic problems also rely on the deterministic corrector functions, which lead to simpler computations than those of the stochastic version of the correctors.

Locally Uniform Birkhoff Ergodic Theorem

Xue-Mei Li
Imperial College London, England

Given a family of diffusion operators satisfying Hormander's conditions or sub-elliptic estimates, we obtain a Birkhoff ergodic theorem (law of large numbers) with rate, the rate is locally uniform in the parameter. These theorems can be used to obtain limit theorems for stochastic differential equations.

Markovian and Non-Markovian Closures for Stochastic PDEs Based on Parameterizations

Honghu Liu
Virginia Tech, USA
Mickael D. Chekroun, James C. McWilliams, Shouhong Wang

In this talk, we discuss a new approach to deal with the parameterization problem of small spatial scales by large ones for stochastic partial differential equations (SPDEs). The approach is variational in nature, and relies on stochastic parameterizing manifolds, which are random manifolds aiming to provide—in a mean square sense—approximate small-scale parameterizations. We will highlight a simple semi-analytic approach to determine such manifolds based on backward-forward auxiliary systems. We will then illustrate the approach on low-dimensional closure problems in the context of a stochastically driven Burgers-type equation in presence of small viscosity and linearly unstable modes. The role of path-dependent, non-Markovian coefficients arising in the related closure systems will also be emphasized. This is joint work with Mickael D. Chekroun (UCLA), James C. McWilliams (UCLA) and Shouhong Wang (IUB).

On Scaling Limit of a Cost in Adhoc Network Model

Yukio Nagahata
Niigata University, Japan

We are interested in giving a mathematical formula of a cost in adhoc network model. In our model, the cost is formulated as an application of first-passage percolation and the motion of devices is random and an asymptotic density of devices is formulated by hydrodynamic limit. In this talk, we give an asymptotic of this cost by means of the cost of first-passage percolation and the solution of the partial differential equation given by hydrodynamic limit.

Diffusion in Coulomb Environment and a Phase Transition

Hirofumi Osada
Kyushu University, Japan

I talk homogenization of diffusion in two-dimensional Euclidian space in a periodic Coulomb environment. That is, we consider a periodic point process in the plane and the diffusion has the repulsive interaction with the two-dimensional Coulomb potential with inverse temperature β to each particle in the periodic point process. We first prove that the diffusion is diffusive with non-degenerated effective diffusion constant γ. We then remove one particle from the environment and consider the diffusive scaling limit of the diffusion. Then its new effective constant depending on the inverse temperature β has a phase transition whose critical point is given explicitly in terms of the original effective diffusion constant γ of the periodic homogenization problem. Using this result, we present explicit bounds for the critical point of the self-diffusion matrices of the two-dimensional strict Coulomb interacting Brownian motions with respect to inverse temperature β.

Variational Solutions to Nonlinear Stochastic Differential Equations in Hilbert Spaces

Michael Roeckner
Bielefeld University, Germany
Viorel Barbu

One introduces a new variational concept of solution for the stochastic differential equation $dX + A(t)X\,dt + \lambda X\,dt = X\,dW$, $t \in (0,T)$; $X(0) = x$ in a real Hilbert space where $A(t) = \partial\varphi(t)$, $t \in (0,T)$, is a maximal monotone subpotential operator in H while W is a Wiener process in H on a probability space $\{\Omega, \mathcal{F}, \mathbb{P}\}$. In this new context, the solution $X = X(t,x)$ exists for each $x \in H$, is unique, and depends continuously on x. This functional scheme applies to a general class of stochastic PDE not covered by the classical variational existence theory and, in particular, to stochastic variational inequalities and parabolic stochastic equations with general monotone nonlinearities with low or superfast growth to $+\infty$. Reference: arXiv:1802.07533

On the Green-Kubo Formula and the Gradient Condition on Currents

Makiko Sasada
The University of Tokyo, Japan

In the diffusive hydrodynamic limit for a symmetric interacting particle system (such as the exclusion process, the zero range process, the stochastic Ginzburg-Landau model, the energy exchange model), a possibly non-linear diffusion equation is derived as the hydrodynamic equation. The bulk diffusion coefficient of the limiting equation is given by Green-Kubo formula and it can be characterized by a variational formula. In the case the system satisfies the gradient condition, the variational problem is explicitly solved and the diffusion coefficient is given from the Green-Kubo formula through a static average only. In other words, the contribution of the dynamical part of Green-Kubo formula is 0. In this talk, we consider the converse, namely if the contribution of the dynamical part of Green-Kubo formula is 0, does it imply the system satisfies the gradient condition or not. We show that if the equilibrium measure is product and L^2 space of its single site marginal is separable, then the converse also holds. The result gives a new physical interpretation of the gradient condition.

As an application of the result, we consider a class of stochastic models for energy transport studied by Gaspard and Gilbert, where the exact problem is discussed for this specific model.

On Stochastic Heat Equations

Shang-Yuan Shiu
National Central University, Taiwan
Davar Khoshnevisan, Kunwoo Kim, Carl Mueller

We consider the following parabolic SPDEs:

$$\frac{\partial}{\partial t}u(t,x;\lambda) = \triangle u(t,x;\lambda) + b(u(t,x;\lambda)) + \lambda\sigma\left(u(t,x;\lambda)\right)\frac{\partial^2}{\partial t \partial x}\xi(t,x),$$

subject to nonrandom initial data $u_0(x)$ where $\xi(t,x)$ is a Gaussian noise. One motivation for studying this model is a physical phenomena: intermittency, so we will introduce it first. In this talk, we will discuss limiting behaviors of the solutions $u(t,x;\lambda)$ in variant $b(x)$ and how λ effects their limiting behaviors.

Metastability for Singular Stochastic PDE

Hendrik Weber
University of Warwick, England
Nils Berglund, Giacomo di Gesù, Pavlos Tsatsoulis

We study the Allen-Cahn equation perturbed by a small space-time white noise. It is by now well known that if the space dimension is larger than one, such a white noise driven equation has to be interpreted in a renormalised sense, i.e. formally infinite counterterms have to be added to make sense of the solution. In this talk I will discuss the small noise behaviour of this renormalised SPDE. I will discuss the validity of the Kramers-Eyring law, which gives a precise description of the asymptotic expected transition times between meta-stable states in the small noise limit. An important part of the proof is a synchronisation result, which states that two solutions which start nearby will never separate with overwhelming probability.

Harnack Inequalities for Reflected SPDEs and Their Applications

Bin Xie
Shinshu University, Japan

We are mainly interested in the dimension-free Harnack inequalities for the stochastic partial differential equation with reflecting walls. The reflected SPDE is one kind of random obstacle problems and is usually regarded as the infinite-dimensional Skorokhod problem. We first establish the Harnack inequalities for the reflected SPDE and then apply them to the study of various properties of Markov semigroup associated with the reflected SPDE, for example, the uniqueness of invariant measures, the entropy-cost inequality and hyperboundedness.

Sharp Interface Limit for Stochastically Perturbed Mass Conserving Allen-Cahn Equation

Satoshi Yokoyama
Waseda University, Japan
Tadahisa Funaki

In this talk, we introduce the sharp interface limit for a mass conserving Allen-Cahn equation with a small parameter $\epsilon > 0$ added an external noise. The stochastic term destroys the precise conservation law, instead the total mass changes like a Brownian motion in time. The evolutional law of the limit hypersurface is described by the mass conserving mean curvature flow with a multiplicative white noise. In order to study the limit, we use the asymptotic expansion method.
However, in our case, when ϵ tends to 0, each term except the leading term appearing in the expansion of the solution in a small parameter ϵ diverges. This is because our equation contains the noise which converges to a white noise and the products or the powers of the white noise diverge when ϵ tends to 0. We introduce how to control those terms when ϵ tends to 0 to obtain our goal.
Reference: T. Funaki and S. Yokoyama, Sharp interface limit for stochastically perturbed mass conserving Allen-Cahn equation, to appear in Ann. Probab.

Transitions in a Genetic Transcriptional Regulatory System Under Lévy Motion

Yayun Zheng
Huazhong University of Science and Technology, Peoples Rep of China

Based on a stochastic differential equation model for a single genetic regulatory system, we examine the dynamical effects of noisy fluctuations, arising in the synthesis reaction, on the evolution of the transcription factor activator in terms of its concentration. The fluctuations are modeled by Brownian motion and α-stable Levy motion. Two deterministic quantities, the mean first exit time (MFET) and the first escape probability (FEP), are used to analyse the transitions from the low to high concentration states. A new geometric concept is introduced to quantify the basin stability of the low concentration region, characterised by the escaping behaviour.

Stochastic Heat Equation with Values in a Riemannian Manifold: Non-Compact Case

Xiangchan Zhu
Beijing Jiaotong University, Peoples Rep of China

In this talk, we will give the existence of martingale solutions to the stochastic heat equation with spatial variable on the whole line and taking values in a non-compact Riemannian manifold, which admits Wiener measure as an invariant measure by using Dirichlet form. Moreover, we establish the log-Sobolev inequality when Ricci curvature is strictly positive which implies exponential ergodicity for the process. Also when the sectional curvature is negative, the process is non-ergodic.

Conservative Stochastic Cahn-Hilliard Equation

Rongchan Zhu
Beijing Institute of Technology, Peoples Rep of China

We consider the stochastic 2-dimensional Cahn-Hilliard equation which is driven by the derivative in space of a space-time white noise. We use two different approaches to study this equation. First we prove that there exists a unique solution Y to the shifted equation, then $X := Y + Z$ is the unique solution to stochastic Cahn-Hilliard equation, where Z is the corresponding O-U process. Moreover, we use Dirichlet form approach to construct the probabilistically weak solution of the original equation below. By clarifying the precise relation between the solutions obtained by the Dirichlet forms approach and X, we can also get the restricted Markov uniqueness of the generator and the uniqueness of martingale solutions to the equation.

Special Session 17: Nonlinear Elliptic and Parabolic Problems

Sze-Bi Hsu, National Tsing-Hua University, Hsinchu, Taiwan
Julian Lopez-Gomez, Complutense University of Madrid, Spain

Nonlinear elliptic and parabolic equations model a great variety of real systems of huge interest from a social perspective. In particular, they are pivotal in population dynamics, though their applications cover all branches of Science and Engineering. This special section tries to gather some of the very best experts in this field in order to discuss the more recent advances, as well as some of the most significant applications in population dynamics. In particular, an important amount of attention will be focused into the role played by spatial and temporal heterogeneities in the dynamics of these models.

A Priori Estimates for Elliptic Equations with a Source Term Involving the Product of the Function and Its Gradient

Marie-Francoise Bidaut-Veron
University Francois Rabelais, Tours, France
Marta Garcia-Huidobro, Laurent Veron

Here we consider the nonnegative solutions of equations in a punctured ball $B(0,R)\setminus\{0\}\subset\mathbb{R}^N$ or in $\mathbb{R}^N$, of the type

$$-\Delta u = u^p|\nabla u|^q$$

where $p+q>1$. We give new a priori estimates on the solutions and their gradient, and Liouville type results, extending the case $q=0$ of the well known Emden-Fowler equation. We use Bernstein technique and Osserman's or Gidas-Spruck's type methods.

Asymptotic Behavior of a Least Energy Solution for Henon Equation with Neumann Boundary Condition

Jaeyoung Byeon
KAIST, Korea

We consider the following equation

$$\Delta u - u + |x|^{\alpha}u^p = 0 \text{ in } \Omega, \frac{\partial u}{\partial n} = 0 \text{ on } \partial\Omega.$$

Here Ω is a domain in the unit ball $B(0,1)$ in $\mathbf{R}^N$ with $\partial\Omega\cap\partial B(0,1)\neq\emptyset$. We are concerned on the least energy solutions of the problem for $p\in(1,(N+2)/(N-2)]$ and large $\alpha>0$. The asymptotic shape of the solutions for large $\alpha>0$ strongly depends on the range of $p\in(1,N/(N-2)], p\in(N/(N-2),(N+2)/(N+2)), p=(N+2)/(N-2)$ and a geometric shape of $\partial\Omega$ near $\partial B(0,1)$.

Resident-Invader Dynamics in Infinite Dimensional Systems

Robert Stephen Cantrell
University of Miami, USA
Chris Cosner, King-Yeung Lam

Motivated by evolutionary biology, we study general infinite-dimensional dynamical systems involving two species - the resident and the invader. Sufficient conditions for competitive exclusion phenomena are given when the two species play similar strategies. Those conditions are based on invasibility criteria, for instance, evolutionarily stable strategies in the framework of adaptive dynamics.
Such questions were first proposed and studied by Geritz and collaborators in the early 2000's for a class of ordinary differential equations. We extend and generalize previous work in two directions. First, we consider analytic semiflows in infinite-dimensional spaces. Secondly, we device an argument based on the Hadamard graph transform method that does not depend on the monotonicity of the two-species system. Our results are applicable to a wide class of reaction-diffusion models as well as models with nonlocal diffusion operators.

Singular Solutions of Weighted Divergence-Form Equations

Ting-Ying Chang
Monash University, Australia
Florica Cirstea

We study the existence and complete classification of the isolated singularities for weighted divergence-form equations of the form

$$\operatorname{div}(\mathcal{A}(|x|)|\nabla u|^{p-2}\nabla u) = b(x)h(u) \quad \text{in } B_1(0)\setminus\{0\}.$$

We assume that $\mathcal{A}\in C^1(0,1]$, $b\in C(B_1\setminus\{0\})$ and $h\in C[0,\infty)$ are positive functions associated with regularly varying functions of index θ, σ, and q at 0, 0, and ∞ respectively.
We reveal how the interplay between these indices affect the classification of the singular solutions near the singularity. We are particularly interested in the so-called critical cases of the indices which are important in the non-power nonlinearity case as they represent the threshold between having a trichotomous classification, a dichotomous classification or no singularities at all. Our results complement a series of

works on removable and non-removable singularities in the framework of pure-power nonlinearities, where the difficulties that arise from regular variation do not appear.

Singular Points Effects in Some Elliptic Equations with Sobolev Exponent and Hardy Potential

Jann-Long Chern
National Central University, Taiwan

In this talk, we will study how the singularities affect the existence, non-existence and structure of solutions for some elliptic equations with Sobolev exponent and Hardy potential.

Sharp Asymptotic Profiles of Singular Solutions to an Elliptic Equation with a Sign-Changing Non-Linearity

Florica Cirstea
University of Sydney, Australia
F. Robert, J. Vètois

In this talk, we provide a full description of the asymptotic profile near zero of the positive solutions to elliptic equations of the form $-\Delta u = |x|^{-s}u^{2^*(s)-1} - \mu u^q$ in $B \setminus \{0\}$, where B denotes the unit ball of $\mathbb{R}^n$ with $n \geq 3$, $s \in (0,2)$, $2^*(s) := 2(n-s)/(n-2)$, $\mu > 0$ and $q > 1$. We show that along with a Caffarelli-Gidas-Spruck type profile, the solution may develop two new singular behaviors at zero due to the interaction between the critical Hardy-Sobolev type potential and the pure power non-linearity. We also prove the existence of all these sharp singular profiles. The results are based on works with F. Robert (University of Lorraine) and J. Vètois (McGill University).

Dynamics of Populations with Multiple Movement Modes

Chris Cosner
University of Miami, USA
Robert Stephen Cantrell, Xiao Yu

Most classical models for the movement of organisms assume that all individuals have the same patterns and rates of movement, but there is empirical evidence that movement rates and patterns may vary among individuals. One way to capture variation in dispersal is to allow individuals to switch between two distinct dispersal modes. We consider models for populations with logistic-type local population dynamics whose members can switch between two different nonzero rates of diffusion. The resulting reaction-diffusion systems can be cooperative at some population densities and competitive at others. We analyze the dynamics of such systems on bounded regions. (Traveling waves and spread rates have been studied by others for similar models in the context of biological invasions.) The analytic methods include ideas and results from reaction-diffusion theory, semi-dynamical systems, and bifurcation/continuation theory.

Existence and Concentration of Positive Solutions for a Class of Kirchhoff Type Equations

David Costa
University of Nevada Las Vegas, USA
J. Jianjun, J.M. do Ò

A result will be presented on existence and concentration of positive solutions for a class of Kirchhoff-type equations on RN involving a potential function, a positive parameter and a non-linearity at critical growth. A basic idea underlying such singular perturbation problems is to obtain, under suitable conditions on the potential functions and the non-linearity, a localized bound state solution concentrating at a local minimum of the potential function as the parameter approaches zero. In particular, in our approach, a monotone-type conditions or the (AR) condition are not required. This is joint work with J. Zhang and J.M. do Ò.

Invasion Speeds in a Competition-Diffusion Model with Mutation

Elaine Crooks
Swansea University, Wales
Luca Borger, Aled Morris

We consider a reaction-diffusion system modelling the growth, dispersal and mutation of two phenotypes. This model was proposed by Elliott and Cornell (PLOS One, 2012), who presented evidence that for a class of dispersal and growth coefficients and a small mutation rate, the two phenotypes spread into the unstable extinction state at a single speed that is faster than either phenotype would spread in the absence of mutation. After first verifying that, under reasonable conditions on the mutation and competition parameters, the spreading speed of the two phenotypes is determined by the linearisation about the extinction state, we prove that the spreading speed is a non-increasing function of the mutation rate (implying that greater mixing between phenotypes leads to slower propagation), determine the ratio at which the phenotypes occur in the leading edge in the small-mutation limit, and discuss the effect of trade-offs between dispersal and growth on the spreading speed of the phenotypes. This is joint work with Luca Börger and Aled Morris (Swansea).

Global Dynamics of Generalized Logistic Equations

Daniel Daners
University of Sydney, Australia
Julian Lopez-Gomez

We consider a parameter dependent parabolic logistic population model with diffusion and degenerate logistic term allowing for refuges for the population. The aim is to remove quite restrictive geometric and smoothness conditions on the refuge commonly used. The key is a simplified construction of a supersolution involving a limit theorem for the principal eigenvalue. The method works for non-selfadjoint operators and does not make use of any regularity condition of the refuge. At the same time we also simplify other arguments commonly used in the literature.

High Multiplicity of Positive Solutions to Indefinite Problems Arising in Population Genetic Models

Guglielmo Feltrin
University of Turin, Italy

We discuss existence of multiple positive solutions to some migration-selection models in population genetics governed by nonlinear differential equations of the form $u'' + q(t)g(u) = 0$, where $q(t)$ is sign-changing and $g\colon [0,1] \to \mathbb{R}$ is continuous with $g(0) = g(1) = 0$, $g(s) > 0$ for $0 < s < 1$ and $\lim_{s\to 0^+} g(s)/s = 0$. Inspired by the works of P. H. Rabinowitz (Indiana Univ. Math. J., 1973/74), Y. Lou, W.M. Ni, L. Su (Discrete Contin. Dyn. Syst., 2010), we obtain new multiplicity results dealing with various boundary conditions, including Dirichlet and Neumann ones. More exactly, using topological techniques based on shooting methods and on Mawhin's coincidence degree theory, we show how the number of positive solutions is affected by the nodal behaviour of $q(t)$. We also focus on radially symmetric solutions to boundary value problems associated with $\Delta u + q(x)g(u) = 0$. This talk is based on joint works with A. Boscaggin (University of Turin) and E. Sovrano (University of Udine).

Single Species Growth Consuming Inorganic Carbon with Internal Storage in a Poorly Mixed Habitat

Sze-Bi Hsu
National Tsing-Hua University, Taiwan
Adrian Lam, Feng-Bi Wang

In this talk we present a PDE system modeling the growth of a single species population consuming inorganic carbon that is stored internally in a poorly mixed habitat. Inorganic carbon takes the forms of CO2 (dissolved CO2 and carbonic acid) and CARB (bicarbonate and carbonate ions), which are substitutable in their effects on algal growth. We first establish a threshold type result on the extinction and persistence of the species in terms of the sign of a principal eigenvalue associated with a nonlinear eigenvalue problem.

Existence of Bubbling Solutions for the Liouville System

Hsin-Yuan Huang
National Chiao-Tung University, Taiwan

In this talk, I will briefly introduce the recent developments on the Liouville system. The system is closely related to several models of chemistry, ecology and physics. The existence result for the bubbling solutions defined on a flat tours will be present.

Eigenvalue Estimate of Nonlinear Schrödinger Equations

Tai-Chia Lin
National Taiwan University, Taiwan

The virial theorem is a nice property for the linear Schrödinger equation in atomic and molecular physics as it gives an elegant ratio between the kinetic and potential energies and is useful in assessing the quality of numerically computed eigenvalues. If the governing equation is a nonlinear Schrödinger equation with power-law nonlinearity, then a similar ratio can be obtained but there seems no way of getting any eigenvalue estimate. It is surprising as far as we are concerned that when the nonlinearity is either square root or saturable nonlinearity (not a power-law), one can develop a virial theorem and eigenvalue estimate of nonlinear Schrödinger equations in R2 with square root and saturable nonlinearity, respectively. Furthermore, the eigenvalue estimate can be used to prove the 2nd order term (which is of order $\ln \Gamma$) of the lower bound of the ground state energy as the coefficient Γ of the nonlinear term tends to infinity.

New Trends in Diffusive Lotka-Volterra Competition

Julian Lopez-Gomez
Complutense University, Madrid, Spain
S. Fernandez-Rincon

In this talk we review some of the very last advances in the theory of competition in spatially heterogeneous landscapes. In particular, we get an optimal uniqueness result sharpenning substantially a previous result of X. He and W.M. Ni. Furthermore, we can characterize the dynamics of the model for sufficiently small diffusion rates.

Uniqueness and Multiplicity of Large Solutions

Luis Maire
Rey Juan Carlos University, Spain
J. Lopez-Gomez

The main goal of this lecture is analyzing the uniqueness and multiplicity of large solutions for the problem $u'' = f(u)$ in $[-T, T]$ and $u(-T) = u(T) = +\infty$, where $f \in \mathcal{C}^1[0, +\infty)$ satisfies $f(0) = 0$. Although this problem has been widely studied in the literature, in order to get uniqueness it is usually assumed that f is increasing. In this talk, it will be shown that this condition is far from being necessary to get uniqueness. Moreover, some examples exhibiting an arbitrarily large number of large solutions will be constructed. These results are optimal for this one-dimensional problem.

Front Propagation Through a Perforated Wall and a De Giorgi Type Theorem

Hiroshi Matano
Meiji University, Japan
Henri Berestycki, Francois Hamel

We consider a bistable reaction-diffusion equation on $\mathbb{R}^N \setminus K$, where K represents an obstacle that can be regarded as an infinite wall of finite thickness with periodically arrayed holes. More precisely, K is a set with smooth surface whose projection onto the x_1-axis is bounded, while it is periodic in the rest of variables $\tilde{x} := (x_2, \ldots, x_N)$. We assume that $\mathbb{R}^N \setminus K$ is connected. Our goal is to study what happens when a planar traveling front coming from $x_1 = +\infty$ encounters the wall K. We first show that there is clear dichotomy between *blocking* and *propagation*, and that there is no intermediate behavior of solutions. To prove this result, we first establish a De Giorgi type theorem for the elliptic equation $\Delta u + f(u) = 0$ on $\mathbb{R}^N$, which may be of interest in its own right. Then we will discuss sufficient conditions for propagation and those for blocking.

The Role of Pathfollowing in Nonlinear Elliptic Problems

Marcela Molina Meyer
Universidad Carlos III, Spain

Pathfollowing has proved to be an important tool in the study of nonlinear elliptic equations and systems. The aim of this talk is to show some of most useful pathfollowing contributions of our research team to the theory of nonlinear elliptic problems during the last three decades.

Oscillatory Bifurcation for Semilinear Eigenvalue Problems

Tetsutaro Shibata
Hiroshima University, Japan

We consider the asymptotic behavior of bifurcation curves for semilinear eigenvalue problems with some oscillatory nonlinear terms.
We treat the case where λ is parameterized by the maximum norm $\alpha = \|u_\lambda\|_\infty$ of the solution u_λ associated with λ. Then λ is represented as $\lambda = \lambda(\alpha)$.
In this talk, we focus on the problems where $\lambda(\alpha) \to \pi^2/4$ as $\alpha \to \infty$.
We establish the asymptotic formulas for $\lambda(\alpha)$ as $\alpha \to \infty$ and $\alpha \to 0$ with the exact second terms. Then we understand well the whole structures of the bifurcation curves.

Some Removability Results for K-Hessian Equation and K-Curvature Equation

Kazuhiro Takimoto
Hiroshima University, Japan

In 1983, Kràl proved the following theorem: *Let Ω be a domain in $\mathbb{R}^n$. If $u \in C^1(\Omega)$ is harmonic in $\Omega \setminus u^{-1}(0) = \{x \in \Omega \mid u(x) \neq 0\}$, then u is harmonic in the whole domain Ω.* This type of removability results has been extensively studied in the literature. Among other things, we have studied the Kràl type removability result for fully nonlinear equations. Our result in this talk is an extension of the previous one for the so-called k-Hessian equation and k-curvature equation.

High Multiplicity of Positive Solutions for Superlinear Indefinite Problems with Neumann Boundary Conditions

Andrea Tellini
Universidad Autonoma de Madrid, Spain

I will present a result of high multiplicity of positive solutions for a class of superlinear indefinite problems in one spatial dimension, with Neumann boundary conditions. Such problems consist of a modified version of the stationary diffusive logistic equation, in which the indefinite nature comes from a sign-changing weight in front of the nonlinearity.
I will also present the structure of the corresponding bifurcation diagrams, using the size of the region where the weight is positive as a main bifurcation parameter.

Hölder Estimates for Second-Order Operators with Mixed Boundary Conditions

Tom Ter Elst
University of Auckland, New Zealand
J. Rehberg

We investigate linear elliptic, second-order boundary value problems with mixed boundary conditions on domains with a rough boundary. Assuming only boundedness/ellipticity on the coefficient function and very mild conditions on the geometry of the domain, including a very weak compatibility condition between the Dirichlet boundary part and its complement, we prove Hölder continuity of the solution. Moreover, Gaussian Hölder estimates for the corresponding heat kernel are derived.
This is joint work with Joachim Rehberg.

Longtime Behavior of Solutions of a SIS Epidemiological Mode

Je-Chiang Tsai
National Tsing Hua University, Taiwan
Xinfu Chen, Yaping Wu

We investigate the longtime behavior of solutions of a susceptible-infectious-susceptible (SIS) epidemiological model, proposed to explain an epidemiological phenomenon that pathogen spread does not necessarily keep pace with its host invasion.

Positivity of Bifurcating Solutions of Indefinite Concave-Convex Problems

Kenichiro Umezu
Ibaraki University, Japan
Uriel Kaufmann, Humberto Ramos Quoirin

In this talk, we consider semilinear elliptic equations of indefinite concave-convex type with Dirichlet and Neumann boundary conditions and prove the existence of loop components of nonnegative solutions bifurcating from a trivial line in some cases. Positivity of nontrivial nonnegative solutions on the loops is also studied by use of a continuation argument.

Initial Trace of Positive Solutions of Semilinear Fractional Diffusion Equations

Laurent Veron
University of Tours, France
Huyuan Chen

In this paper, we prove the existence of an initial trace $\mathcal{T}_u$ for any positive solution u to the semilinear fractional diffusion equation (H)

$$\partial_t u + (-\Delta)^\alpha u + f(t,x,u) = 0 \quad \text{in} \quad (0,+\infty)\times\mathbb{R}^N,$$

where $N \geq 1$, the operator $(-\Delta)^\alpha$ with $\alpha \in (0,1)$ is the fractional Laplacian, $f : \mathbb{R}_+ \times \mathbb{R}^N \times \mathbb{R}_+ \to \mathbb{R}$ is a Caratheodory function satisfying $f(t,x,u)u \geq 0$ for all $(t,x,u) \in \mathbb{R}_+ \times \mathbb{R}^N \times \mathbb{R}_+$ and $\mathbb{R}_+ = [0,+\infty)$. We define the regular set of the trace $\mathcal{T}_u$ as an open subset $\mathcal{R}_u \subset \mathbb{R}^N$ carrying a nonnegative Radon measure ν_u such that

$$\lim_{t\to 0}\int_{\mathcal{R}_u} u(t,x)\zeta(x)\,dx = \int_{\mathcal{R}_u}\zeta\,d\nu_u, \quad \forall\,\zeta \in C_0^2(\mathcal{R}_u),$$

and the singular set $\mathcal{S}_u = \mathbb{R}^N \setminus \mathcal{R}_u$ as the set points a such that

$$\limsup_{t\to 0}\int_{B_\gamma(a)} u(t,x)\,dx = +\infty \quad \text{for any } \gamma > 0.$$

We also study the reverse problem of constructing a positive solution to (H) with a given initial trace $(\mathcal{S},\nu)$, where $\mathcal{S} \subset \mathbb{R}^N$ is a closed set and ν is a positive Radon measure on $\mathcal{R} = \mathbb{R}^N \setminus \mathcal{S}$ and develop the case $f(t,x,u) = t^\beta u^p$ with $\beta > -1$ and $p > 1$.

A Reaction-Diffusion Model of Harmful Algae and Zooplankton in an Ecosystem

Feng-Bin Wang
Chang Gung University, Taiwan

In this talk, we investigate an unstirred chemostat system modeling the interactions of two essential nutrients (e.g., nitrogen and phosphorus), harmful algae (e.g., P.parvum and cyanobacteria), and a small-bodied zooplankton in an ecosystem. To obtain a weakly repelling property of a compact and invariant set on the boundary, we introduce an associated elliptic eigenvalue problem. It turns out that the model system admits a coexistence steady state and is uniformly persistent provided that the trivial steady state, two semi-trivial steady states and a global attractor on the boundary are all weak repellers.
This talk is based on my recent work joint with Drs. Sze-Bi Hsu and Xiao-Qiang Zhao.

Global Existence of Solutions to the Cauchy Problem for an Attraction–Repulsion Chemotaxis System in Two Dimensions in the Attractive Dominant Case

Tetsuya Yamada
National Institute of Technology, Fukui College, Japan
Toshitaka Nagai

We consider the Cauchy problem for an attraction–repulsion chemotaxis system with the chemotactic coefficient of the attractant and that of the repellent. In the case the repulsion dominates or cancels the attraction, the nonnegative solutions to the Cauchy problem exist globally in time. On the other hand, in the case the attraction dominates, there are blowing-up solutions in finite time under the suitable assumption on the total mass of the nonnegative initial data. In this talk, we shall discuss the global existence of nonnegative solutions to the Cauchy problem in the case the attraction dominates.

Maximization of the Total Population in a Reaction-Diffusion Model with Logistic Growth

Eiji Yanagida
Tokyo Institute of Technology, Japan
Kentaro Nagahara

This talk is concerned with a nonlinear optimization problem that naturally arises in population biology. We consider the effect of spatial heterogeneity on the total population of a biological species at a steady state, using a reaction-diffusion logistic model. Our objective is to maximize the total population when resources are distributed in the habitat to control the intrinsic growth rate, but the total amount of resources is limited.
It is shown that under some conditions, any local maximizer must be of "bang-bang type, which gives a partial answer to the conjecture addressed by Ding et al. (2010).
To this purpose, we compute the first and second variations of the total population.
When the growth rate is not of bang-bang type, it is shown in some cases that the first variation becomes nonzero and hence the resource distribution is not a local maximizer.
When the first variation becomes zero, we prove that the second variation is positive.
These results implies that the bang-bang property is essential for the maximization of total population.

Complex Dynamics in a ODE Model Related to Phase Transition

Fabio Zanolin
University of Udine, Italy
Duccio Papini

Motivated by some recent studies on the Allen–Cahn phase transition model with a periodic non-autonomous term, we prove the existence of complex dynamics for the second order equation

$$-\ddot{x} + (1 + \varepsilon^{-1} A(t))G'(x) = 0,$$

where $A(t)$ is a non-negative T-periodic function and $\varepsilon > 0$ is sufficiently small. In particular, we find a full symbolic dynamics made by solutions which oscillate between any two different strict local minima x_0 and x_1 of $G(x)$. Our approach is based on a variant of the theory of topological horseshoes.

Special Session 18: Emergence and Dynamics of Patterns in Nonlinear Partial Differential Equations and Related Fields

Danielle Hilhorst, CNRS and University Paris-Sud Paris-Saclay, France
Yoshihisa Morita, Ryukoku University, Japan
Junping Shi, College of William and Mary, USA

The solution structures of many nonlinear partial differential equations reveal the emergence and the evolution of very exciting patterns. Such nonlinear models come from various fields of mathematical science, including material science as well as life sciences. In this session, we will bring together recent studies on solutions of nonlinear partial differential equations related to pattern formation, dynamics, and bifurcations, presenting new aspects of solutions capturing nonlinear phenomena together with underlying solution structures.

Higher-Order, Unidirectional Models for Surface Water Waves. Resolution into Solitary Waves.

Jerry Bona
University of Illinois at Chicago, USA

Discussed will be a class of higher-order models for the one-way propagation of long crested water waves. Such models have appeared often in the literature, but it has proven difficult to provide global well-posedness results for them in the same way as is done for the lower-order KdV- and BBM-type models. Within this class, one discerns a special subclass of Hamiltonian models. It is shown that these do indeed possess the desired well-posedness theory. These models are further investigated numerically and interesting conjectures emerge.
The report is based on joint work with Xavier Carvajal, Hongqiu Chen, Youngjoon Hong, Ohannes Karakashian, Mahendra Panthee and Marcia Scialom.

Elliptic Schroedinger Systems with Large Interaction Forces

Jaeyoung Byeon
KAIST, Korea

We consider the elliptic Schroedinger system with three components on a bounded domain or the entire space When the inter-species interaction forces are very large and the self intra-species interaction forces are fixed, there are many interesting pattern formation phenomena. In this talk, I would like to introduce some results when the inter-species interaction forces are all positive and large.

Travelling Waves in Anisotropic Smectic C* Liquid Crystals

Elaine Crooks
Swansea University, Wales
Michael Grinfeld, Geoff McKay

We consider minimality conditions for the speed of monotone travelling waves in a model of a sample of smectic C* liquid crystal subject to a constant electric field, dealing with both isotropic and anisotropic cases. Such conditions are important in understanding switching properties of a liquid crystal, and our focus is on understanding how the presence of anisotropy can affect the speed of switching. Through a study of travelling-wave solutions of a quasilinear parabolic equation, we obtain an estimate of the influence of anisotropy on the minimal speed, sufficient conditions for linear and non-linear minimal speed selection mechanisms to hold in different parameter regimes, and a characterisation of the boundary separating the linear and non-linear regimes in parameter space. This is joint work with Michael Grinfeld and Geoff McKay (Strathclyde).

On a Dispersal Model for Farmers and Hunter-Gatherers in the Neolithic Transition

Jan Elias
University of Graz, Austria
Danielle Hilhorst, M. Humayun Kabir, Masayasu Mimura, Yoshihisa Morita

The Neolithic migration of farmers in regions previously inhabited by hunter-gatherers is a long time studied ecological problem. We propose a new reaction-diffusion model that consists of equations governing the spatio-temporal evolution of sedentary and migrating farmers and hunter-gatherers in the Neolithic transition. Ecologically, the model stems from the fact that a lifestyle of agriculture and settlement, as it allows for a larger population, is evolutionary advantageous than hunting and gathering. Therefore, in our modelling framework, we assume that farmers do not migrate unless the population density pressure forces them. The population density pressure is, in a sense, linked with a certain level of development of farming and food-producing technology and associated sedentary lifestyle traits such as pottery making, domestication of various plants and animals, and related social and cultural changes, security, trading. We prove the global well-posedness of the system and show numerically that for a suitable value of a stay-or-migrate-threshold the model reproduces the spread of farming that corresponds to the archeological findings in Europe. Moreover, due to different time-scales between the intra-population dynamics (slow) and the stay-or-migrate decision making (fast) we consider a singular limit of the proposed problem when the conversion stay-or-migrate-rate grows above all the limits. We show that in the limit we obtain a nonlinear diffusion problem of interesting spatio-temporal properties such as the breakdown of radial symmetry of the solution.

Forced Waves of the Fisher-KPP Equations in a Shifting Environment

Jian Fang
Harbin Institute of Technology, Peoples Rep of China

This talk concerns the Fisher-KPP equation in a shifting environment. We are interested in the questions whether such a forced moving KPP nonlinearity from behind can give rise to traveling waves with the same speed and how they attract solutions of initial value problems when they exist. Under a sublinearity condition, we obtain the complete existence and multiplicity of forced traveling waves as well as their attractivity except for some critical cases. In these cases, we provide examples to show that there is no definite answer unless one imposes further conditions depending on the heterogeneity of the nonlinearity. This talk is based on joint works with Henri Berestycki, and with Yijun Lou and Jianhong Wu.

On Singular Limits in Fluid Mechanics

Eduard Feireisl
Czech Academy of Sciences, Czech Rep

We consider several singular limits arising in the study of viscous/inviscid fluids. We suggest a new approach where the solutions of the primitive systems are considered in the most general class possible while the target system is supposed to admit a smooth solution. Examples of singular limits for the compressible Euler system are discussed.

Oscillating Wavetrains in a FitzHugh-Nagumo Type RD System

M. Osman Gani
Jahangirnagar University, Bangladesh
Toshiyuki Ogawa

It is important to study spatially periodic traveling wave solutions for many partial differential equations in order to understand the mechanism of pattern formation in the higher-dimensional problems. In this study, we introduce a reaction-diffusion system for excitable media to mimic the cardiac cell activities. We investigate numerically the existence and stability of periodic traveling wave solutions in a two-dimensional parameter plane. Our results show two types of stability change in the periodic traveling waves: Eckhaus type and Hopf type. There are two families of periodic traveling waves: fast and slow. The fast family is stable in the case of standard FitzHugh-Nagumo system. However, we observe that the fast family becomes unstable in our model. As a result, it bifurcates to an "oscillating PTW". We explain this phenomenon by calculating the essential spectra of the periodic traveling wave solutions numerically. In two-dimensions, we show spiral wave breakup in a one-parameter family of solutions as a consequence of the stability of periodic traveling wave solutions.

Pattern Formation in High-Order PNP-Type Systems

Nir Gavish
Technion, Israel
Doron Elad, Arik Yochelis

The Poisson-Nernst-Planck (PNP) theory is one of the most widely used analytical methods to describe electrokinetic phenomena for electrolytes. The model, however, considers isolated charges and thus is valid only for dilute ion concentrations. The key importance of concentrated electrolytes in applications has led to the development of a large family of generalized PNP models.
However, the wide family of generalized PNP models fails to capture key phenomena recently observed in experiments and simulations, such as self-assembly and under-screening in concentrated electrolytes.
In this talk, we present a thermodynamically consistent mean-field model for concentrated solutions that goes beyond the PNP framework. We show that the model describes bulk and interfacial pattern-formation, map the parameter regimes of distinct self-assembly behaviors and the relevant bifurcation associated with them, and consider their effect on electrostatic screening and transport. In particular, we reveal a novel mechanism of under-screening.

Propagating Terraces: Existence and Properties

Thomas Giletti
University of Lorraine, France

In this talk we will discuss the dynamics of solutions of one-dimensional reaction-diffusion equations, where space-time transitions from one equilibrium to another typically occur. We will consider the general case when the profile of the propagation is not characterized by a single front, but by a layer of several fronts. This means, intuitively, that transition between equilibria may occur in several successive steps involving intermediate stationary states. In joint works with Arnaud Ducrot and Hiroshi Matano, we deal with such a situation by using the so-called zero number argument, which consists in using the number of zeros of the solution (of a linear parabolic equation) as a discrete Lyapunov function. We will show that the large-time behavior of solutions is described by a family of travelling fronts which we call a propagating terrace.

Concentration and Singular Waves in a Nonlocal Reaction-Diffusion Equation

Quentin Griette
Meiji University, Japan

I consider a reaction-diffusion equation modelling the propagation of a species that possesses a continuum of phenotypic traits. The spatial dynamics of the individuals is modelled by a diffusion process, and the population undergoes a reproduction-mutation-competition dynamics at each spatial point, which is modelled by a nonlocal operator acting on the bounded domain representing the phenotypic space. Under some conditions on the fitness function, the mutation rate and the dimensionality of the domain, a concentration phenomenon is known to happen for the linearized equation, meaning that a singular measure part exists in the principal eigenfunction. I will discuss the validity of this phenomenon for the full (nonlinear) equation, with a particular attention to homogeneous stationary states and traveling waves. In particular, I will talk about the techniques used to construct weak (possibly singular) traveling waves.

Wave Interaction for Reaction-Diffusion Equations with a Multiple-Well Potential

Chih-Chiang Huang
National Taiwan University, Taiwan

In this talk, I would like to introduce a variational approach to construct traveling waves for reaction-diffusion equations. We study a classical combustion model which has a triple-well potential with three stable constant equilibrium 0, 1 and 2. In some situations, a traveling wave connecting 0 to 2(called 0-2 wave) can be viewed as an interaction between 0-1 wave and 1-2 wave. We will give a criterion to determine whether 0-2 wave exists. Moreover, some equations with a multiple-well potential are also studied.

Quasi-Periodic Solution in a Dynamical System for the Motion of a Single Particle

Kota Ikeda
Meiji University, Japan
Hiroyuki Kitahata, Yuki Koyano, Tomoyuki Miyaji, Natsuhiko Yoshinaga

A single self-driven particle exhibits various types of periodic motions like oscillatory, rotational and quasi-periodic motions through interaction with external forces. An oil droplet exhibits various periodic motions in sodium dodecyl sulfate aqueous solution by the Marangoni flow induced by the chemical reaction as described by Tanaka et al. (2015), where a mathematical model was proposed to analyze such phenomena theoretically. Such mathematical models generate periodic solutions via Hopf bifurcation. In those cases we can deduce a canonical system of an ODE form. Actually, under a suitable condition, there are quasi-periodic solutions which connect oscillatory and rotational solutions in a bifurcation diagram. Generally speaking, it is difficult to prove the existence and stability of quasi-periodic solutions in a rigorous manner. Thus we rewrite the canonical system into a lower dimensional system, which we name the RVF system. In this talk, we verify the existence and stability of a periodic solution in the RVF system. Moreover, we will state that the periodic solution in the RVF system may correspond to a quasi-periodic solution in the canonical system.

On a Nonlocal System for Vegetation in Drylands

Hirofumi Izuhara
University of Miyazaki, Japan
Matthieu Alfaro, Masayasu Mimura

Several mathematical models are proposed to understand spatial patchy vegetation patterns arising in drylands. In this talk, we consider the system with nonlocal dispersal of plants proposed by Pueyo et al. as a model for vegetation in water-limited ecosystems. For this system, we consider the stationary problem from the viewpoint of vegetated pattern formation, and show a transition of vegetation patterns when parameter values in the system vary.

Traveling Wave Solutions of a Reaction-Diffusion System for Neolithic Transition in Europe

Muhammad Humayun Kabir
Meiji University, Japan
Masayasu Mimura, Je-Chiang Tsai

The Neolithic transition, a demographic shift from hunter-gatherers to farmers, which is one of the most significant single developments in human civilization. Around 10,000 years ago, Neolithic transition occurred in Europe and its archeological evidence indicates that expanding velocity of farmers is roughly constant [1,2]. To understand such phenomenon, many theoretical attempts have been progressed through mathematical modeling [2].
Existing modeling approaches on Neolithic transition demonstrates that expanding velocity is faster than the observed one. For understanding of this difference, we propose a three-component reaction-diffusion system by introducing the influence of farming technology as an environmental effect on the spread of farmers into the hunters‘ region.

From the viewpoint of transient behaviors of farmers and hunter-gatherers, we investigate the occurrence of traveling wave solutions with minimal and non-minimal velocities of the model. In this talk, we discuss the traveling wave solutions with minimal velocity depending on the development of farming technology. Numerical result reveals that the minimal velocity of traveling wave solutions becomes slower when farming technology is suitably developed [4]. This is a joint work with Masayasu Mimura (Musashino Univ., Japan) and Je-Chiang Tsai (National Tsing Hua Univ., Taiwan).

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Pattern of Predator-Prey System by Extinction Dynamics

Yong-Jung Kim
KAIST, Korea

Most of population models, if not all of them, do not have the extinction dynamics. The Lotka-Volterra ordinary differential equations are such cases and solutions never become zero if they are not initially zero. PDE extensions also have the same property and solutions are always positive everywhere. However, the population often becomes zero locally in space and the survival of a species is a global phenomenon. The spatial pattern of biological organisms reflects its history of extinction and growth and the extinction is a key ingredient of spatial patterns. In this paper Lotka-Volterra equations equipped with extinction dynamics are introduced. We will see beautiful patterns of life generated by the newly added extinction dynamics.

Construction and Stability Analysis of Stationary Solutions to the Schnackenberg Model with Heterogeneity

Kazuhiro Kurata
Tokyo Metropolitan University, Japan
Yuta Ishii

In this talk, we consider the 1-dimensional Schnackenberg model on the interval $(-1, 1)$ with heterogeneity $g(x)$ in front of the nonlinearity under the Neumann boundary condition. Here, $g(x)$ is a given positive, symmetric, i.e. $g(x) = g(-x)$, and Lipschitz continuous function. For a certain singular perturbation regime, first we construct a symmetric 1-peak stationary solution by using the contraction mapping theorem with precise asymptotic behaviors. Next, for more smooth $g(x)$, we study the stability of the constructed stationary solution. Especially, we reveal the effect of the heterogeneity $g(x)$ on the stability. This is a joint work with Dr. Yuta Ishii.

Spectral Stability of Traveling Waves for Keller-Segel Chemotactic Model

Yi Li
California State University, Northridge, USA
Yong Li (Beijing University of Technology) and Yaping Wu (Capital Normal University)

The stability of traveling waves for Keller-Segel model with chemo- taxis is considered. In addition, we will discuss 1) Known results on existence and stability of waves; 2) Local existence and uniqueness of classical solutions; 3) Spectral analysis in weighted spaces; 4) Existence and analyticity of Evans function; 5) Theoretical estimates and numerical simulation on Evans Function.

Bifurcation of Positive Solutions to Scalar Reaction-Diffusion Equations with Nonlinear Boundary

Ping Liu
Harbin Normal University, Peoples Rep of China
Junping Shi

The bifurcation of non-trivial steady state solutions of a scalar reaction-diffusion equation with nonlinear boundary conditions is considered using several new abstract bifurcation theorems. The existence and stability of positive steady state solutions are proved using a unified approach. The general results are applied to a Laplace equation with nonlinear boundary condition and bistable nonlinearity, and an elliptic equation with superlinear nonlinearity and sublinear boundary conditions.

Generation of Interface for Solutions of the Mass Conserved Allen–Cahn Equation

Hiroshi Matano
Meiji University, Japan
Danielle Hilhorst, Thanh Nam Nguyen, Hendrik Weber

I will discuss the generation of interface for solutions of the mass conserved Allen-Cahn equation of the form

$$u_t = \Delta u + \varepsilon^{-2}\left(f(u) - \langle f(u)\rangle\right)$$

on a bounded domain Ω in $\mathbb{R}^N$, where f is an Allen–Cahn type nonlinearity such as $u - u^3$, while ε is a small parameter and $\langle\cdot\rangle$ denotes the avarage over Ω. The main goal is to show that, given a virtually arbitrary initial function $u(x,0)$, the solution generally develops a steep transition layer of thickness $O(\varepsilon^\gamma)$ for some $0 < \gamma \le 1$ at a very early intial stage of $t = O(\varepsilon^2|\ln\varepsilon|)$. The location of this interface is in an $O(\varepsilon)$ neighborhood of the set

$$\Gamma_0 = \{x \in \Omega \mid f(u_0(x)) = \langle f(u_0)\rangle,\, f'(u_0(x)) > 0\}.$$

No interface develops if this set is empty.

Cahn-Hilliard Models with Logarithmic Nonlinear Terms

Alain Miranville
University of Poitiers, France

Our aim in this talk is to discuss the Cahn-Hilliard equation and some of its variants with the physically relevant logarithmic nonlinear terms.

A Limit Equation and Bifurcation Diagrams of Semilinear Elliptic Equations with General Supercritical Growth

Yasuhito Miyamoto
The University of Tokyo, Japan

We study radial solutions of a semilinear elliptic equation under rather general growth conditions on the nonlinear term. We construct a radial singular solution and study the intersection number between the singular solution and a regular solution. An application to bifurcation problems of elliptic Dirichlet problems is given. To this end, we derive a certain limit equation from the original equation at infinity, using a generalized similarity transformation. We see by a certain transformation that all the limit equations can be reduced into two typical cases, i.e., the pure power nonlinearity and the exponential nonlinearity.

On an Interface Equation with Exponential Curvature

Harunori Monobe
Okayama University, Japan
Tetsuya Ishiwata

An interface equation with exponential curvature appears in some mathematical models. For example, mean curvature flow proposed by Mullins is a well-known interface equation describing the formation of grain boundaries. This mathematical model is originally derived from an interface equation with exponential curvature. Thus it is important to analyze an interface equation with exponential curvature. For this original equation, the existence and uniqueness of solutions have been already shown by Hamamuki. Moreover, a free boundary problem with exponential curvature was studied by Abergel et. al.. In this talk, from mathematical point of view, we consider a simple interface equation with exponential curvature and introduce some results related to the behavior of solutions.

A Mathematical Model of Cell-Cell Adhesion and Its Application

Hideki Murakawa
Kyushu University, Japan

Cell-cell adhesion and cell sorting processes are essential in organ formation during embryonic development and in maintaining multicellular structure. We propose a nonlocal advection-diffusion system as a possible continuous mathematical model for these phenomena. Numerical experiments illustrate that the proposed model is able to replicate different types of phenomena observed in cell-cell adhesion experiments. Furthermore, we will give an application to medical science.

Effect of Diffusion Type on the Propagation Speed of Traveling Fronts in Periodic Environments

Ken-Ichi Nakamura
Kanazawa University, Japan

We will discuss the homogenized speed of traveling fronts in periodic environments under three types of diffusion for the densities of biological populations — neutral transition, repulsive transition and attractive transition.

Stability of Front Solutions of the Bidomain Equation on a Strip

Mitsunori Nara
Iwate University, Japan
Hiroshi Matano, Yoichiro Mori

We study the initial value problem of the bidomain equation with bistable type nonlinearity on the strip. The bidomain equations originate from the study of electrical properties of cardiac tissue. It contains a pseudo-differential operator instead of the Laplacian, while it has a planar traveling wave, similar to the Allen-Cahn equation. Our objective is to study its stability, especially to analyze how the stability of planar fronts depends on the width of the strip.

Motion by Surface Diffusion

Amy Novick-Cohen
Technion, Israel

Results are presented with regard to evolution and steady states in the context of motion by surface diffusion.

Rippling Rectangular Waves for a Modified Benney Equation

Toshiyuki Ogawa
Meiji University, Japan
Tomoyuki Miyaji, Ayuki Sekisaka

One parameter family of rectangular periodic traveling wave solutions are known to exists in a perturbed system of the modified KdV equation which was obtained from the traffic jam model. The rectangular periodic traveling wave consists basically of front and back transitions. It turns out that the rectangular traveling wave becomes unstable as its period becomes large. More precisely, torus bifurcation occurs successively along the branch of the rectangular traveling wave solutions. And, as a result, a "rippling rectangular wave" appears. It is roughly the rectangular traveling wave on which small pulse wave trains are superimposed. The bifurcation branch is constructed by a numerical torus continuation method. The instability is explained by using the accumulation of eigenvalues on the essential spectrum around the stationary solutions. Moreover, the critical eigenfunctions which correspond to the torus bifurcation can be characterized theoretically.

Existence and Structure of Precipation Patterns in Fast-Reaction Limit of the Keller-Rubinow Model for Liesegang Rings

Marcel Oliver
Jacobs University, Germany
Zymantas Darbenas

The Keller-Rubinow model for Liesegang rings reduces to a single scalar reaction-diffusion equation in the fast-reaction limit, albeit with a non-local non-Lipshitz reaction term. In this talk, we present evidence that the pattern of successive precipitation regions and precipitation-free interrings breaks down in finite time, either by rings accumulating at finite locus or by degeneration of the ignition condition. We discuss the question of continuation after breakdown, time-asymptotic profile, and interpretation of the solution after breakdown as a precipitation probability density function. Finally, we discuss uniqueness of solutions and possibly scenarios for non-uniqueness.

Bounded and Unbounded Spatial Patterns to Some Reaction-Diffusion-ODE Systems

Kanako Suzuki
Ibaraki University, Japan

We discuss systems of a single reaction-diffusion equation coupled with an ordinary differential equation. Such systems of equations are often used as models of pattern formation phenomena, which arise, for example, from modeling of interactions between cellular processes such as cell growth, differentiation or transformation and diffusion signaling factors. We show that all continuous spatially heterogeneous stationary solutions can be unstable in most cases. Therefore, even if it is a model of pattern formation, there is no stable spatial pattern, which is very different from classical reaction-diffusion models. Moreover, we see that space inhomogeneous solutions of some reaction-diffusion-ODE systems become unbounded in either finite or infinite time, even if space homogeneous solutions are bounded uniformly in time. We would like to understand a mechanism of the growing solutions in infinite time. These are joint works with A. Marciniak-Czochra (University of Heidelberg) and G. Karch (University of Wroclaw).

Radon Measure-Valued Solutions of First Order Hyperbolic Conservation Laws

Alberto Tesei
Sapienza University of Roma, Italy
Michiel Bertsch, Flavia Smarrazzo, Andrea Terracina

We study nonnegative solutions of the Cauchy problem

$$u_t + \varphi(u)_x = 0 \text{ in } \mathbf{R} \times (0,T),\ u = u_0 \geq 0 \text{ in } \mathbf{R} \times 0,$$

where u_0 is a Radon measure and $\varphi(u) : [0,\infty) \to \mathbf{R}$ is a globally Lipschitz continuous function. We construct suitably defined entropy solutions in the space of Radon measures. Under some additional conditions on φ, we prove their uniqueness if the singular part of u_0 is a finite superposition of Dirac masses. In terms of the behaviour of φ at infinity we give criteria to distinguish two cases: either all solutions are function-valued for positive times (an instantaneous regularizing effect), or the singular parts of certain solutions persist until some positive *waiting time* (in the linear case $\varphi(u)) = u$ this happens for all times). In the latter case we describe the evolution of the singular parts.

Gradient Structures for Flows of Concentrated Suspensions

Marita Thomas
Weierstrass Institute Berlin, Germany
Dirk Peschka, Barbara Wagner

We discuss a two-phase model of Stokes-Stokes type for concentrated non-Brownian suspensions with yield-stress behaviour. We establish a PDE formulation in terms of a gradient structure, featuring dissipative coupling terms between fluid and solid phase as well as different driving forces.
Routes of mathematical analysis in the framework of generalized gradient flows and extentions of this structure to applications in Geosciences are outlined.

Structural and Practical Identifiability Analysis of Zika Epidemiological Models

Necibe Tuncer
Florida Atlantic University, USA
Maia Martcheva

The Zika virus (ZIKV) epidemic has caused an ongoing threat to global health security and spurred new investigations of the virus. Use of epidemiological models for arbovirus diseases can be a powerful tool to assist in prevention and control of the emerging disease. In this article, we introduce six models of ZIKV, beginning with a general vector-borne model and gradually including different transmission routes of ZIKV. These epidemiological models use various combinations of disease transmission (vector and direct) and infectious classes (asymptomatic and pregnant), with addition to loss of immunity being included. The disease induced death rate is omitted from the models. We test the structural and practical identifiability of the models to find whether unknown model parameters can uniquely be determined. The models were fit to obtained time series data of cumulative incidences and pregnant infections from the Florida Department of Health Daily Zika Update Reports. The average relative estimation errors (ARE) were computed from the Monte Carlo simulations to further analyze the identifiability of the models. We show that direct transmission rates are not practically identifiable, however, fixed recovery rates improve identifiability overall. We found ARE low for each model (only slightly higher for those that account for a pregnant class), and help to confirm a reproduction number greater than one at the start of the Florida epidemic.

On a Numerical Method for Estimating Blow-Up Rates for Nonlinear Evolution Equations

Takeo Ushijima
Tokyo University of Science, Japan
Anada Koichi, Tetsuya Ishiwata

Many nonlinear evolution equations possess solutions which blow up in finite time. It is one of the most interesting issues concerning blow-up phenomena how the solutions develop singularities. To determine the blow-up rate is a major problem in such a direction. Depending on the equations, blow-up rates take the forms of various types such as simple power type and logarithmic type, and even more complex ones. There are vast amounts of literature deeply considering how solutions blow up. To our best knowledge, however, only a few studies have investigated numerical methods for blow-up rates. In this talk, we propose a simple but effective numerical method which estimates blow-up rates for a class of nonlinear evolution equations. Here, we consider the class of equations which satisfy a scaling invariance. Thanks to this scaling invariance, we adopt the rescaling algorithm and construct a sequence whose behavior leads to the blow-up rate. Applying the method to several nonlinear equations, we examine the effectiveness of the method. We show that our method is applicable to not only the simple power type blow-up rates but also more complex ones.

Nonlocal Dispersal Equations in Time-Periodic Media: Principal Spectral Theory, Bifurcation and Asymptotic Behaviors

Hoang Hung Vo
Vietnam National University, Vietnam
Zhongwei-Shen

This talk is devoted to the investigation of a nonlocal dispersal equation in a bounded and connected domain of RN with smooth boundary, with time-periodic nonlinear function of generalized KPP type. We first study the principal spectral theory of a suitable linearized operator. We establish an easily verifiable, general and sharp sufficient condition for the existence of the principal eigenvalue as well as important sup-inf characterizations of the principal eigenvalue. We next study the influences of the principal eigenvalue on the global dynamics and confirm that the principal eigenvalue being zero is critical. It is then followed by the study of the effects of the dispersal rate D and the dispersal range characterized by sigma on the principal eigenvalue and the positive time-periodic solution, and prove various asymptotic behaviors of the principal eigenvalue and the positive time-periodic solution when D and sigma tend to infinity. To achieve these, we develop new techniques to overcome substantial difficulties caused by the lack of the usual L2 variational formula for the principal eigenvalue, the lack of the regularizing effects of the semigroup generated by the nonlocal dispersal operator, and the presence of the time-dependence of the nonlinearity f. Finally, we establish the maximum principle for the time-periodic nonlocal operator.

Traveling Wave and Aggregation in a Flux-Limited Keller-Segel Model

Shugo Yasuda
University of Hyogo, Japan
Vincent Calvez, Benoit Perthame

Flux-limited Keller-Segel (FLKS) model has been recently derived from kinetic transport models for bacterial chemotaxis and shown to represent better the collective movement observed experimentally. Recently, associated to the kinetic model, a new instability formalism has been discovered related to stiff chemotactic response. This motivates our study of traveling wave and aggregation in population dynamics of chemotactic cells based on the FLKS model with a population growth term. Our study includes both numerical and theoretical contributions. In the numerical part, we uncover a variety of solution types in the one-dimensional FLKS model additionally to standard Fisher/KPP type traveling wave. The remarkable result is a counter-intuitive backward traveling wave, where the population density initially saturated in a stable state transits toward an unstable state in the local population dynamics. Unexpectedly, we also find that the backward traveling wave solution transits to a localized spiky solution as increasing the stiffness of chemotactic response. In the theoretical part, we obtain a novel analytic formula for the minimum traveling speed which includes the counter-balancing effect of chemotactic drift vs. reproduction/diffusion in the propagating front. The front propagation speeds of numerical results only slightly deviate from the minimum traveling speeds, except for the localized spiky solutions, even for the backward traveling waves.

Dynamical Behaviors of a Reaction-Diffusion SIR Epidemic Model Describing the Population Dynamics of Fox Rabies

Fengqi Yi
Harbin Engineering University, Peoples Rep of China
Y. Tang, N. Tuncer, Y. Zhang

In this talk, I will present our recent results on the dynamical behaviors of a class of reaction-diffusion SIR models which are used to model the population dynamics of fox rabies epidemic diseases observed in Europe. We not only consider the original nondegenerate system, but also consider its two degenerate counterparts: one is PDE-ODEs system and the other is the shadow system. Dynamical behaviors, including the existence of global-in-time solutions, the stability and the bifurcations of the steady states, are investigated in great details. The talk is based on the joint works with Y. Tang, N. Tuncer and Y. Zhang.

Special Session 20: Attractors and Their Applications

Xiaoying Han, Auburn University, USA
Tomas Caraballo, University of Seville, Spain
Meihua Yang, Huazhong University of Science and Technology, Peoples Rep of China

Attractors are invariant compact sets that attract all trajectories of an underlying dynamical system, and hence are crucial to the analysis of dynamical systems. On the one hand, attractors are typically not regular surfaces in the state space but rather more complicated kinds of sets. On the other hand, when the trajectories are complex, the attractor may eventually exhibit strange and chaotic structures. Their interesting and complicated nature motivates extensive studies on attractors in the past decade. The theory of global attractors, that capture the asymptotic behavior of autonomous systems, has been well established over the past decades. More recently a new type of attractors, called pullback attractors, were proposed to investigate asymptotic behavior of non-autonomous and random dynamical systems. The goal of this special session is to present recent developments in the studies of various type of attractors such as global attractors, pull back attractors, random attractors, forward attractors, and strange attractors, etc. The topics include not only general existence and continuity properties of attractors, but also their detailed geometrical structures. A special emphasis will be given to attractors for dynamical systems arising in the applied sciences and engineering.

Attractors of Operator Sequences on KB-Spaces

Nazife Erkursun Ozcan
Hacettepe University, Turkey

The concept of an attractor was used by several mathematicians to characterize the asymptotic behavior of operators. In this talk we show that a positive operator sequence on a KB-space is mean ergodic if the operator sequence has a weakly compact attractor. Moreover if a weakly compact attractor is an order interval, then a positive operator sequence converges strongly onto the finite dimensional fixed space. As a consequence we investigate also stability of Markovian operator sequences and the existence of lower bound functions on KB-spaces.

Asymptotic Behavior of Random Navier-Stokes Equations Driven by Colored Noise

Anhui Gu
Southwest University, Peoples Rep of China
Bixiang Wang

We investigate the asymptotic behavior of the solutions of the two-dimensional stochastic Navier-Stokes equations driven by colored noise. We prove the existence and uniqueness of tempered pullback attractors for the random equations with a general diffusion term. Under certain conditions, we also prove the convergence of solutions and random attractors of the approximate equations when the step size of approximations approaches zero.

Criteria on the Existence and Stability of Pullback Exponential Attractors and Their Applications

Yanan Li
Zhengzhou University, Peoples Rep of China
Zhijian Yang

In this talk, we are concerned with the existence and stability of pullback exponential attractors for a non-autonomous dynamical system. (i) We propose two new criteria for the discrete dynamical system and continuous one, respectively. (ii) By applying the criteria to the non-autonomous Kirchhoff wave models with structural damping and supercritical nonlinearity we construct a family of pullback exponential attractors which are stable with respect to perturbations.

Hyperspectral Image Classification Using Functional Data Analysis

Hong Li
Huazhong University of Science and Technology, Peoples Rep of China

The large number of spectral bands acquired by hyperspectral imaging sensors allows us to better distinguish many subtle objects and materials. Unlike other classical hyperspectral image classification methods in the multivariate analysis framework, in this talk, a novel method using functional data analysis (FDA) for accurate classification of hyperspectral images has been proposed. The central idea of FDA is to treat multivariate data as continuous functions. From this perspective, the spectral curve of each pixel in the hyperspectral images is naturally viewed as a function. This can be beneficial for making full use of the abundant spectral information. The relevance between adjacent pixel elements in the hyperspectral images can also be utilized reasonably. Functional principal component analysis is applied to solve the classification problem of these functions. Experimen-

tal results on three hyperspectral images show that the proposed method can achieve higher classification accuracies in comparison to some state-of-the-art hyperspectral image classification methods.

Longtime Dynamics of the Quasi-Linear Wave Equations with Structural Damping and Supercritical Nonlinearities

Zhiming Liu
Zhengzhou University, Peoples Rep of China
Zhijian Yang

We consider the well-posedness and the longtime dynamics of the quasilinear wave equations with structural damping and supercritical nonlinearities. We show that even for the supercritical nonlinearities, the solutions of the mentioned equations still possess parabolic-like characteristics because of the effectiveness of the structural damping. Furthermore, we obtain the existence of the global and exponential attractors (rather than the weak ones) in natural energy space.

Uniform Global Attractors and Trajectory Attractors for the Nonautonomous 3D Navier-Stokes Equations

Songsong Lu
Sun Yat-Sen University, Peoples Rep of China

We obtain the existence of the weak uniform (with respect to the initial time) global attractor and construct a trajectory attractor for the 3D Navier-Stokes equations (NSE) with a fixed time-dependent force satisfying a translation boundedness condition. Moreover, we show that if the force is normal and every complete bounded solution is strongly continuous, then the uniform global attractor and the trajectory attractor are strong, strongly compact, respectively. As a consequence, we obtain the strong equicontinuity of all bounded complete trajectories and the finite strong uniform tracking property that for any fixed accuracy and time length, a finite number of trajectories on the global attractor are able to capture in strong metric all trajectories after sufficiently large time. Our method is based on a new established framework called evolutionary system, whose trajectories are solutions to the nonautonomous 3D NSE. The framework is general and can also be applied to other nonautonomous dissipative partial differential equations for which the uniqueness of solutions might not hold. It is not known whether previous frameworks can also be applied in such cases as we indicate in open problems related to the question of uniqueness of the Leray-Hopf weak solutions.

Strongly Exponentially Separated Linear Systems

Kenneth Palmer
National Taiwan University, Taiwan
Flaviano Battelli

An important concept in the study of nonautonomous linear differential systems is that of exponential separation. It is closely related to the concept of exponential dichotomy. Also it has played a key role in the theory of Lyapunov exponents. Usually in the study of exponential separation, it is assumed that the coefficient matrix is bounded in norm. Our first aim here is to develop a theory of exponential separation which applies to unbounded systems. It turns that in order to have a reasonable theory, it is necessary to add the assumption that the angle between the two separated subspaces is bounded below. This is what is meant by strong exponential separation. Our second aim is to show that if a bounded linear Hamiltonian system is exponentially separated into two subspaces of the same dimension, then it must have an exponential dichotomy.

Semilinear Parabolic Equations with Unbounded Attractors

Juliana Pimentel
Universidade Federal do ABC, Brazil

We consider non-dissipative semilinear parabolic equations on a bounded interval. We review the recently developed theory for the autonomous version of this class of problems and present a characterization for the associated noncompact global attractor.

Expoenential Attractors for Random Dynamical Systems in Banach Spaces

Stefanie Sonner
Radboud University Nijmegen, Netherlands
Tomas Caraballo

Exponential attractors of infinite dimensional dynamical systems are compact, semi-invariant sets of finite fractal dimension that attract all bounded subsets at an exponential rate.
They contain the global attractor and, due to the exponential rate of convergence, are generally more stable under perturbations than global attractors.
In the autonomous setting exponential attractors have been studied for several decades and their existence has been shown for a large variety of dissipative equations. More recently, using the pullback approach the theory has been extended to nonautonomous and random problems. We present general existence results for random pullback exponential attractors in Banach spaces and derive explicit estimates on their fractal dimension. As an application a stochastic semilinear damped wave equation is considered.

Well-Posedness for the Incompressible Hall-MHD Equations in Low Regularity Spaces

Yanbin Tang
Huazhong University of Science and Technology, Peoples Rep of China
Xing Wu

In this talk we first establish the local well-posedness of strong solutions to the Cauchy problem of the incompressible viscous resistive Hall-MHD equations in $H^s(\mathbb{R}^3)$ $(\frac{3}{2} < s \leq \frac{5}{2})$, and then we prove that the local solution is global when the initial data is small enough.

Global Attractor for the BBM Equation in Low Regularity Spaces with Low Regularity Forces

Ming Wang
China University of Geosciences, Peoples Rep of China
Anping Liu

In a given phase space, it is very interesting to find out the critical exponent of growth order for the nonlinear term, such that the nonlinear system has a global attractor. However, the problem does not exists for the Benjamin-Bona-Mahony (BBM) equation, since the nonlinear term is given by uu_x. In this case, an "equivalent" question is to find out the lowest regularity space in which the global attractor exists. In this talk, we shall give some results for the global attractor for the BBM equations in low regularity spaces with low regularity forces.

Compactly Generated Shape Index Theory in Infinte-Dimensional Dynamical Systems

Jintao Wang
Huazhong University of Science and Technology, Peoples Rep of China
Desheng Li, Jinqiao Duan

We establish a theory of compactly generated shape index for local semiflows on complete metric spaces via very ordinary shape index pairs. The main advantage is that the quotient space N/E is not necessary to be metrizable for the shape index pair (N, E). In the new shape theory, we can calculate the shape index of a compact isolated invariant set K in any closed subset that contains a local unstable manifold of K, and define the shape cohomology index of K to develop the Morse equations. This provides more effective ways to calculate shape indices and Morse equations theoretically and specifically for infinite dimensional systems, without particular requirements on the index pairs or the unstable manifolds.

A Determining Form for a Nonlocal System

Meihua Yang
Huazhong University of Science and Technology, Peoples Rep of China
Lu Bai

This work is concerned with constructing a finite dimensional form (named determining form) by adding a feedback control term through an interpolation operator. The dynamics of the determining form is consistent with those of the original system.

Stability of Exponential Attractors for a Family of Semilinear Wave Equations with Gentle Dissipation

Zhijian Yang
Zhengzhou University, Peoples Rep of China
Zhiming Liu

In this talk, we are concerned with the stability of exponential attractors for a family of semilinear wave equations with gentle dissipation: $u_{tt} - \Delta u + \gamma(-\Delta)^{\alpha} u_t + f(u) = g(x)$, with $\alpha \in (0, 1/2)$. We propose a new criterion on the existence and stability of a family of exponential attractors depending on the perturbation parameters. By applying this criterion to the equations, we construct a family of exponential attractors $\mathcal{A}^{\alpha}_{exp}$ and show their stability on the dissipative exponent α.

Dynamics for a Stochastic Reaction-Diffusion Equation with Dynamical Boundary Condition

Lu Yang
Lanzhou Universtiy, Peoples Rep of China

In this talk, we study the dynamic behavior of a stochastic reaction-diffusion equation with dynamical boundary condition. Some higher-order integrability of the difference of the solutions near the initial time, and the continuous dependence result with respect to initial data were established. As a direct application, we can obtain the existence of pullback random attractor immediately.

Special Session 22: Regularity of PDE

Dongsheng Li, Professor, Xian Jiaotong University, Peoples Rep of China
Sun-Sig Byun, Professor, Seoul National University, Korea
Lihe Wang, Professor, University of Iowa, USA

Regularity theory for partial differential equations

A Geometric Proof of L^P Estimates for the Fractional Laplace Equations

Yi Cao
Shaanxi Normal University, Peoples Rep of China

We study the interior L^p estimates to the equation $(-\triangle)^{\sigma/2}u = f$ in B_1 by a geometric method, where $(-\triangle)^{\sigma/2}$ stands for the fractional Laplace operator. Our results generalize the classical L^p estimates established by Calderòn and Zgymund. The analytical tools used in this paper are the Hardy-Littlewood maximum functions, energy estimates and the Besicovich's covering lemma. The main difficulty of our works is that the fractional Laplace operator is a nonlocal elliptic operator.

Turing Patterns in Parabolic Systems of Conservation Laws

Soyeun Jung
Kongju National University, Korea
Balke Barker, Kevin Zumbrun

Turing patterns on unbounded domains have been widely studied in systems of reaction-diffusion equations. However, up to now, they have not been studied for systems of conservation laws. Here, we derive conditions for Turing instability in conservation laws and use these conditions to find families of periodic solutions bifurcating from uniform states numerically.

Regularity of Elliptic Equations in Composite Materials

Youchan Kim
University of Seoul, Korea

In this talk, we obtain regularity results such as potential type estimates and Calderòn-Zygmund type estimates for elliptic equations in composite materials.

On L^P-Estimates for Coercive and Noncoercive Elliptic Equations

Hyunseok Kim
Sogang University, Korea

We consider the Dirichlet problem for second-order linear elliptic equations with the first-order term given by a singular vector field u. $W^{1,p}$-estimates for weak solutions are derived for the case when $u \in L^n$, where $n \geq 3$ is the spatial dimension. We also discuss the case of more singular u in the weak-L^n space but having nonnegative divergence.

A Radial Symmetry and Liouville Theorem for Systems Involving the Fractional Laplacian

Zhenjie Li
Nanjing Tech University, Peoples Rep of China
Dongsheng Li

In this paper, we investigate the nonnegative solutions of systems involving the fractional Laplacian:

$$\begin{cases} (-\Delta)^{\alpha}u_i(x) = f_i(u), \quad x \in \mathbb{R}^N, \quad i = 1, 2, \cdots, m, \\ u(x) = (u_1(x), u_2(x), \cdots, u_m(x)), \end{cases}$$

Recent Advances in Global Stability of Minkowski Space-Time with the Presence of Massive Scalar Field

Yue Ma
Xian Jiaotong University, Peoples Rep of China
P. LeFloch

In this talk we will present some recent work about the system of Einstein equation coupled with a massive scalar field. We remove the condition of "being compact perturbation of Schwarzschild metric" on initial data which seems to be restrictive in previous work. The key technique which allows this generalization is called the "Euclidian-hyperbolic foliation". Compared with the hyperboloidal foliation method introduced by the author's team in previous work, this new framework has the capacity of treating noncompact supported initial data.

Global Regularity for Quasilinear Parabolic Equations Involving Measure Data

Jung-Tae Park
Korea Institute for Advanced Study, Korea
Sun-Sig Byun, Pilsoo Shin

In this talk we present global Calderón-Zygmund type estimates for parabolic problems with p-growth in bounded nonsmooth domains when the right-hand side is a signed Radon measure with finite mass. We define suitable solutions and provide conditions which guarantee regularity results for such measure data problems. This is joint work with Sun-Sig Byun and Pilsoo Shin.

Global Calderón-Zygmund Theory of Nonlinear Parabolic Operators Over Irregular Domains

Pilsoo Shin
Seoul National University, Korea
Sun-Sig Byun, Dian K. Palagachev

We develop a global Calderón–Zygmung theory for quasilinear divergence form parabolic equations over Reifenberg flat domain with nonlinearity depending also on the weak solution u. The nonlinear term behaves as the p-Laplacian with respect to the spatial gradient Du, its discontinuity in the independent variables is measured in small-BMO seminorm, while only Hölder continuity is required with respect to the variable u.

Boundary Hölder Regularity for Elliptic Equations

Kai Zhang
Northwestern Polytechnical University, Peoples Rep of China
Guanghao Hong, Dongsheng Li

This talk investigates the relation between the boundary geometric properties and the boundary regularity of the solutions of elliptic equations. We prove by a new unified method the pointwise boundary Hölder regularity under proper geometric conditions. “Unified” means that our method is applicable for the Laplace equation, linear elliptic equations in divergence and non-divergence form, fully nonlinear elliptic equations, the p-Laplace equations and the fractional Laplace equations etc. In addition, these geometric conditions are quite general. In particular, for local equations, the measure of the complement of the domain near the boundary point concerned could be zero. The key observation in the method is that the strong maximum principle implies a decay for the solution, then a scaling argument leads to the Hölder regularity.

Special Session 23: Stochastic Partial Differential Equations

Benjamin Gess, MPI MIS Leipzig, Germany
Michael Röckner, University of Bielefeld, Germany

Stochastic Partial Differential Equations (SPDE) and their applications is a relatively young field of mathematics. In the past two decades and it has, however, become one of the main research directions of Probability Theory, with rising activity across its entire spectrum. In particular, modern SPDE techniques and their combination with ideas from rough path theory led to Martin Hairer's theory of regularity structures for which he was awarded the Fields medal in 2014. Since then this theory has been significantly extended, for example, by applications to the geometrical evolution of random loops on manifolds. The field of SPDE combines the classical area of Partial Differential Equations (PDE) with modern branches of Probability Theory, in particular, Stochastic Analysis, and thus constitutes one of the most prominent contact points between Analysis and Stochastics. Besides various other connections to pure mathematics (e.g. Differential Geometry, Dynamical Systems) one main focus of SPDE are its applications to the Sciences, in particular Physics, but also Biology and Chemistry. Another main area of applications is economics, in particular mathematical finance. The aim of the session is to give an update on recent developments on SPDE and at the same time identify new frontiers with challenging open problems for the field, with emphasis on both theory and applications.

Modulation Equations for SPDEs on Unbounded Domains

Dirk Blömker
Universität Augsburg, Germany
Luigi A. Bianchi, Guido Schneider

We consider the approximation via modulation equations for nonlinear stochastic partial differential equations (SPDEs) like the stochastic Swift-Hohenberg (SH) equation on the unbounded domain with additive space time white noise. Close to a bifurcation of a single mode a small band of infinitely many eigenvalues changes stability. Thus solutions of SH are well described by a modulated wave, where the amplitude solves a stochastic Ginzburg-Landau (GL) equation with space time white noise.
In the one-dimensional case due to the weak regularity of solutions the standard deterministic methods for modulation equations fail, as we need weighted spaces that allow for unboundedness at infinity of solutions, which is natural for translation invariant noise. Moreover, solutions of GL are only Hölder continuous, which gives just enough regularity to obtain the approximation result.
In the two-dimensional case one runs into the problem that solutions of GL with space-time white noise are no longer well defined.

Entropy Solutions for Stochastic Porous Media Equations

Konstantinos Dareiotis
Max Planck Institute, MIS, Germany
Máté Gerencsér, Benjamin Gess

We provide an entropy formulation for porous media-type equations with a stochastic, non-linear, spatially inhomogeneous forcing. Well-posedness and L_1-contraction is obtained in the class of entropy solutions. Our scope allows for porous medium operators $\Delta u^{[m]}$ for all $m \in (1,\infty)$, and Hölder continuous diffusion nonlinearity with exponent 1/2.

Quasilinear Stochastic PDEs with Two Obstacles

Laurent Denis
Le Mans University, France
A. Matoussi, J. Zhang

We consider a backward parabolic SPDE on $\mathbb{R}^d$ whose solution is forced to stay between two barriers $\underline{v}$ and $\bar{v}$. More precisely, we consider the following SPDE with obstacles:

$$\begin{cases} du_t(x) + [\ \frac{1}{2}\Delta u_t(x) + f_t(x, u_t(x), \nabla u_t(x)) + \mathrm{div} g_t\left(x, u_t\left(x\right), \nabla u_t\left(x\right)\right)\ ]\, dt \\ \qquad\qquad + h_t(x, u_t(x), \nabla u_t(x)) \cdot \overleftarrow{dB}_t = -\nu^+(dt, dx) + \nu^-(dt, dx) \\ u_T = \xi, \\ \underline{v}(t,x) \le u_t(x) \le \bar{v}(t,x) \quad dt \times dx\ a.e. \end{cases}$$

with the Skhorod minimal conditions:

$$\int_0^T \int_{\mathbb{R}^d} \left(\tilde{u}_s\left(x\right) - \underline{v}_s\left(x\right)\right) \nu^+\left(dsdx\right) = \int_0^T \int_{\mathbb{R}^d} \left(\overline{v}_s\left(x\right) - \tilde{u}_s\left(x\right)\right) \nu^-\left(dsdx\right) = 0,\ a.s.$$

where $\tilde{u}$ is a *quasi-continuous* version of u. In this equation, the random measures ν^+ and ν^- pushes the solution when it reaches the barriers. We prove existence and uniqueness of the solution under usual Lipschitz hypotheses.
The proof is based on a probabilistic interpretation of the Laplacian and the divergence term thanks to a forward Brownian motion W independent of B. Namely we prove that the solution (u, ν^+, ν^-) of our SPDE with obstacles can be expressed in terms of the solution $\left(Y_t, Z_t, K_t^+, K_t^-\right)_{t\in[0,T]}$ of the following doubly reflected backward doubly stochastic differential equation:

$$\begin{cases} Y_t = \Phi\left(W_T\right) + \int_t^T f_r\left(W_r\right) dr + \frac{1}{2}\int_t^T g_r\left(W_r\right) * dW + \int_t^T h_r\left(W_r\right) \cdot \overleftarrow{dB}_r \\ \qquad - \sum_i \int_t^T Z_{i,r} dW_r^i + K_T^+ - K_t^+ - K_T^- + K_t^- \\ Y_T = \imath(W_T) \end{cases} \tag{1}$$

with $\underline{v}(t,W_t) = L_t \le Y_t \le S_t = \overline{v}(t,W_t)$, $\forall\, t \in [0,T]$, $\left(K_t^+\right)_{t\in[0,T]}$ and $\left(K_t^-\right)_{t\in[0,T]}$ are increasing continuous processes s.t.

$$\int_0^T (Y_s - L_s)dK_s^+ = \int_0^T (S_t - Y_t)dK_t^- = 0. \qquad (2)$$

Kolmogorov Equations Associated to Stochastic Fluid Dynamic Models

Franco Flandoli
Scuola Normale Superiore of Pisa, Italy

We describe recent results on Kolmogorov and Fokker-Planck equations (transport and continuity equations in the deterministic case) associated mainly to 2D Euler equations with random initial conditions. In the case of deterministic 2D Euler equations, we solve the Kolmogorov equation in a space of LlogL function (joint work with G. Da Prato and M. Roeckner). In the stochastic case, we mainly consider a transport type noise, which give rise to a moderately degenerate elliptic operator. We prove existence of solutions satisfying a suitable gradient estimate (joint work with D. Luo). We also investigate a special limit from transport type noise with small space correlation to additive white noise.

Coupled KPZ (Kardar-Parisi-Zhang) Equation

Tadahisa Funaki
Waseda University, Japan
Masato Hoshino

We consider the coupled KPZ equation on a one-dimensional unit interval with periodic boundary condition. This equation is ill-posed in a usual sense. We discuss the local well-posedness, convergence of solutions of two types of approximating equations, invariant measures and global well-posedness under the trilinear condition (T) for the coupling constant, the role of the condition (T) in view of invariant measures and renormalizations for 4th order terms, an example which does not satisfy (T) but has an invariant measure, and the cross-diffusion case.

Quasilinear Singular SPDEs Within Regularity Structures

Mate Gerencser
IST Austria, Austria
Martin Hairer

We present an approach for quasilinear singular SPDEs that enables one to fit them into the theory of regularity structures. This way many general results available can be leveraged, and in particular a solution theory can be obtained for an essentially optimal class of equations.

Parabolic Partial Differential Equations with Rough Dirichlet Boundary Conditions

Beniamin Goldys
Sydney University, Australia
Szymon Peszat

We will consider partial differential equations of parabolic type with very rough Dirichlet boundary conditions, including the space-time white noise. We will reformulate the equation as a stochastic evolution equation in an L^p-space with non-standard weight.

Optimal Portfolio Problems Driven by Path Dependent SDEs

Fausto Gozzi
Luiss University, Roma, Italy
Enrico Biffis, Cecilia Prosdocimi, Jun Sekine

We present a family of optimal portfolio problems where the wealth is governed by path dependent dynamics. A way to solve such problems is to solve suitable infinite dimensional PDEs (HJB equations). We discuss the main features of such equations, the methods of solve them, and how to recover from them the optimal portfolio policies. We then present some recent and ongoing results on two specific problems: - portfolio optimization with path dependent labor income; - portfolio optimization with a path dependent volatilty.

Parameter Estimation for a Fractional Linear Stochastic Schrödinger Equation

Wilfried Grecksch
Martin-Luther-University Halle-Wittenberg, Germany

We introduce the following linear stochastic equation of Schrödinger type

$$dX(t,x) = (1+a\,i)\Delta_x X(t,x)dt + \sigma\, i \sum_{k=1}^{\infty} \mu_k (X(t,\cdot), h_k(\cdot))_H h_k(x) dB_k^h(t),$$

with $x \in [0,1]$, $X(0) = X_0 \in H = L^2[0,1]$, $X(t,0) = X(t,1) = 0$. $(B_k^h(t))_{t\ge 0}$, $(k = 1,2,\dots)$ denote independent fractional Brownian motions with Hurst index $h \in]1/2, 1[$. Let $h_k(\cdot)$ $(k = 1,2,\dots)$ be the eigenfunctions of Laplacian operator with above boundaries. Finally, let be $\mu_k \in R^1$ with $\sum_{k=1}^{\infty} \mu_k^2 < \infty$. The solution process is defined in mild sense and is approximated by the Galerkin approximation. We get a sequence of one dimensional problems. A mean square estimation criterion is used for these equations to estimate a. It is proved that the estimation is unbiased and weakly consistent for the original problem.

Integration by Parts on the Law of the Modulus of the Brownian Bridge

Martin Grothaus
TU Kaiserslautern, Germany
Robert Vosshall

We prove an infinite dimensional integration by parts formula on the law of the modulus of the Brownian bridge from 0 to 0. The main motivation for all this is the construction of an SPDE whose invariant measure would be the law of the reflecting Brownian bridge, a problem which is still open despite the recent fantastic advances in very difficult SPDEs, thanks to regularity structures and, or paraproducts. It seems that the SPDE which motivates this integration by parts formula is even more difficult than KPZ, since it contains a local time which is not covered by the new theories yet.

The Stochastic Gray Scott System

Erika Hausenblas
Montanuniversitaet Leoben, Austria
Mechtild Thalhammer, Paul Razafimandimby

Reaction and diffusion of chemical species can produce a variety of patterns, reminiscent of those often seen in nature. The Gray Scott system is a coupled equation of reaction diffusion type, modelling these kind of patterns. Depending on the parameter, stripes, waves, cloud streets, or sand ripples may appear. These systems are the macroscopic model of microscopic dynamics. Here, in the derivation of the equation the random fluctuation of the molecules are neglected. Adding a stochastic noise, the inherit randomness of the microscopic behaviour is modelled. In particular, we add a time homogenous spatial Gaussian random field with given spectral measure. In the talk, first the Gray Scott system is motivated. Then, the stochastic system is introduced and the question of existence and uniqueness are addressed. In the second part of the talk, we present some numerical simulations and the corresponding adapted approximation scheme is explained. Finally, some extension and generalization of the result are presented.

Global Solutions to Elliptic and Parabolic Φ^4 Models in Euclidean Space

Martina Hofmanova
University Bielefeld, Germany

I will present some recent results on global solutions to singular SPDEs on $\mathbb{R}^d$ with cubic nonlinearities and additive white noise perturbation, both in the elliptic setting in dimensions $d = 4, 5$ and in the parabolic setting for $d = 2, 3$. A motivation for considering these equations is the construction of scalar interacting Euclidean quantum field theories. The parabolic equations are related to the Φ^4_d Euclidean quantum field theory via Parisi–Wu stochastic quantization, while the elliptic equations are linked to the Φ^4_{d-2} Euclidean quantum field theory via the Parisi–Sourlas dimensional reduction mechanism. We prove existence for the elliptic equations and existence, uniqueness and coming down from infinity for the parabolic equations. Joint work with Massimiliano Gubinelli.

The Stochastic Landau-Lifshitz-Bloch Equation

Ngan Le
Monash University, Australia
Zdzislaw Brzezniak, Beniamin Goldys, Kim Ngan Le

The stochastic Landau–Lifshitz–Bloch equation perturbed by a multiplicative space-dependent noise is studied for a ferromagnet filling a bounded domain $D \subset \mathbb{R}^d$, $d = 1, 2, 3$. We show the existence of strong solutions in the sense of partial differential equation for $d = 1, 2, 3$. Furthermore, in case $d = 1, 2$ we prove the existence of invariant measures and the existence of the unique strong solution in the sense of the theory of stochastic differential equations.

Brownian Motions, Brownian Bridges and All That ...

Xue-Mei Li
Imperial College London, England

Brownian bridges are conditioned Brownian motions. They are given an SDE with a singular drift obtained from the heat kernel. They provide a natural probability measure on the space of loops/pinned paths. Are there other natural probability measures on such spaces? We will discuss this and elementary examples.

Quasi-Linear Stochastic Partial Differential Equations with Time-Fractional Derivatives

Wei Liu
Jiangsu Normal University, Peoples Rep of China
Michael Roeckner, Jose Luis da Silva

In this talk we present a method to solve (stochastic) evolution equations on Gelfand triples with time-fractional derivative based on monotonicity techniques. Applications include deterministic and stochastic quasi-linear partial differential equations with time-fractional derivatives, including time-fractional (stochastic) porous media equations (including the case where the Laplace operator is also fractional) and p-Laplace equations as special cases.

On Some Smoothening Effects of Transition Semigroups

Szymon Peszat
Jagiellonian University, Cracow, Poland

An important problem in the theory of Markov processes is the uniqueness of an invariant measure and its ergodicity. This holds if the transition semigroup exhibits some smoothening effects: for example if it is strong Feller and irreducible or asymptotic strong Feller. In the talk I will derive the so-called Bismut-Elworthy-Li formula for diffusions. This formula guarantees the strong Feller property. Later, I recall the classical result of Hawkes for Lévy semigroups. It relates the strong Feller property with absolute continuity of the process. The last part of the talk will be devoted to the absolute continuity of Lévy processes.

Fluctuations for Point-Vortex Models

Marco Romito
Università di Pisa, Italy
C. Geldhauser

The first part is a short review of the statistical mechanics theory for point vortex models with random intensity for the 2D Euler equation. In the second part we outline an extension of the theory to a slightly more general class of models with singular interaction.

A Variational Approach to Some Classes of Singular SPDEs

Luca Scarpa
University College London, England
Carlo Marinelli

Well-posedness is proved for some classes of singular second-order SPDEs on a smooth bounded domain D in $\mathbb{R}^n$ in the form

$$du(t)-\operatorname{div}\gamma(\nabla u(t))\,dt+\beta(u(t))\,dt \ni B(t,u(t))\,dW(t)\,,$$

with $u(0)=u_0$. The drift is associated to two maximal monotone operators γ and β on $\mathbb{R}^n$ and $\mathbb{R}$, respectively, on which neither growth nor smoothness assumptions are imposed. Moreover, the noise is given by a cylindrical Wiener process W on a Hilbert space U, with a stochastic integrand B taking values in the Hilbert-Schmidt operators from U to $L^2(D)$: classical Lipschitz-continuity hypotheses for the diffusion coefficient are assumed. The proof consists in approximating the equation, finding uniform estimates both pathwise and in expectation on the approximated solutions, and then passing to the limit using monotonicity and compactness arguments. This study is based on a joint work with Carlo Marinelli (University College London).

Parabolic Equations with Rough Coefficients and Singular Forcing

Scott Smith
Max Planck Institute Leipzig, Germany
Felix Otto, Jonas Sauer, Hendrik Weber

This talk is concerned with linear parabolic equations with rough diffusion coefficients and singular forcing which are ill-posed in the classical sense of distributions. Working within the philosophy of rough paths and regularity structures, we establish an existence and uniqueness theory under the hypothesis that the diffusion coefficients have an enhanced description as a modelled distribution. This is joint work with Felix Otto, Jonas Sauer, and Hendrik Weber.

On Convergence of Discretisations for Filtering SPDEs

Alexander Veretennikov
University of Leeds, England

In this talk diffusion filtering model is considered. It will be shown that under the appropriate conditions, the scheme with appropriately discretized observations leads to discretised filtering equations, the solutions of which converge to the solution of the limiting filtering equation, as the parameter of partitions approaches zero.

Strong Localisation for Singular SPDEs

Hendrik Weber
University of Warwick, England
Augustin Moinat

Hairer's theory of regularity structures provides a powerful tool to define and construct solutions to extremely irregular SPDEs, e.g. white noise driven equations from mathematical physics. The theory is tailored to provide local in time existence and uniqueness by formalising the mild formulation of the equation. In this talk I will discuss how to obtain a priori bounds which lead to global existence within the framework of regularity structures. I will concentrate in the ϕ^4 model.

Path-By-Path Solutions of Hilbert Space-Valued SDEs

Lukas Wresch
Bielefeld University, Germany

Consider the following SDE in $\mathbb{R}^d$ or a separable Hilbert space

$$\mathrm{d}X_t = -AX_t\mathrm{d}t + f(t,X_t)\mathrm{d}t + \mathrm{d}W_t,$$

where A is a positive, linear operator, f is a bounded Borel measurable function and W a cylindrical Wiener process. If the components of f decay to 0 in a faster than exponential way we establish path by path uniqueness for mild solutions of this SDE. This extends A.M. Davies famous result from $\mathbb{R}^d$ to Hilbert space-valued stochastic differential equations. In this talk we consider the so-called path-by-path approach where the above SDE is considered as a random integral equation with parameter $\omega \in \Omega$. We show that there exists a set Ω' of full measure such that for every $\omega \in \Omega'$ the corresponding integral equation for this ω has exactly one solution. This notion of uniqueness (called path-by-path uniqueness) is much stronger than the usual pathwise uniqueness considered in the theory of SDEs.

Optimal Bilinear Control of Stochastic Nonlinear Schroedinger Equations

Deng Zhang
Shanghai Jiao Tong University, Peoples Rep of China
Viorel Barbu, Michael Roeckner, Deng Zhang

In this talk we consider the optimal bilinear control problem of quantum mechanical systems with final observation governed by a stochastic nonlinear Schroedinger equation with linear multiplicative noise. The existence of an open loop optimal control and first order Lagrange optimality conditions are derived, via Skorohod's representation theorem, Ekeland's variational principle and the existence for the linearized dual backward stochastic equation. The approach in particular applies to the deterministic case. This is a joint work with Viorel Barbu and Michael Roeckner.

Stochastic Navier-Stokes Equations

Xiangchan Zhu
Beijing Jiaotong University, Peoples Rep of China

In this talk we establish two results: One is the strong Feller property for the Markov semigroups associated to the two or three dimensional Navier-Stokes equations driven by space-time white noise using the theory of regularity structures introduced by Martin Hairer. In the 2D case this implies ergodicity and global well-posedness of the Navier-Stokes equations driven by space-time white noise starting from every initial point in $(\mathcal{C}^\eta)^2$ for $\eta \in (-\kappa, 0)$ for κ small enough. The other is the existence and uniqueness of the global solutions to the stochastic Navier-Stokes equations in 3D case for the small initial data independent of time, with the stochastic integration being understood in the sense of the integration of controlled rough path which can be viewed as a stochastic version of the Kato-Fujita result.

Stochastic Heat Equations with Values in a Manifold

Rongchan Zhu
Beijing Institute of Technology, Peoples Rep of China

In this paper, we prove the existence of martingale solutions to the stochastic heat equation taking values in a Riemannian manifold, which admits Wiener (Brownian bridge) measure on the Riemannian path (loop) space as an invariant measure using a suitable Dirichlet form. Using the Andersson-Driver approximation, we heuristically derive a form of the equation solved by the process given by the Dirichlet form.
Moreover, we establish the log-Sobolev inequality for the Dirichlet form in the path space. In addition, some characterizations for the lower or uniform bounds of the Ricci curvature are presented related to the stochastic heat equation.

Special Session 24: Nonlinear Dispersive Waves

J. Bona, University of Illinois at Chicago, USA
Hongqiu Chen, University of Memphis, USA
Min Chen, Purdue University, USA
S.M. Sun, Virginia Tech, USA

This session concerns recent developments in the theory of nonlinear wave propagations in a media where dispersion and/or dissipation cannot be ignored. Mathematical problems connected with nonlinear dispersive waves arise in many different practical applications, such as surface or internal waves in various fluid and Plasma flows. Over the last thirty years, the field of nonlinear dispersive waves is very active and many new ideas and techniques have emerged. We expect the session to feature a range of work, including modeling, mathematical theory, numerical simulation and applications. Fundamental questions of well-posedness and long-time asymptotics will be a focus of some of the presentations, while comparisons between various models and with laboratory and field studies will also be featured. On the mathematical side, both initial-value problems and initial-boundary-value problems will be dealt with.

Singular Solutions of a Boussinesq System for Water Waves

Min Chen
Purdue University, USA
Jerry Bona

We will investigate the fundamental issue of determining conditions on the initial data that ensure the solution exists for all time for the Boussinesq system

$$\eta_t + u_x + (\eta u)_x + a u_{xxx} - b\eta_{xxt} = 0,$$

$$u_t + \eta_x + \frac{1}{2}(u^2)_x + c\eta_{xxx} - d u_{xxt} = 0.$$

This system has been used in theory and practice as a model for small-amplitude, long-crested water waves. The investigation proceeds by way of numerical simulations using a computer code based on a a semi-implicit, pseudo-spectral code.
It turns out that larger amplitudes or velocities do seem to lead to singularity formation in finite time, indicating that the problem is not globally well posed.

Lie Analysis to the Kuramoto-Sivashinsky-Type Equation

Zhaosheng Feng
University of Texas-Rio Grande Valley, USA

In this talk, we develop a connection between an ordinary differential equation that is cubic in the unknown function, and the generalized Kuramoto-Sivashinsky equation, a partial differential equation that occupies a prominent position in describing some physical processes in motion of turbulence and other unstable process systems. By means of the Lie symmetry reduction method and the Maximum Principle, some implicit wave solutions are demonstrated.

Analyticity of the Global Attractor for Damped Forced Periodic Korteweg-de Vries Equation

Olivier Goubet
Université de Picardie Jules Verne, France

We consider here a weakly damped forced periodic KdV equation. We prove that if the forcing term is analytic in space, then the global attractor is also contained into a space of analytic functions.

Propagation of Long-Crested Water Waves

Colette Guillopé
Université Paris-Est Créteil, France
Jerry L. Bona, Thierry Colin

In this talk we will present some results about the propagation of waves such as those sometimes observed in canals and in near-shore zones of large bodies of water. A special interest will also be on waves arising in bore propagation, when a surge of water invades an otherwise constantly flowing river. The results are developed in the context of Boussinesq models, so they are applicable to waves that have small amplitude and long wave length when compared with the undisturbed depth. We will discuss the theory of well-posedness results on the long, Boussinesq time scale. In the case of bore propagation, where the mass of water has an infinite energy a priori, we will show how to use suitable approximations with which to compare the full solution.

Numerical Solution of the Serre Equations

Dimitrios Mitsotakis
Victoria University of Wellington, New Zealand

We solve numerically the Serre-Green-Naghdi (SGN) system using stable, accurate and efficient fully discrete numerical schemes based on Galerkin/finite element methods. Although the SGN equations contain third-order derivatives, a modified Galerkin/finite element method allows the use of Lagrange finite elements and combined with explicit Runge-Kutta

schemes for the discretization in time approximate solutions of the SGN system with variable bottom efficaciously. After reviewing the convergence properties of the new numerical scheme, a detailed study of the dynamics of the solitary waves of the SGN system over variable bottom topographies is presented. The effects of surface tension are also reviewed and new solutions are presented.

Three-Dimensional Internal Waves

Dag Nilsson
Lund University, Sweden

We consider three-dimensional inviscid irrotational flow in a two layer fluid under the effects of gravity and surface tension. By using a spatial dynamics approach together with the center-manifold theorem we are able to prove the existence of waves that are periodic in some horizontal direction z, with a bounded profile in some other horizontal direction x. In addition we also find waves that are periodic in both x and z.

Control of a Boussinesq System of KdV-KdV Type on a Bounded Interval

Lionel Rosier
MINES ParisTech, France
Roberto A. Capistrano-Filho, Ademir F. Pazoto

We consider a Boussinesq system of KdV-KdV type introduced by J. Bona, M. Chen and J.-C. Saut as a model for the motion of small amplitude long waves on the surface of an ideal fluid. This system of two equations can describe the propagation of waves in both directions, while the single KdV equation is limited to unidirectional waves. We are concerned here with the exact controllability of the Boussinesq system by using some boundary controls. By reducing the controllability problem to a spectral problem which is solved by using the Paley-Wiener method, we determine explicitly all the critical lengths for which the exact controllability fails for the linearized system, and give a complete picture of the controllability results with one or two boundary controls of Dirichlet or Neumann type. The extension of the exact controllability to the full Boussinesq system is derived in the energy space in the case of a control of Neumann type. It is obtained by incorporating a boundary feedback in the control in order to ensure a global Kato smoothing effect.

Capillary-Gravity Surface Waves on Water of Finite Depth with Small Surface Tension

Shu-Ming Sun
Virginia Tech, USA
Shengfu Deng

The talk discusses the exact theory for the existence of single or multi-hump surface waves with small oscillations at infinity on a layer of fluid with finite depth. The fluid is assumed to be incompressible and inviscid with a constant density (one common example is water) and the flow is irrotational. The surface wave is propagating with a constant speed on the free surface under gravity and small surface tension. If the wave speed is near its critical value, it has been shown that the fully nonlinear governing equations, also called the Euler equations, have solitary-wave solutions of elevation with small oscillations at infinity, known as generalized solitary waves. In this talk, the existence of two-hump solutions with small oscillations at infinity for the Euler equations will be considered. (This is a joint work with S. Deng).

Lifespan Estimate for the Partial Nonlinear Radiation Problems

Xin Yang
University of Cincinnati, USA
Zhengfang Zhou

This talk is about the lifespan estimate for the heat equation $u_t = \Delta u$ in a bounded domain Ω in $\mathbb{R}^n (n \geq 2)$ with positive initial data u_0 and partial nonlinear radiation boundary conditions. First, the local existence and uniqueness of the classical solution will be discussed. Secondly, both upper and lower bounds of the lifespan will be shown. Finally, the asymptotic behavior of the bounds concerning the nonlinearity power q, the initial data u_0 and the area of the boundary part where the nonlinear radiation occurs will be explored. This is a joint work with Zhengfang Zhou.

Nonhomogeneous Initial Boundary Value Problems of the Fifth-Order KdV Equations Posed on a Bounded Domain

Deqin Zhou
Chongqing University, Peoples Rep of China
Bingyu Zhang

We study the non-homogeneous initial-boundary-value problem (IBVP)of the fifth-order KdV equations posed on a bounded domain $(0, L)$, in the classical Sobolev space $H^s(0, L)$. We show that this problem is locally well-posed in the space $H^s(0, L)$ with $s \geq 0$ and appropriate five boundary conditions. Moreover, the corresponding solution mapping is analytic.

Special Session 25: Celestial Mechanics and N-Body Problem

Kuo-Chang Chen, National Tsing Hua University, Taiwan
Mitsuru Shibayama, Kyoto University, Japan

Celestial mechanics and n-body dynamics have been studied by many prominent mathematicians and physicists for centuries. With developments of mathematical and computational tools, there are exciting progresses during the past two decades. These progresses include variational approaches, stability, chaotic phenomenon, integrability, central configurations, solar system, space mission designs, planetary formations, among many others. In this special session we aim at providing a forum for researchers in this field to share latest developments and exchange ideas.

Central Configurations and Central Measures

Kuo-Chang Chen
National Tsing Hua University, Taiwan
Bo-Yu Pan, Ku-Jung Hsu

In this talk we introduce the concept of central measures which generalizes central configurations to include continuum mass distributions. We show that concentric spherical shells can be properly arranged so that their mass distributions are central measures. For any pair of adjacent shells, the ratio of outer and inner radii is between cubic root of 2 and infinity, and this bound is sharp. This provides a bound for outer and inner radii of concentric spheres if the system explodes or collapses homothetically.

Linear Stability and Morse Index for the Figure-Eight and $K = 5$ Slalom Solutions Under Homogeneous Potential

Toshiaki Fujiwara
Kitasato University, Japan
Hiroshi Fukuda, Hiroshi Ozaki

The figure-eight solution and slalom solutions are periodic solutions to equal-mass planar three-body problem with vanishing angular momentum. They are called 'choreographic' because three bodies chase each other on a single orbit with equal time delay. Slalom solutions are classified by an integer k. The $k = 5$ slaloms belong to the same homotopy class with five iterations of the figure-eight.
Three $k = 5$ slalom solutions are known under Newton potential. We continue the figure-eight and slalom solutions to homogeneous potential $1/r^{\alpha}$. Linear stability and Morse index of them are investigated. Behavior of action integral near bifurcation points are closely examined.

Periodic Solutions Around the Figure-Eight Choreography for the Equal Mass Three-Body Problem

Hiroshi Fukuda
Kitasato University, Japan
Toshiaki Fujiwara, Hiroshi Ozaki

We found a relationship between figure-eight choreography for the equal mass three-body problem under homogeneous potential $-1/r^a$ and H solution found by Simò through Morse index $N(a)$. The H solution is periodic and consists of three different eight-shaped orbits symmetric in both x and y axes. It coincides with figure-eight choreography at $a = 0.9966$ where $N(a)$ changes. In general, changes of Morse index predict several periodic solutions close to figure-eight choreography. At $a = 1.3424$, $N(a)$ changes and a periodic solution H', less symmetric than the H, symmetric only in y axis, is suggested. For the system under Lenard-Jones-type (LJ) potential $1/r^{12} - 1/r^6$, the figure-eight choreography with period T is not scalable. Changes of its Morse index in T predicts another type of periodic solution 8', an eight-shaped choreography not symmetric either in x or y axis but at origin. Numerically the H solution can be found with an isosceles triangle configuration at $t = 0$ and Euler configuration at $t = T/4$, the H' with incongruent isosceles triangle configurations at $t = 0$ and $t = T/2$, and 8' with incongruent Euler configurations at $t = 0$ and $t = T/6$. We found numerically the H and the 8' solution under LJ but did not find the H' either under homogeneous or LJ.

Taiwan's National Space Program Past and Future

Feng-Tai Hwang
National Space Organization, Taiwan

Taiwan has implemented two phases of space technology long-term plans since 1991. As a newly emerging space country, Taiwan has gradually established its own space technology. After 26 years, Taiwan first indigenous FORMOSAT-5 was successfully launched and operated on August 25, 2017, making Taiwan to be one of the few countries with the capability of self-building a high resolution remote sensing satellite. FORMOSAT-7 program with 6 satellites, a joint space program between Taiwan and USA, will be also launched in this year. Currently, Taiwan is planning its the third phase space technology development plan from 2019 to 2028. In the next decade, 10 satellite programs will be implemented, and all of them will be built domestically. In addition to introducing the past and future of Taiwan's national space program, I will also talk about the orbit selections of Taiwan's remote sensing satellites as well as the constellation deployment method of FORMOSAT-7 6 satellites.

Variational Proof of the Existence of Brake Orbits in the 2-Center Problem

Yuika Kajihara
Kyoto University, Japan

The 2-center problem is an integrable Hamiltonian system. The first integrals are complicated functions and it is difficult to know what kind of periodic orbits exist. Brake orbits are simple periodic orbits with symmetry. In this talk, we prove the existence of brake orbits in the 2-center problem by using the variational method.

Orbital Elements Distribution of the Invariant Manifolds Associated to the Lyapunov Family of Periodic Orbits Around L_1 and L_2

Masaya Saito
The Institute of Statistical Mathematics, Japan

One of the key questions on the origin of Jovian irregular satellites, characterized by their highly eccentric orbit and small mass, is how they are transferred to the present place. These satellite are believed to be captured by Jupiter, rather than formed in situ. We employ Lyapunov periodic orbits (LOs) as a formal definition of the vicinity of Jupiter and numerically track the orbital distribution of the invarianet manifold of a LO, varying Jacobi constant C_J. The numerical tracking of the manifold is carried out directly via repeated Poincaré mapping of points initially allocated densely on a fragment of the manifolds near the fixed points, with the assistance of MPFR multiprecision arithmetics. The numerical computations shows that the distribution of semi-major axis of points on the manifolds are quite heavy tailed while its median spans roughly 1-2x the Jovian orbital radius.

Variational Existence Proof of Periodic and Connecting Orbits in the Planar Sitnikov Problem

Mitsuru Shibayama
Kyoto University, Japan

We consider the existence of solutions in the planar Sitnikov problem, realizing given symbolic sequences by based on variational method. We also prove the existence of various periodic solutions and connecting orbits between them.

On the Linear Stability of Relative Equilibrium in N-Body Problem

Shanzhong Sun
Capital Normal University, Peoples Rep of China
Zhiping Huang, Yiming Long

We review the linear stability of relative equilibrium generated by central configurations in N-body problem, especially our recent work towards the understanding of the rings of asteroids.

Counting Central Configurations at the Bifurcation Points

Ya-Lun Tsai
National Chung Hsing University, Taiwan

Enumeration problems for the central configurations of the Newtonian n body problem are hard for $n > 3$ in $\mathbb{R}^2$ and $n > 4$ in $\mathbb{R}^3$. These are problems in finding the numbers of classes of the central configurations for all the masses in a parameter space of positive dimensions. Many results are obtained generically. That is, rigorous proofs of the counting problems only exists for parameters not at the bifurcation points. For the bifurcation points, only numerical evidences are provided due to the complexity of the problems. In this talk, I will propose an algorithm that rigorously proves results on counting central configurations for all masses in one dimensional parameter spaces. Some examples on central configuration problems solved by our method will be presented.

Super Central Configurations and the Number of Central Configurations of N-Body Problem

Zhifu Xie
The University of Southern Mississippi, USA

Let the configuration $q = (q_1, q_2, \cdots, q_n)$ be a central configuration of n-body for masses $m = (m_1, m_2, \cdots, m_n)$. The central configuration q for m is called a super central configuration if q is also a central configuration for a permutation of the same mass vector. There are different ways to count the number of central configurations due to the different definitions of equivalence classes. In this talk, we discuss how the existence of super central configurations affects the number of central configurations under different equivalence classes.

Nonintegrability of Hamiltonian Systems with Homogeneous Potentials of Degree -2

Junji Yamada
Kyoto University, Japan

We discuss nonintegrability of Hamiltonian systems with homogeneous potentials of degree -2. Morales-Ramis theory provides sufficiently conditions for non-integrability of homogeneous potential systems, but it can not be apply to the case that degree of potential is -2. By focusing on remarkable properties of systems, we obtain sufficiently conditions for the non-integrability of these systems. As an application, we prove the nonintegrability of the planer three-body problem with inverse-square potential.

Geometric Properties of Minimizing Orbits in the 3-Body Problem

Duokui Yan
Beihang University, Peoples Rep of China
Wentian Kuang, Yiming Long, Tiancheng Ouyang, Zhifu Xie

In this talk, we introduce new geometric properties for action minimizers connecting two free boundaries in the three body problem. As its application, we can show that the action minimizer connecting a collinear configuration with a binary collision and an isosceles configuration must be one dimensional and it must coincide with the Schubart orbit.

Design of Low Energy Earth-Moon Transfers in the 4-Body System

Hiroaki Yoshimura
Waseda University, Japan

We present the design of a low energy transfer from the Earth to the Moon by using the coupled 3-body system with perturbations for modeling the restricted planar 4-body system of the Sun-Earth-Moon-spacecraft system. In particular, to apply the tube dynamics, we regard the 4-body system as the coupled system of the Sun-Earth-spacecraft 3-body system perturbed by the Moon and the Earth-Moon-spacecraft 3-body system perturbed by the Sun. In both perturbed systems, analogs of stable and unstable manifolds are numerically analyzed and we show how to obtain a family of non-transit orbits departing from a low Earth orbit in the Moon-perturbed system as well as a family of transit orbits arriving into a low lunar orbit in the Sun-perturbed system. Finally, we construct a low energy transfer from the Earth to the Moon from both families and patching these trajectories so that the required Delta V can be minimized under given conditions. This talk is based on the joint work with Kaori Onozaki and Shane Ross.

Application of Morse Index in Weak Force N-Body Problem

Guowei Yu
University of Turin, Italy

Due to collision singularities, the Lagrange action functional of the N-body problem generally is not differentiable in the space of Sobolev paths. Because of this, the usual critical point theory can not be applied to this problem directly. In this paper, we introduce a notion called weak critical point for such an action functional, as a generalization of the usual critical point. A corresponding definition of Morse index for such a weak critical point will also be given. Moreover it will be shown that the Morse index gives an upper bound of the number of possible binary collisions in a weak critical point of the N-body problem with weak force potentials including the Newtonian potential.

Special Session 26: Recent Trends in Navier-Stokes Equations, Euler Equations and Related Problems

Sarka Necasova, Institut of Mathematics, Academy of Sciences, Czech Rep
Reimund Rautmann, Paderborn University, Germany
Werner Varnhorn, University of Kassel, Germany

The active research on this field in many places around the world has led to important new results in diverse directions: e.g. convex integration, measure-valued solutions, weak-strong uniqueness, relative entropy inequality, local smoothness criteria for 3D- solutions, regularity by properties of the vorticity or of one component of the velocity field, singular solutions, self-similar solutions, lower bounds to blowing up solutions, norm inflation, analyticity in general spaces, Lagrangean representation of flows, fluid flow with Chemotaxis, fluid-structure interaction, motion of fluids around rigid or elastic bodies, problem of collisions, nonstandard boundary conditions. The aim of our special session is to bring together researchers working in different directions in order to initiate fruitful discussions.

Global Well-Posedness of the Two-Dimensional Exterior Navier-Stokes Equations for Non-Decaying Data

Ken Abe
Osaka City University, Japan

We consider the two-dimensional Navies-Stokes equations in an exterior domain, subject to the Dirichlet boundary condition. We report some global well-posedness result for bounded initial data with a finite Dirichlet integral, and unique existence of asymptotically constant solutions for large Reynolds numbers.

Interior Regularity to the Steady Incompressible Fluids with Non-Standard Growth

Hyeong-Ohk Bae
Ajou University, Korea
H. So, Y. Young

We consider weak solutions to the equations of stationary motion of a class of non-Newtonian fluids which includes the power law model. The power depends on the spatial variable, which is motivated by electrorheological fluids. We prove the existence of second order derivatives of weak solutions in the shear thinning cases.

Strong Solutions of the Navier-Stokes Equations with Various Types of Boundary Conditions

Petr Kucera
Czech Academy of Sciences, Czech Rep

We solve systems of the Navier-Stokes equations with various types of boundary conditions, e.g., no-slip boundary conditions, boundary conditions of Navier's type and Navier's boundary conditions. We study some properties of strong solutions to these systems, e.g., regularity, local in time existence of strong solutions and robustness. We present criterion for local in time existence of strong solutions to the system of the Navier-Stokes equations with Navier's boundary conditions.

Body with a Cavity Filled with a Compressible Fluid

Vaclav Macha
Czech Academy of Sciences, Czech Rep
G. P. Galdi, S. Necasova

We study the dynamics of a system composed by a rigid body containing a visous compressible fluid. The emphasis is laid upon the analysis of the long time behavior of the whole system. We show that for small initial data the whole system tends to a permanent rotation similarly as in the incompressible case. On the other hand, we highlight some problems coming from compressibility which do not allow to prove the same for solutions emanating from arbitrary initial data.

Bi-Fluid: Existence of Weak Solutions

Piotr Mucha
University of Warsaw, Poland

I plan to talk about the issue of existence of weak solutions to a simple model of a bi-fluid. In rough words, we consider a fluid characterized by two densities of ingredients, and the mathematical model is set by two continuity equations and the Stokes law defining a potential velocity in terms of pressure described by two densities of the fluid. Adapting a brand new tool of compactness developed by D. Bresch and P.E. Jabin we are able to construct solutions, using a novel approach based on the Lagrangian coordinates. What is important to underline, there is no parabolic regularization of continuity equations, and the form of the pressure does not allow to use the standard Lions-Feireisl approach. The talk is based on results joint with Didier Bresch and Ewelina Zatorska.

Weak-Strong Uniqueness for a Fluid-Rigid Body Interaction Problem

Boris Muha
University of Zagreb, Croatia
N.V. Chemetov, S. Necasova

We consider a coupled PDE-ODE system describing the motion of the rigid body in a container filled with the incompressible, viscous fluid. The fluid and the rigid body are coupled via Navier's slip boundary condition. We prove that the local in time strong solution is unique in the larger class of weak solutions on the interval of its existence. To the best of our knowledge this is the first weak-strong uniqueness result in the area of fluid-structure interaction.

Influence of Pressure and Bulk Viscosity in Congestion Phenomena

Sarka Necasova
Academy of Sciences, Institute of Math., Czech Rep
D. Bresch. C. Perrin

We analyze in this lecture macroscopic models for heterogeneous media (mixtures, suspensions, crowd) in dense regimes. These regimes exhibit interesting behaviors such as transition phases (congestion, jamming, glass transition, etc.). Two different approaches are generally considered in the literature to model congestion phenomena at the macroscopic level. The first one, usually called *hard approach*, consists in coupling compressible dynamics in the free domain $\{\rho < \rho^*\}$, with incompressible dynamics in the congested domain $\{\rho = \rho^*\}$. The second one which, by opposition, is called *soft approach*, prevents the apparition of congested phases by introducing in the compressible dynamics repulsive forces which become singular as ρ approaches ρ^*. An intuitive link can be made between the two approaches: if the scope of action of the repulsive forces tends to 0, one expects that the soft congestion model degenerates towards a hard congestion model. The goal of this lecture is to investigate rigorously this link, more precisely we aim at understanding how the choice of the constitutive laws (pressure and/or viscosities as functions of the density) in the soft model impacts the behavior of the limit hard system in congested regions.

On Regularity of a Weak Solution to the Navier-Stokes Equations in Dependence on Some Particular Quantities

Jiri Neustupa
Czech Academy of Sciences, Czech Rep

We deal with a suitable weak solution (v, p) to the Navier–Stokes equations, where $v = (v_1, v_2, v_3)$ is the velocity and p is the pressure. We show that the regularity of (v, p) at a space-time point (x_0, t_0) is essentially determined by the Serrin–type integrability of the positive part of a certain linear combination of v_1^2, v_2^2, v_3^2 and p in a backward neighborhood of (x_0, t_0). An appropriate choice of the coefficients in the linear combination leads to the Serrin–type condition on on the positive part of the Bernoulli pressure $\frac{1}{2}|v|^2 + p$, or the negative part of p, or one component of v, etc.

On Strong Dynamics of a Two Component Mixture Flow

Tomasz Piasecki
University of Warsaw, Poland
Yoshihiro Shibata, Ewelina Zatorska

The mathematical modeling of the flow of chemically reactive mixtures gas been gaining increasing interest the recent years. We investigate the model described in the monograph of V. Giovangigli. The model is composed of compressible Navier-Stokes equations coupled with a system of reaction-diffusion equations describing the evolution of fractional masses. The main difficulty lies in the lack of symmetry of the cross-diffusion matrix. For this reason, the existence results for the system require usually certain simplifying assumptions such as diagonality of the cross-diffusion matrix (Fick Law) or assumption of equal molar masses. More recently, the global existence of weak solutions admitting general diffusion matrix and different molar masses has been obtained by Mucha, Pokorny and Zatorska. However, for strong solutions the only global existence results were available in the whole space, assuming that solutions are close to equilibrium.
In my talk I would like to present our recent result with Y. Shibata and E. Zatorska on the existence of strong solutions in a bounded domain for a two component flow under constant temperature, however for a non-diagonal cross-diffusion matrix and different molar masses. The proof consist in rewriting the problem in new unknowns which lead to a symmetric system. Then we apply the L_p-L_q maximal regularity theory and estimates for the nonlinearities in order to obtain local in time solutions. Finally, using appropriate exponential decay estimate we are able to prolong this solutions for arbitrary times under additional smallness assumptions. The result can be considered a first step in the proof of analogous results for a multispecies model with variable temperature.

Stable Mild Navier-Stokes Solutions

Reimund Rautmann
Paderborn University, Institute of Mathematics, Germany

We consider a variant of the classical Navier-Stokes approximation schemes due to Fujita-Kato and to Giga-Miyakawa: By iterative solution of linear singular Volterra integral equations on any compact time interval J, in any smoothly bounded domain $\Omega \subset \mathbb{R}^n$, $2 \leq n$, again we find the existence of a unique mild Navier-Stokes solution under smallness conditions, but moreover we get the stability of each (possibly large) mild solution, inside a scale of Banach spaces which are imbedded in some spaces $C^0(J, L^r(\Omega))$, $1 < r < \infty$.

Boundary Control of a Self-Propelled Body in a Navier-Stokes Liquid

Ana Silvestre
Instituto Superior Tècnico, Universidade de Lisboa, Portugal
Takeo Takahashi (INRIA Nancy Grand-Est, France), Toshiaki Hishida (Nagoya University, Japan)

Consider a rigid body S moving through a Navier-Stokes liquid L which fills the three-dimensional domain exterior to it, and the steady state regime of the system $\{S, L\}$, as seen by an observer attached to the solid. Self-propulsion means that the total net force and torque, external to the system $\{S, L\}$, acting on S, are identically zero, and may be produced by drawing fluid inwards across portions of the boundary and expelling it from others, or by moving tangentially portions of the boundary. We will see that the problem of finding appropriate boundary values that generate a prescribed motion of S can be solved as a control problem. We will also investigate how to minimize the work needed to overcome the drag exerted by the fluid on the solid.

Navier-Stokes Equations - a Survey of Recent Regularity Results in Terms of ∇U_3, $\partial_3 U$ and $\partial_1 U_3$

Zdenek Skalak
Mathematical Institute CAS, Czech Rep

As is well known, the regularity of the weak solutions of the Navier-Stokes equations (NSE) is still an open problem. In our talk we present a survey of some recent results on the conditional regularity of solutions to NSE in the whole three-dimensional space and focus maily on the Prodi-Serrin conditions imposed only on ∇u_3, $\partial_3 u$ and $\partial_1 u_3$, where $u = (u_1, u_2, u_3)$ stands for the velocity. We are especially interested in optimal results.

About Time Periodic Solutions to the Stokes System in a Layer

Maria Specovius-Neugebauer
University of Kassel, Germany
Konstantin Pileckas, Sebastian Rauchhaus

We consider the time periodic Stokes problem in a layer $\Omega = \mathbb{R}^2 \times (0,1) \ni x = (y, z)$:

$$u_t - \Delta u + \nabla p = f, \quad \operatorname{div} u = g \text{ in } \Omega$$
$$u|_{z=1} = h, \quad u|_{z=0}, \quad u|_{t=0} = u|_{t=2\pi},$$

where the data f, g, h are also time periodic and smooth with bounded support for simplicity and present results about existence and the asymptotic behavior of the solutions in space.

Some Remarks on Leray's Structure Theorem

Werner Varnhorn
Kassel University, Germany
Reinhard Farwig, Hermann Sohr

Flow of Heat Conducting Fluid in a Time Dependent Domain

Aneta Wròblewska-Kamiǹska
Polish Academy of Sciences / Imperial College London, Poland

We consider a flow of heat conducting fluid inside a moving domain whose shape in time is prescribed by a given velocity field. The flow in this case is governed by the compressible Navier-Stokes-Fourier system consisting of equation of continuity, momentum balance, entropy balance and energy equality. The velocity is supposed to fulfil the full-slip boundary condition and we assume that the fluid is thermally isolated. In the presented article we show the existence of a variational solution. To this end we construct proper penalising approximation. This result is a joint work with Ondrej Kreml, Vaclav Macha, and Sarka Necasova.

Special Session 27: Geometry and Dynamics

Konstantinos Efstathiou, University of Groningen, The Netherlands
Andrea Giacobbe, University of Catania, Italy
Tudor Ratiu, Shanghai Jiaotong, China

The aim of this special session on geometry and dynamics is to bring together experts working in a selected range of topics in the forefront of research in this area. The topics that will be covered include the geometry of mechanical systems, including Hamiltonian and non-holonomic systems; symmetries, reduction, and relative equilibria; dynamics and geometry of systems near resonance; and the geometry and topology of integrable Hamiltonian systems.

New Insights in the Geometry and Interconnection of Port-Hamiltonian Systems

María Barbero Liñán
Universidad Politécnica de Madrid, Spain
H. Cendra, E. García-Toraño, D. Martín de Diego

We discuss a new geometric construction of port-Hamiltonian systems by using Dirac structures. The composition of port-Hamiltonian systems can be intrinsically defined by means of the forward and backward of Dirac structures. Using this framework, we revisit the notion of interconnection providing it with an intrinsic description. Some illustrative examples such as electric circuits or multi-body mechanical systems will be discussed.

Optimization by Numerical Algorithms on the Symplectic Group

Petre Birtea
West University of Timisoara, Romania
Ioan Casu, Dan Comanescu

By embedding the real symplectic group $\mathrm{Sp}(2n)$ in $\mathbb{R}^{4n^2}$, we construct the gradient embedded vector field, which is the gradient vector field of a cost functional defined on the symplectic group, and we write it in the ambient coordinates. We present necessary and sufficient conditions for critical points of the cost functional. We further give an explicit formula for the Hessian operator on the symplectic group written in the ambient coordinates. As an application, we explicitly describe the steepest descent and Newton algorithms on $\mathrm{Sp}(2n)$.

Symplectic Invariants of Integrable Hamiltonian Systems: the Case of Degenerate Singularities

Alexey Bolsinov
Moscow State University, Russia

The nature of symplectic invariants for non-degenerate singularities of integrable Hamiltonian systems has been studied and clarified (both in the local and semi-local setting) in fundamentally important papers by Vey, Eliasson, Dufour, Toulet, Miranda, Zung and San Vu Ngoc. The talk is devoted to some new ideas and techniques that can be used for studying symplectic invariants of degenerate singularities. As an example, I would like to discuss normal forms and symplectic invariants of parabolic orbits and cuspidal tori in integrable Hamiltonian systems with two degrees of freedom. Such singularities appear in many integrable systems in geometry and mathematical physics and can be considered as the simplest example of degenerate singularities.

Integrability and Dynamics of the Multi-Dimensional Veselova System

Luis Garcia-Naranjo
UNAM, Mexico
Francesco Fasso, James Montaldi

We analyse the dynamics of the n-dimensional generalisation of the classical nonholonomic Veselova system introduced by Fedorov and Kozlov and treated before by Fedorov and Jovanovic. Our main contribution is to show that under certain symmetric assumptions on the mass distribution of the body, the dynamics is integrable. We proceed by performing a detailed symmetry analysis that allows us to reduce the system to a singular semi-algebraic space where the dynamics is periodic. Our approach allows us to recover all the known cases of integrability that correspond to physical inertia tensors (that turn out to be axi-symmetric bodies) and to determine new ones (that we term cylindrical bodies). Moreover, in both cases we conclude quasi-periodicity of the flow in the natural time variable and without the need of a time reparametrisation. The result is particularly interesting for the cylindrical case considering that the system does not seem to allow a Chaplygin Hamiltonization.

On the Twisting Index of Semitoric Systems

Sonja Hohloch
University of Antwerp, Belgium

A semitoric integrable Hamiltonian system, briefly a semitoric system, is given by two autonomous Hamiltonian systems on a 4-dimensional manifold whose flows Poisson-commute and induce an $(\mathbb{S}^1 \times \mathbb{R})$-action that has only nondegenerate, nonhyperbolic singularities. Semitoric systems have been symplectically classified a couple of years ago by Pelayo & Vu Ngoc by means of five invariants. One of these five invariants is the so-called twisting index which compares the "distinguished" torus action given near each focus-focus singular fiber to the global toric "background action". Although the abstract definition of the twisting index is not very difficult, it was only very recently calculated for the first time, namely for systems with 1 focus-focus singularity, see Alonso & Dullin & Hohloch [arXiv:1712.06402 + 1 preprint in preparation]. In this talk, we present the (results of the) ongoing project with J. Alonso (Antwerp) and J. Palmer (Rutgers): (a) Calculation of the twisting index for the family of semitoric systems admitting 2 focus-focus singularities in Hohloch & Palmer [arXiv:1710.05746]. (b) Putting the twisting index in relation with well-known notions from classical dynamical systems like rotation number, winding number, intersection number etc. (c) Observing the change of the twisting index under changes of semitoric systems.

Lagrangian Averaging with Geodesic Mean

Marcel Oliver
Jacobs University, Germany
Sergiy Vasykevych

We revisit the derivation of the Lagrangian averaged Euler (LAE), or Euler-α equations in the light of an intrinsic definition of the averaged flow map as the geodesic mean on the volume preserving diffeomorphism group. Under the additional assumption that first-order fluctuations are statistically isotropic and transported by the mean flow as a vector field, averaging of the kinetic energy Lagrangian of an ideal fluid yields the LAE Lagrangian. The derivation presented here assumes an Euclidean spatial domain without boundaries.

Families of Semitoric Systems

Joseph Palmer
Rutgers University, USA
Y. Le Floch, S. Hohloch

Semitoric integrable systems are a special class of 4 dimensional integrable systems for which one of the integrals generates a periodic flow. In this talk we discuss recent efforts to construct new examples of these systems and understand their properties. In particular, we focus on semitoric systems which can be obtained via deformations that produce focus-focus singular points.

Nonholonomic Systems with Affine Constraints and Moving Energies

Nicola Sansonetto
University of Verona, Italy
Francesco Fasso, Luis Garcia-Naranjo

Nonholonomic systems with constraints that are affine functions of the velocities do not typically conserve energy. Nevertheless there might exists modifications of the energy, called moving energies, that under certain conditions are first integrals of the systems. In this talk we will discuss the conservation of the energy for nonholonomic systems with affine constraints and investigate the existence of moving energies if the energy is not preserved, furthermore we will relate the existence of moving energies to the presence of symmetries. Then we will apply the obtained results to a class of nonholonomic systems with affine constraints.

Stability Analysis for Generalized Free Rigid Body Dynamics on Real Semi-Simple Lie Algebras

Daisuke Tarama
Ritsumeikan University, Japan
Tudor S. Ratiu

This talk deals with the stability analysis for equilibria of Euler equations on real semi-simple Lie algebras. Around 1980, Mishchenko and Fomenko have introduced a class of generalized Euler equations on arbitrary semi-simple Lie algebras and proved their complete integrability. Precisely speaking, for an arbitrary Cartan subalgebra in the real semi-simple Lie algebra, there is associated a family of Euler equations. Mishchenko and Fomenko have proved that the restriction of such Euler equations to a generic adjoint orbit is completely integrable in the sense of Liouville. Rather recently, the stability of equilibria has been analyzed for Mishchenko-Fomenko rigid body dynamics on real semi-simple Lie algebras of type A, on compact real Lie algebras, and on split real form of complex semi-simple Lie algebras. In this talk, the stability analysis is performed for isolated equilibria on a generic adjoint orbit for Euler equation on any real semi-simple Lie algebra associated with an arbitrary Cartan subalgebra. The stability property of the equilibria is characterized by the types of the roots corresponding to the complexification of the Cartan subalgebra.

Defect in the Joint Spectrum of Hydrogen Due to Monodromy

Holger Waalkens
University of Groningen, Netherlands
Holger R. Dullin

In addition to the well known case of spherical coordinates the Schrödinger equation of the hydrogen atom separates in three further coordinate systems. Separating in a particular coordinate system defines a system of three commuting operators. We show that the joint spectrum of the Hamilton operator, the z-components of the angular momentum, and an operator involving the z-component of the quantum Laplace-Runge-Lenz vector obtained from separation in prolate spheroidal coordinates has quantum monodromy for energies sufficiently close to the ionization threshold. The presence of monodromy means that one cannot globally assign quantum numbers to the joint spectrum. Whereas the principal quantum number n and the magnetic quantum number m correspond to the Bohr-Sommerfeld quantization of globally defined classical actions a third quantum number cannot be globally defined because the third action is globally multi-valued.

Integral Affine Structure on the Base Space of Integrable Systems

Christophe Wacheux
IBS-CGP, Pohang, Korea
T. Ratiu, N.T. Zung

In this talk we will give a sheaf-like presentation of an (integral) affine structure. In that framework, one can give a notion of intrinsic convexity.
The celebrated theorem of Action-Angle coordinates by Liouville-Arnold-Mineur gives as a byproduct the existence of an integral affine structure on the base space of the associated Lagrangian fibration. We will discuss local and global convexity properties for some integrable systems, and present examples and counter-examples.

Special Session 28: Patterns, Traveling Wave Solutions and Symbolic Dynamics

Song-Sun Lin, National Chiao Tung University, Taiwan
Jung-Chao Ban, National Dong Hwa University, Taiwan

This session brings experts of theoretical and numerical dynamical systems from several areas, to discuss the progress on the pattern generation problem, traveling wave solutions in differential equations and synchronization phenomena of the chaotic systems. The aim of this session is to exchange ideas and discuss recent advances in these fields and offer opportunities for potential cooperation in the future.

A Criterion for Almost Periodicity of Substitutive Systems

Shigeki Akiyama
Institute of Mathematics, University of Tsukuba, Japan
Paul Mercat

To show the pure discrete spectrum of a given substitutive dynamics, several coincidence conditions are studied by Ito-Rao and Barge-Kwapisz. In this talk, we give a new formulation of such coincidence conditions for irreducible Pisot substitution, which gives an easy access to the Pisot substitution conjecture. We give several applications of this condition. This is a joint work with Paul Mercat.

The Entropy of Shifts of Finite Type on Cayley Graph

Jung-Chao Ban
National Dong Hwa University, Taiwan
Chih-Hung Chang

In this paper, we provide the effective method to compute the entropy of shifts of finite type (SFT) which is defined on the Cayley graph. Moreover, we present an algebraic characterization of their entropies, which shows that they are equal to the logarithm of the Perron numbers. These results extend the classical results of Z^1 SFTs, and SFTs on Cayley trees, to Cayley graphs.

Degree of Symbolic Dynamics on Finitely Generated Monoids

Chih-Hung Chang
National University of Kaohsiung, Taiwan

In this talk, we consider the complexity of shift spaces on finitely generated monoids. We focus on topological degree rather than topological entropy. It is seen that positive entropy implies full degree.

Nonemptiness Problems of Wang Tiles with Three Colors

Hung-Hsun Chen
National Chiao Tung University, Taiwan
Wen-Guei Hu, De-Jan Lai, Song-Sun Lin

This investigation studies nonemptiness problems of plane edge coloring with three colors. In the edge coloring (or Wang tiles) of a plane, unit squares with colored edges that have one of p colors are arranged side by side such that the touching edges of the adjacent tiles have the same colors. Given a basic set B of Wang tiles, the nonemptiness problem is to determine whether or not $\Sigma(B) \neq \emptyset$, where $\Sigma(B)$ is the set of all global patterns on $\mathbb{Z}^2$ that can be constructed from the Wang tiles in B. Wang's conjecture is that for any B of Wang tiles, $\Sigma(B) \neq \emptyset$ if and only if $P(B) \neq \emptyset$, where $P(B)$ is the set of all periodic patterns on $\mathbb{Z}^2$ that can be generated by the tiles in B. When $p \geq 5$, Wang's conjecture is known to be wrong. When $p = 2$, the conjecture is true. This study proves that when $p = 3$, the conjecture is also true. If $P(B) \neq \emptyset$, then B has a subset $B^{'}$ of minimal cycle generators such that $P(B^{'}) \neq \emptyset$ and $P(B^{''}) = \emptyset$ for $B^{''} \subsetneq B^{'}$. This study demonstrates that the set $C(3)$ of all minimal cycle generators contains 787,605 members that can be classified into 2,906 equivalence classes. $N(3)$ is the set of all maximal non-cycle generators : if $B \in N(3)$, then $P(B) = \emptyset$ and $P(\bar{B}) \neq \emptyset$ for $\bar{B} \supsetneq B$. Wang's conjecture is shown to be true by proving that $B \in N(3)$ implies $\Sigma(B) = \emptyset$.

Stability of Traveling Wavefronts for a Discrete Diffusive Competition System

Cheng-Hsiung Hsu
National Central University, Taiwan
Guang-Sheng Chen, Shi-Liang Wu

This talk is concerned with the nonlinear stability of monostable traveling wavefronts for a discrete three species competition diffusion system. Applying the the weighted energy method combining with the comparison principle, we first show that the traveling wavefronts with large speed are exponentially stable when the initial perturbation around the traveling wave decays exponentially. Then, choosing different weight function, we improve the stability result to any traveling wavefronts with speed greater than the critical wave speed.

Zeta Functions for Two-Dimensional Finite-To-One Sofic Shifts

Wen-Guei Hu
Sichuan University, Taiwan
Song-Sun Lin

This talk is concerned with dynamical zeta functions for two-dimensional finite-to-one sofic shifts, a natural generalization of shifts of finite type. Firstly, the trace operators and rotational matrices are introduced to compute the numbers of periodic patterns for finite-to-one sofic shifts. Then, the zeta function can be represented as an infinite product of rational functions. The results also hold in the coordinates of any unimodulor transformation in general linear groups over integers. Therefore, there is a family of zeta function representations, which is useful to study the natural boundary of zeta functions. This method can apply to higher-dimensional cases.

Intermittent Behaviors in Weakly Coupled Map Lattices

Tiexiang Li
Southeast University, Peoples Rep of China
Wen-Wei Lin, Yiqian Wang, Shing-Tung Yau

In this work, we study intermittent behaviors of coupled piecewise-expanding map lattices with two nodes and a weak coupling. We show that the successive phase transition between ordered and disordered phases occurs for almost every orbit. That is, we prove $\liminf_{n\to\infty} |x_1(n) - x_2(n)| = 0$ and $\limsup_{n\to\infty} |x_1(n) - x_2(n)| \geq c_0 > 0$, where $x_1(n), x_2(n)$ correspond to the coordinates of two nodes at the iterative step n. We also prove the same conclusion for weakly coupled tent-map lattices with any multi-nodes.

Flocking Motions in Cucker-Smale Models

Yu-Hao Liang
National Chiao Tung University, Taiwan
Jonq Juang

Phenomena of collective dynamics such as flocking of birds and schooling of fishes are ubiquitous in the world. Several models have been established to show us insight into how the dynamical behaviors occur. Among them, the one introduced by Cucker and Smale in 2007 has gained much attention. This is due to the fact that their models can be not only used to explain the emergence of flocking dynamics but also applied to design several mobile autonomous agent systems such as UAV and robot systems. In this talk, we will first give a brief survey on Cucker-Smale models. Then we will focus on one of the most important and non-negligible problem, collision avoidance, on the models under an asymmetric network topology. Finally, some open and important issues such as pattern formation and cluster flocking shall be addressed. The talk is based on a paper with Prof. Jonq Juang, recently accepted by SIAP.

Intermittent Behaviors of Coupled Map Lattices

Wen-Wei Lin
National Chiao Tung University, Taiwan
Tiexiang Li, Yiqian Wang, Shing-Tung Yau

In this talk, we study intermittent behaviors of coupled piecewise-expanding map lattices with two nodes and a weak coupling. We show that the successive phase transition between ordered and disordered phases occurs for almost every orbit. That is, we prove that $\limsup |x_1(n) - x_2(n)| \geq c_0 > 0$ and $\liminf |x_1(n) - x_2(n)| = 0$, as $n \to \infty$, where $x_1(n)$, $x_2(n)$ correspond to coordinates of two nodes at the iterative step n. We also prove the same conclusion for weakly coupled tent-map lattices with any multi-node.

Stability Analysis of Traveling Wave Solutions for a Class of Lattice Differential Equations

Jian-Jhong Lin
National Taipei University of Technology, Taiwan
Cheng-Hsiung Hsu

In this talk, we are concerned with the exponential stability of traveling wave solutions for a class of lattice differential equations. By the comparison principles, we can prove that any solution of the Cauchy problem for the lattice differential equations converges exponentially to a traveling wave solution provided that the initial perturbations belonging to suitable weighed spaces.

Dynamics for Mira Maps Near Anti-Integrable Limits

Chen-Chang Peng
National Chiayi University, Taiwan

In this talk, first we introduce that why we study Mira maps and some results about chaos near anti-integrable limits. Near anti-integrable limits, the invariant subsets are hyperbolic repellers but are not all conformal. We prove that these repellers for Mira maps coincide with the attractors of the related iterated function systems and estimate the upper and lower bounds of the fractal dimensions of the attractors for the iterated function systems.

Approximate Synchronization in Coupled Systems

Jui-Pin Tseng
Chengchi University, Taiwan
Chih-Wen Shih

This talk presents a framework to investigate the approximate synchronization of coupled non-identical systems under general coupling schemes with delays. Both delay-dependent and delay-independent criteria for approximate synchronization are derived, based on an approach termed sequential contracting. It is elucidated that the synchronization error decreases with decreasing difference between subsystems, and variations of row sums of the connection matrix and coupling time delays. This error vanishes when these factors decay to zero, and approximate synchronization becomes identical synchronization for the coupled system.
Several examples with numerical simulations are given to illustrate the present theory.

On Smooth Quasiperiodic Schrödinger Operators

Yiqian Wang
Nanjing University, Peoples Rep of China
Linrui Ge, Jinhao Liang, Jiahao Xu, Jiangong You, Zhenghe Zhang

A lot of deep results on (discrete) quasiperiodic Schrödinger operators (QPSO) have been obtained under an analytic condition. However, many of them depend heavily on analytic conditions and are difficult to be extended to smooth situations. In this talk, we survey the area of smooth QPSO and introduce a new dynamical system method on it. Based on it, we obtain a series of positive results only under smooth conditions. Some of them are new even in analytic topology.

Standing Waves in Near-Parallel Vortex Filaments

Chiru Yang
Shantou University, Peoples Rep of China
Walter Craig, Carlos Garcia

In this talk, we consider the model for n almost parallel vortex filaments in a 3 dimensional fluid which takes in consideration the interaction between different filaments and an approximation for the self-induction. We will apply Nash-Moser theorem to prove the existence of periodic standing waves.

Existence and Exponential Stability of Traveling Waves for Delayed Reaction-Diffusion Systems

Tzi-Sheng Yang
Tunghai University, Taiwan
Cheng-Hsiung Hsu, Zhixian Yu

The purpose of this work is to investigate the existence and exponential stability of traveling wave solutions for general delayed multi-component reaction-diffusion systems. Following the monotone iteration scheme via an explicit construction of a pair of upper and lower solutions, we first obtain the existence of monostable traveling wave solutions connecting two different equilibria. Then, applying the techniques of weighted energy method and comparison principle, we show that all solutions of the Cauchy problem for the considered systems converge exponentially to traveling wave solutions provided that the initial perturbations around the traveling wave fronts belong to a suitable weighted Sobolev space.

Special Session 29: Nonlinear Evolution Equations and Related Topics

Mitsuharu Otani, Waseda University, Japan
Tohru Ozawa, Waseda University, Japan

This session will focus on the recent developments in the theory of Nonlinear Evolution Equations and Related Topics including the theory of abstract evolution equations in Banach spaces as well as the studies (the existence, regularity and asymptotic behaviour of solutions) of various types of Nonlinear Partial Differential Equations.

On Global Bounds for Sobolev Norms of Time Global Solutions of Parabolic Problem Involving Critical Sobolev Exponent

Michinori Ishiwata
Osaka University, Japan

In this talk, the existence of time-global bounds of the Sobolev norm of time global solutions for the following semilinear parabolic equation involving critical Sobolev exponent will be discussed:

$$(\mathrm{P})\quad \begin{cases} \partial_t u = \Delta u + u|u|^{p-2} & \text{in } \mathbb{B}R^N \times (0, T_m), \\ u|_{t=0} = u_0 & \text{in } \mathbb{B}R^N, \end{cases}$$

where $N \geq 3$, $u_0 \in H^1(\mathbb{B}R^N) \cap L^\infty(\mathbb{B}R^N)$ (for the simplicity) and T_m denotes the maximal existence time of classical solution of (P) and we only consider the case $T_m = \infty$.

On the Existence of the Global Solutions of the Viscous Cahn-Hilliard Equation

Keiichiro Kagawa
Waseda University, Japan
Mitsuharu Ôtani

We consider an initial boundary value problem for the viscous Cahn-Hilliard equation. The Cahn-Hilliard equation is proposed in 1958 to describe the phase separation in an alloy. When one takes into account internal microforces, some viscous term appears in the Cahn-Hilliard equation, which is called the viscous Cahn-Hilliard equation. In Bui, et al. (2014), they proved the existence of a strong solution under the condition that the nonlinear term $\varphi(u)$ appearing in the chemical potential satisfies $\varphi(u)u \geq 0$ for all $u \in \mathbb{R}$ and Sobolev subcritical growth condition. In this talk, we exclude these conditions by decomposing the nonlinear term $\varphi(u)$ into the sum of a monotone function and a locally Lipschitz perturbation and show the existence of a strong solution. In physics, $\varphi(u) = u^3 - u$ is often used as a typical example. However, the previous result cannot cover this case. Our framework can cover not only this case but also more general cases $\varphi(u) = |u|^{p-2}u - |u|^{q-2}u$ with $p > q \geq 2$.

On Some Parabolic Systems Arising from a Nuclear Reactor Model with Nonlinear Boundary Conditions

Kosuke Kita
Waseda University, Japan
Mitsuharu Ôtani, Hiroki Sakamoto

In this talk, we consider the following initial-boundary value problem for a reaction diffusion system:

$$(\mathrm{NR})\begin{cases} \partial_t u_1 - \Delta u_1 = u_1 u_2 - b u_1, & x \in \Omega,\ t > 0, \\ \partial_t u_2 - \Delta u_2 = a u_1, & x \in \Omega,\ t > 0, \\ \partial_\nu u_1 + \alpha u_1 = \partial_\nu u_2 + \beta |u_2|^{\gamma-2} u_2 = 0, & x \in \partial\Omega,\ t > 0, \\ u_1(x,0) = u_{10}(x) \geq 0,\ u_2(x,0) = u_{20}(x) \geq 0, & x \in \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, ν denotes the unit outward normal vector on $\partial\Omega$ and $\partial_\nu u_i = \nabla u_i \cdot \nu \quad (i = 1, 2)$. Moreover, u_1, u_2 are real-valued unknown functions, a, $b > 0$, $\alpha \geq 0$, $\beta > 0$, $\gamma \geq 2$ are parameters and u_{10}, $u_{20} \in L^\infty(\Omega)$ are given initial data. This system describes diffusion phenomena of neutrons and heat in nuclear reactors, introduced by Kastenberg and Chambrè. In this model, the unknown functions u_1 and u_2 represent the neutron density and the temperature in nuclear reactors respectively. From physical point of view it would be natural to consider the nonlinear boundary condition for the temperature. Indeed, it is well known that the power type nonlinearity on the boundary condition for u_2 is justified by Stefan-Boltzmann's law. We show the existence of positive stationary solutions and investigate some threshold property of them to determine blow-up or global existence of solutions to (NR).

Nonlinear Evolution Equations and Application to a Chemotaxis Model

Akisato Kubo
Fujita Health University, Japan

In this talk we consider initial-Neumann boundary value problem of nonlinear evolution equations with strong dissipation and proliferation arising from mathematical biology and physics formulated as

$$(\mathrm{NE})\begin{cases} u_{tt} = D\triangle u_t + \nabla\cdot(\chi(u_t, e^{-u})\nabla u) + \mu u_t(1-u_t), & (x,t) \in \Omega \times (0,\infty) \\ \partial_\nu u|_{\partial\Omega} = 0 \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x) \end{cases}$$

where constants D, μ are positive, Ω is a bounded domain in R^n with a smooth boundary $\partial\boldsymbol{\Omega}$ and ν is the outer unit normal vector. We show the global existence in time and asymptotic behavior of the solution. Then we apply the result of (NE) to a chemotaxis model proposed by J.I. Tello and A. Kubo, which is a competitive system of a parabolic equation with a logistic term and ODE describing the behavior of two biological species, and discuss the property of the solution.

Global Existence of the Solutions for the Complex Ginzburg-Landau Equations with P-Laplacian

Takanori Kuroda
Waseda University, Japan
Ôtani Mitsuharu

In this talk we consider the following complex Ginzburg-Landau equation

$$\frac{\partial u}{\partial t}(t,x)-(\lambda+i\alpha)\Delta_p u+(\kappa+i\beta)|u|^{q-2}u-\gamma u=f, \quad (t,x)\in[0,T]\times\Omega, \qquad (\mathrm{CGL})_p$$

subject to the homogeneous Dirichlet boundary condition and suitable initial values. Here $-\Delta_p u = -\operatorname{div}(|\nabla u|^{p-2}\nabla u)$ denotes the p-Laplace operator. The parameters in our equation are $\lambda,\kappa > 0$ and $\alpha,\beta,\gamma \in \mathbb{R}$; $i=\sqrt{-1}$ is the imaginary unit; $\Omega \subset \mathbb{R}^N$ is a possibly unbounded domain and $f:[0,T]\times\Omega \to \mathbb{C}$ is an external force with $T>0$. This equation is a generalization of the well-known case $p=2$. We establish the global existence of solutions for $(\mathrm{CGL})_p$ under $\max\{1,2N/(N+2)\}<p$, $2\le q$ and the suitable conditions on parameters $\lambda,\kappa,\alpha,\beta$ subject to q.

Right-Differentiability of Solution to Nonlinear Evolution Equations with Perturbation

Shun Uchida
Waseda University, Japan

We consider the following nonlinear evolution equation in a real Hilbert space.

$$\frac{du}{dt}(t)+\partial\varphi(u(t))+B(t,u(t))\ni f(t) \qquad t\in[0,T],$$

where $\partial\varphi$ is the subdifferential operator generated by a proper lower semi-continuous convex functional φ, B is a perturbation term, and f is a given external force. When $B\equiv 0$, it is well known that the right-differentiability of u at $t_0\in[0,T]$ is equivalent to the property that $u(t_0)$ belongs to the domain of $\partial\varphi$. This is a useful tool for establishing higher order estimates in the analysis of evolution equations. In this talk, we give a sufficient condition of perturbation B to obtain the same characterization of right-differentiability as above.

On a Maximizing Problem of the Sobolev Embedding Related to the Space of Bounded Variation

Hidemitsu Wadade
Kanazawa University / Institute of Science and Engineering, Japan
Michinori Ishiwata

In this talk, we consider the maximizing problem associated with Sobolev embedding related to the space of bounded variation of BV-functions, which is a substitute of the Sobolev space of the marginal case. In our setting of the maximizing problem, we suffer from the non-compactness due to the vanishing phenomenon and the non-reflexivity of the space of BV-functions. In order to overcome these difficulties, we use the fact that the family of maximizers of the Sobolev embedding with BV-functions is the set of characteristic functions on balls. Simultaneously, we give a characterization of maximizers of our problem to prove that the maximizers must form characteristic functions on balls and specify their radii and heights exactly. This is a joint work with Prof. Michinori Ishiwata in Osaka University.

Stability Issues for the Nonlinear Schrödinger Equation Coupled with the Maxwell Equation

Tatsuya Watanabe
Kyoto Sangyo University, Japan
Tatsuya Watanabe

In this talk, we consider stability issues of solitary waves for the nonlinear Schrödinger equation coupled with Maxwell equation. First we study the existence and the uniqueness of minimizers with prescribed charge. Then by adapting the Cazenave-Loins type stability result, we study the orbital stability of standing waves.

Special Session 30: Mathematical Modeling and Computation in Systems and Quantitative Biology

Lei Zhang, Peking University, Peoples Rep of China
Ching-Shan Chou, Ohio State University, USA
Qing Nie, University of California, Irvine, USA

One of the central problems in biology is to understand the principles of complex biological systems. According to the level of biological details include, mathematical and computational models of biological processes involve different levels of mathematical complexity. In this special session, we focus on recent findings of mathematical modeling and computation in systems and quantitative biology. We will discuss the insights gained on the complexity of cellular behaviors and functions, and also the comparison with the experimental observations.

Coupled Mechanochemical Multi-scale Model to Study the Growth Regulation and Morphogenesis During Tissue Development

Weitao Chen
University of California, Riverside, USA
Ali Nematbakhsh

Growth regulation and pattern formation are two main problems in developmental biology. How cells know when to stop growing at certain tissue size with specific shape is an important question in both developmental biology and regenerative medicine, and it is still an unsolved mystery in many systems. During the growth, tissues and organs always exhibit self-government to some extent. Cells stop proliferation precisely when the intended size of the tissue or organ is achieved. Meanwhile, differential cell shapes in space are integrated to give rise to well-organized overall structure. Uncontrolled growth of the cells in tissues or organs will lead to abnormal development or fatal diseases such as cancer. Therefore, developing an extensible predictive mathematical model for exploring the mechanisms involved in the tissue development is significant for understanding the fundamental principles in developmental biology, with a broad range of applications from tissue engineering to biomanufacturing and biotech industry. Experimental data suggests that mechanical properties of cells and chemical signals in both intracellular and extracellular domains play critical roles in size control and shape formation. Here we develop a multi-scale, mechochemical coupled model of tissue growth control. This first-of-class modeling approach provides sub-cellular details to both mechanical properties and chemical signaling during tissue growth. This model is applied to test competing hypotheses in the field to resolve the highly debated question of how tissues reach their final size, as well as how the tissue shape is determined simultaneously.

Fluctuating-Rate Model of Single-Cell Dynamics and Its Applications

Hao Ge
Peking University, Peoples Rep of China

Stochastic processes become more and more popular to model the mesoscopic biophysical dynamics, especially in single-cell biology. We proposed a fluctuating-rate model for the stochastic biochemical dynamics in a single cell, which is indeed piecewise deterministic Markov process. We also found that the fluctuating-rate model yields a nonequilibrium landscape function, which, similar to the energy function for equilibrium fluctuation, provides the leading orders of fluctuations around each phenotypic state, as well as the transition rates between the two phenotypic states. The rigorous proof needs to integrate the well-known Donsker-Varadhan theory and Feidlin-Wentzell theory in such an averaging case. We further apply this model to Lac operon, and show that the stochastic gene-state switching can significantly broaden the environmental parameter ranges for the existence of bistability induced by positive feedback, which can be beneficial dealing with unpredictable environmental changes. We also demonstrate that the phenotype transition rates can help to distinguish two categories of bistability.

A Landscape View on the Interplay Between EMT and Cancer Metastasis

Chunhe Li
Fudan University, Peoples Rep of China
Gabor Balazsi

The epithelial-mesenchymal transition (EMT) is a basic developmental process that converts epithelial cells to mesenchymal cells. Although EMT might promote cancer metastasis, the molecular mechanisms for it remain to be fully clarified. To address this issue, we constructed an EMT-metastasis gene regulatory network model and quantified the potential landscape of cancer metastasis-promoting system computationally. We identified four steady state attractors on the landscape, which separately characterize anti-metastatic (A), metastatic (M), and two other intermediate (I1 and I2) cell states. The tetrastable landscape and the existence of intermediate states are consistent with recent single cell measurements. We identified one of the two intermediate states I1 as the EMT state. From a minimum action path (MAP) approach, we found that for metastatic progression, cells need to first undergo EMT (enter the I1 state), and then become metastatic (switch from the I1 state to the M state). Specifically, for metastatic progression, EMT genes (such as ZEB) should be activated before metastasis genes (such as BACH1). This suggests that temporal order is important for the activation of cellular programs in biological systems, and provides a possible mechanism of EMT promoting cancer metastasis. To identify possible therapeutic targets from this landscape view, we performed sensitivity analysis for individual molecular factors, and identified optimal interventions for landscape control. We found that minimizing transition actions more effectively identifies optimal combinations of targets that induce transitions between attractors than single factor sensitivity analysis. Overall, the landscape view not only suggests that intermediate states increase plasticity during cell fate decisions, providing a possible source for tumor heterogeneity that is critically important in metastatic progress, but also provides a way to identify therapeutic targets for preventing cancer progression.

Mathematical Modeling of Interleukin-35 Promoting Tumor Growth and Angiogenesis

Kang-Ling Liao
Tamkang University, Taiwan
Xue-Feng Bai, Avner Friedman

Interleukin-35 (IL-35), a cytokine from the Interleukin-12 cytokine family, has been considered as an anti-inflammatory cytokine which promotes tumor progression and tumor immune evasion. It has also been demonstrated that IL-35 is secreted by regulatory T cells. Recent mouse experiments have shown that IL-35 produced by cancer cells promotes tumor growth via enhancing myeloid cell accumulation and angiogenesis, and reducing the infiltration of activated $CD8^+$ T cells into tumor microenvironment. In this work, we develop a mathematical model based on these experimental results. We include in the model an anti-IL-35 drug as treatment. The extended model (with drug) is used to design protocols of anti-IL-35 injections for treatment of cancer. We find that with a fixed total amount of drug, continuous injection has better efficacy than intermittent injections in reducing the tumor load while the treatment is ongoing. We also find that the percentage of tumor reduction under anti-IL-35 treatment improves when the production of IL-35 by cancer is increased.

A Link Between Cell Polarization and Colony Formation in Budding Yeast

Wing-Cheong Lo
City University of Hong Kong, Hong Kong
Ching-Shan Chou

Robust cell polarity is critical for cell survival and normal tissue development. Budding yeast, which undergoes polarized growth during budding and mating, has been an excellent model system to study cell polarization. Here we will use budding yeast models to discuss several mechanisms of cell polarization. Also, we will discuss a budding yeast population model to study how budding and mating are involved in colony formation. Our results provide a mathematical framework to study cell polarization in other multicellular organisms.

Scalable Inference of Transcription Dynamics from Single-Cell RNA-Sequencing Data

Jiajun Zhang
Sun Yat-Sen University, Peoples Rep of China

Gene expression levels vary greatly from cell to cell, leading to significant consequences in many biological process from bacterial decision-making to mammalian development. The underlying processes responsible for generating expression variability are poorly understood. Single-cell RNA-seq provides an unprecedented opportunity to decipher this phenomenon, and statistical methods need to be developed to interpret stochasticity in gene expression. We propose a scalable computational pipeline (BMA) to infer the kinetics of stochastic gene expression from single-cell RNA-seq data. Given an underlying model of gene expression, BMA uses a binomial moments approximation method to identify predictive models of transcriptional dynamics. We generate gene transcription models with varying complexity and use BMA to select the most predictive model, then apply BMA to single-cell RNA-seq data of hematopoietic progenitor cells. These results illustrate that BMA provides a flexible and efficient way to investigate the kinetics of transcription.

Quantifying the Biological Functions in Gene Regulatory Networks

Lei Zhang
Peking University, Peoples Rep of China

Gene regulatory network in biology plays a critical role in achieving accurate biological functions. In this talk, I will start with exploration of the topologies for dual function networks by achieving both adaptation and noise attenuation. We show the three-node networks are not able to buffer noise while implementing a good adaptation due to the tradeoff. Thus, we construct a four-node network topology achieves dual functions, in which the fluctuation in input is dampen

significantly in the upstream reactions and the downstream reactions accomplish the adaptive behavior. Secondly, I will present the dual role of Nanog during stem cell differentiation and reprogramming. The low-Nanog state enhances cell differentiation through serving as an intermediate state to reduce the energy barrier of transition. On the contrary, the existence of low stemness low-Nanog state will slow down the reprogramming process, and additional Nanog activation is revealed to be essential to attain fully reprogrammed cell state faster.

Special Session 31: Dissipative Systems and Applications

Georg Hetzer, Auburn University, USA
Wenxian Shen, Auburn University, USA
Feng Cao, Nanjing University of Aeronautics and Astronautics, Peoples Rep of China
Lourdes Tello, Universidad Politecnica de Madrid, Spain

Dissipative Systems arise in many applications. The special session will feature talks from infinite dimensional dynamical systems and random dynamical systems theory to evolutionary partial differential equations and numerical simulation. The scope of applications covers reaction-diffusion systems with local and nonlocal dispersal, ecology and climate modeling.

Spreading Speeds and Traveling Waves for Space-Time Periodic Nonlocal Dispersal Cooperative Systems

Xiongxiong Bao
Changan University, Peoples Rep of China
Wenxian Shen, Zhongwei Shen

In this talk, I am concerned with the spatial spreading speeds and traveling wave solutions of cooperative systems in space-time periodic habitats with nonlocal dispersal. First, we will introduce some principal eigenvalue theory for space-time periodic linear cooperative systems with nonlocal dispersal. Then this cooperative system has a finite spreading speed interval, and the system also has a single spreading speed under certain condition. Next, we show the existence of space-time periodic traveling wave solutions for this system. Finally, we apply the above results to nonlocal monostable equations and two-species competitive systems with nonlocal dispersal and space-time periodicity. This is a joint work with W. Shen and Z. Shen.

Transition Waves of Lattice KPP Equations in Heterogeneous Media

Feng Cao
Nanjing University of Aeronautics and Astronautics, Peoples Rep of China
Wenxian Shen

The study of front propagation dynamics of Fisher-KPP type equations with general time and/or space dependence is attracting more and more attention due to the presence of general time and space variations in real world problems. In this talk, we investigate the stability and uniqueness of transition wave solutions of lattice Fisher-KPP equations with general time and space dependence and discuss the applications on the existence, stability and uniqueness of periodic traveling wave solutions of lattice Fisher-KPP equations in time and space periodic media, and the existence, stability and uniqueness of transition wave solutions of lattice Fisher-KPP equations in time heterogeneous media.

Asymptotic Behavior of a Neural Field Lattice Model with a Heaviside Operator

Xiaoying Han
Auburn University, USA
Peter E. Kloeden

Motivated by the importance of discrete structures of neuron networks, a neural field lattice system arising from the discretization of neural field models in the form of integro-differential equations is studied. The neural field lattice system is first formulated as a differential inclusion on a weighted space of infinite sequences, due to the switching effects. Then the existence of solutions of the resulting differential inclusion is proved by a series of sequential finite-dimensional approximations. The solutions are shown to generate a nonautonomous set-valued dynamical system which possesses a pullback attractor. Forward omega limit sets for the set-valued dynamical system are also discussed.

Quasiconvergence in Parabolic Equations in One Dimensional Space

Fang Li
Shanghai Normal University, Peoples Rep of China

In this talk we first prove a quasiconvergence result (that is, the ω-limit set of a solution consists entirely of steady states) for bounded solutions of general quasilinear parabolic equations in unbounded or variable domains in one dimensional space. Then, we use this result to study a one dimensional heterogeneous reaction diffusion equation with combustion type of nonlinearity. We prove a trichotomy result on the asymptotic behavior of solutions of the Cauchy problem: any nonnegative solution converges as $t \to \infty$ to either a positive steady state, or the ignition point, or the trivial solution 0.

Global Stability of Feedback Systems with Multiplicative Noise on the Nonnegative Orthant

Xiang Lv
Shanghai Normal University, Peoples Rep of China
Jifa Jiang

We investigate the dynamical behavior of pull-back trajectories for feedback systems with multiplicative noise and prove that there exists a globally stable positive random equilibrium in the nonnegative orthant $\mathbb{R}^d_+$, where the global stability means that all pull-back trajectories originating from nonnegative orthant converge to this positive random equilibrium almost surely. Our results can be applied to well-known stochastic Goodwin negative feedback system, Othmer-Tyson positive feedback system and Griffith positive feedback system as well as other stochastic cooperative, competitive and predator-prey systems. This is a joint work with Prof. Jifa Jiang.

Cross-Diffusion-Driven Instability for Reaction-Diffusion Systems on Evolving Domains and Surfaces: Models, Analysis and Simulations

Anotida Madzvamuse
University of Sussex, England
Raquel Barreira

In this talk, I will present cross-diffusion and domain-growth induced pattern formation for reaction-diffusion systems with linear cross-diffusion on evolving domains and surfaces. Our major contribution is that by selecting parameter values from spaces induced by cross-diffusion, domain and surface evolution, patterns emerge only when cross-diffusion and domain growth are present. Such patterns do not exist in the absence of cross-diffusion, domain and surface evolution. In order to compute these cross-diffusion domain-induced parameter spaces, linear stability theory is employed to establish the necessary conditions for domain-growth induced cross-diffusion-driven instability for reaction-diffusion systems with linear cross-diffusion. Model reaction-kinetic parameter values are then identified from parameter spaces induced by cross-diffusion and domain-growth only; these exist outside the classical standard Turing space on stationary domains and surfaces. To exhibit these patterns we employ the surface finite element method for solving reaction-diffusion systems with cross-diffusion on continuously evolving domains and surfaces.

On a Nonlocal Evolution Interior Bernoulli-Type Problem

Juan Francisco Padial
Universidad Politecnica de Madrid, Spain

Our aim is to study the existence of solutions for a nonlocal evolution interior Bernoulli-type free boundary problem with a unknown measure data. We observed that semilinear problems that can be written as $-\Delta u(x) = F(x, u(x))$, $x \in \Omega$ (are given open bounded set in $\mathbb{R}^N$), with boundary conditions, are intensively studied in the literature when F is a function from $\Omega \times \mathbb{R}$ into $\mathbb{R}$. But, we realize that some models in Physics, or Mechanics can be expressed as $-\Delta u(x) = \mu(x, u)$ in $\mathcal{D}'(\Omega)$, where $\mu(x, u)$ is a Radon measure depending also on the own solution u. The problem arises in several nonlinear flow laws and physical situation. The elliptic problem was studied by J.I. Dìaz, J.F. Padial and J.M. Rakotoson in "On some Bernoulli free boundary type problems for general elliptic operators", Proceedings of the Royal Society of Edinburgh, 137A (2007), 895-911. To look for a weak solution to the associated evolution problem, we introduce a semi-implicit time differencing in order to obtain a family of elliptic problems. For each one of this problems, we find weak solution by applying a general mountain pass principle due to Ghoussoub- Preiss for a sequence of approximate nonsingular problems. Finally, apriori estimates allow us to obtain the solution by passing to the limit. (Joint work with J.M. Rakotoson).

Traveling Curved Waves in Two Dimensional Excitable Media

Chang-Hong Wu
National University of Tainan, Taiwan
Hirokazu Ninomiya

Wave propagation phenomenon can occur in various area such as physics, biology, chemical kinetics, and so on. In particular, excitable media can support abundant spatiotemporal dynamics. In this talk, we focus on a free boundary problem in two-dimensional excitable media arising from a singular limiting problem of a FitzHugh-Nagumo-type reaction-diffusion system. The existence, uniqueness and stability of traveling curved waves will be discussed. This is a joint work with Hirokazu Ninomiya (Meiji University).

Geometric Singular Approach to Poisson-Nernst-Planck Models for Ionic Flows Through Membrane Channels: Effects from Ion Sizes

Mingji Zhang
New Mexico Institute of Mining and Technology, USA
Peter W. Bates, Weishi Liu, Hong Lu

We analyze a one-dimensional steady-state Poisson-Nernst-Planck type model for ionic flows through ion channels with oppositely charged ion species. A local hard-sphere potential is included to account for ion size effects on ionic flows. Our analysis is based on the geometric singular perturbation theory but, most importantly, on specific structures of this model. The existence of solutions to the boundary value problem for small ion sizes is established. Treating ion sizes as small parameters, we derive approximations of the individual fluxes, the I-V relation and identify some critical potentials for ion size effects. Under electroneutrality conditions, each of these critical potentials separates the potential into two regions over which ion size effects are qualitatively opposite to each other. Without electroneutrality conditions, the qualitative effects of ion sizes will depend not only on the critical potentials but also on boundary concentrations and relative ion valences. The flow properties of interest depend on multiple physical parameters such as boundary conditions and diffusion coefficients, in addition to ion sizes and valences. For the relatively simple setting and assumptions of the model, we are able to characterize the distinct effects of the nonlinear interplay between these physical parameters.

Dynamics of Almost Periodic Parabolic Equations on the Circle

Dun Zhou
Nanjing University of Science and Technology, Peoples Rep of China
Wenxian Shen, Yi Wang

In this talk, we consider the skew-product semiflows generated by almost periodic semilinear parabolic equations on the circle. Structures of Omega limit sets of bounded solutions of these equations were thoroughly investigated. Our results show that there are some essential differences between general non-autonomous systems and autonomous systems, or even periodic systems; moreover, almost-periodically forced circle flows may occur in these systems. This is a joint work with Prof. Wenxian Shen and Prof. Yi Wang.

Special Session 32: Control and Optimization: New Developments and Applications

Monica Motta, University of Padua, Italy
Alexander J. Zaslavski, The Technion- IIT, Israel
Hong-Kun Xu, Hangzhou Dianzi University, Peoples Rep of China
Jen-Chih Yao, Kaohsiung Medical University, Taiwan

This special session on new developments in control and optimization and their applications will bring together a selected group of experts in these areas. The growing importance of control and optimization has been realized in recent years. This is due not only to theoretical developments, but also because of numerous applications to engineering, economics and life sciences. The topics which will be discussed include turnpike phenomenon, averaging in optimal control, impulsive optimal control problems, necessary and sufficient optimality conditions, qualitative and quantitative aspects of optimal control, control and stabilization of PDEs.

Necessary Optimality Conditions for Impulsive Optimal Control Problems

Maria Soledad Aronna
Escola de Matematica Aplicada (FGV), Brazil
Monica Motta, Franco Rampazzo

We study control problems governed by nonlinear ordinary differential equations of the form

$$\dot{x}(t) = f(x(t), u(t), v(t)) + \sum_{\alpha=1}^{m} g_\alpha(x(t), u(t))\dot{u}_\alpha(t),$$

for $t \in [0, T]$ where $x : [0, T] \to \mathbb{R}^n$ is the *state variable,* $u : [0, T] \to \mathbb{R}^m$ is the *impulsive control* and $v : [0, T] \to \mathbb{R}^l$ is the *ordinary control.* The control u is allowed to be a (discontinuous) bounded variation function, which gives the system an impulsive character. For this class of equations, we adopt the concept of *graph completion solution,* that was introduced by A. Bressan and F. Rampazzo in the 90's.
We consider an optimal control problem in the Mayer form, with general control and final state constraints, for which we prove a maximum principle and higher-order necessary conditions in terms of the adjoint state and the Lie brackets of the involved vector fields.

Necessary Optimality Conditions and Constant Rank

Joel Blot
Université Paris 1 Panthéon-Sorbonne, France

We present new results of the theory of the necessary optimality conditions for problems of Calculus of Variations and of Optimal Control by using conditions of constancy of the rank of the differentials of the constraints mappings. We apply them to infinite-dimensional infinite-horizon dynamical problems.

Recent Advances on Brezis-Nirenberg Type Theorems on Local Minimizers

Leszek Gasinski
Jagiellonian University, Poland

Since 1993, when the paper of H. Brezis and L. Nirenberg on local minimizers was published, many versions, generalizations and competitive proofs of their theorem have appeared. A short survey of these results together with their applications will be presented.

On the Hamilton-Jacobi-Bellman Equation of Macroeconomic Dynamics

Yuhki Hosoya
Kanto-Gakuin University, Japan
Susumu Kuwata, Hiroyuki Ozaki

We consider an extension of the classical capital accumulation model, and show that the value function is the unique classical solution of the Hamilton-Jacobi-Bellman equation in some class of functions. Moreover, we propose a construction method of the solution of this model from the value function and the Hamilton-Jacobi-Bellman equation. The class of models we treat in this paper is sufficiently wide, and includes the Ramsey-Cass-Koopmans model with unbounded instantaneous utility function that admits some unbounded growth paths. By using our result, we solves some linear technology model in which the utility function is, for example, logarithmic.

Intrinsic and Apparent Singularities in Differentially Flat Systems, and Application to Global Motion Planning

Yirmeyahu Kaminski
Holon Institute of Technology, Israel
Yirmeyahu Kaminski, Jean Levine, Francois Ollivier

In this talk, we study the singularities of differentially flat systems, in the perspective of providing global or semi-global motion planning solutions for such systems: flat outputs may fail to be globally defined, thus potentially preventing from planning trajectories leaving their domain of definition, the complement of which we call singular. Such singular subsets are classified into two types: apparent and intrinsic. A rigorous definition of these singularities is introduced in terms of atlas and local charts in the framework of the differential geometry of jets of infinite order and Lie-Backlund isomorphisms. We then give an inclusion result allowing to effectively compute all or part of the intrinsic singularities. Finally, we show how our results apply to the global motion planning of the celebrated example of non holonomic car.

Barriers in Nonlinear Control Systems with Mixed Constraints, Some Applications and Open Questions

Jean Lévine
Mines-ParisTech and FSMP, France
W. Esterhuizen

In this talk, we extend previous results, obtained by the authors, on the determination of the *admissible set*, namely the subset of the state space where the *mixed multidimensional constraints*, i.e. constraints involving both the state and input vectors, can be satisfied for all times. We prove that the boundary of this admissible set may be divided in two parts, one of them being called *barrier*, a semipermeable surface that must satisfy a minimum-like principle involving the Karush-Kühn-Tucker multipliers associated to the constraints and endpoint conditions describing the geometric way this barrier ends when touching the constraint boundary. The proof uses a similar argument, in the context of mixed constraints and without optimality requirements, as in the Pontryagin-Boltyansky-Gamkrelidze-Mischenko construction of extremals by needle perturbations, without restricting the extremal integral curves to remain on a constraint boundary and without restriction on the regularity of the extremal control vectors. We then give a quick outline of some applications and open questions.

Lipschitz Regularity of the Minimizers for Non Autonomous Problems with Slow Growth of the Calculus of Variations

Carlo Mariconda
University of Padua, Italy
Piernicola Bettiol

We consider the Lipschitz regularity of the minimizers to the constrained problem of the calculus of variations:

$$\min \int_a^b L(t, x(t), x'(t))\, dt,\ x \in W^{1,1}([a,b]),$$
$$x(a) = A, x(b) = B, x(t) \in \Sigma \subset R^n.$$

A celebrated result by F. Clarke and R. Vinter asserts the Lipschitz regularity of the minimizers when the Lagrangian L is *autonomous*, locally Lipschitz and convex in the velocity variable. We prove that the result holds true for *non autonomous* lagrangians by assuming merely that the Lagrangian is *Borel*, locally Lipschitz (just) in t, under a *weak growth condition* that is slower than superlinearity. The proof relies upon a new extension of the Du Bois-Reymond necessary condition, obtained via Clarke's Extended Maximum Principle.

Properties of Weak Solutions to Stochastic Inclusions and Their Applications to Optimization Problems

Mariusz Michta
University of Zielona Gora, Poland

In the talk we study the problem of existence of weak solutions to stochastic inclusions. Further we present main topological properties of such solutions and discuss their applications to optimization in some economic problems.

Normality and Gap Phenomena in Optimal Unbounded Control

Monica Motta
Dep. of Mathematics, University of Padua, Italy
Franco Rampazzo, Richard Vinter

Optimal unbounded control problems with affine control dependence may fail to have minimizers in the class of absolutely continuous state trajectories. For this reason, extended impulsive versions –which cannot be of measure-theoretical type– have been investigated, in which the domain is enlarged to include discontinuous state trajectories of bounded variation, and for which existence of minimizers is guaranteed. It is of interest to know whether the passage from the original optimal control problem to its extension introduces an infimum gap. This paper provides sufficient conditions for the absence of an infimum gap based on normality of extremals. In certain cases, the normality conditions reduce to simple verifiable criteria, which improve on earlier, directly-derived sufficient conditions for no infimum gap.

A Stochastic Model of Optimal Debt Management and Bankruptcy

Tien Khai Nguyen
North Carolina State University, USA
A. Marigonda, A. Bressan and M. Palladino

Consider a problem of optimal debt management which is modeled as a non-cooperative game between a borrower and a pool of risk neutral lenders. Since the debtor may go bankrupt, lenders charge a higher interest rate to offset the possible loss of part of their investment. In this talk, I will present results on existence and properties of optimal strategies, both in a deterministic and in a stochastic framework.

Metric Regularity Under Gâteaux Differentiability with Applications to Optimization and Stochastic Optimal Control Problems

Francisco Jose Silva Alvarez
XLIM, DMI, Université de Limoges, France
A. Jourani

In this talk, we consider the existence of Lagrange multipliers for infinite dimensional problems under Gâteaux differentiability assumptions on the data. Our investigation follows two main steps: the proof of the existence of Lagrange multipliers under a calmness assumption on the constraints and the study of sufficient conditions, which only involve the Gâteaux derivative of the function defining the constraint, that ensure this assumption. We apply the abstract results to recover in a direct manner the optimality systems associated to two types of standard stochastic optimal control problems.

Turnpike Conditions for Optimal Control Problems

Alexander Zaslavski
The Technion - Israel Institute of Technology, Israel

We study turnpike properties of approximate solutions of optimal control systems. To have these properties means that the approximate solutions of the problems are determined mainly by the objective functions, and are essentially independent of the choice of intervals and endpoint conditions, except in regions close to the endpoints. We discuss necessary and sufficient turnpike conditions.

Special Session 33: Dynamics of Parabolic Type Equations in Life Sciences and Physics

Wan-Tong Li, Lanzhou University, Peoples Rep of China
Guo Lin, Lanzhou University, Peoples Rep of China
Zhi-Cheng Wang, Lanzhou University, Peoples Rep of China

This special session focuses on the spatial dynamics of parabolic type equations. Due to the backgrounds of these equations in life sciences and physics as well as the importance in mathematical literature, we shall pay attention to the dynamics formulated by transition dynamics and other spatial-temporal modes. This session will try to present as much information as possible and offer opportunities for potential cooperation.

Analysis and Control of Population Dynamics Models

Yuan He
Lanzhou University, Peoples Rep of China
Bedreddine Ainseba

In this talk, we consider a multistage, physiologically structured, population model describing one of the most important grapevine insect pests. Growth of the population at each stage is modeled considering the climatic variations and the grape variety. We obtain the existence and uniqueness of solutions for the model, and also prove the existence of a global attractor for the trajectories of the dynamical system defined by the solutions of the model. Then we analyze the exact null and approximate controllability of the lobesia botrana model with nonlocal boundary condition. The main result is established by combining some estimations and the Carleman inequality for the backward system related to an optimal control problem.

Traveling Waves of Two Species Competition System with Nonlocal Dispersal in Periodic Habitats

Wantong Li
Lanzhou University, Peoples Rep of China
Xiongxiong Bao, Wenxian Shen

This paper is concerned with space periodic traveling wave solutions of the following Lotka-Volterra competition system with nonlocal dispersal and space periodic dependence,

$$\begin{cases} \frac{\partial u_1}{\partial t} = \int_{\mathbb{R}^N} \kappa(y-x)u_1(t,y)dy - u_1(t,x) \\ \quad +u_1(a_1(x) - b_1(x)u_1 - c_1(x)u_1),\ x \in \mathbb{R}^N \\ \frac{\partial u_2}{\partial t} = \int_{\mathbb{R}^N} \kappa(y-x)u_2(t,y)dy - u_2(t,x) \\ \quad +u_2(a_2(x) - b_2(x)u_1 - c_2(x)u_2),\ x \in \mathbb{R}^N. \end{cases}$$

Under suitable assumptions, the system admits two semitrivial space periodic equilibria $(u_1^*(x), 0)$ and $(0, u_2^*(x))$, where $(u_1^*(x), 0)$ is linearly and globally stable and $(0, u_2^*(x))$ is linearly unstable with respect to space periodic perturbations. By sub- and supersolution techniques and comparison principals, we show that, for any given $\xi \in S^{N-1}$, there exists a continuous periodic traveling wave solution of the form $(u_1(t,x), u_2(t,x)) = (\Phi_1(x - ct\xi, ct\xi), \Phi_2(x - ct\xi, ct\xi))$ connecting $(u_1^*(\cdot), 0)$ and $(0, u_2^*(\cdot))$ and propagating in the direction of ξ with speed $c > c^*(\xi)$, where $c^*(\xi)$ is the spreading speed of the system in the direction of ξ. Moreover, for $cc^*(\xi)$, we also prove the asymptotic stability and uniqueness of traveling wave solution using squeezing techniques.

Propagation Dynamics of Time Periodic Systems

Guo Lin
Lanzhou University, Peoples Rep of China

In this talk, I shall present some results on traveling wave solutions and asymptotic spreading. These systems do not generate monotone semiflows and are reducible at the unstable steady state. On the traveling wave solutions, we focus on minimal wave speed by presenting existence or nonexistence of traveling wave solutions. When the asymptotic spreading is concerned, we try to estimate different spreading speeds of different unknown functions governed by the same systems.

Transition Fronts of Combustion Reaction Diffusion Equations in $\mathbb{R}^N$

Zhi-Cheng Wang
Lanzhou University, Peoples Rep of China

This paper is concerned with combustion transition fronts in $\mathbb{R}^N$ $(N \geq 1)$. Firstly, we prove the existence and the uniqueness of the global mean speed which is independent of the shape of the level sets of the fronts. Secondly, we show that the planar fronts can be characterized in the more general class of almost-planar fronts. Thirdly, we show the existence of new types of transitions fronts in $\mathbb{R}^N$ which are not standard traveling fronts. Finally, we prove that all transition fronts are monotone increasing in time, whatever shape their level sets may have.

Stability of Monostable Traveling Waves of Reaction-Diffusion Equations with Delay

Yunrui Yang
LanZhou JiaoTong University, Peoples Rep of China

In this talk, we are concerned with the stability of monostable traveling waves of scalar reaction-diffusion equations and systems with delay. Especially, it is more difficult to investigate the stability of critical waves of delayed systems without quasi-monotonicity for the facts that comparison principle does not hold for the scarcity of quasi-monotonicity in equations; the normally weighted energy method can not be applied to monostable traveling waves in the case of critical speed for their asymptotic behaviors at positive infinity and it is not easy to get the decay estimates of elementary solutions of ordinary functional differential equations because of the coupling of different unknown functions, and thus it is not suitable any longer by using the frequent methods and normal theory to solve the stability of traveling wave solutions for classical reaction-diffusion equations. Therefore, it brings a new challenge to develop and improve theory of traveling waves of reaction-diffusion equations with delay.

Spatio-Temporal Propagation of Nonlocal Anisotropic Dispersal Equations

Li Zhang
Changan University, Peoples Rep of China
Wan-Tong Li, Zhi-Cheng Wang

We mainly focus on the entire solutions of a nonlocal dispersal equation with asymmetric kernel function and different reaction terms, such as monostable, bistable and ignition nonlinearities. Compared with symmetric case, the asymmetry of the dispersal kernel function has a great influence on the profile of the traveling waves and the sign and size of the wave speeds, which further makes the types and properties of the entire solution more diverse. Accoding to different reaction terms, we established many different front-like entire solutions and investigated their qualitative features. Before this, we characterize the precise asymptotic behaviors of the traveling wave solutions at infinity since they play an important role in obtaining entire solutions.

Threshold Dynamics of a Reaction-Diffusion Epidemic Model with Stage Structure

Liang Zhang
Lanzhou University, Peoples Rep of China
Zhi-Cheng Wang

In this talk, we investige a time-delayed reaction-diffusion epidemic model with stage structure and spatial heterogeneity, which describes the dynamics of disease spread only proceeding in the adult population. We establish the basic reproduction number for the model system, which gives the threshold dynamics. Furthermore, it is shown that there is at least one positive steady state when the basic reproduction number is large than unity. Finally, in terms of general birth function for adult individuals, through introducing two numbers, we establish sufficient conditions for the persistence and global extinction of the disease, respectively.

Special Session 34: Modeling and Computational Methods for Dynamics on Networks and their Applications

Xiaojing Ye, Georgia State University, USA
Haomin Zhou, Georgia Institute of Technology, USA

In recent years, there has been high demand for novel and accurate mathematical models, and fast, stable and scalable computational techniques to address problems emerging from applications on real-world networks, such as online social networking, smart power grids, Internet of Things. In these applications, big data is often generated, collected, stored and processed in large-scale heterogeneous networks. New models and computational methods must tackle challenges of inhomogeneous structures of networks, randomness of time-varying dynamics, and noise in data. This mini-symposium focuses on the recent advances of mathematical modeling and numerical methods as well as their applications.

Decentralized Consensus Algorithms for Distributed Optimization

Yunmei Chen
University of Florida, USA
Chenxi Chen, Xiaojing Ye

We propose two decentralized consensus algorithms for solving a class of distributed optimization problems. The first one is based on deterministic primal-dual method that can achieve the rate of convergence O(1/N) for both objective and consensus, where N is the number of iterations, The total cost of communication is O(Nmd), where m is the number of nodes in the network, and d is the degree of the network. To reduce the communication cost, we propose the second algorithm based on randomized primal-dual algorithm. This algorithm maintains the same rate of convergence for both objective and consensus at O(1/N), but with O(d) cost of communication for each iteration. Compared to the cost of communication of deterministic primal-dual based method, randomized primal-dual based method is capable to reduce the total communication cost by O(m). The numerical results indicate the effectiveness of the proposed methods.

PDE-Net: Learning PDEs from Data

Bin Dong
Peking University, Peoples Rep of China
Zichao Long, Yiping Lu, Xianzhong Ma, Bin Dong

Partial differential equations (PDEs) play a prominent role in many disciplines such as applied mathematics, physics, chemistry, material science, computer science, etc. PDEs are commonly derived based on physical laws or empirical observations. However, the governing equations for many complex systems in modern applications are still not fully known. With the rapid development of sensors, computational power, and data storage in the past decade, huge quantities of data can be easily collected and efficiently stored. Such vast quantity of data offers new opportunities for data-driven discovery of hidden physical laws. Inspired by the latest development of neural network designs in deep learning, we propose a new feed-forward deep network, called PDE-Net, to fulfill two objectives at the same time: to accurately predict dynamics of complex systems and to uncover the underlying hidden PDE models. The basic idea of the proposed PDE-Net is to learn differential operators by learning convolution kernels (filters), and apply neural networks or other machine learning methods to approximate the unknown nonlinear responses. Comparing with existing approaches, which either assume the form of the nonlinear response is known or fix certain finite difference approximations of differential operators, our approach has the most flexibility by learning both differential operators and the nonlinear responses. A special feature of the proposed PDE-Net is that all filters are properly constrained, which enables us to easily identify the governing PDE models while still maintaining the expressive and predictive power of the network. These constrains are carefully designed by fully exploiting the relation between the orders of differential operators and the orders of sum rules of filters (an important concept originated from wavelet theory). We also discuss relations of the PDE-Net with some existing networks in computer vision such as Network-In-Network (NIN) and Residual Neural Network (ResNet). Numerical experiments show that the PDE-Net has the potential to uncover the hidden PDE of the observed dynamics, and predict the dynamical behavior for a relatively long time, even in a noisy environment.

Optimal Transport on Graphs with Applications

Wuchen Li
UCLA, USA
Wuchen Li

In recent years, optimal transport has witnessed a lot of applications in statistics, image processing, and machine learning. It provides a solid metric among histograms that incorporate the geometry (ground metric) of features. In this talk, we introduce the optimal transport on finite graphs, from which the probability simplex set forms a Riemannian manifold. We call it probability manifold. Various developments related to the optimal control problems in the probability manifold, e.g. discrete Fokker-Planck equation, Schrödinger equation/bridge problem and generalized Hopf-Lax formula will be sketched. Their connections with Shannon-Boltzmann entropy and

Fisher information on graphs will be emphasized. Many applications will be discussed, including L1 Monge-Kantorovich problem, image segmentation, and population games.

Analysis and Simulation of Multiscale Stochastic Intracellular BioChemical Reacting Networks

Di Liu
Michigan State University, USA

Intracellular reacting networks involving gene regulation often exhibits multiscale properties. That includes multiple reacting rates, multiple population magnitudes and multi-stability. Direct Stochastic Simulation Algorithm (SSA) would turn out to be inefficient dealing with such systems. Schemes such as Nested SSA and Tau-leaping method have proved to be effective for certain asymptotic regimes. Based on the framework of transition path theory (TPT), we extended the probability current between two adjacent reacting states to single reacting states as well as reacting trajectories, thereby give the definition of transition state (TS) as states with maximum velocity strength. I will present recent results on the convergence analysis and applications of the algorithms.

Deep Coevolutionary Network

Le Song
Georgia Tech, USA
Le Song

Recommender systems often use latent features to explain the behaviors of users and capture the properties of items. As users interact with different items over time, user and item features can influence each other, evolve and co-evolve over time. The compatibility of user and item's feature further influence the future interaction between users and items. Recently, point process based models have been proposed in the literature aiming to capture the temporally evolving nature of these latent features. However, these models often make strong parametric assumptions about the evolution process of the user and item latent features, which may not reflect the reality, and has limited power in expressing the complex and nonlinear dynamics underlying these processes. To address these limitations, we propose a novel deep coevolutionary network model (DeepCoevolve), for learning user and item features based on their interaction graph. DeepCoevolve use neural network over evolving networks to define the intensity function in point processes, which allows the model to capture complex mutual influence between users and items, and the feature evolution over time. We also develop an efficient procedure for training the model parameters, and show that the learned models lead to significant improvements in recommendation and activity prediction compared to previous state-of-the-arts parametric models.

Cyber-Physical Data Analytics and Security in Energy and Environment Systems

Wenzhan Song
University of Georgia, USA

This talk will present several research opportunities and case studies of Cyber-physical Data Analytics and Security in energy and environment systems. We will present an innovative Real-time In-situ Seismic Imaging (RISI) system design that has vast application in oil/gas extraction and environment safety. It is a smart sensor network that senses and computes the 3D subsurface imaging in real-time and continuously. Instead of data collection then post processing, the mesh network performs the distributed data processing and tomographic inversion computing under the severe bandwidth and resource constraints, and generates an evolving 3D subsurface image as more data arrives. A RISI system is essentially a “Subsurface Camera” that is a groundbreaking technology and has never been attempted before. We will also briefly discuss several smart grid analytics and security problems. With the integration of advanced computing and communication technologies, Smart Grid holds the promise as the next-generation energy critical infrastructure - efficient, resilient and sustainable. To achieve that end, significant research challenges and opportunities need to be addressed, such as security attacks and countermeasures, fault identification and restoration, demand response and microgrid cooperative controls.

Autonomous Visual Exploration of Unknown Domains Aided by Machine Learning

Richard Tsai
The University of Texas at Austin, USA
Louis Long Ly

In this talk, we consider the problem of exploring and reconstructing an a priori unknown environment based on range (visual) data from a single, moving observer. The observer is to roam around a piece of unknown domain and reconstruct it, using as few observations of the environment as possible. We present a greedy and iterative algorithm for designing the observation locations based on observations made in the previous iteration. The choice of each new observation location is aided by a convolution neural network, which is trained with a suitable database as a prior to the unknown environment to be explored. We will demonstrate the performance of the new algorithm for exploring realistic 2D and 3D settings.

Decentralized Consensus Algorithms with Network Independent Stepsize

Ming Yan
Michigan State University, USA
Zhi Li, Wei Shi

We consider the problem of decentralized optimization with a composite objective containing smooth and non-smooth terms. To solve the problem, a proximal-gradient scheme is studied. Specifically, the smooth and nonsmooth terms are dealt with by gradient update and proximal update, respectively. The studied algorithm is closely related to a previous decentralized optimization algorithm, PG-EXTRA, but has a few advantages. First of all, in our new scheme, agents use uncoordinated step-sizes and the stable upper bounds on step-sizes are independent from network topology. The step-sizes depend on local objective functions, and they can be as large as that of the gradient descent. Secondly, for the special case without non-smooth terms, linear convergence can be achieved under the strong convexity assumption. The dependence of the convergence rate on the objective functions and the network are separated, and the convergence rate of our new scheme is as good as one of the two convergence rates that match the typical rates for the general gradient descent and the consensus averaging. We also provide some numerical experiments to demonstrate the efficacy of the introduced algorithms and validate our theoretical discoveries.

Deep Mean Field Games on Graphs for Population Behavior Modeling

Xiaojing Ye
Georgia State University, USA
Jiachen Yang, Rakshit Trivedi, Huan Xu, Hongyuan Zha

We consider the problem of representing collective behavior of large populations and predicting the evolution of a population distribution over a discrete graph. A discrete time mean field game (MFG) is motivated as an interpretable model founded on game theory for understanding the aggregate effect of individual actions and predicting the temporal evolution of population distributions. We achieve a synthesis of MFG and Markov decision processes (MDP) by showing that a special MFG is reducible to an MDP under certain assumptions. This enables us to infer MFG models of large real-world systems via deep inverse reinforcement learning. Our method learns both the reward function and forward dynamics of an MFG from real data, and we report the first empirical test of a mean field game model of a real-world social media population.

Special Session 35: Evolutions of Single and Set-Valued Dynamical Systems and their Applications

Jerzy Motyl, University of Zielona Góra, Poland
Michta Mariusz, University of Zielona Góra, Poland
Stanislaw Migorski, Jagiellonian University in Krakow, Poland

The session will focus on recent developments in deterministic and stochastic nonlinear dynamical systems and related topics including real life problems of economics, biology, mechanics and other areas. Main topics of the session include, but are not limited to, nonlinear evolution equations and inclusions, control and optimization problems, deterministic and stochastic differential equations and inclusions, nonsmooth systems, variational methods, convex and nonconvex problems as well as their various applications.

Nonhomogeneous Dirichlet Problems with Dependence on the Gradient

Yunru Bai
Jagiellonian University, Poland
Leszek Gasinski

The goal of the present paper is to explore the existence of positive solutions for a nonlinear elliptic equation driven by a nonhomogeneous partial differential operator with Dirichlet boundary condition and a convection term in which the reaction term is not required to satisfy any global growth condition. Our approach is based on the Leray-Schauder alternative principle, truncation and comparison approaches, and nonlinear regularity theory.

Numerical Analysis for a Class of Dynamic Variational Inequalities Involving Clarke Subdifferential

Krzysztof Bartosz
Jagiellonian University, Poland

We consider a dynamic visco-elastic contact problem of a non-clamped body governed by a normal damped response condition with a unilateral constraint. The mechanical problem in its weak formulation reduces to a variational-hemivariational inequality involving a multivalued term generated by a Clarke subdifferential of a locally Lipschitz potential. We study a fully discrete numerical scheme for the variational problem based on finite difference appriximation for time variable and the finite element approximation for the spatial one. Our goal is to derive an error estimate for the studied scheme with respect to the time and spatial discretization parameters.

Existence of Mild Solutions for a Class of Second-Order Differential Inclusions

Aurelian Cernea
University of Bucharest, Romania

We study the following problem

$$x''(t) \in A(t)x(t) + \int_0^t K(t,s)F(s,x(s))ds, \qquad (1)$$

with $x(0) = x_0, x'(0) = y_0$ where $F : [0,T] \times X \to \mathcal{P}(X)$ is a set-valued map, X is a Banach space, $\{A(t)\}_{t\geq 0}$ is a family of linear closed operators from X into X that generates an evolution system of operators $\{\mathcal{U}(t,s)\}_{t,s\in[0,T]}$, $\Delta = \{(t,s) \in [0,T] \times [0,T]; t \geq s\}$, $K(.,.) : \Delta \to \mathbf{R}$ is continuous and $x_0, y_0 \in X$. The general framework of evolution operators $\{A(t)\}_{t\geq 0}$ that define problem (1) has been developed by Kozak ([2]) and improved by Henriquez ([1]). We consider this problem in the case when the set-valued map is not convex valued, but is Lipschitz in the second variable. We obtain several existence results for mild solutions of this problem using fixed point techniques and using classical selection results as Kuratowsky and Ryll-Nardzewski, Bressan and Colombo, De Blasi and Pianigiani.

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Nonlinear Elliptic Problems with Dependence on the Gradient

Leszek Gasinski
Jagiellonian University, Poland

We consider a quasilinear Neumann problem with a differential operator and a reaction term, both dependent on a function and its gradient. Using topological methods together with suitable truncation and comparison techniques, we show the existence of a smooth positive solution.

A Bipolynomial Fractional Dirichlet-Laplace Problem

Dariusz Idczak
University of Lodz, Poland

In the paper, we derive an existence result for a nonlinear nonautonomous partial elliptic system on an open bounded domain with Dirichlet boundary conditions, containg fractional powers of the weak Dirichlet-Laplace operator that are meant in the Stone-von Neumann operator calculus sense. We apply a variational method which gives strong solutions of the problem under consideration.

On the Continuous Dependence of Solutions to a Bipolynomial Fractional Dirichlet-Laplace Problem

Marek Majewski
University of Lodz, Poland
Marek Majewski

We consider a problem of continuous dependence of solutions on functional parameter to a fractional bipolynomial Dirichlet-Laplace problem. The main tool applied here is a variational method. However, since the solution to a fractional problem is not unique in general, therefore we use some set-valued approach. The obtained results can be applied to a Lagrange problem.

Stochastic Differential Inclusions and Set-Valued Stochastic Differential Equations

Mariusz Michta
University of Zielona Gora, Poland

In the talk we present main properties of strong solutions to stochastic inclusions and set-valued stochastic differential equations. In particular we show that every solution to stochastic inclusion is a continuous selection of a multivalued solution to associated set-valued stochastic equation and that attainable sets generated by solutions to inclusions are contained in values of multivalued solutions to set-valued stochastic equations.

New Results on Evolution History-Dependent Variational-Hemivariational Inequalities with Applications

Stanislaw Migorski
Jagiellonian University in Krakow, Poland
Stanislaw Migorski

We deal with a new class of variational-hemivariational inequalities with history-dependent operators. An existence and uniqueness result is delivered, and its proof is based on a surjectivity theorem for abstract first order evolution inclusions with the Clarke generalized gradient. Two applications to inequality problems in Contact and Fluid Mechanics are discussed. First, a dynamic frictional viscoelastic contact problem with the friction described by a version of the Coulomb law of dry friction and the friction bound depending on the total slip is investigated. Second, an application to a nonstationary Oseen model for the incompressible fluid flow with a multivalued and nonmonotone slip boundary condition is provided.

Upper Separated Multifunctions and Their Applications to Set-Valued Dynamical Systems

Jerzy Motyl
University of Zielona Gora, Poland

Let X be a Banach space while $(Y, \leq)$ a Banach lattice. One of valuable problems in set valued analysis is concerned in the existence of regular selections of set-valued functions acting from X into nonempty subsets of Y. Investigating nonlinear dynamical systems, continuous, Lipschitz, differentiable or bounded variation selections are considered most often.
In the talk we introduce the class of upper separated set-valued functions and investigate the problem of the existence of order-convex selections of F. First, we present necessary and sufficient conditions for the existence of such selections. Next we discuss the problem of the existence of the Carathéodory-order-convex type selections and apply obtained results to investigation of the existence and properties of solutions of set-valued deterministic and stochastic dynamical systems, like stability or lower-upper bounds. In second part of the talk we will discuss the applicability of obtained selection results to optimal control problems. Some examples will be presented also.

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Evolution Hemivariational Inequalities with History-Dependent Operators and Their Applications

Anna Ochal
Jagiellonian University in Krakow, Poland

We present recent results on the existence and uniqueness of the solution to the hemivariational inequality of first order with the history-dependent operator. The proof is based on arguments of surjectivity for pseudomonotone operators and the Banach fixed point theorem.
We study also the continuous dependence of the solution to the considered inequality w.r.t. the operators, functions and initial data involved in the problem. The interest in continuous dependence of the solution on the perturbed data is twofold. First, the associated regularized problems can be used in numerical methods. Second, it can be the first step in studying of optimal control and identiffcation problems. Finally, we consider an example which shows how the abstract result is applicable to the model of the contact problem.
This is a joint contribution with Stanislaw Migorski and Mircea Sofonea.

Existence of Solutions for Coupled Systems of Parabolic Hemivariational Inequalities

Zijia Peng
Guangxi University for Nationalities, Peoples Rep of China
Dumitru Motreanu

Hemivariational inequality has been widely used to nonsmooth and noncovex energy functional problems. This talk concerns an initial-boundary value problem for a quasilinear parabolic system of hemivariational inequalities which is not necessarily coercive. The system is given in inclusion forms and doubly coupled on both the source and multivalued terms. The existence of weak solutions is established within a trapping region based on sub-supersolutions and nonsmooth analysis. Moreover, we show the existence of positive solutions by effectively constructing the sub-supersolutions of the coupled system.

Impulsive Quantum Difference Inclusions

Jessada Tariboon
King Mongkut's University of Technology North Bangkok, Thailand
Sotiris K. Ntouyas

By applying new concepts of quantum calculus on finite intervals, we study initial value problems for impulsive first- and second-order quantum-difference inclusions. We establish new existence results, when the right hand side is convex valued, by using the nonlinear alternative of Leray-Schauder type. Some illustrative examples are also presented.

Turnpike Phenomenon in the Calculus of Variations and Optimal Control

Alexander Zaslavski
The Technion - Israel Institute of Technology, Israel

We study the structure of approximate solutions of variational and optimal control problems on large intervals, and show that a turnpike phenomenon holds for large classes of problems. To have this property means, roughly speaking, that the approximate optimal trajectories are determined mainly by the integrand, and are essentially independent of the choice of time intervals and data, except in regions close to the endpoints of the time interval.

Evolutionary Variational-Hemivariational Inequalities with Constraints and Their Applications

Biao Zeng
Jagiellonian University in Krakow, Peoples Rep of China
Stanislaw Migorski

The talk investigates a class of evolutionary variational-hemivariational inequalities of parabolic type with constraints. Based on the theory of mixed equilibrium problems, we provide a result on existence and uniqueness of solution to this class of inequalities. An application to a diffusion semipermeability problem with constraint illustrates the abstract results.

A Class of Fractional Differential Hemivariational Inequalities with Application to Contact Problem

Shengda Zeng
Jagiellonian University, Poland

In this paper we study a class of generalized differential hemivariational inequalities of parabolic type involving the time fractional order derivative operator in Banach spaces. We use the Rothe method combined with surjectivity of multivalued pseudomonotone operators and properties of the Clarke generalized gradient to establish existence of solution to the abstract inequality. As an illustrative application, a frictional quasistatic contact problem for viscoelastic materials with adhesion is investigated, in which the friction and contact conditions are described by the Clarke generalized gradient of nonconvex and nonsmooth functionals, and the constitutive relation is modeled by the fractional Kelvin-Voigt law.

Special Session 36: Analytical and Numerical Approaches in Soliton Theory

Michail Todorov, Technical University of Sofia, Bulgaria
Rossen Ivanov, Dublin Institute of Technology, Ireland

Recently we have celebrated the 50th anniversary of the discovery of the Inverse Scattering Transform method for the soliton equations. In the last half century the research on solitary waves and solitons remains one of the most vibrant areas of mathematics and physics. The proposed session will cover talks dedicated to the development of both analytical and numerical methods for some well known integrable PDEs (Nonlinear Schrödinger equation, Camassa-Holm equation etc.) as well as related integrable equations and their multicomponent generalizations. Another accent of the session will be numerical approaches to nonintegrable dynamical systems like generalized Schrödinger equations, Boussinesq Paradigm, etc.

The Modelling of Hydrodynamic Solitons and Breathers

Amin Chabchoub
The University of Sydney, Australia
Hung-Chu Hsu, Yang-Yih Chen, Meng-Syue Li, Olivier Kimmoun, Hubert Branger, Christian Kharif, Norbert Hoffmann, Miguel Onorato, Takuji Waseda, Edmund Kelleher, Bertrand Kibler, Goery Genty, John Dudley, Nail Akhmediev

The uni-directional propagation of stationary and unstable wave packets can be deterministically described and modelled within the framework of the nonlinear Schroedinger equation or modified nonlinear Schroedinger equation. We will report a number of observations attributed to soliton and breather dynamics, measured in different hydrodynamic wave facilities, and discuss the applicability and limitations as well as the role of weakly nonlinear evolution equations in the modelling of such localized and coherent structures on the water surface.

On the Inverse Spectral Method for Solving the Camassa–Holm Equation

Jonathan Eckhardt
University of Vienna, Austria

The Camassa–Holm equation is a nonlinear partial differential equation that models unidirectional wave propagation on shallow water. I will show how to integrate this equation by means of solving an inverse spectral problem for a Sturm–Liouville problem with an indefinite weight. The global conservative (weak) solutions obtained in this way form into a train of (in general infinitely many) peakons in the long-time limit.

On Some Geodesic Flows on Fréchet–Lie Groups

Joachim Escher
Leibniz University Hannover, Germany
Boris Kolev

It is the aim of this talk to present a geometric method to study various model equations appearing in mathematical hydrodynamics as geodesic flows of right-invariant metrics induced by suitable Fourier multipliers on the Fréchet–Lie group of all diffeomorphisms of the n-dimensional torus and the Euclidean n-space. This approach covers in particular right-invariant metrics induced by Sobolev norms of fractional order. It is shown that the corresponding initial value problem is well-posed in the smooth category and that the Riemannian exponential mapping is a smooth local diffeomorphism, provided that the symbol complies with certain mild structural conditions.

Camassa-Holm Equation - Soliton and Cuspon Solutions and Their Interactions

Rossen Ivanov
Dublin Institute of Technology, Ireland
Tony Lyons, Nigel Orr

We present the derivation of the soliton and cuspon solutions of the Camassa-Holm equation by the implementation of the dressing method. The form of the one and two soliton and cuspon solutions as well as soliton-cuspon interactions will be discussed.

Generalized Indefinite Strings and the Camassa–Holm Equation

Aleksey Kostenko
University of Vienna, Austria

In this talk, we review the direct and inverse spectral theory for generalized indefinite strings, the object introduced in J. Eckhardt and A. Kostenko, *Invent. Math.* (2016), and relate it to the conservative Camassa–Holm flow. As one of our main results we are going to present the indefinite analog of M.G. Krein's celebrated solution to the string density problem. The talk is based on joint work with Jonathan Eckhardt.

Semi-Discrete Central-Upwind Schemes for Elasticity in Heterogeneous Media

Alexander Kurganov
Southern University of Science and Technology, China and Tulane University, USA, Peoples Rep of China

We develop new central-upwind schemes for nonlinear elasticity equations in a heterogeneous medium. Finite volume central-upwind schemes consist of three steps: reconstruction, evolution, and projection onto the original grid. In our new method, the evolution is performed in the standard way by integrating the system over the space-time control volumes. However, the reconstruction and projection are performed in a special manner by taking into account the fact that the conservative variables (strain and momentum) are discontinuous across the material interfaces, while the flux variables (velocity and strain) are continuous across these material interfaces. The new reconstruction and projection procedures lead to the central-upwind scheme with extremely small numerical diffusion so that in long time calculations, the new scheme outperforms existing upwind alternatives. In addition, the proposed scheme can be made positivity preserving. To achieve this goal, the system is rewritten in terms of auxiliary variables and the local propagation speeds of the system are adjusted accordingly. Our numerical experiments demonstrate that the developed scheme is capable of accurately resolving waves with dispersive behavior that over a long period of time evolve into solitary waves while remaining nonnegative.

Asymptotic Analysis on the Modelling of the Shallow-Water Waves with the Coriolis Effect

Yue Liu
University of Texas at Arlington, USA
Guilong Gui

In this talk, a mathematical model of long-crested water waves propagating mainly in one direction with the effect of Earth's rotation is derived by following the formal asymptotic procedures. Such a model equation is analogous to the Camassa-Holm approximation of the two-dimensional incompressible and irrotational Euler equations and has a formal bi-Hamiltonian structure. Its solution corresponding to physically relevant initial perturbations is more accurate on a much longer time scale. It is shown that the deviation of the free surface can be determined by the horizontal velocity at a certain depth in the second-order approximation. The effects of the Coriolis force caused by the Earth rotation and nonlocal higher nonlinearities on blow-up criteria and wave-breaking phenomena are also investigated. Our refined analysis is approached by applying the method of characteristics and conserved quantities to the Riccati-type differential inequality.

Solitons of the Kaup-Boussinesq Equation

Tony Lyons
Waterford Institute of Technology, Ireland
Jack Haberlin

The Kaup-Boussinesq equation is a well known weakly nonlinear equation arising as an approximate model for shallow water flows. The equation is known to be integrable, in the sense that it may be written as a compatibility condition imposed on a pair of linear spectral problems. A particularly interesting aspect of the Lax pair associated with this fluid model is its energy dependence, whereby the potential of the spectral problem depends on the spectral parameter. In this talk we will present a recently developed pair of energy-dependent conjugate spectral problems associated with the Kaup-Boussinesq equation and briefly outline how this pair spectral problems is used to construct a topological solution of the system, via the Inverse Scattering Transform method. Following this, we will present a recent extension of this work, in which soliton solutions of the Kaup-Boussinesq equation have been constructed via the Inverse Scattering Transform.

GBDT Version of Darboux Transformation and Explicit Solutions of Dynamical Systems and Nonlinear Wave Equations

Alexander Sakhnovich
University of Vienna, Austria

GBDT version of Darboux transformation was introduced in [1] and actively developed later on (see, e.g., [2–6] and references therein). It is an important approach to the construction of explicit solutions of nonlinear integrable equations, there are interesting connections with the inverse spectral transform, and we will present also recent results on explicit solutions of dynamical systems constructed via GBDT.

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Long-Time Evolution and Interaction of Localized Solutions of Nonlinear Wave and Envelope Systems. Consistency vs. Integrability

Michail Todorov
Technical University of Sofia, Bulgaria

Boussinesq's equation was the first model for the propagation of surface waves over shallow inviscid fluid layer. Boussinesq found an analytical solution of his equation and thus proved that the balance between the steepening effect of the nonlinearity and the flattening effect of the dispersion maintains the shape of the wave. This discovery can be properly termed 'Boussinesq Paradigm.' Apart from the significance for the shallow water flows, this paradigm is very important for understanding the particle-like behavior of nonlinear localized waves. The localized solutions (with finite energy in infinite region) which can retain their identity during the interaction are called quasi-particles if some mechanical properties (such as mass, energy, momentum) are conserved by the governing equations. Special interest are the generalized wave equations containing both a nonlinearity and a dispersion as well as the nonlinear evolution equations. The original Boussinesq equation is fully integrable but incorrect in sense of Hadamard. 2D Boussinesq paradigm is correct but fully nonintegrable. The Manakov system is fully integrable but models only an elastic interaction. As it should have been expected, most of the physical systems are not fully integrable (even in one spatial dimension) and only a numerical approach can lead to unearthing the pertinent physical mechanisms of the interactions. In this paper we study numerically the soliton dynamics of Manakov system with gain/loss, cross modulation, and external potentials (vector Schrödinger equation). The system is not integrable and admits only 3 conservation laws. The results obtained shed the mechanism of soliton interactions.

Numerical Study of Nonlinear Wave Equation Using Lattice Boltzmann Method

Hidekazu Tsuji
Kyushu University, Japan

More efficient numerical scheme of nonlinear wave equation is required especially for solving two-dimensional problem (and sometimes with complex boundary). From this point of view, application of the lattice Boltzmann method, which has been used for solving some equations including Navier-Stokes equation, is promising. As a preliminary study, a new scheme for solving Burgers equation with low dissipation is proposed. Its application to shock problem shows the preferable results compared to the standard scheme.

Pseudo-Hermitian Reductions of a Matrix Generalized Heisenberg Ferromagnet Equation

Tihomir Valchev
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria
A. B. Yanovski

We shall introduce and study in this talk new $1+1$ dimensional system of nonlinear partial differential equations integrable through inverse scattering transform. The integrable system under consideration has a linear bundle Lax pair related to Hermitian symmetric space of the series **A.III** according to Cartan's classification. Thus, it represents a Mikhailov's type reduction of a matrix generalization of classical $1+1$ dimensional integrable Heisenberg ferromagnet equation. We shall describe an integrable hierarchy connected to the matrix system in terms of generating operators and discuss integrable anisotropic (local) deformations related to a rational bundle Lax pair for the same type of symmetric space.

The Initial-Boundary Value Problems for a Class of Sixth Order Nonlinear Wave Equation

Runzhang Xu
Harbin Engineering University, Peoples Rep of China

In this talk we consider the initial boundary value problem of solutions for a class of sixth order 1-D nonlinear wave equations. We discuss the probabilities of the existence and nonexistence of global solutions and give some sufficient conditions for the global and non-global existence of solutions at three different initial energy levels, i.e., sub-critical level, critical level and sup-critical level.

Special Session 37: Nonlinear PDEs Modeling Fluid Dynamics

Kazuo Yamazaki, University of Rochester, USA
Juan-Ming Yuan, Providence University, Taiwan
Lizheng Tao, University of California, Riverside, USA
Jiahong Wu, Oklahoma State University, USA

Despite the effort of mathematicians over generations ever since the pioneering work of Leray in 1930s, the Navier-Stokes problem, specifically whether a solution initiated from sufficiently smooth data preserves its regularity or experiences a finite-time blowup, remains unsolved. Nevertheless, very recently, remarkable progress has been made on various topics. For example, numerical analysis led to a wide range of toy models that have much similarity to the Navier-Stokes equations and yet display a finite-time blowup; harmonic analysis allowed us to obtain well-posedness and regularity criteria that are unthinkable with classical methods; stochastic analysis revealed interesting effect of random noise that may regularize the solution or potentially cause finite-time singularity. The following related equations have also attracted much attention from researchers: Euler equations, Boussinesq system, KdV equations, micropolar fluid system, hydrodynamics models such as magnetohydrodynamics system, geophysics models such as the surface quasi-geostrophic equations. The purpose of this special session is to invite researchers devoted to analysis on fluid mechanics but possibly with different expertise and wide range of background and encourage deep discussions from unconventional approach.

Axisymmetric Flows in the Exterior of a Cylinder

Ken Abe
Osaka City University, Japan
Gregory Seregin

We consider the three-dimensional Navier-Stokes equations for axisymmetric initial data. It is known that the Cauchy problem is globally well-posed for large axisymmetric initial data in L_3 with finite energy, if the swirl component of initial velocity is identically zero (with no swirl). However, unique solvability is unknown in general for the case with swirl. In this talk, we study axisymmetric flows with swirl in the exterior of an infinite cylinder subject to the slip boundary condition. We report unique existence of global solutions for large axisymmetric data in L_3 with finite energy, satisfying a decay condition of the swirl component.

A Numerical Algorithm for Blow-Up Problems

Chienhong Cho
National Chung Cheng University, Taiwan

In many evolution equations, solutions may become unbounded in finite time. This phenomenon is often called blow-up and the finite time is called the blowup time. To numerically reproduce the finite-time blow-up phenomenon, schemes with adaptive time meshes were considered to be necessary. Since the numerical blow-up time is defined by an infinite sum, which implies that one needs to compute infinite times to achieve blow-up, this method cannot be carried out in real computation. As a consequence, we revisit an algorithm accomplished by schemes with uniform time meshes for the computation of blow-up solutions. In this talk, we are concerned with a question: to what extent can this algorithm be applied to compute the blow-up solutions and reproduce the blow-up behavior? Our computational results for the semilinear wave equation and the generalized Proudman-Johnson equation will be reported.

Determining Modes for the NSE

Mimi Dai
University of Illinois at Chicago, USA
Alexey Cheskidov

In this talk we will review classical results on determining modes for fluid equations and present a slightly different approach where we start with a time-dependent determining wavenumber defined for each individual trajectory and then study its dependence on the force. While in some cases this wavenumber has a uniform upper bound, it may blow up when the equation is supercritical. A bound on the determining wavenumber provides determining modes, which in some sense measure the number of degrees of freedom of the flow, or resolution needed to describe a solution.
For the 3D Navier-Stokes equations, we obtain a uniform bound on the time average of this wavenumber, which we estimate in terms of the Kolmogorov dissipation number and Grashof constant.

An Approximation of Riesz Transform and Its Application

Quansen Jiu
Capital Normal University, Peoples Rep of China
Huan Yu

Numerical Study of Dispersive Regularisations of Burgers' Equation

Christian Klein
IMB, France
F. Linares R. Peter, D. Pilod, J.-C. Saut

We present a numerical study of various issues arising in dispersive regularizations of Burgers' equations, soliton stability, dispersive shock waves and blow-up. These questions are considered for generalised Korteweg-de Vries (KdV) and Kadomtsev-Petviashivili equations as well as for equations with nonlocal dispersion as fractional KdV and Whitham equation.

Linear and Nonlinear Continuous Data Assimilation for Fluid Equations

Adam Larios
University of Nebraska-Lincoln, USA
Animikh Biswas, Elizabeth Carlson, Joshua Hudon, Yuan Pei

A major difficulty in accurately simulating turbulent flows is the problem of determining the initial state of the flow. For example, weather prediction models typically require the present state of the weather as input. However, the state of the weather is only measured at certain points, such as at the locations of weather stations or weather satellites. Data assimilation eliminates the need for complete knowledge of the initial state. It incorporates incoming data into the equations, driving the simulation to the correct solution. The objective of this talk is to discuss new computational and mathematical methods to test, improve, and extend a promising new class of algorithms for data assimilation in turbulent flows and related PDEs, stemming from the pioneering work of Azouani, Olson, and Titi.

Global Well-Posedness of the Anisotropic Primitive Equations

Jinkai Li
The Chinese University of Hong Kong, Hong Kong
Chongsheng Cao, Edriss S. Titi

The motion of the large-scale atmospheric and oceanic flows is governed by the primitive equations (PEs), which are derived from the Navier-Stokes equations by using the Boussinesq and hydrostatic approximations. The strong horizontal turbulent mixing, which creates the horizontal eddy viscosity, leads us to consider the PEs with horizontal viscosity. It will be shown that the 3D PEs with horizontal viscosity admits a unique global strong solution, for arbitrary sufficient smooth initial data, as long as one still has the horizontal or vertical thermal diffusivity. These are joint works with Chongsheng Cao and Edriss S. Titi.

Recent Progress on Partial Viscosity MHD Equation

Jingna Li
Jinan University, Peoples Rep of China
Boqing Dong, Jiahong Wu

This talk is concerned with a system of the 2D MHD equations with the kinematic dissipation given by the fractional operator and the magnetic diffusion by partial Laplacian. We are able to show that this system with any $\alpha > 0$ always possesses a unique global smooth solution when the initial data is sufficiently smooth. In addition, we also give the optimal large-time decay rates. The second part is devoted to existence and regularity for a system of the two-dimensional (2D) magnetohydrodynamic (MHD) equations with only directional hyperresistivity. More precisely, the equation of b_1 (the horizontal component of the magnetic field) involves only vertical hyperdiffusion while the equation of b_2 (the vertical component) has only horizontal hyperdiffusion. We prove that for derivative index great than 1, this system always possesses a unique global-in-time classical solution when the initial data is sufficiently smooth.

Decay and Vanishing of Some Axially Symmetric D-Solutions of the Navier-Stokes Equations

Xinghong Pan
Nanjing University of Aeronautics and Astronautics, Peoples Rep of China
Bryan Carrillo, Qi S. Zhang

We study axially symmetric D-solutions of the 3 dimensional Navier-Stokes equations. The first result is an a priori decay estimate of the velocity for general domains. The second is an a priori decay estimate of the vorticity in $\mathbb{R}^3$, which improves the corresponding results in the literature. Next we turn to D-solutions which are periodic in the third variable and prove vanishing result under a reasonable condition. As a corollary we prove that axially symmetric D-solutions in the slab $\mathbb{R}^2 \times I$ with suitable boundary condition is 0. Here I is any finite interval. To the best of our knowledge, this seems to be the first vanishing result on a 3 dimensional D-solution without extra integral or decay or smallness assumption on the solution.

Continuous and Discrete Nonlinear Schrödinger Equations and Applications in Mechanics

Qing Pan
The University of Hong Kong, Peoples Rep of China
H.N. Chan, K.W. Chow

The nonlinear Schrödinger equation (NLSE) describes the propagation of wave packets for fluids of finite depth. NLSE incorporates second order dispersion in the space-time domain and cubic nonlinearity. Solitons (exponentially decaying) and rogue waves (algebraically localized) solutions have been established. These modes have also been observed experimentally too. Third order dispersion needs to be restored for sufficiently short waves. Exact solutions can be obtained for special parameter regimes but numerical schemes will be necessary for arbitrary hydrodynamic configurations. Remarkably discrete versions of NLSE can be formulated and are applicable to lattice dynamics and optical fibers. The objective here is to construct discrete analogue of a third order NLSE where analytical advances are possible. More precisely, discrete breathers (pulsating modes) and rogue waves are derived by the Hirota bilinear transform. In contrast to the continuous case, rogue waves for discrete equations oscillate in amplitude in the growth and decay stages. Spatially periodic breathers are demonstrated and are utilized to verify low order conservation laws. Discrete breathers are of interest to many branches in physics and an analytical example will be tremendously valuable.

Some Results on the Stochastic Langrangian Averaged Euler Equations and Grade Two Fluid on Non-Smooth Domain: Well-Posedness and Regularity

Paul Andre Razafimandimby
University of Pretoria, So Africa

In this talk we will present some results obtained from the analysis of the stochastic Lagrangian Averaged Euler equations and grade two fluid filling a bounded Lipschitz domain O. We will mainly talk about the existence of $H^1(O)$ weak martingale solution on any bounded Lipschitz domain O. Furthermore, we show that when O is a convex polygon the solution $\mathbf{u}$ lives in the Sobolev space $W^{2,r}(O)$ for some $r > 2$. We also prove that the *vorticity* $curl(\mathbf{u} - \alpha\Delta\mathbf{u})$, where $\mathbf{u}$ is the solution, is continuous in $L^2(\mathcal{O})$ with respect to the time variable.

Short Time, Eventual and Far Field Regularity of Infinite Energy Solutions of Navier-Stokes Equations

Tai-Peng Tsai
University of British Columbia, Canada
Kyungkeun Kang, Hideyuki Miura, Zachary Bradshaw

We consider infinite energy solutions of 3D incompressible Navier-Stokes equations with initial data not square integrable. We establish short time regularity near a point where the initial data is sufficiently regular, and eventual and far field regularity for initial data with suitable decay.

Stochastic Hall-Magneto-Hydrodynamics System in Three and Two and a Half Dimensions

Kazuo Yamazaki
University of Rochester, USA
Manil Mohan

I will review recent developments on the Hall-magnetohydrodynamics system perturbed by noise. This system is second-order quasi-linear when added by a diffusion, and thus well-known results for the Navier-Stokes equations which is second-order semilinear may not be extended. We discuss results such as global existence of martingale solutions, local existence of a unique solution, global existence of unique solution for small initial data, irreducibility, and more if time permits.

Particle Dynamics in the KP Approximation

Juan-Ming Yuan
Providence University, Taiwan
Y. Shimaby, J.H. Chang, H. Kalisch, J.M. Yuan

The KP equation arises in the model of the evolution of long ion-acoustic waves of small amplitude propagating in plasma under the effect of long transverse perturbations. The KP equation also can be treated as an extension of the KdV equation with two spatial dimensions and be applied to model for surface and internal water waves, shallow water waves, nonlinear optics. Enclosed in the KP model are relations that may be used to reconstruct the velocity field in the fluid below a given surface wave. In this talk, velocity fields associated to exact solutions of the KP equation are found, and particle trajectories are computed numerically.

Special Session 38: Harmonic Analysis and Partial Differential Equations

Armin Schikorra, University of Pittsburgh, USA
Daniel Spector, National Chiao Tung University, Taiwan

This session will bring together international experts in analysis to speak on selected topics related to the "harmonic" theme in analysis. The focus is on the theoretical side as well as on the applications e.g. in partial differential equations. With this common focus, the diverse backgrounds will benefit each other to see the perspective of others on mathematical topics which are topologically close.

Finite Time Singularities of Geometric Evolution Equations and a Reverse Isoperimetric Inequality

Simon Blatt
University Salzburg, Austria

The classification of singularities in both stationary as well as time dependent solutions of partial differential equations is and has been throughout the last decades in the centre of mathematical research. A natural question of course is, whether or not such singularities exist.
In the talk I will first present some well-known results in this direction in the context of geometric evolution equations. We will then discuss some new results for the constrained Willmore flow and –time permitting– similar equations in the context of the Möbius energy. Parts of them rely on the fact that every sphere eversions has a quadruple point and a new reverse isoperimetric inequality.
Our approach extends and simplifies results due to McCoy and Wheeler.

Harmonic Analysis in Bi-Free Probability Theory

Hao-Wei Huang
National Sun Yat-Sen University, Taiwan

In free probability the notion of free convolution of probability distributions on $\mathbb{R}$ has played an important role since its inception by D. Voiculescu some 30 years ago. In 2013, Voiculescu generalized the notion of free independence to study left and right actions on reduced free product spaces simultaneously, known as bi-free independence. One generalization of the free convolution to the bi-free setting is the bi-free convolution of planar probability distributions. In this talk, we will explain that the bi-freely infinitely divisible laws, and only these laws, can be used to approximate the distributions of sums of identically distributed bi-free pairs of commuting faces. We will also talk about bi-free Lèvy-Khintchine representations from an infinitesimal point of view. The proofs depend on the bi-free harmonic analysis machinery that we developed for integral transforms of two variables. If time permits, some recent developments in this direction will also be discussed.

On the Smoothing Property of the Gain Term of the Boltzmann Collision Operator

Jin-Cheng Jiang
National Tsing Hua University, Taiwan

We will review the smoothing property of the gain term of the Boltzmann collision operator whose proof is closely connected to the estimate of the Fourier integral operator. Then we will present a new result in this aspect.

Harnack Inequality in Sub-Riemannian Settings

Alessia Kogoj
University of Urbino "Carlo Bo", Italy
Sergio Polidoro

We consider nonnegative solutions $u : \Omega \longrightarrow \mathbb{R}$ of second order hypoelliptic equations

$$\mathcal{L}u(x) = \sum_{i,j=1}^{n} \partial_{x_i} \left(a_{ij}(x)\partial_{x_j} u(x)\right) + \sum_{i=1}^{n} b_i(x)\partial_{x_i} u(x) = 0,$$

where Ω is a bounded open subset of $\mathbb{R}^n$ and x denotes the point of Ω. For any fixed $x_0 \in \Omega$, we prove a Harnack inequality of this type

$$\sup_K u \leq C_K u(x_0) \qquad \forall\ u \ \text{ such that } \ \mathcal{L}u = 0, u \geq 0,$$

where K is any compact subset of the interior of the $\mathcal{L}$-propagation set of x_0 and the constant C_K does not depend on u. The result presented are obtained in collaboration with Sergio Polidoro.

Jump Detection in Besov Spaces Via a New BBM Formula. Applications to Aviles-Giga-type functionals

Arkady Poliakovsky
Ben-Gurion University onf the Negev, Beer-Sheva, Israel

About 15 years ago, Bourgain, Brezis and Mironescu proposed a new characterization of BV and $W^(1, q)$ spaces (for $q > 1$) using a certain double integral functional involving radial mollifiers. We study what happens when one changes the power of $|x-y|$ in the denominator of the integrand from q to 1. It turns out that for $q > 1$ the corresponding functionals see only the jumps of the BV-function. We further iden-

tify the function space relevant to the study of these functionals as an appropriate Besov space. We also present applications to the study of singular perturbation problems of Aviles-Giga type.

Two Weight T1 and Tb Theorems for the Hilbert Transform

Chun-Yen Shen
National Taiwan University, Taiwan

We present our recent advances showing that the two weight boundedness of the Hilbert transform is equivalent to the so-called testing conditions and A_2 condition. Moreover, we will discuss the difficulties for higher dimensional singular integrals.

Two Theorems on Vortex Patches

Joan Verdera
Universitat Autònoma de Barcelona, Spain
T. Hmidi, J. Mateu, A. Bertozzi, J. Garnett, T. Laurent

The vorticity form of the planar Euler equation says that vorticity is constant along particle trajectories. A vortex patch is a weak solution of the vorticity equation with initial condition the characteristic function of a domain D_0. Thus at time t vorticity is the characteristic function of a domain D_t. Simulations show that the evolution of D_t is extremely complicated. In spite of this general fact there are some special domains, called V-states, whose evolution is just rotation around the center of mass with constant angular velocity. Ellipses are examples of V-states. I will discuss Burbea's proof of existence of other V-states and then I will discuss the smoothness of their boundary (joint work with Hmidi and Mateu). For general vortex patches, if the initial condition is the characteristic function of a domain with boundary of class $C^{1+\gamma}$, then the boundary of D_t conserves the regularity for all times (Chemin's theorem). I will mention a similar result for the aggregation equation in higher dimensions (joint work with Bertozzi, Garnett and Laurent).

Sobolev Embeddings and Approximation

Po Lam Yung
The Chinese University of Hong Kong, Hong Kong

About a decade ago, Bourgain and Brezis established an approximation theorem for functions in the Sobolev space $W^{1,n}$ on $\mathbb{R}^n$, when $n \geq 2$. This has some far-reaching consequences, and we will survey some of that in this talk. We will also discuss some new results along this line obtained in the past few years. The new results are from joint work with Sagun Chanillo, Jean Van Schaftingen, and also joint work with Pierre Bousquet, Emmanuel Russ and Yi Wang.

Special Session 41: Revealing the Mathematical Complexity of Cell Migration and Pattern Formation: From Modelling to Applications

Anotida Madzvamuse, University of Sussex, England

The aim of this special session is bring knowledge-exchange between applied theoreticians and numerical analysts working at the forefront of problems emanating from the life sciences with the goal of revealing the complexity of cell migration and pattern formation. Advances in the state-of-the-art experimental designs and techniques, genetic manipulation techniques, and high resolution multicolor recordings in living cells and cutting-edge imaging techniques and analysis tools enable experimentalists to generate large amounts of datasets for cell migration and pattern formation. In this session, topical presentations will emphasise theoretical results of models inspired by experimental observations, with the goal of exhibiting novel mathematical approaches and techniques from physics, differential geometry, mathematical modelling and numerical analysis to help unravel current problems in the areas of 2D and 3D cell migration and pattern formation. Understanding mechanisms for cell migration and pattern formation has important implications in embryo development, immune response, wound healing, tissue differentiation, tissue regeneration, inflammation, tumor invasion and metastasis formation. As such, the results of this symposium will impact areas of mathematical modelling, numerical analysis and HPC scientific computing, developmental biology, cell motility, biophysics, biomedicine, and material sciences.

The Mechanisms of Keratin Dynamics: Solving an Inverse Problem in Mathematical Biology

Eduard Campillo-Funollet
University of Sussex, England
Eduard Campillo-Funollet, Stephanie Portet, Anotida Madzvamuse

Keratins are a family of proteins involved in several cellular processes. They are a crucial element of the cytoskeleton, and they contribute to many of the mechanical properties of the cell. In the experiments, we cannot observe all the aspects of the keratin dynamics. In turn, we use a mathematical model, adjusted using experimental data, to study the mechanisms underlying the keratin structure.

We present a one-dimensional reaction-diffusion model for the dynamics of keratin. We derive the model from first principles and from assumptions based on the current knowledge in biology.

We use a Bayesian approach to perform the parameter identification. In order to prescribe sensible priors, we incorporate the assumptions provided by biologists. Several parameters depend on the spatial position within the cell, and therefore we define the priors on functional spaces. We use a parallel Metropolis-Hastings sampler to obtain numerical approximations of the posterior distributions.

The posterior distribution for the parameters reveals the underlying mechanisms of the keratin dynamics. The location of the keratin assembly and disassembly regions in the cell matches results obtained independently using image processing techniques. The results also show the existence of a limiting assembly and disassembly reaction rate.

A 3d Bulk-Surface Model for Cell Polarization

Davide Cusseddu
University of Sussex, England
Anotida Madzvamuse, Stephanie Portet

Several cellular activities, such as directed migration, are coordinated by an intricate network of biochemical reactions which lead to a polarised state of the cell, in which cellular symmetry is broken, causing the cell to have a well defined front and back. Balancing biological complexity with mathematical tractability is one of the aims of the works by Mori *et al* (2008), Holmes and Edelstein-Keshet (2016), in which they propose a famous minimal model for polarisation, known as *wave pinning* model.

We will present an extension of these works in a bulk-surface setting for 3-dimensional domains. We will show how a local perturbation of the surface component can trigger a propagative activation over the membrane. The interplay with the bulk component generates a stable profile with only a portion of the membrane strongly active. Depending on the type of proteins, these might represent the front and the back of the cell, or *viceversa*.

Furthermore, we will present the *Coupled Bulk Surface Finite Element Method*, used to solve the model, together with a predictor-corrector method for the temporal discretisation. Numerical results will be presented that validate the mathematical predictions.

References

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Crosstalk Between Rac and Rho GTPases Promote Morphological Heterogeneity Among Motile Cells

William Holmes
Vanderbilt University, USA

Networks of Rho-family GTPases regulate eukaryotic cell polarization and motility. In particular, the mutually inhibitory interactions between Rac and Rho are known to be critical in regulating a number of migratory processes. Here I discuss a simple model of these dynamics and show that this motif can account for diverse observations related to heterogeneity of cell shapes (contracted, spread, of polarized for example), motility type (amoeboid versus mesenchymal), as well as migratory persistence of cells. These results support the notion that Rac and Rho affect a multitude of migratory decisions and serve as a central regulatory hub interpreting an array of input signals.

Modelling, Analysis and Simulations of Rho-Myosin Spatio-Temporal Dynamics

Victor Ogesa Juma
Sussex University, England
Anotida Madzvamuse, Stephanie Portet, Leif Dehmelt, Perihan Nalbant

Rho GTPases-RhoA (hence Rho), Rac and cdc42 control enormous variety of processes within the cell. Rho controls stress fibre assembly, contractility and focal adhesion dynamics. We will focus on Rho activity dynamics linked to cellular contractility.
In this talk, model formulation, model analysis, numerical analysis, and bifurcation analysis as well as Rho amplitude analysis is presented that seeks to describe the Rho activity dynamics as observed experimentally (**MPI/UDE**). Rho GTPase reaction kinetics are modeled using first principles based on experimental observations and new mathematical models are obtained. Rigorous mathematical and numerical analysis is carried out to provide robust models and numerical results that exhibit the spatio-temporal dynamics as observed experimentally. Detailed numerical bifurcation and numerical simulation results are obtained and these confirm theoretical analytical results as well as confirming experimental observations.

A Non-Local Problem Describing Pattern Formation for the Shadow System of the Gierer-Meinhardt System on an Evolving Domain

Nikos Kavallaris
University of Chester, England
Nikos Kavallaris, Anotida Madzamuse

The purpose of the current talk is to contribute to the comprehension of the dynamics of the shadow system of an activator-inhibitor system known as a Gierer-Meinhardt model on an evolving domain. Shadow systems are intended to work as an intermediate step between single equations and reaction-diffusion systems. In the case where the inhibitor's response to the activator's growth is rather weak, then the shadow system of the Gierer-Meinhardt model is reduced to a single though non-local equation whose dynamics is thoroughly investigated. The main focus is on the derivation of blow-up results for this non-local equation which can be seen as instability patterns of the shadow system. In particular, a diffusion driven instability (DDI), or Turing instability, in the neighbourhood of a constant stationary solution, which it is destabilised via diffusion-driven blow-up, is observed. The latter actually indicates the formation of some unstable patterns, whilst some stability results of global-in-time solutions towards non-constant steady states guarantee the occurrence of some stable patterns.

Physical Driving Forces of Epithelial Organisation

Marco Kokic
ETH Zurich, Switzerland
Antonella Iannini, Gema Villa-Fombuena, Fernando Casares, Dagmar Iber

Epithelial cells adhere tightly, forming a polygonal lattice. The resulting cell packing exhibits striking universal regularities, regardless of the organism and organ. Several theories have been formulated to explain epithelial organisation, but none explains Lewis' almost 100-year-old observation of a linear relationship between the average number of cell neighbours and the average apical cell area. We define the physical driving forces that lead to the emergence of Lewis' law, and that more generally explain apical epithelial organisation. Our work provides a link between epithelial organisation, cell division, and growth, and establishes computational frameworks such as LBIBCell as appropriate tools to simulate epithelial dynamics.

A Mechanochemical Model for Cell Motility

Laura Murphy
University of Sussex, England
Anotida Madzvamuse

The way that cells move is key to the creation and development of most organisms on earth. Consequently a deeper understanding of cell motility is likely to have significant applications to medicine. The aim of this work is to implement an efficient numerical method to study cell deformation and movement. The model we employ considers the actomyosin filament network as a viscoelastic and contractile gel. The mechanical properties are modeled with with a force balancing equation for displacement. This is coupled to two reaction-diffusion equations describing the actin and myosin biochemical dynamics. We carry out linear stability analysis to determine key bifurcation parameters and find analytical solutions close to bifurcation points. We then use a finite element scheme to produce numerical solutions in 2 and 3 dimensions. We see that the solutions predicted from linear stability theory are replicated in the early stages of movement. Subsequently we see significant deformations, some of which are consistent with shapes found in experiments. The model we describe could be thought of as the initial deformation which leads to movement.

On the Way of Modelling and Simualting Cell Monolayers: the FBLM-FEM Approach

Nikolaos Sfakianakis
Heidelberg University, Germany
C. Schmesier, D. Peuridhard, A. Brunk

The cytoskeleton is a cellular skeleton found inside the cytoplasm of fast moving live cells, and in particular inside cancer cells. The front of the cytoskeleton, also known as lamellipodium, is the driving mechanism of cell motility and is mostly comprised of long double helix polymers of actin protein, termed actin-filaments. The actin-filaments polymerize/depolymerize and exhibit a series of mechanical properties such as elasticity, friction with the substrate, crosslink binding, repulsion, myosin-driven contractility, nucleation, fragmentation, capping and more. Our approach is to employ the previously developed Filament Based Lamelipodium Model (FBLM) and the corresponding Finite Element Method (FEM) to model and simulate the motility of cells as an emerging property of the dynamics of the actin-filament network. In more detail, we embed the FBLM-FEM on an active environment of variable chemical and adhesive properties. Moreover, we include a number of cells and the interactions between them i.e. collisions and adhesions. Finally, we present our results on the constructing cell clusters and monolayers.

Multiscale Modelling of Adhesion Within the Cancer Invasion Process

Dumitru Trucu
University of Dundee, Scotland

Understanding the complex mechanisms of growth and spread of cancer in the human body remains one of the greatest scientific challenges. At the hearth of this challenge stays the natural multiscale character of cancer development which is conferred by the double-feedback link between the complex molecular signaling enabling local inter-cellular interactions (at cell-scale) and the collective dynamics of cell population (at tissue-scale). In conjunction with enzymatic activities, increased cancer cell motility due to changes in cell-adhesion properties (via dynamically formed adhesion junctions and cell-surface binding from ECM ligands) further exacerbates the invasion. The cell-scale activities are however in a dynamic cross-talk with critical tissue-scale processes reflected in the evolving morphology and migration patterns of the cancer cell population. In this talk we will present a novel multiscale moving boundary approach for cancer invasion that accounts for cell-adhesion in the context of the multiphase nature of the ECM dynamics. This connects the tissue-scale macro-dynamics with both the proteolytic cell-scale dynamics occurring at the tumour invasive edge and the micro-scale ECM fibres dynamic degradation and realignment occurring inside the tumour domain. The new modelling framework, will be accompanied by details of the computational approach and a discussion of the numerical simulation results.

Coupling Surface Finite Elements with Stokes' Elements in Modeling Cell Blebbing

Necibe Tuncer
Florida Atlantic University, USA

In this work, we introduce a mathematical model of cell blebbing. Cells, while migrating in an extracellular matrix, form a rounded membrane protrusions called "cell blebbing." We develop a mathematical model of cell blebbing by coupling the Stokes equation in the cell (bulk) to the surface partial differential equation on the membrane to model cell membrane deformation due to tension or bending. We present a finite element method where the surface elements are coupled with the bulk finite elements.

Cell-Based Modelling in Cancer and Wound Contraction

Fred Vermolen
Delft University of Technology, Netherlands

In this talk we consider cell-based modelling in the framework of applications in tumor growth and initiation and in the formation of contractions associated with deep tissue injuries. The simulation framework is based on a combination of continuum-scale chemical and mechanical modelling and on particle(agent)-based modelling. Migration of cells is modelled using a system of coupled stochastic differential equations. Cell proliferation, differentiation and death are modelled as stochastic processes. We will consider simulations in R2 and in R3. The framework has a generic nature and can be applied to various processes in organ development and tissue engineering. Since the model contains a stochastic nature, and since many input parameters are hard to obtain, Monte Carlo simulations are needed to reveal the parameter sensitivity and the correlation between the various input and output parameters. In the presentation, statistical issues will be dealt with as well. If time permits, a newly developed model for skin cancer will be presented along with its statistical assessment.

Special Session 42: Dynamical Systems on Ecology, Epidemiology and Immunology

Yasuhiro Takeuchi, Aoyama Gakuin University, Japan
Malay Banerjee, Indian Institute of Technology Kanpur, India
Hisashi Inaba, The University of Tokyo, Japan
Chang-Yuan Cheng, National Pingtung University, Taiwan

This special session on new developments in Dynamical Systems on Ecology, Epidemiology and Immunology will bring together a selected group of experts in these areas. We can find many interesting phenomena in biological world which are essentially nonlinear and have rich dynamical properties, but in general it is very hard to understand the mechanism to produce the phenomena. Then we try to constitute a (mathematical) model which includes the essential mechanism thought to cause the phenomena. If the model does not show the phenomena we are interested in, then we revise the model by choosing the other mechanism to produce the phenomena. This session will give a good chance to discuss on several mathematical problem in the field of ecology, epidemiology and immunology. The following scientists are planned to be invited to give a talk in the session in AIMS 12.

Viral Dynamics Model in Heterogeneous Environments Incorporating Antiretroviral Therapy

Chang-Yuan Cheng
National Pingtung University, Taiwan
Y. Dong, Y. Takeuchi

I will first introduce basic virus reproductions via virus-to-cell infection and cell-to-cell transmission respectively, and then explore an virus model in heterogeneous environments, which imitates the complexity of the human body (like an ecological system). How the two infection routes and heterogeneous environments affect the viral dynamics and even the drug efficacy are significant in administrating a drug therapy. Accordingly, the study provides a method determining the optimum to distribute drugs in the multi-compartmental environment.

Delayed Feedback Controls in an Escherichia Coli and Tetrahymena System

Yueping Dong
Aoyama Gakuin University, Japan
Moitri Sen, Malay Banerjee, Yasuhiro Takeuchi, Shinji Nakaoka

We develop a mathematical model to investigate the interaction between Shiga-toxin producing Escherichia coli and Tetrahymena with delayed feedback controls by production of Shiga-toxin and recruitment of neutrophils. By applying the quasi steady state approximation, the proposed model can be reduced to a Lotka-Volterra (LV) predator-prey type system with two discrete delays. By investigating the distributions of the roots of the characteristic equation, the local stability as well as Hopf-bifurcation are well studied. We provide a clear classification framework to detect the possibility of Hopf-bifurcation when two delays are present. Numerical simulations are carried out to verify the analytical results. Our findings reveal that the instability regions of coexistence equilibrium in two delay parameters plane always enlarge as the increase of negative feedback control coefficients, and especially the feedback controls on Tetrahymena population play a dominant role in the destabilization of coexistence equilibrium. Besides, we observe some interesting phenomena such as peak-adding bifurcation, quasi-periodic oscillation and chaos.

Complete Global Stability of an SEIS Model with Delays

Yoichi Enatsu
Tokyo University of Science, Japan
Yoichi Enatsu

We consider the dynamical behavior of an SEIS model with delay denoting an incubation time. In Paulhus, Wang (2015), global stability of equilibria is investigated by constructing suitable Lyapunov functionals. They mention that a disease-free equilibrium E_0 is globally asymptotically stable if $R_0 \leq 1$, and the endemic equilibrium E_* is global asymptotically stable if $R_0 > 1$, where R_0 is the basic reproductive ratio. However, in the proof of global stability of E_0, an additional condition is required to construct Lyapunov functional in Paulhus, Wang (2015). We prove stability of E_0 under an another condition by constructing a new Lyapunov functional. Combining with their results, we claim that any conditions are not necessary under $R_0 \leq 1$ to prove global asymptotic stability of E_0.

Basic Reproduction Number R0 in Time-Heterogeneous Environments: Revisited

Hisashi Inaba
The University of Tokyo, Japan

In the previous paper (JMB 65, 309-348, 2012), I proposed a new general definition of the basic reproduction number R0 for structured populations in general time-heterogeneous environments. Using the idea of cone spectral radius, we show that our R0 is the spectral radius of the generation evolution operator (GEO) in the time-state space. Next we show that the sign relation holds between R0 and the growth bound of the evolutionary family associated with the

linearized population evolution equation, which implies that this R0 is the invasion threshold. We also discuss that this R0 can work as a threshold value for existence of periodic endemic solution for epidemic models in periodic environments.

Sustained Oscillations in Three-Dimensional Nonlinear Iteroparous Leslie Matrix Models

Ryusuke Kon
University of Miyazaki, Japan

A Leslie matrix model is an age-structured population model described by a system of difference equations. The model is categorized as semelparous or iteroparous, depending on reproduction strategies. The semelparous model is well studied under the assumption that the basic reproduction number $\mathcal{R}_0$ is slightly larger than the unity. Such studies reveal that it is unlikely that the semelparous model with two or three age-classes exhibits sustained oscillations if $\mathcal{R}_0 \approx 1$. In the semelparous model, individuals of the same age is assumed to reproduce strictly in the same age. However, it is unlikely in nature. In this talk, we relax the assumption of semelparity and consider the iteroparous model. Then we find that the iteroparous model with three age-classes can exhibit sustained oscillations even if $\mathcal{R}_0 \approx 1$.

Development and Applications of an Optimization Method to Detect Neighbor Equilibria of a Trajectory for Lotka-Volterra Systems

Shinji Nakaoka
JST PRESTO, The University of Tokyo, Japan
Shinji Nakaoka

In this study, we construct a generalized Lotka-Volterra system, a mathematical model describing population dynamics of living organisms, to describe dynamics of compositional change in a bacterial community. An optimization method in combination with numerical integration of differential equations is newly proposed in order to detect neighbor equilibria of a trajectory which can be an indicator of compositional change in a bacterial community. The method will be applied to demonstrate the ability to extract predisposing factors of allergic disorders from real datasets.

Distinct Palatability Between Two Plant Species and Nutrient Recycling Differently Drive Deer-Plant Dynamics

Toshiyuki Namba
Osaka Prefecture University, Japan
Rena Isono, Yoshiji Fujiwara

Recently, many ungulate populations become overabundant worldwide. Overpopulated ungulates often extirpate preferred plants and alter plant communities into those dominated by less palatable ones. We study effects of herbivory and nutrient recycling on dynamics of nutrient, plants and herbivore. We consider a simple dynamical model of a nutrient, palatable and unpalatable plants, and an herbivore, assuming that (i) the nutrient is supplied from outside at a constant rate; (ii) the unpalatable plant is less efficient in resource acquisition; (iii) the herbivore preferentially feeds on palatable plant; (iv) nutrient recycling is accelerated by excretion of the herbivore. If nutrient inflow and herbivore grazing rates on palatable plant are low, unpalatable plant becomes extinct. Although unpalatable plant can survive and two plant species and herbivore coexist if these rates are high, further increases in the rates destabilize the coexistence and population oscillations occur. Rapid mineralization of nutrients also makes the system unstable, but it causes very different types of oscillations from those driven by differences in palatability of plants. We also show that improvement in quality of less palatable plants and subsequent increase in grazing intensity on the plant may cause extirpation of not unpalatable but more palatable plants.

Maturation Delay for the Predators Can Enhance Stable Coexistence for a Class of Prey-Predator Models

Yasuhiro Takeuchi
Aoyama Gakuin University, Japan
Malay Banerjee

Maturation time delay for the predators is introduced in prey-predator models to implicitly model the stage-structure of predators. Most of the prey-predator models with maturation delay are known to exhibit destabilization of coexistence steady-state. This is due to the introduction of time delay with lack of ecological justification. The main objective of the present work is to show the stabilizing role of maturation delay for a class of delayed prey-predator model. To be specific, we consider prey-predator models with strong and weak Allee effects in prey growth and Michaelis-Menten type functional response. We provide ecological justification for the introduction of maturation delay parameter in predator's growth equation. We obtain the conditions for stable and oscillatory coexistence of prey and their specialist predator in case of strong as well as weak Allee effect for non-delayed and delayed models. Our analytical and numerical findings reveal that delay is not always a destabilizing factor rather the stable coexistence in the presence of time delay depends upon the formulation of the delayed model. The biological implications of the current investigation are provided in the conclusion section. We also explain the validity of obtained results for other types of prey-predator models with a specialist predator.

Special Session 45: Randomness Meets Life

Jasminee Foo, University of Minnesota, USA
Peter Hinow, University of Wisconsin-Milwaukee, USA
Blerta Shtylla, Pomona College, USA

We will bring together researchers that employ stochastic methods to model a rich set of biological phenomena. Stochastic effects are important when modeling small scale phenomena such as biochemical reactions, intra-cellular interactions, cellular pattern formation and transport due to the small scale of the environment. In these cases stochasticity is not a mere afterthought but the engine that mediates exquisitely complex phenomena. Stochastic effects are also important at larger scales, as in animal dispersion, spread of epidemics, mathematical immunology and cancer. Despite the large difference in scales the techniques used for model development and analysis are often similar. We intend to bring researchers together that have worked at both large and small scales in order to stimulate the exchange of ideas. An important aspect of the session is that we will encourage speakers to give large picture ideas of their respective fields which is important for cross pollination and also for young researchers interested in work on mathematical modeling of new biological applications.

Numerical Methods for Random Ordinary Differential Equations with Time Delay

Yusuke Asai
Hokkaido University, Japan
Peter E. Kloeden

Mathematical modeling by differential equations plays an important role to understand natural sciences. In particular, required data are frequently not obtained in biology and medicine and simulation using mathematical models helps us to analyze system behavior under various scenarios. Deterministic models have been long investigated and applied to natural phenomena, however, some factors might be ignored in model building process or we encounter random effect from environment in practice. To handle such uncertain effect, random ordinary differential equations (RODEs) can be an ideal tool because of its simplicity in model building as well as the regularity of the corresponding noise processes. Recently, several classes of numerical methods for RODEs have been developed and applied to real problems, yet time delay has been ignored in those applications. To capture and understand system behavior more accurately, we need to handle both of randomness and time delay simultaneously. In this talk, numerical methods for RODEs with time delay are systematically constructed and their convergence are discussed. The developed methods are applied to virus dynamics model with target cells, infected cells and viruses compartments, and eclipse phase, the time elapsed between cell infection and virus production, and their behavior will be investigated.

Structured Population with Fast Dynamics - Patches and Networks

Jacek Banasiak
University of Pretoria, So Africa

We consider structured population models in which the population is subdivided into states according to a certain feature of the individuals. We consider various rules allowing individuals to move between the states – it may be physical migration between geographical patches, or the change of the genotype by mutation during mitosis. Such models are often referred to as piecewise deterministic but at the level of densities, depending on the migration rule, they may vary from a system of nonlocally coupled McKendrick equations to a system of transport equations on a graph. We address the well-posedness of such problems, classical in the first case and more challenging in the second. The main interest, however, is the asymptotic state aggregation that, in the presence of different time scales of the processes driving the evolution, allows for a significant simplification of the equations. Interestingly, the aggregated equations vary widely, from a scalar transport equation to systems of ordinary differential equations.

Data-Driven Multiscale Mathematical Models of Signaling in the Maintenance of Transcription Factor Distribution in Stem Cell Homeostasis

Weitao Chen
University of California, Riverside, USA
Venugopala Gonehal, Alex Plong, Kevin Rodriguez, Andy Snipes, Albert Do

The regulation and interpretation of transcription factor levels is critical in spatiotemporal regulation of gene expression in development biology. However, concentration-dependent transcriptional regulation, and the spatial regulation of transcription factor levels are poorly studied in plants. WUSCHEL, a stem cell-promoting homeodomain transcription factor was found to activate and repress transcription at lower and higher levels respectively. The differential accumulation of WUSCHEL in adjacent cells is critical for spatial regulation on the level of CLAVATA3, a negative regulator of WUSCHEL transcription, to establish the overall gradient. Experiments show that subcellular partitioning and protein destabilization control the WUSCHEL protein level and spatial distribution. Meanwhile the destabilization of WUSCHEL also depends on the protein concentration which in turn is influenced by intracellular processes. However, the roles of extrinsic spatial cues in maintaining differential accumulation of WUSCHEL are not well understood. Moreover, the utilization of transcriptional regulatory domains for sensing hormones in regulating protein concentration forms a feedback which is difficult to understand in experiments alone. We develop a 3D cell-based mathematical model which integrates sub-cellular partition with cellular concentration across the spatial domain to analyze the regulation of WUS and stem cell homeostasis in quantitative level. By using this model, we investigate the machinery of the maintenance of WUS gradient within the tissue. We also incorporate cell division in this model to study the shoot apical meristem growth under chemical signaling network.

Stationary Distributions and Convergence Rates for Semistochastic Processes

Alexander Grigo
University of Oklahoma, USA
James Broda, Nikola Petrov

We consider a semistochastic continuous-time continuous-state space random process $\{X(t)\}_{t\geq 0}$ that models deterministic growths (governed by an autonomous ODE) that is subject to (downward) jumps due to random severity events occurring at random times. The times of occurrence of the disturbances are modeled by a Poisson process whose rate Λ is allowed to depend on the value of $X(t)$. At each time t a random event occurs the value $X(t^-)$ is multiplied by a continuous random variable ("severity") supported on $[0,1]$, which gives the value of $X(t)$. An important example of such a process is the dynamics of the carbon content of a forest whose deterministic growth is interrupted by natural disasters (fires, droughts, insect outbreaks, etc.). The time-dependent distribution of the process $\{X(t)\}_{t\geq 0}$ satisfies an integro-differential PDE. We derive explicit expressions for the stationary distribution of the random process, and we develop a method for giving an upper bound of the rate at which the distribution of $X(t)$ approaches the stationary distribution. This is a collaboration with James Broda and Nikola Petrov.

Dynamics of Zika Virus Epidemic Under Random Environments

Xiaoying Han
Auburn University, USA
Yusuke Asai, Peter E. Kloeden

A mathematical model for Zika virus dynamics under randomly varying environment conditions is developed, in which the birth and death rates for mosquitoes, and the transmission rate of Zika virus between humans are modeled as random processes. The resulting system of random ordinary differential equations is studied by the theory of random dynamical systems. In particular, the existence, uniqueness and positiveness of solutions are first discussed. Then the long term dynamics in terms of existence and geometric structures of random attractors are investigated.

Modelling Antimicrobial Deescalation in ICUs

Xi Huo
Univeristy of Miami, USA
J. Hughes, L. Falk, A. Hurford, B. Coburn, A. Morris, J. Wu

Antimicrobial de-escalation aims to minimize resistance to high-value broad-spectrum empiric antimicrobials by switching to alternative drugs when testing confirms susceptibility. Though widely practiced in ICUs, the effects of de-escalation are not well understood. We develop a high-dimensional ODE model to assess the effect of de-escalation on a broad range of outcomes, and clarify expectations. In this talk, I will present the medical background and conclusions of our first modelling results, and show how we numerically analyze the model output with a broad range of undetermined parameters and limited data.

Multiscale Stochastic Reaction-Diffusion Algorithms for Biochemical Networks

Hye-Won Kang
University of Maryland Baltimore County, USA
Radek Erban (University of Oxford)

A Markov chain model has become popular to present the discrete nature of the molecular copy numbers and inherent stochasticity in reaction-diffusion systems, but its computation can be expensive. A possible approach to reduce computational cost is to approximate a part of the model by some coarse-grained methods. In this talk, I will introduce two multiscale algorithms coupling the suitably discretized stochastic partial differential equations (SPDEs) and the Markov chain model, which provide good approximations to the solutions obtained by the Markov chain model applied in the entire spatial domain. Two coupling methods of the Markov chain model and the SPDEs across the interface will be discussed. This is joint work with Radek Erban at the University of Oxford.

Spatial Stochastic Models for Molecular Motors Attaching and Detaching from Parallel Microtubules

Peter Kramer
RPI, USA
Abhishek Choudhary, Joe Klobusicky, John Fricks

Intracellular transport is conducted largely by molecular motor proteins such as kinesin and dynein, which walk along microtubule or actin filaments, from which they can attach or detach. The progress of cargo carried by these molecular motors is therefore determined by the effective transport of these motors over several cycles of attachment and detachment. We employ a framework of switched stochastic differential equations to model this process, with the switching rates influenced by the spatial configuration of the motors and cargo. Mean first passage time calculations are also developed to provide a model for how attachment rates depend on the geometry of the cargo and microtubule network.

Bayesian Uncertainty Quantification for Particle-Based Simulation of Lipid Bilayer Membranes

Anastasios Matzavinos
Brown University, USA

A number of problems of interest in applied mathematics and biology involve the quantification of uncertainty in computational and real-world models. A recent approach to Bayesian uncertainty quantification using transitional Markov chain Monte Carlo (TMCMC) is extremely parallelizable and has opened the door to a variety of applications which were previously too computationally intensive to be practical. In this talk, we first explore the machinery required to understand and implement Bayesian uncertainty quantification using TMCMC. We then describe dissipative particle dynamics, a computational particle simulation method which is suitable for modeling biological structures on the sub-cellular level, and develop an example simulation of a lipid membrane in fluid. Finally, we apply the algorithm to a basic model of uncertainty in our lipid simulation, effectively recovering a target set of parameters (along with distributions corresponding to the uncertainty) and demonstrating the practicality of Bayesian uncertainty quantification for complex particle simulations.

O'Hara's Knot Energies and $W^{1/P,P}$-Harmonic Maps Into Spheres

Armin Schikorra
U Pittsburgh, USA
S. Blatt, Ph. Reiter.

I will report on advances in the regularity theory for minimizers and critical points of a class of knot energies defined by Jun O'Hara. When parametrized by arclength the tangent field of these knots are critical points of a $W^{1/p,p}$-type energy, and we employ arguments from the regularity theory of $W^{1/p,p}$-harmonic maps into the sphere.

Stochastic Dynamics on Networks: Complexity Reduction Via Importance Ranking of Noise Sources in Network Models

Deena Schmidt
University of Nevada, Reno, USA

Markov processes are widely used to model the dynamics of biological processes evolving on networks. Complexity reduction for such models aims to capture the essential dynamics of the process via a simpler representation, with minimal loss of accuracy. The stochastic shielding approximation is a novel dimension reduction method that has been used to simplify stochastic network models arising in neuroscience, such as randomly gated ion channel models, but applies broadly to many biological systems. In this talk, I will describe the stochastic shielding approximation and our related edge importance measure which allows us to rank each noise source according to its contribution to the observed variability. The approximation works by replacing the lowest ranked Markovian transitions with deterministic ones, and doesn't significantly affect the variability of the observed variables. I will also explore the robustness of the method under conditions of timescale separation and population sparsity.

Mathematical Models for Mechanisms Driving Asymmetric Cell Division

Blerta Shtylla
Pomona College, USA

In early development, embryo cells can opt to divide asymmetrically and produce daughters that have distinct fates. At the cell scale, asymmetric division is achieved with the help of exquisitely fine-tuned mechanistic processes that control the location of the cell division plane. In this talk we will introduce two mechanisms for how a dividing cell might achieve asymmetric division. First, we start with the simple single cell bacteria, Caulobacter Crescentus and show how a single cell employs spatio-temporal protein localization patterns to estimate the location of the division site. Using advection-reaction-diffusion PDE models we show how modulation of ATP-ase reaction rates can affect the spatial location of several proteins and transition the cell from a symmetric to an asymmetric division program. We also show how Brownian dynamics simulations can be used to verify some of the conclusions of the PDE models. A second example of how asymmetric division is achieved will be illustrated in the early stages of C. elegans embryo division. In this case, the cell employs more complicated biopolymer networks to achieve proper asymmetric division plane placement. We use stochastic models and experimental data to show how division plane placement can be controlled robustly. In both cases, model results will be compared with data and general mechanisms for division plane placement will be discussed.

Invariance Criteria for Stochastic Systems and Applications in Biology

Stefanie Sonner
Radboud University Nijmegen, Netherlands
Jacky Cresson, Messoud Efendiev, Benedicte Puig

Invariance is a crucial property for many mathematical models of biological or biomedical systems, meaning that the solutions necessarily take values within a given range. While invariance criteria are well-known for systems of ODEs and semilinear parabolic PDEs and most deterministic models respect invariance, such results are less known for stochastic systems. In fact, several recent stochastic model extensions violate this fundamental property.
We recall explicit invariance criteria for deterministic models and present the corresponding extensions in the stochastic framework. The results can be used to characterize the class of stochastic models preserving invariance, and hence, for model validation. This is illustrated in various biological examples.

Transit Times and Mean Ages for Non-Autonomous and Autonomous Compartmental Systems Modeling the Terrestrial Carbon Cycle

Ying Wang
University of Oklahoma, USA
Yiqi Luo et al.

We develop a theory for transit times and mean ages for non-autonomous compartmental systems. Using the McKendrick-von Forster equation, we show that the mean ages of mass in a compartmental system satisfy a linear non-autonomous ordinary differential equation that is exponentially stable. We then define a non-autonomous version of transit time as the mean age of mass leaving the compartmental system at a particular time and show that our non-autonomous theory generalizes the autonomous case. We apply these results to study a nine-dimensional non-autonomous compartmental system modeling the terrestrial carbon cycle, which is a modification of the Carnegie-Ames-Stanford approach model, and we demonstrate that the non-autonomous versions of transit time and mean age differ significantly from the autonomous quantities when calculated for that model.

3D Structure Prediction of Alpha-1,4 Fucosyltransferase (Preliminary)

Chi-Jen Wang
National Chung Cheng University, Taiwan
Ching-Ching Yu

Fucosyltransferase (FucT) is an enzyme that transfers fucose sugar to a sugar or protein, and Helicobacter plori in human causes both gastric and duodenal ulcers. Helicobacter plori α-1,3 FucT is involved in the biosynthesis of the Lewis X trisaccharide of the lipopolysaccharide O-antigen. The function and structure of α-1,3 FucT has been reported in these years, however the structure of α-1,4 FucT has not. The prior experiment shows that α-1,4 FucT shares the function of α-1,3 FucT, further the gene sequence of α-1,4 FucT also indicates this enzyme might have two functions. Our goal is to predict the structures of α-1,4 FucT, and compare these structures to other FucTs to verify the possibility of the dual functionalities of α-1,4 FucT.

Principal Component Analysis in the Space of Phylogenetic Trees

Ruriko Yoshida
Naval Postgraduate School, USA
Tom Nye, Xiaoxian Tang, Grady Weyenberg

Principal component analysis is a popular method of reducing high-dimensional data to a low-dimensional representation that preserves much of the sample's structure. However, the space of all phylogenetic trees on a fixed set of species does not form a Euclidean vector space, and methods adapted to tree-space are needed. Previous work introduced the notion of a principal geodesic in this space, analogous to the first principal component. Here we propose a geometric object for tree-space similar to the kth order principal component in Euclidean space: the locus of the weighted Fréchet mean of $k+1$ vertex trees when the weights vary over the k-simplex. We establish some basic properties of these objects, in particular that they have dimension k. We propose algorithms for projection onto these surfaces and for finding the principal locus associated with a sample of trees. Simulation studies demonstrate that these algorithms perform well, and analysis of two empirical data sets, containing Apicomplexa and African coelacanth genomes respectively, reveal important structure from the second-order principal components.

Special Session 46: Dynamical Systems with Applications to Population Biology

Yijun Lou, The Hong Kong Polytechnic University, Hong Kong
Hongying Shu, Tongji University, Peoples Rep of China
Xiang-Sheng Wang, University of Louisiana at Lafayette, USA
Xiaotian Wu, Shanghai Maritime University, Peoples Rep of China

Over the past decades, the areas of dynamical systems and population biology have gained an increasing attention and received a growing momentum. In this special session, we will bring together researchers from various backgrounds to work together and contribute to the study of dynamical systems with applications to population biology. It will serve as a platform to present recent progresses, exchange research ideas, extend academic networks, and seek future cooperation. Speakers and talks are carefully selected to make the session interesting and attractive to a diverse audience.

On a Reaction Diffusion Model for Competition Species with Allee Effects

Wei Feng
University of North Carolina Wilmington, USA
Xin Lu

In this talk, we study a reaction-diffusion model for two competing biological species (u and v) with Allee effects. Under one-side or two-side Allee effect, the model demonstrates complexity on its coexistence steady states. The conditions for persistence and competitive exclusion of the species are obtained through asymptotic stability with attraction regions and convergent rates depending on the biological parameters. Using the upper-lower solution method, we further prove that for a family of wave speeds, there exist traveling wave solutions connecting various steady-state solutions. Finally, numerical simulations are also presented to illustrate the theoretical results and population dynamics.

Mathematical Analysis of a Vector-Borne Disease Model with Imperfect Quarantine

Daozhou Gao
Shanghai Normal University, Peoples Rep of China
Xiulei Jin, Shuwan Jin

Quarantine and isolation are public health interventions that are widely implemented in containing the spread of infectious diseases by separating exposed or ill persons from the healthy population. Quite a few mathematical models of isolation and quarantine have been proposed and studied in the last two decades, but few on indirectly transmitted diseases. We develop a deterministic mosquito-borne disease model where imperfect quarantine is implemented to reduce the disease transmission from infected humans to susceptible mosquitoes. The basic reproduction number R_0 is computed and the model equilibria and their stabilities are analyzed. We prove that a subcritical (backward) bifurcation is possible at $R_0 = 1$. Numerical simulations show that the quarantine regime can make a significant contribution in avoiding a large disease outbreak.

HIV Epidemics in Chongqing and Shenzhen, China

Daihai He
Hong Kong Polytechnic University, Hong Kong
S. Yang, A. Chiu, Q. Lin, Z. Zeng, Y. Li, Y. Zhang, Z. Yang, L. Yang

Men who have sex with men (MSM) and heterosexuals are the populations with the fastest growing HIV infection rates in China. We characterize the epidemic growth and age patterns between these two routes from 2004 to 2015 in Chongqing and Shenzhen, China.
The MSM epidemics grew significantly in both cities. Chongqing experienced quadratic growth in HIV reported cases with an estimated growth rate of 0.086 per week and a deceleration rate of 0.673. HIV reported cases of MSM in Shenzhen grew even more drastically with a growth rate of 0.033 per week and deceleration rate of 0.794. The new infections are mainly affecting the ages of 18 to 30 in Chongqing and ages of 20 to 35 in Shenzhen. They peaked in early 1990's and mid-1990's birth cohorts in Chongqing and Shenzhen respectively. The HIV epidemic among heterosexuals grew rapidly in both cities.

The Complex Dynamics of a Diffusive Prey-Predator Model with an Allee Effect in Prey

Yun Kang
Arizona State University, USA
Feng Rao

This paper investigates complex dynamics of a predator-prey interaction model that incorporates: (a) an Allee effect in prey; (b) the Michaelis-Menten type functional response between prey and predator; and (c) diffusion in both prey and predator. We provide rigorous mathematical results of the proposed model including: (1) the stability of non-negative constant steady states; (2) sufficient conditions that lead to Hopf/Turing bifurcations; (3) a prior estimates of positive steady states; (4) the non-existence and existence of non-constant positive steady states when the model is under zero-flux boundary condition. We also perform completed analysis of the corresponding ODE model to obtain a better under-

standing on effects of diffusion on the stability. Our analytical results show that the small values of the ratio of the prey's diffusion rate to the predator's diffusion rate are more likely to destabilize the system, thus generate Hopf-bifurcations and Turing instability that can lead to different spatial patterns. Through numerical simulations, we observe that our model, with or without Allee effect, can exhibit extremely rich pattern formations that include but not limit to strips, spotted patterns, symmetric patterns. In addition, the strength of Allee effects also plays an important role in generating distinct spatial patterns.

Dynamics of Novel Delay Logistic Equations

Gergely Röst
University of Oxford, England

The delay logistic equation, originating from Hutchinson, has played a crucial role in the theory of nonlinear delay differential equations, inspiring the development of a variety of techniques. However, the equation has received criticisms from biological modellers due to the lack of a mechanistic derivation. In the talk we present a new delay logistic equation with clear biological underpinning from cell population dynamics. We give a global analysis of the equation showing global convergence to the positive equilibrium. However, there exist very long transients with oscillatory patterns of various shapes. We also show that if we add an instantaneous positive feedback term to the classical delay logistic equation, then local stability does not imply global stability so a Wright-type conjecture is not valid any more.

Wavefront Solutions of Degenerate Quasilinear Reaction-Diffusion Systems with Mixed Quasi-Monotonicity

Weihua Ruan
Purdue University Northwest, USA

We study the existence of wavefront solutions in a system of degenerate quasilinear reaction-diffusion equations of mixed quasi-monotone properties.

Special Session 47: Bifurcations and Asymptotic Analysis of Solutions of Nonlinear Models

Jann-Long Chern, Central University, Taiwan
Yoshio Yamada, Waseda University, Japan
Shoji Yotsutani, Ryukoku University, Japan

The aim of this special session is to exchange recent results, ideas and techniques on nonlinear elliptic and parabolic PDEs, including reaction-diffusion systems and free boundary problems, from mathematical physics, chemical reactions, mathematical biology, medical science and some other fields. In particular, we are interested in the global bifurcation structure for such models. Combinations of numerical simulations and theoretical approaches with asymptotic analysis will be very useful to understand the nonlinear phenomena together with underlying structure of solutions. We will give opportunities to both established and junior researchers working in the related area to present their recent results.

Partial Separation of Positive Entire Solutions for Semilinear Elliptic Equations

Soohyun Bae
Hanbat National University, Korea

We study partial separation of positive solutions for semilinear elliptic equations. Under some integral conditions on indefinite coefficient in the equations, we show that positive solutions are separated if the uniform norms are sufficiently small. The asymptotic behavior of the solutions is the main tool to verify the phenomena.

Spiral Waves in Circular and Spherical Geometries: the Ginzburg-Landau Paradigm

Jia-Yuan Dai
Free University of Berlin, Germany

We prove the existence of m-armed spiral wave solutions for the complex Ginzburg-Landau equation in the circular and spherical geometries. Instead of applying the shooting method in the literature, we establish a functional approach and adopt global bifurcation analysis to generalize the known results of existence for rigidly-rotating spiral waves. Moreover, we prove the existence of two new patterns: frozen spirals in the circular and spherical geometries, and 2-tip spirals in the spherical geometry.

Asymptotic Behavior of Solutions to Nonlinear Diffusion Problems with Dirichlet and Free Boundary Conditions

Maho Endo
Waseda University, Japan
Yoshio Yamada

We consider a nonlinear diffusion equation $u_t = du_{xx} + f(u)$ in $[0, h(t)]$ with zero Dirichlet boundary condition at $x = 0$ and a free boundary condition at $x = h(t)$. Here $f(u)$ belongs to a certain class of bistable nonlinearity which allows two positive stable equilibrium states as an ODE model. This problem models the invasion of a biological species and the free boundary $x = h(t) > 0$ represents the spreading front of its habitant. Our main interest is to study large-time behaviors of solutions for the free boundary problem. We obtain a trichotomy result for vanishing and two types of spreading and sharp thresholds for the asymptotic behavior of the solutions by the phase plane analysis and the zero number arguments.

Eigenvalues of the Laplace-Beltrami Operator on a Zonal Domain in the Unit Sphere

Yoshitsugu Kabeya
Osaka Prefecture University, Japan

We consider the distribution and the asymptotic behavior of eigenvalues of the Laplace-Beltrami operator on a zonal domain in the unit sphere. The domain is defined on the unit sphere with small two holes. As the diameters of holes goes to zero, we discuss what will happen the asymptotic behavior of eigenvalues.

Properties of Spreading Solutions to a Free Boundary Problem with Dirichlet Boundary Conditions

Yuki Kaneko
Waseda University, Japan
Yoshio Yamada

We discuss a free boundary problem for a reaction-diffusion equation with Dirichlet boundary conditions on both fixed and free boundaries. This model, proposed by Du-Lin (2010) under the Neumann boundary condition on the fixed one, models the spreading of biological (new or invasive) species. It is known that, when the free boundary goes to infinity as time tends to infinity (i.e. spreading occurs), the solution converges to a stationary solution in any compact set. In this talk we will consider other properties of spreading solutions such as convergence near the free boundary, spreading speeds and profiles of the solutions. This is a joint work with Professor Yoshio Yamada (Waseda University).

Bifurcation Structure of a Prey-Predator Model with Population Flux by Attractive Transition

Kousuke Kuto
University of Electro-Communications, Japan
Kazuhiro Oeda

This talk considers coexistence steady states of a prey-predator model with a strongly coupled diffusion term describing a predators‘ ability to chase preys. The first topic is the bifurcation structure of steady states and the second topic is the asymptotic behavior of steady states as a coefficient of the strongly coupled diffusion term tends to infinity. A main result asserts that coexistence steady states of the prey-predator model can approach coexistence steady states of an equal diffusive competition model as the coefficient tends to infinity.

The Fisher–KPP Equation As an Asymptotic Limit of a Chemotaxis System with Logistic Source

Masaaki Mizukami
Tokyo University of Science, Japan
Johannes Lankeit

This work is concerned with the question of "how far does small chemotactic interaction perturb the Fisher–KPP dynamics?" A chemotaxis system with logistic source was studied by e.g., Winkler (2010, 2014) and Zheng (2016). However, there are still many open problems about the chemotaxis system. On the other hand, the Fisher–KPP system has been studied extensively. Thus the development of this work will enable us to see new properties of solutions for the chemotaxis system. The main result of this talk gives convergence of solutions for the chemotaxis system to solutions for the Fisher–KPP system on bounded convex domains.

Existence, Nonexistence, Multiplicity, and Numerical Stability of Solutions for the SKT Cross-Diffusion Stationary Limiting Equation

Tatsuki Mori
Osaka University, Japan
Yuan Lou, Hisashi Matusbara, Wei-Ming Ni, Seiya Sukekuni, Shoji Yotsutani

We are interested in SKT cross-diffusion equation. The equation is proposed by Shigesada, Kawasaki and Teramoto in 1979 to investigate segregation phenomena of two competing species with each other in the same habitat area. The effect of cross-diffusion affects the population pressure between two different kinds. Lou and Ni derived limiting equations to see whether this effect may give rise to a spatial segregation or not, and to clarify its mechanism. In this talk, we introduce a new representation of solutions to a stationary limiting problem modified from representation by Lou, Ni and Yotsutani. Moreover, we show existence, nonexistence, multiplicity, and numerical stability of stationary solutions for the limiting equation by analyzing the representation. This is joint work with Y. Lou, H. Matsubara, W.-M. Ni, S. Sukekuni and S. Yotsutani.

Turing-Type Instability of Diffusion Equations with Mass Transport Through the Boundary

Yoshihisa Morita
Ryukoku University, Japan
Kunimochi Sakamoto

Motivated by models of cell polarity, we consider diffusion equations in a domain and on the boundary, where those equations are coupled with mass transport. We discuss the Turing-type instability of a constant solution to the equations.

On the Asymptotic Form of Solutions of the Tadjbakhsh-Odeh's Variational Problem

Minoru Murai
Osaka City University, Japan
Shoji Yotsutani

Let Γ be a plane closed elastic curve with length $L > 0$. We denote the arc-length and the curvature by s and $\kappa(s)$, respectively.
Let $\mathcal{A}(\Gamma)$ be the signed area of the domain bounded by Γ. We consider the variational problem: Find a curve Γ (the curvature $\kappa(s)$) which minimize the functional $\frac{1}{2}\int_0^L \kappa(s)^2 ds + p\mathcal{A}(\Gamma)$ subject to $\int_0^L \kappa(s)ds = 2\pi$, where p is a given positive constant. This variational problem was first considered by Tadjbakhsh-Odeh to study the equilibrium states of an elastic inextensible ring under a uniform pressure $p > 0$. S. Okabe showed the asymptotic form of solutions of the Euler-Lagrange equation for sufficient large p and derived the second variation. He showed the uniqueness and the profile of the minimizer for sufficiently large p. In this talk, we will give new proofs of the asymptotic form of solutions of the Euler-Lagrange equation for sufficient large p.

Theoretical Analysis of a Mathematical Model for a Self-Propelled Motion

Masaharu Nagayama
Hokkaido University, Japan
Mamoru Okamoto, Yusuke Yasugahira, Hiroyuki Kitahata, Satoshi Nakata

Mathematical modelling can not only be used to clarify the mechanism of characteristic features of self-propelled motion, but also to design an original self-

propelled system. In this talk, we explain how spatio-temporal features of self-propelled motion can be reproduced by a mathematical model. The model is composed of a reaction-diffusion equation for camphor molecule layer on water surface, and a equation of motion for a self-propelled object. We next treat the existence and stability of a constant velocity solution of the mathematical model, and finally report the motion of multiple self-propelled objects using a numerical simulation and a mathematical analysis.

Applications of Generalized Trigonometric Functions to Nonlocal Boundary Value Problems

Shingo Takeuchi
Shibaura Institute of Technology, Japan

Generalized trigonometric functions (GTFs) are simple generalization of the classical trigonometric functions. GTFs are deeply related to the p-Laplacian, which is known as a typical nonlinear differential operator, and there are a lot of works on GTFs concerning the p-Laplacian. However, few applications to differential equations unrelated to the p-Laplacian are known. In this talk, we will apply GTFs with two parameters to nonlinear nonlocal boundary value problems without p-Laplacian.

Characterization on Eigenvalues and Eigenfunctions for 1-Dimensional Linearized Scalar Field Equations

Tohru Wakasa
Kyushu Institute of Technology, Japan

In this talk we will investigate the linearized eigenvalue problems associatied with periodic solution of 1-dimensional scaler field equation with Neumann boundary condition. More precisely, when the exponent p is either 2 or 3, we will show the expressions of every eigenvalue and eigenfunctions by using elliptic functions and the special solution of the characteristic equation which can be written as a special form of elliptic integral of the third kind.

Dynamics of Hot Spots in the One-Dimensional Logarithmic Diffusion Equation

Eiji Yanagida
Tokyo Institute of Technology, Japan

This talk is concerned with the behavior of positive solutions to the logarithmic diffusion equation on the whole real line. It is known that due to fast diffusion of the equation, the extinction of solutions may occur in finite time. I will discuss the behavior of hot spots (maximum points) of solutions near the extinction time in the one-dimensional case.
By applying the intersection number principle, it is shown that the hot spot typically converges to a point or goes to spatial infinity. However, in some cases, the hot spot remains bounded but does not converge to any particular point.

Remarks on Boundedness and Stabilization in a Fully Parabolic Keller-Segel System with Signal-Dependent Sensitivity

Tomomi Yokota
Tokyo University of Science, Japan

In this talk we present recent results on boundedness and stabilization in the fully parabolic Keller-Segel system with signal-dependent sensitivity. The result on boundedness is based on a joint work with Masaaki Mizukami (Math. Nachr. 2017; 290:2648-2660) and the result on stabilization is based on a joint work with Michael Winkler (Nonlinear Anal., to appear).

Special Session 48: Nonlinear Water Waves

Christian Kharif, IRPHE, France
Hung-Chu Hsu, Tainan Hydraulics Laboratory, NCKU, Taiwan

This session is devoted to mathematical, theoretical and numerical aspects of nonlinear waves propagating on water of arbitrary depth from shallow to infinite depths. Recent results on rotational and potential waves as well are concerned involving gravity, capillary-gravity and capillary waves. The aim of this session is to focus on recent findings in nonlinear water waves, namely in the presence of vorticity.

Shallow Water Dynamics on Linear Shear Flows and Plane Beaches

Maria Bjoernestad
University of Bergen, Norway
Henrik Kalisch

Long waves in shallow water propagating over a background shear flow towards a sloping beach are being investigated. The classical shallow-water equations are extended to incorporate both a background shear flow and a linear beach profile, resulting in a non-reducible hyperbolic system. Nevertheless, it is shown how several changes of variables based on the hodograph transform may be used to transform the system into a linear equation which may be solved exactly using the method of separation of variables. This method can be used to investigate the run-up of a long wave on a planar beach including the development of the waterline.

Drifting Modulation Instability Dynamics

Amin Chabchoub
The University of Sydney, Australia
Norbert Hoffmann, Takuji Waseda and Nail Akhmediev

One possible mechanism to explain the formation of extreme nonlinear waves in uni-directional and narrow-banded seas is the modulation instability (MI). The classical MI process in physical space is characterized by the emergence of extreme periodic water wave groups, that can reach substantial wave heights, when initially slightly perturbing periodically a regular Stokes wave train. The MI can be triggered in the Fourier domain, when side-bands around the main frequency are excited and subsequently, follow an exponential growth. We present an experimental study on MI wave groups that propagate with a velocity that is higher than the group velocity within the context of exact Akhmediev breather solutions of the nonlinear Schroedinger equation (NLSE). It is shown that when this additional velocity to the wave groups is small, a good agreement with NLSE is reached. Otherwise, a significant deviation is observed. The latter could be explained when the water wave modelling accuracy exceeds the NLSE limitations.

Modeling Extreme Waves Due to Typhoon by Coupled Current and Wave Simulations

Wei-Ting Chen
National Sun Yat-Sen University, Taiwan
H.C. Yu, Jason C.S. Yu

Taiwan suffered several typhoons every single year. The change in pressure and strong winds caused by typhoons have a great impact on the wave motion characteristics near Taiwan. Current can have significant influences to the wave characteristics. Tree consequtive typhoons, i.e. Meranti(12-15/09/2016), Malakas(15-18/09/2016) and Megi(25-28/09/2016), passing through the southern tip of Taiwan in September 2016. An extreme wave was recorded by an off-shore bouy in the seas of Nanwan Bay. In order to study the wave-current interactions during these typhoon period, a wave model(WWM) is coupled with a current model (SCHISM). The coupled model is based on an unstructured grid (tri-angular FEM) system which is suitable for high-resolution coastal studies. The wind field data from the weather forecasts(WRF) done by Central Weather Bureau. The coupled model results are shown to be able to capture the wave-current interactions in Nanwan bay. Results show that the wavelength becomes shorter in the opposing current, which leads to the increasing of wave number. Consequently, wave height and wave steepness are also getting higher due to the changes of the wave number.

Nonlinear Waves Over Deep Water Shear Currents and Stokes Drift

Christopher Curtis
San Diego State University, USA
John Carter, Henrik Kalisch

We investigate the effect of constant vorticity background shear on the properties of wavetrains in deep water. We derive a higher-order nonlinear Schrödinger equation in the presence of shear and surface tension. We show that the presence of shear induces a strong coupling between the carrier wave and the mean surface displacement. The effects of the background shear on the modulational instability of plane waves is also studied, where it is shown that shear can suppress instability, though not for all carrier wavelengths in the presence of surface tension. Using a modification of the Generalized Lagrangian Mean theory, explicit, asymptotic approximations for the Stokes drift velocity are obtained for plane-wave and Jacobi elliptic function solutions of the nonlinear

Schrödinger equation. We show that background currents have significant effects on the mean transport properties of waves. In particular, certain combinations of background shear and carrier wave frequency lead to the disappearance of mean surface mass transport. These results provide possible explanations for a several still puzzling oceanographic measurements.

Vorticity in the Viscous Gravity Water-Wave Problem

Debbie Eeltink
University of Geneva, Switzerland
A. Armaroli, M. Brunetti, J. Kasparian

We outline the role of vorticity in the Euler-like system for weakly viscous water waves, specifically with respect to the mass conservation of this system. The derivation of a viscous Euler system was performed by Ruvinksky et al. in 1991 and Longuet-Higgins in 1992. In 2008, Dias, Dyachenko and Zakharov adapted this system to express the viscosity contribution in terms of the surface elevation η. This Euler-like system has been widely used to perform higher order spectral method-based simulations, or as a starting point to derive Nonlinear Schrödinger (NLS) like propagation equations. However, in the above cases small vorticity terms (of higher order in wave steepness ϵ and viscosity ν) that arise in the derivation when moving from Navier Stokes to the Euler-like system are neglected. In our findings, omitting these terms is not always justified, namely when the thickness of the boundary layer where vorticity is active becomes smaller than the wave amplitude. Moreover, their omission breaks the conservation of mass. We derive an expression for these 'lost' vorticity terms in terms of the velocity potential ϕ.

Multi-Dimensional Shear Shallow Water Flows

Sergey Gavrilyuk
Aix-Marseille University, France
K. Ivanova, N. Favrie

The multi-dimensional equations of shear shallow water flows represent a $2D$ hyperbolic non-conservative system of equations which is reminiscent of generic Reynolds averaged equations for barotropic turbulent fluids. The model has three families of characteristics corresponding to the propagation of surface waves, shear waves and average flow.

I present a new splitting technique to define the weak solutions to this non-conservative system of equations. The full system is split into several subsystems for which the notion of the weak solution is almost classical. Each split subsystem contains only one family of waves (either surface or shear waves) and contact characteristics. The accuracy of such an approach is tested on the exact $2D$ solutions describing the flow where the velocity is linear with respect to the space variables, and on the solutions describing $1D$ roll waves. The capacity of the model to describe multi-D experimentally observed phenomena was shown : "fingering" of plane one-dimensional wave fronts, and the formation of singularities in radially convergent flows. I will also discuss the perspectives of such a "splitting approach" for the description of $3D$ compressible turbulent flows. This is joint work with K. Ivanova and N. Favrie.

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[1] (2017) S. Gavrilyuk, K. Ivanova, N. Favrie, Multi-dimensional shear shallow water flows : problems and solutions (submitted).

Particle Trajectory in the Wave-Current Interaction

Hung-Chu Hsu
National Cheng Kung University, Taiwan
Wu, Hsien Chang, Cheng-Da Lee, Yang-Yih Cheng

Traditionally, a particle trajectory in a wave tank is measured by setting a limpid grid-film on the flume for calibration and recording by a high speed camera. But this method may be sheltered from equipment and waves, and the data loss may be occurred. In general, the accuracy of image position is effected form wave gage, strength gage, surface line, artificial particles, etc. Besides, it is hard to obtain particle postion accurately, because of grid paper image may have a distortion from lens. These reason are all made the traces and identify error. So, we would use the image treatment method substituted for the traditional method to study and analyse the particle trajectory. In the case of image distortion are image optical-error and the geometry physical error. The image optical-error including refraction and reflection by light passing through the flume and water in a camera, distortion of image are difference in the side expanding rate. The geometry physical error occured including viewpoint, and the geometry position error in the space, this paper can obtain correct experiment position values, and reduce the image error and get the better result. Comparing above two method, it is satisfied that the RMSE of present method is below 10% much better than 160% passed method.

Stokes Waves in a Constant Vorticity Flow

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Stokes in his classical memoir made formal but far-reaching considerations about periodic traveling waves at the surface of water, subject to the force of gravity, and conjectured that the wave of greatest height exhibits a 120 degree's corner at the crest. For zero vorticity, Amick, Fraenkel and Toland proved that such a limiting wave exists. But, for nonzero vorticity, the situation is much more complicated. I will present a recent work, with Sergey Dyachenko (Illinois), where we formulate the Stokes wave problem, permitting constant vorticity and finite depth, via a conformal mapping as a nonlinear pseudo-differential equation, involving a periodic Hilbert transform for a strip, and solve by the Newton-GMRES method and a fast Fourier transform. We find overhanging and touching waves for strong positive vorticity, and Crapper's limiting wave and the rigid body rotation of a fluid disk at the large vorticity limit.

Dispersive Shock Water Waves. Experiments and numerical comparisons

Olivier Kimmoun
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Hung-Chu Hsu, Amin Chabchoub

Shock waves are non-stationary wave trains characterized by a localized steep gradient across the shock front. In classical shock wave, dissipation plays a role as regulation mechanism. A different type of shock wave is the dispersive shock waves (DSW) that are also non-stationary wave trains that form spontaneously in weakly dispersive media. It corresponds to a sharp soliton front connected to a rapidly varying oscillatory wave with structure envelope. In this case, dispersion is the regularization mechanism. These DSW are usually observed in shallow water and they are called undular bores. DSW are also observed in various other. The experimental observations of such waves was reported first in 1970 for plasma. Recently Fatome et al. (2014) have shown experimentally and numerically for light propagation in optical fibers that wave breaking leads to the observation of multiple optical dispersive shocks and their interaction under different for-wave mixing configurations. In this paper, we report the results of an experimental campaign performed in a 200m wavetank with a bathymetry that consists with two flat parts connected by a 1/20 slope. When breathers propagates over this bottom, dispersive shock waves are observed that are very similar to those showed by Fatome et al. (2014).

Multi-Modal and Non-Symmetric Steady Water Waves with Vorticity

Evgeniy Lokharu
Lund University, Sweden
Evgeniy Lokharu, Vladimir Kozlov

We consider the nonlinear problem of steady gravity-driven waves on the free surface of a two-dimensional flow of an incompressible fluid. We neglect the surface tension and assume a flow to be rotational and also allow interior stagnation points. In the talk we will discuss a recent progress in the theory of small-amplitude steady water waves with vorticity. Such waves are small perturbations of uniform streams with a several counter-currents. In contrast to the irrotational case of zero vorticity, when only Stokes and Solitary waves exist, the class of small-amplitude waves with vorticity includes multi-modal and non-symmetric waves and, possibly, waves with even more complicated geometry.

Pressure Fields Beneath Intense Surface Water Wave Groups and Solitons: Weakly Nonlinear Vs Strongly Nonlinear Results

Alexey Slunyaev
Institute of Applied Physics, Russia
E. Pelinovsky, H.-C. Hsu

A weakly-nonlinear potential theory is developed for the description of deep penetrating pressure fields caused by single and colliding wave groups of collinear waves due to the second-order nonlinear interactions [1]. The result is applied to the representative case of groups with the sech-shape of envelope solitons in deep water. When solitary groups experience a head-on collision, the induced due to nonlinearity dynamic pressure may have magnitude comparable with the magnitude of the linear solution. It attenuates with depth with characteristic length of the group, which may greatly exceed the individual wave length. In general the picture of the dynamic pressure beneath intense wave groups looks complicated. The qualitative difference in the structure of the induced pressure field for unidirectional and opposite wave trains is emphasized. The results of the weakly nonlinear approach are compared with strongly nonlinear numerical simulations of the potential Euler equations, which are performed with the help of the High Order Spectral Method. The focus on extreme events in the pressure field is made.
The support from RFBR grant No. 16-55-52019 and also from grant 105-2923-E-006-002-MY3 is acknowledged.
[1]. A. Slunyaev, E. Pelinovsky and H.-C. Hsu, The pressure field beneath intense surface water wave groups. Eur. J. Mech. B / Fluids 67, 25-34 (2018).

Hydraulic Jumps in Shallow Water with Background Shear

Vincent Teyekpiti
University of Bergen, Norway
Henrik Kalisch

Modelling of surface wave motion in a fluid is normally based on classical systems which are obtained in the framework of irrotational flow. In such a context, the influence of vorticity is completely disregarded in the formulation of the governing equations. Although this consideration is justified in many circumstances, there are observed cases in the field of meteorology and hydrology in which this approach is unsuitable due to the presence of currents.
The natural action of currents invariably create shear at the bottom of the channel and in the realm of shallow-water theory, the shear effect can become a dominant feature in the wave dynamics.
Recently, the classical shallow water equations have been modified to incorporate the action of vorticity in the flow dynamics leading to the system

$$\partial_t H + \partial_x \left(\frac{\Gamma}{2} H^2 + uH \right) = 0,$$

$$\partial_t \left(\frac{\Gamma}{2} H^2 + uH \right) + \partial_x \left(\frac{\Gamma^2}{3} H^3 + \Gamma u H^2 + u^2 H + \frac{1}{2} g H^2 \right) = 0.$$

A steady state solution of a flow with horizontal shear current is investigated. In particular, we analyse a stationary jump of a linear shear flow by using the Froude number which is defined in terms of depth-averaging integral in other to account for the average flow velocity over the entire fluid depth. It will be shown that in this way, the flow is uniquely determined by three parameters, namely, the total fluid depth, the Froude number and a non-dimensional measurable quantity which determines the strength of the vorticity.

Analysis of an Instability in Stratified Shear Flows

Vishal Vasan
International Centre for Theoretical Sciences, TIFR, India

Shear flows are a common model for large-scale fluid motions in the ocean and atmosphere. Such flows are susceptible to instability leading to momentum transport as well as turbulent mixing. It is well known that the spectral analysis of shear flows may not be well described by an eigenvalue analysis alone. Often, even (spectrally) stable shear flows can undergo significant short-term growth. In this talk, I will present some new ideas and tools to deduce the pseudospectra of shear flows. I will also present the application of these ideas to some standard shear flows. This work is part of an ongoing project to develop tools to analyse instabilities in geophysical fluids.

Investigation on Velocity and Acceleration Characteristics of Undular Bore Traveling Over a Mild Beach

Wei-Ying Wong
National Chung Hsing University, Taiwan
Chang Lin, Ming-Jer Kao, Yi-Po Shao

The characteristics of velocity distribution and acceleration of an undular bore propagating over a 1:20 sloping bottom are investigated experimentally. The target bore with a water-depth ratio of h1/h0 = 1.70 is tested in a flume, and the investigation area is focused on the swash zone. A flow visualization technique using particle trajectory photography method and a high-speed particle image velocimetry (HSPIV) system for velocity measurement are conducted with the employment of high-speed digital camera and laser sheet. The results of the acceleration are also calculated from the measured velocity value by using Navier-Stokes equation. The investigation on the velocity shifting in the internal flow of the generated bore due to the effect of leading waves is reported. In addition, the variation features of the depth-averaged values of the local and convective accelerations as well as pressure gradient are also discussed. Moreover, the characteristic of flow reversal and formation of shear layer near the sloping bottom are explored. Hence, the generation of a vortex train (or street) is reported due to the existence of shear layer between two opposite currents with one heading in onshore and another in offshore direction near the sloping bottom.

Special Session 49: Integrable Systems and their Applications

Bao-Feng Feng, The University of Texas Rio Grande Valley, USA
Jyh-Hao Lee, Institute of Mathematics, Academia Sinica, Taiwan
Ken-ichi Maruno, Waseda University, Japan
Peter Miller, The University of Michigan, USA

The discovery of the complete integrability of the Korteweg-de Vries equation is just 50 years old and the influence of this discovery has generated a new field of research, called soliton theory, with wide-ranging and unexpected applications. By gathering a group of researchers from all over the world, this session will highlight some current contributions to this field, touching on such topics as integrable models in physics and their discrete analogues, exotic exact solutions for integrable nonlinear equations, asymptotic analysis of integrable nonlinear wave problems, and connections with the theory of special functions and statistical physics.

A Robust Inverse Scattering Transform for the Focusing Nonlinear Schrödinger Equation

Deniz Bilman
University of Michigan, USA
Peter D. Miller

We propose a modification of the standard inverse scattering transform for the focusing nonlinear Schrödinger equation (also other equations by natural generalization) formulated with nonzero boundary conditions at infinity. The purpose is to deal with arbitrary-order poles and potentially severe spectral singularities in a simple and unified way. As an application, we use the modified transform to place the Peregrine solution and related higher-order "rogue wave" solutions in an inverse-scattering context for the first time. This allows one to directly study properties of these solutions such as their dynamical or structural stability, or their asymptotic behavior in the limit of high order. The modified transform method also allows rogue waves to be generated on top of other structures by elementary Darboux transformations, rather than the generalized Darboux transformations in the literature or other related limit processes.

Nonintersecting Brownian Bridges on the Unit Circle with Drift

Robert Buckingham
University of Cincinnati, USA
Karl Liechty

Nonintersecting Brownian bridges on the unit circle form a determinantal stochastic process exhibiting random matrix statistics for large numbers of walkers. We investigate the effect of adding a drift term to walkers on the circle conditioned to start and end at the same position. We compute the asymptotic distribution of total winding numbers in the scaling regime in which the expected total winding is finite. Furthermore, we show that an appropriate double scaling of the drift and return time leads to a generalization of the tacnode process expressed in terms of generalized Hastings-McLeod functions. Our results follow from Riemann-Hilbert analysis of a family of discrete orthogonal polynomials with a complex weight.

High-Order Rogue Waves of a Long Wave-Short Wave Model

Junchao Chen
Lishui University, Peoples Rep of China
Bao-Feng Feng, Ken-ichi Maruno

The long wave-short wave model describes the interaction between the long wave and the short wave. Exact higher-order rational solution expressed by determinants is calculated via the Hirota's bilinear method and the KP hierarchy reduction. It is found that the fundamental rogue wave for the short wave can be classified into three different patterns: bright, intermediate and dark ones, whereas the rogue wave for the long wave is always bright type. The higher-order rogue waves correspond to the superposition of fundamental rogue waves. The modulation instability analysis show that the condition of the baseband modulation instability where an unstable continuous-wave background corresponds to perturbations with infinitesimally small frequencies, coincides with the condition for the existence of rogue-wave solutions. Numerical simulations are provided to confirm that rogue waves can be excited in the regime of the baseband instability.

The Inverse Scattering Theory of Perturbed Kadomtsev-Petviashvili II Line Solitons

Wu Derchyi
Academia Sinica, Taipei, Taiwan

We will report recent progress on the inverse scattering theory of perturbed Kadomtsev-Petviashvili II line solitons.

Gap Probabilities in Q-Racah Tiling Model and Discrete PainlevÈ Equations

Anton Dzhamay
University of Northern Colorado, USA
Alisa Knizel

It is well-known that important statistical quantities, such as gap probabilities, in various discrete probabilistic models of random matrix type satisfy the

so-called discrete Painlevé equations, which provides an effective way to computing gap probabilities. In this talk we discuss this correspondence for a particular class of models, known as boxed plane partitions (equivalently, lozenge tilings of a hexagon). For uniform probability distribution, this is one of the most studied models of random surfaces. Borodin, Gorin, and Rains showed that it is possible to assign a very general elliptic weight to the distribution, with various degenerations of this weight corresponding to the degeneration cascade of discrete polynomial ensembles, such as Racah and Hahn ensembles and their q-analogues. This also correspond to the degeneration scheme of discrete Painlevé equations, due to Sakai. The connection between the two is given by the isomonodromy theory. In this project we use geometry to study this correspondence, as well as the degeneration. In particular, we show that gap probabilities in the q-Racah tiling model can be computed in terms of discrete Painlevé equation of type $E_7^{(1)}$. This, in turn, gives us a new symmetric Lax pair for this equation.

Integrable Discretization of Log-Aesthetic Curves in Industrial Design

Kenji Kajiwara
Institute of Mathematics for Industry, Kyushu University, Japan
J. Inoguchi, K.T. Miura, W.K. Schief

A class of plane curves called the log-aesthetic curves (LAC) and their generalization used in the industrial design and the computational geometric design is considered. We give its new characterization based on the integrable deformation theory of the plane curves in the similarity geometry, together with the variational formulation by introducing the fairing energy. This implies that LAC can be regarded as the similarity geometric analogue of the celebrated Euler's elastic curves. Based on this formulation, we propose the discrete LAC by applying the integrable discretization of the plane curve deformation in the similarity geometry. We also introduce the discrete fairing energy and formulate it by using the discrete variational principle.

Combinatorial Statistics of Soliton Automaton

Saburo Kakei
Rikkyo University, Japan

We discuss combinatorial aspects of Takahashi-Satsuma soliton automaton, which is an automaton model that exhibits solitonic behavior. In our recent work, we have introduced an elementary description of linearization property of the model. Using this technique, we consider combinatorial statistics associated with the states of soliton automaton.

Semiclassical Limit of the Davey-Stewartson II Equation

Christian Klein
IMB, France
O. Assainova, K. McLaughlin, P. Miller, K. Roidot, N. Stoilov

We present a numerical study of solutions of the Davey-Stewartson (DS) II equation in the semiclassical limit. We study the appearance of dispersive shock waves, zones of rapid modulated oscillations. The inverse scattering approach to DS equations in this limit is addressed. We discuss the appearence of blow-up in the solutions. A comparison with non-integrable DS equations is presented.

Generalized Hypergeometric Functions and Integrable Hydrodynamic Type Equations

Yuji Kodama
The Ohio State University, USA
Yuji Kodama, Boris Konopelchenko

I begin to present a brief introduction of the Aomoto-Gelfand generalized hypergeometric (AGHG) functions and their confluences by means of the action of the centralizers of regular elements. The confluence then implies that the AGHG functions are now defined in a degenerate cell of the Grassmannian. I will then construct integrable hydrodynamic type systems defined on such cells.

The Cauchy Two-Matrix Model, C-Toda Lattice and CKP Hierarchy

Chunxia Li
Capital Normal University, Peoples Rep of China
Shi-Hao Li

Starting from the symmetric reduction of Cauchy biorthogonal polynomials, we are able to derive the C-Toda lattice as well as its Lax pair by introducing time flows. Matrix integral solutions to the C-Toda lattice are extended to give solutions to the CKP hierarchy. It is remarkable that the time-dependent partition function of the Cauchy two-matrix model is nothing but the $tau-$function of the CKP hierarchy. As a consequence, the correlation between the Cauchy two-matrix model and the CKP hierarchy is established by virtue of orthogonal polynomial theory and Toda-type equations. Moreover, the connection between the Cauchy two-matrix model and Bures ensemble is built from the viewpoint of integrable systems.

The Extended Equation of Long Waves: Integrability and the Related Systems

Qing Ping Liu
China University of Mining and Technology, Peoples Rep of China

An extension of the equation of long waves was proposed by Boris Kupershmidt. We will construct a Lax representation for this system and show that it is related to a system associated with the energy-dependent Schrödinger problem of Antonowicz and Fordy. Possible modifications and dual systems are also worked out.

DKP Solitons and Networks

Kenichi Maruno
Waseda University, Japan
Ken-ichi Maruno, Shinya Kido, Yuta Tanaka, Yasuyuki Watanabe, Saburo Kakei

The theory of classification of soliton interactions of the KP equation has been developed recently by Kodama and his collaborators. One of the important tools for the KP solitons is networks. In contrast to the classification of KP solitons, the classification of soliton interactions of the DKP equation (coupled KP equation) is still far from the goal. We need to develop tools for the DKP solitons. In this talk, we develop networks for the DKP solitons.

Rational Solutions of the Painlevé-III Equation

Peter Miller
University of Michigan, USA
Thomas Bothner, Yue Sheng

Most solutions of the famous Painlevé differential equations are highly transcendental, yet all but the Painlevé-I equation admit particular solutions that are elementary rational functions. These particular solutions are important in diverse applications, including the description of equilibrium patterns of fluid vortices, universal phenomena in nonlinear wave theory, electrochemistry, and string theory. The generic Painlevé-III equation has two free parameters and admits a rational solution whenever at least one of them is an integer. This talk will describe recent asymptotic results on the properties of rational solutions for large integer parameter.

Symmetries and Conservation Laws of Semi-Discrete Equations

Linyu Peng
Waseda University, Japan

Symmetry method provides significant insights for finding exact analytic solutions of differential equations; their extensions to difference as well as semi-discrete equations have been greatly investigated during last decades. In this talk, we are mainly focused on general prolongation formulae for continuous symmetries of semi-discrete equations, which is essential for symmetry analysis but has been unclear for quite some time. These formulae will allow us to establish the semi-discrete version of Noether's theorem connecting variational symmetries and conservation laws.

Stability of Peakons and Wave Breaking for the Higher-Order Modified Camassa-Holm Equations

Changzheng Qu
Ningbo Univeristy, Peoples Rep of China
Fu Ying, Liu Xiaochuan

In this talk, we study a higher-order modified Camassa-Holm equation, which can be viewed as a natural generalization of the modified Camassa-Holm equation. It has been known that the modified Camassa-Holm equation enjoys several nontrivial properties, of different character than those of the Camassa-Holm equation. Our primary goal is to understand how higher-order nonlinearities affect the dispersive dynamics. First, we investigate invariant properties and conservation laws of the equation, and establish the local well-posedness of the Cauchy problem in Besov and Sobolve spaces. Second, we study the orbital stability of peaked traveling wave solutions of the equation. Finally, We study the formation of singularities and provide sufficient conditions on initial data that lead to the finite time blow-up of the second-order derivative of the solution.

Completion of Integrable Couplings and Bi-Hamiltonian Structures

Shoufeng Shen
Zhejiang University of Technology, Peoples Rep of China

Integrable couplings are associated with non-semisimple Lie algebras. We propose a new method to generate new integrable systems through making perturbation in matrix spectral problems for integrable couplings, which is called the "completion process of integrable couplings". The idea of construction is applied to the AKNS, KN, WKI and Volterra integrable coupling. Each equation in the resulting hierarchies has a bi-Hamiltonian structure furnished by the component-trace identity.

Stability of Relativistic Solitons

Yusuke Shimabukuro
Academica Sinica, Taiwan

Quantum mechanics and special relativity were brought together after the discovery of the Dirac equation. The massive Thirring model (MTM), a family of the nonlinear Dirac equation, is known to be integrable. Our discussion will focus on applications of integrable techniques to stability of soliton, well-posedness, etc, of the MTM. We point out difficulty of soliton stability analysis rising in the nonlinear Dirac equation, which can be resolved in the case of the MTM. In the end, we plan to give new developments.

Nonlocal Problems for Linear Evolution Equations

Dave Smith
Yale-NUS College, Singapore

Linear evolution equations, such as the heat equation, are commonly studied on finite spatial domains via initial-boundary value problems. In place of the boundary conditions, we consider “multipoint conditions“, where one specifies some linear combination of the solution and its derivative evaluated at internal points of the spatial domain, and “nonlocal” specification of the integral over space of the solution against some continuous weight.

From Davey-Stewatson to Electric Impedance Tomography

Nikola Stoilov
Université te Bourgogne, France
Christian Klein

Electric Impedance Tomography (EIT) is a method that allows us to reconstruct the inner body by measuring a patient’s response to electric potential applied to the skin. Mathematically this is a Dirichlet to Neumann problem which can be recast as a d-bar problem. Remarkably, the same d-bar problem appears in the scattering treatment of members of the Davey-Stewartson (DS) family of equations. One of the best-known models of multidimensional integrable systems, DS appears as a two-dimensional generalisation of the infamous nonlinear Schrödinger equation. It finds various applications in physics, where in general it models the interaction of long and short waves. In this paper, we present analytical results allowing numerical treatment based on spectral methods for both DS and EIT problems and discuss implementation on graphical processing units (GPUs). Our findings offer the possibility of significant improvements in the speed, cost, and accuracy of medical imaging done with EIT, with further applications in a wide range of areas in physics.

Initial Value Problem for Max Equations

Daisuke Takahashi
Waseda University, Japan

Max equation is a piecewise linear type of difference equation constructed from operators “max”, “min”, “+” and “-”. Such equations are studied intensively in recent years after the discovery of ultradiscrete soliton equations, for example, ultradiscrete Lotka-Volterra equation (= box and ball system), ultradiscrete Burgers equation (= elementary cellular automaton of rule number 184), and so on. We discuss the initial value problem for various integrable or non-integrable max equations. Some has conserved quantity or Lyapunov function and asymptotic behavior of solutions can be obtained from those quantities. We can show exact explicit solutions to some equations using formulas on max operators and classify them by their complexity. We review these results for 1+1D max equations including filter type and higher order type.

Hurwitz Numbers and Integrable Hierarchy of Volterra Type

Kanehisa Takasaki
Kindai University, Japan

The Hurwitz numbers count the topological types of finite branched coverings of a compact Riemann surface. The Hurwitz numbers of the Riemann sphere $\mathbf{CP}^1$ can be assembled in a generating function $\tau(t_1, t_2, \ldots)$ that is known to be a tau function of the KP hierarchy. This tau function can be extended to a tau function $\tau(s, t_1, t_2, \ldots)$, $s \in \mathbf{Z}$, of the modified (or discrete) KP hierarchy, in which s plays the role of a lattice coordinate.
The Lax equations of the modified KP hierarchy, like the Toda hierarchy, are formulated in the language of difference operators as

$$\frac{\partial L}{\partial t_k} = [B_k, L], \quad k = 1, 2, \ldots,$$

where L is a pseudo-difference operator of the form $L = e^{\partial_s} + u_1 + u_2 e^{-\partial_s} + \cdots$, and B_k is the part $(L^k)_{\geq 0}$ of non-negative powers of e^{∂_s} in L^k.
Actually, the variable s in the Hurwitz tau function may be thought of as a continuous variable.
The formal expression e^{∂_s} of the shift operator thereby acquires a literal meaning as the exponential of $\partial_s = \partial/\partial s$. In this interpretation, the logarithm $\log L$ of L turns out to take the special form

$$\log L = \partial_s - u e^{-\partial_s}, \quad u = u(s, t_1, t_2, \ldots),$$

and satisfy the Lax equations of the same form as L. This subsystem of the modified KP hierarchy is a large-N limit of the N-step hungry Volterra (aka Bogoyavlensky-Itoh) hierarchy realized on the fractional lattice $N^{-1}\mathbf{Z} \subset \mathbf{R}$, hence may be called a *continuous* Volterra hierarchy.

Generalized Volterra Lattices: Binary Darboux Transformations and Self-Consistent Sources

Kouichi Toda
Toyama Prefectural University, Japan
Folkert Mueller-Hoissen, Oleksandr Chvartatskyi

We study two families of (matrix versions of) generalized Volterra (or Bogoyavlensky) lattice equations. For each family, the equations arise as reductions of a partial differential-difference equation in one continuous and two discrete variables, which is a realization

of a general integrable equation in bidifferential calculus. This allows to derive a binary Darboux transformation and also self-consistent source extensions via general results of bidifferential calculus. Exact solutions are constructed from the simplest seed solutions.

Co-Primeness Preserving Extensions of Discrete Integrable Equations

Tetsuji Tokihiro
University of Tokyo, Japan

We introduce a class of recursions defined over the d-dimensional integer lattice. The discrete equations we study are interpreted as higher dimensional extensions to the discrete Toda lattice equation and the discrete KdV equation. These equations have polynomial forms which can be regarded as extensions of the bilinear forms of the integrable equations, and their τ function analogues show the Laurent phenomena.
We shall prove that the equations satisfy the coprimeness property, which is one of integrability detectors analogous to the singularity confinement test. In spite of the fact that the degree of their iterates grows exponentially, they exhibit pseudo-integrable nature in terms of the coprimeness property. We also prove that the Toda-like equations can be expressed as mutations of a seed in the sense of the Laurent phenomenon algebra.

Quantum Walks on Graphs and Spin Networks

Satoshi Tsujimoto
Kyoto University, Japan
Hiroshi Miki, Luc Vinet

Two-dimensional spin lattices exhibiting perfect state transfer and fractional revival are obtained from projection of quantum walks on graphs generalizing the hypercube and belonging to the ordered Hamming scheme. Single excitation eigenfunctions are given in terms of many-variables Krawtchouk polynomials.

Mixed Soliton Solutions of the Nonlocal Nonlinear Schrödinger Equation

Tao Xu
China University of Petroleum-Beijing, Peoples Rep of China
Sha Lan, Min Li

In 2013, Ablowitz and Musslimani first proposed an integrable nonlocal nonlinear Schrödinger (NNLS) model. By using the two-fold Darboux transformation, we obtain two families of mixed exponential and rational soliton solutions for the NNLS model. It is revealed that the first family of solutions can display a large variety of interactions among two exponential solitons and two rational solitons, in which the elastic soliton interaction properties are preserved and each soliton could be either the dark or anti-dark type. By developing the asymptotic analysis technique, we also find that the second family of solutions can exhibit the elastic interactions among four mixed solitons. But in sharp contrast to the common solitons, the mixed solitons have the t-dependent velocities and the phase shift of the interacting solitons before and after interaction grows with $|t|$ in the logarithmical manner. In addition, we discuss the degenerate cases when the four-soliton interaction reduces to a three-soliton or two-soliton interaction.

Asymptotics for the Focusing Integrable Discrete Nonlinear Schroedinger Equation

Hideshi Yamane
Kwansei Gakuin University, Japan

The focusing integrable discrete nonlinear Schroedinger equation can be solved by inverse scattering. In the reflectionless case, it admits a (multi-)soliton solution under generic assumptions. Phase shifts are determined by the eigenvalues. We consider what happens if the reflection coefficient does not vanish identically. The soliton resolution conjecture is valid. Namely, the solution is asymptotically a sum of 1-solitons. If $|n/t|$ is less than 2, the phase shifts depend on the eigenvalues and the reflection coefficient. If $|n/t|$ is not less than 2, they are independent of the reflection coefficient. Details can be found in arXiv:1512.01760 [math-ph].

General N-Dark Soliton Solutions of the Multi-Component Maccari System

Yi Zhang
Zhejiang Normal University, Peoples Rep of China

In this talk, we will consider the general dark-dark N-soliton solutions of the multi-component Maccari system by virtue of the KP hierarchy reduction technique. The dynamics of single and two solitons are discussed in detail. Asymptotic analysis shows that two solitons undergo elastic collision accompanied by a position shift.

On a Coupled Focusing-Defocusing Complex Short Pulse Equation

Zuonong Zhu
Shanghai Jiao Tong University, Peoples Rep of China

Nonlinear Schrödinger (NLS) equation, short pulse (SP) equation and complex short pulse (CSP) equation have important applications in nonlinear optics. They can be derived from the Maxwell equation. In this talk, we will address some topics on the CSP equation. We will report our results for a coupled focusing-defocusing CSP equation. The bright-

bright, bright-dark and dark-dark soliton, breather, and rogue wave solutions of the coupled focusing-defocusing CSP equation will be discussed. This talk is based on the joint work with B.F. Feng, L.M. Ling and J. Yang.

Special Session 50: Recent Advances of Differential Equations with Applications in Life Sciences

Ping Liu, Harbin Normal University, Peoples Rep of China
Ying Su, Harbin Institute of Technology, Peoples Rep of China
Fengqi Yi, Harbin Engineering University, Peoples Rep of China

Differential equations have been playing important roles in explaining the rich phenomena arising from life sciences. This special session is to concentrate on the recent advances of differential equations of various types (with or without delay) with applications in life sciences. We will invite researchers in this field from around the world to Taipei for the purpose of providing an excellent forum to exchange ideas, create new research collaborations, and rekindle old connections. Speakers and talks are carefully selected to make the session attractive to a diverse audience.

Rich Dynamics in a Delayed HTLV-I Infection Model: Stability Switch, Multiple Stable Cycles, and Torus

Yuming Chen
Wilfrid Laurier University, Canada

In this joint work with Xuejun Pan and Professor Hongying Shu, we investigate the impact of time delay in CTL immune response on an HTLV-I infection model. We use the basic reproduction number for viral infection R_0 and the basic reproduction number for CTL response R_{CTL} to characterize the model dynamics. In particular, we obtain the global dynamics if $R_0 \leq 1$ or $R_{CTL} \leq 11$. However, the model dynamics become much rich when $R_{CTL} > 1$. In this case, we use the time delay as a bifurcation parameter to obtain stability switch result on the positive equilibrium and global bifurcation diagrams for the model system. We also conduct higher-order normal form analysis and apply center manifold theory to classify the rich model dynamics near the double Hopf bifurcation points. Our analysis indicates that time delay in CTL immune response can induce not only Hopf bifurcation and double Hopf bifurcation, but also quasi-periodic orbits (torus) and coexistence of multiple stable periodic solutions.

Dynamics and Asymptotic Profiles of Steady States of an Epidemic Model in Advective Environments

Renhao Cui
Harbin Normal University, Peoples Rep of China
King-Yeung Lam, Yuan Lou

We study the dynamics of a SIS epidemic model of reaction-diffusion-advection type. The persistence of infected and susceptible populations and the global stability of the disease free equilibrium are established when the basic reproduction number is greater than or less than or equal to one, respectively. We further consider the effects of diffusion and advection on asymptotic profiles of endemic equilibrium: When the advection rate is relatively large comparing to the diffusion rates of both populations, then two populations persist and concentrate at the downstream end. As the diffusion rate of the susceptible population tends to zero, the density of the infected population decays exponentially for positive advection rate but linearly when there is no advection. Our results suggest that advection can help speed up the elimination of disease.

Modelling the Mitigation Strategies for Dengue Virus Transmission Using Wolbachia

Zhaobing Fan
Harbin Engineering University, Peoples Rep of China

Dengue is a viral infectious disease, mainly transmitted by Aedes aegypti and Aedes albopictus. It causes high morbidity and mortality in tropical and subtropical regions. Wolbachia is considered as a biological weapon against dengue virus because mosquitoes infected with Wolbachia is less capable of transmitting dengue virus. In this talk, I present modelling the mitigation strategies of dengue virus transmission using Wolbachia. I analyzed the stability of the disease free equilibrium and endemic equilibrium. Sensitivity analysis determined how sensitive the basic reproduction number, and the population at disease free equilibrium and endemic equilibrium respond to the change of the parameters. Numerical simulations compared different release strategies. Our model and analysis provided guidance for releasing Wolbachia-infected mosquitoes.

The Survey on Multiscale Inversion of Wave Equations in Porous Medium

Feng Guofeng
Harbin Engineering University, Peoples Rep of China
Xu Yingying

The paper focuses on the mathematical model of wave equations in porous media and the discretized model by the Biot theory. Due to the researched question being ill-posed and high in computation cost, the paper introduce multiscale framework with regularization and make an inversion algorithm. The numerical experiment results show that the algorithm can conveniently and effectively solve the two-dimensional, nonlinear ill-posed inversion of wave equations in porous media.

Stochastic Spatiotemporal Diffusive Predator-Prey Systems

Guanqi Liu
Harbin Normal University, Peoples Rep of China
Yuwen Wang

In this talk, we discuss the existence of a global solution of a spatiotemporal diffusive predator-prey system with Holling type-III and its long time behaviour. At first, we prove that the system satisfies the condition of existence theorem for the local solution, and then, by the method of constructor a Lyapunov function, we show that the local solution must be a global solution. Finally, we obtain some conditions under which the system is extinction and stability in mean square.

Acute Perturbation Bounds of Weighted Moore-Penrose Inverse

Haifeng Ma
Harbin Normal University, Peoples Rep of China

It is well known that the upper bounds of the weighted Moore-Penrose inverse

$$\|\bar{A}^{\dagger}_{M,N}\|_{N,M} \leq \frac{\|A^{\dagger}_{M,N}\|_{N,M}}{1 - \|A^{\dagger}_{M,N}\|_{N,M} \cdot \|\Delta A\|_{M,N}}, \quad \bar{A} = A + \Delta A$$

play a fundamental role in the perturbation analysis for the weighted linear least squares problem.

Convexity Estimates for the Solutions of Elliptic Partial Differential Equations

Shujun Shi
Harbin Normal University, Peoples Rep of China

In this talk, we are concerned on the convexity estimates for solutions of some elliptic problems obtained in the pass few years. By the classical maximum principle, we give the convexity estimates for the solution to the torsion problem, for the first eigenfunction and the Green's function of the Laplace operator in a convex bounded domain. In addition, we also give some results about the curvature estimates for the level sets of solutions to Monge-Ampére equation in $\mathbb{R}^n$ or space form.

Dynamical Behaviors of the Diffusive SIR Model Describing the Rabies Epidemic Disease: Shadow System Approach

Yuxin Zhang
Harbin Engineering University, Peoples Rep of China
Fengqi Yi

A reaction-diffusion SIR model describing the overall population dynamics of the fox rabies is considered. The model was presented by R.M. Anderson et al. to explain the epidemiological patterns observed in Europe, including 3 to 5 year cycle in fox populations infected with rabies. We study the dynamics of both the original system and its shadow system, a limiting system of the original SIR model in which the diffusion rate of the rabid foxes tends to infinity. In addition, we show the convergence property between solutions of the original system and its shadow system. Moreover, the basic reproduction number of the shadow system is defined and shown to give a sharp threshold that determines whether or not the disease dies out.

Existence of Ground States for Ambrosetti Type Linearly Coupled Schrödinger System

Jing Zhang
Harbin Normal University, Peoples Rep of China

We consider Ambrosetti type linearly coupled Schrödinger equations with critical exponent and subcritical exponent. For different ranges of the parameter and the exponent, we obtain the existence and nonexistence of ground state solutions via some quite different methods, which all are radially symmetric. It turns out that the least energy level depends heavily on the relations among parameters which make the study via variational methods rather complicated.

Special Session 51: Recent Developments in Conservation Laws and Related Topics

Dongjuan Niu, Capital Normal University, Peoples Rep of China
Ronghua Pan, Georgia Institute of Technology, USA
Dehua Wang, University of Pittsburgh, USA

This session brings together experts in the area of nonlinear partial differential equations arising in compressible and incompressible flows, magnetohydrodynamics, kinetic theory, and related fields to exchange new results and ideas, to explore new research directions and topics in analysis and applications, and to foster new collaborations and connections.

Kinetic and Hydrodynamic Descriptions for the Collective Dynamics of Many-Body Systems

Seung-Yeal Ha
Seoul National University, Korea

Self-organization of complex many-body systems has received lots of attention in scientific disciplines such as applied mathematics, biology, control theory of multi-agent system, statistical physics due to many recent applications in cooperative robot system, unmanned aerial vehicles such as drones and sensor networks etc. In this talk, we will discuss several kinetic and hydrodynamic equations arising from the study of classical and quantum many-body systems and present their well-posedness and emergent dynamics.

MHD Equations with Partial Viscosity

Quansen Jiu
Capital Normal University, Peoples Rep of China

In this talk, we will present some recent progresses on global well-posedness of MHD equations with partial viscosity. In particular, we will focus on the following two cases: 2D MHD equations with magnetic viscosity only and 3D axisymmetric MHD with vertical velocity viscosity only.

Smooth Solutions of the Euler Equations and Blowup Phenomena

Tianhong Li
Chinese Academy of Mathematics and System Science, Peoples Rep of China

For the problem of spherically symmetric solutions to the multi-dimensional Euler equations, $x = 0$ is a singular point for the equations. It is unknown that whether solutions can blow up. In this talk, we study smooth solutions and blowup phenomena.

Euler-Poisson Equations of Semiconductor Model with Sonic Boundary

Ming Mei
Champlain College & McGill University, Canada
Jingyu Li, Ming Mei, Guojing Zhang, Kaijun Zhang

In this talk, we study the well-posedness/ill-posedness and regularity of stationary solutions to the hydrodynamic model of semiconductors represented by Euler-Poisson equations with sonic boundary. When the doping profile is subsonic, we prove that, the steady-state equations with sonic boundary possess a unique interior subsonic solution, and at least one interior supersonic solution, and if the relaxation time is large and the doping profile is a small perturbation of constant, then the equations admit infinitely many transonic shock solutions, while, if the relaxation time is small enough and the doping profile is a subsonic constant, then the equations admits infinitely many C1 smooth transonic solutions, and no transonic shock solution exists. When the doping profile is supersonic, we show that the system does not hold any subsonic solution, furthermore, the system doesn't admit any supersonic solution or any transonic solution if such a supersonic doping profile is small or the relaxation time is small, but it has at least one supersonic solution and infinitely many transonic solutions if the supersonic doping profile is close to the sonic line and the relaxation time is large. The interior subsonic/supersonic solutions all are global C12 Hölder-continuous, and the exponent 12 is optimal. The non-existence of any type solutions in the case of small doping profile or small relaxation time indicates that the semiconductor effect for the system is remarkable and cannot be ignored. The proof for the existence of subsonic/supersonic solutions is the technical compactness analysis combining the energy method and the phase-plane analysis, while the approach for the existence of multiple transonic solutions is artfully constructed. The results obtained significantly improve and develop the existing studies.

Some Results on Incompressible Flows with Helical Symmetry

Dongjuan Niu
Capital Normal University, Peoples Rep of China

In this talk, I will present some new results about the well-posedness and vanishing viscosity limit of three-dimensional Euler equations with helical symmetry.

Stability of Planar Rarefaction Wave to the Multi-Dimensional Viscous Conservation Laws

Yi Wang
Chinese Academy of Sciences, Peoples Rep of China

The talk is mainly concerned with the time-asymptotic stability and vanishing viscosity limit of the planar rarefaction wave to the 2D/3D viscous conservation laws, e. g., compressible Navier-Stokes equations, Boltzmann equation and related physical models.

On Steady Euler Flows with Large Vorticity and Characteristic Discontinuities

Tianyi Wang
Wuhan University of Technology, Peoples Rep of China
G.-Q. Chan, F.-M. Huang, W. Xiang

In this talk, we want to present the existence and uniqueness of smooth solutions with large vorticity and weak solutions with vortex sheets/entropy waves for the steady Euler equations for both compressible and incompressible fluids in arbitrary infinitely long nozzles. A new approach is introduced for the existence of smooth solutions without assumptions on the sign of the second derivatives of the horizontal velocity, or the Bernoulli and entropy functions, at the inlet for the smooth case. Then the existence for the smooth case can be applied to construct approximate solutions to establish the existence of weak solutions with vortex sheets/entropy waves by the compensated compactness argument. This is the first result on the global existence of solutions of the multidimensional steady compressible full Euler equations with free boundaries, which are not necessarily small perturbations of piecewise constant background solutions. The subsonic-sonic limit of the solutions is also shown. Finally, through the incompressible limit, the existence and uniqueness of incompressible Euler flows is established in arbitrary infinitely long nozzles for both the smooth solutions with large vorticity and the weak solution with vortex sheets. This is the joint work with Gui-Qiang G. Chen, Fei-Min Huang, and Wei Xiang.

Uniqueness for Shock Reflection Problem

Wei Xiang
City University of Hong Kong, Hong Kong
Gui-Qiang Chen, Mikhail Feldman

We discuss the uniqueness of shock reflection problem governed by the potential flow equation in a natural class of self-similar solutions. The approach is based on a version of method of continuity. A property of solutions important for the proof of uniqueness is the convexity of the free boundary. So we will also discuss our recent result on the convexity. This talk is based on joint work with G.-Q. Chen and Mikhail Feldman.

Global Solutions to the Boltzmann Equation in an Infinitely Expanding Ball

Wenbin Zhao
City University of Hong Kong, Hong Kong
Huicheng Yin

In this paper, we are concerned with the global existence and large time behavior of mild solutions to the Boltzmann equation in an infinitely expanding ball. The gas in the expanding ball becomes rarefied and eventually tends to a vacuum state. A constructive L^2 decay is derived for the pure specular-reflection boundary condition.

Special Session 52: Recent Progress in Nonlinear Dispersive PDE

Benjamin Dodson, Johns Hopkins University, USA
Jason Murphy, Missouri University of Science and Technology, USA

The purpose of this session is to report on recent progress on nonlinear dispersive equations, including wave equations, Schrödinger equations, geometric dispersive equations, etc. A wide range of topics are welcome, e.g. scattering theory, blowup analysis, analysis of solitons, and so on.

Nonlinear Schrödinger Equations on Cylinders Or with Partial Harmonic Potentials

Xing Cheng
Monash University, Australia
Zihua Guo

In this talk, I will show the scattering of the quintic defocusing nonlinear Schrödinger equations on cylinders $R \times T$. By establishing an abstract linear profile decomposition, we can then reduce the scattering to the scattering of an quintic resonant Schrödinger system. By using the argument developed by B. Dodson in the mass-critical nonlinear Schrödinger equation, we can prove the scattering of the Schrödinger system. Meantime, we also deal with scattering of the quintic nonlinear Schrödinger equations with partial harmonic potentials by using the arguments developed in the scattering of the nonlinear Schrödinger equations on cylinders.

Global Well-Posedness and Scattering for the Cubic Wave Equation

Benjamin Dodson
Johns Hopkins, USA

In this paper we discuss a recent proof of scattering for the defocusing, cubic wave equation with radial data lying in a critical Besov space.

Scattering of the Defocusing Generalized Benjamin-Ono Equations

Soonsik Kwon
Korea Advanced Institute of Science and Technology, Korea
Kihyun Kim, Soonsik Kwon

I present my recent work with Kihyun Kim on a scattering problem of the defocusing generalized Benjamin-Ono equations. Due to a dispersion relation, linear Benjamin-Ono flow travels to the right direction and the higher frequency waves travel faster. This gives rise to a monotonicity formula for linear flow and is expected to hold true for the defocusing generalized Benjamin-Ono flow. We prove the monotonicity. Namely, the center of energy moves faster than the center of mass. This type of monotonicity was first observed by Tao in the defocusing gKdV equations. Later, Dodson used it to prove the scattering of the defocusing mass-critical gKdV equations. Similarly, for gBO equations, we use the monotonicity in the setting of concentration-compactness argument to prove the large data scattering in the energy space $H^{1/2}$.

Asymptotic Behavior of Solutions to Nonlinear Schrödinger Equation with a Critical Homogeneous Nonlinearity

Satoshi Masaki
Osaka University, Japan
Hayato Miyazaki, Kota Uriya

In this talk, we consider nonlinear Schrödinger equation with a general homogeneous nonlinearity of the critical order. Here, the critical means that the order is the borderline of the short range case and the long range case. The aim here is to investigate large time behavior of a solution. It was known for polynomial type nonlinearities that the behavior depends on the shape of the nonlinearity. We introduce new technique to treat non-polynomial nonlinearities.

Energy Subcritical Nonlinear Wave Equations

Dana Mendelson
University of Chicago, USA
B. Dodson, A. Lawrie and J. Murphy

In this talk we'll describe joint work with B. Dodson, A. Lawrie and J. Murphy on the energy subcritical radial cubic wave equation. We prove that all solutions scatter as long as the critical norm of the evolution stays bounded using techniques inspired by the work of Kenig and Merle and Duyckaerts, Kenig, and Merle. We'll focus on the new methods we introduced to treat the energy subcritical case in the non-radial setting.

Conditional Scattering for Nonlinear Dispersive PDE

Jason Murphy
Missouri S&T, USA

In this talk, we will discuss recent results on conditional scattering for nonlinear dispersive PDE (e.g. nonlinear Schrödinger and nonlinear wave equations).

Homogenization for the Cubic Nonlinear Schrödinger Equation on $\mathbb{R}^2$

Maria Ntekoume
UCLA, USA

In this talk we consider the defocusing inhomogeneous cubic nonlinear Schrödinger equation on $\mathbb{R}^2$

$$iu_t + \Delta u = g(nx)|u|^2 u$$

for initial data in $L^2(\mathbb{R}^2)$. We obtain sufficient conditions to ensure existence and uniqueness of global solutions for n sufficiently large, as well as homogenization.

Long-Term Behavior of Solutions in Generalized Hartree Equations

Svetlana Roudenko
George Washington University, USA
Anudeep Kumar Arora

We consider a generalization of the focusing Hartree equation, a nonlinear Schrödinger-type equation with convolution nonlinearity, and discuss basic properties of solutions in the intercritical regime. In particular, similar to the standard nonlinear Schrödinger equation, we address the well-posedness questions, dichotomy for globally existing in time and scattering solutions vs finite time blow-up. To obtain scattering we employ methods of concentration compactness of Kenig-Merle and a recently developed one by Dodson-Murphy.

Modified Scattering for the 1d Cubic Nonlinear Schrödinger Equation with a Repulsive Delta Potential

Jun-Ichi Segata
Tohoku University, Japan
Satoshi Masaki, Jason Murphy

We consider the initial value problem for the cubic nonlinear Schrödinger equation (NLS) with a repulsive delta potential in one space dimension. Our goal is to describe the long-time decay and asymptotics of small solutions to (NLS). From the linear scattering theory, we expect that (NLS) will not scatter to the solution to the linear equation. We prove that if the initial data is sufficiently small in a weighted Sobolev space, then there exists a unique global solution to (NLS) that decays in L^∞ and exhibits modified scattering.

Sharp Local Smoothing Estimates for Fourier Integral Operators

Christopher Sogge
Johns Hopkins University, USA
David Beltram, Jonathan Hickman

We prove sharp $L^p(dx) \to L^p(dtdx)$ local smoothing estimates for Fourier integral operators using the decoupling estimates of Bourgain and Demeter for the range $p \geq 2(n+1)/(n-1)$. These include bounds for solutions of wave equations on manifolds. We also show that for general FIOs satisfying the cinematic curvature assumption in our theorem, our results cannot be improved.

Equivariant Schrödinger Maps from Two Dimensional Hyperbolic Space

Lifeng Zhao
University of Science and Technology of China, Peoples Rep of China
Jiaxi Huang

We consider the equivariant Schrödinger map from $\mathbb{H}^2$ to $\mathbb{S}^2$ which converges to the north pole of $\mathbb{S}^2$ at the origin and spatial infinity of the hyperbolic space. If the energy of the data is less than 4π, we show that the local existence of Schrödinger map. Furthermore, if the energy of the data sufficiently small, we prove the solutions are global in time. This is based on joint work with Jiaxi Huang.

Strichartz Estimate for Schrödinger and Wave Equations on Metric Cones

Jiqiang Zheng
Université de Nice, France
Junyong Zhang

Consider the metric cone $X = C(Y) = (0,\infty)_r \times Y$ with the metric $g = dr^2 + r^2 h$ where the cross section Y is a compact $(n-1)$-dimensional Riemannian manifold (Y,h). Let Δ_g be the Friedrich extension positive Laplacian on X and let Δ_h be the positive Laplacian on Y, and consider the operator $\mathcal{L}_V = \Delta_g + V_0 r^{-2}$ where $V_0 \in C^\infty(Y)$ such that $\Delta_h + V_0 + (n-2)^2/4$ is a strictly positive operator on $L^2(Y)$. We prove the global-in-time Strichartz estimates without loss for the Schrödinger and wave equation associated with the operator $\mathcal{L}_V$. This work is jointed with Junyong Zhang.

Special Session 53: The Movement of Infectious Disease

Chengjun Sun, Kunming University of Science and Technology, Peoples Rep of China
Jane Heffernan, York University, Canada
Yongli Song, Hangzhou Normal University, Peoples Rep of China

Infectious diseases are mobile creatures. Due to migration and transportation, infectious diseases can spread over very large distances in relatively short periods of time. As a consequence, this also allows for more contact between humans and animals globally, also allowing for greater chances in the evolution of more virulent strains. This special session aims to address recent advances of differential equations and dynamical systems with applications in the areas of spatio-temporal spread of epidemics, pandemic influenza, vector-borne diseases, immuno-epidemiology, and other problems in epidemiology. We will bring together researchers in these fields from around the world, to stimulate major progress in mathematical modelling, analyses, and simulations of related biological systems, and to discuss and initiate policies for the mitigation of disease movement locally and globally. The session provides a unique opportunity for researchers to present their recent results, to interact and exchange scientific ideas, and to collaborate with one another in a productive and sustained way.

Travel Frequency and Infectious Disease

Daozhou Gao
Shanghai Normal University, Peoples Rep of China
Xianyun Chen, Jifa Jiang

In this talk, we will propose a multipatch epidemic model to study how the difference in travel frequency affects disease spread. The basic reproduction number R_0 is derived which completely governs the global dynamics of the model system. Lower and upper bounds of the reproduction numbers are given and the disease can become endemic or extinct even though it dies out or persist in each isolated patch. Both analytical and numerical approaches show that the model without distinguishing the difference in travel frequency tends to underestimate the risk of infection. Numerical examples are presented to illustrate the impact of changes in modern travel on disease spread.

Ascertaining the End of Ebola Virus Disease Epidemic

Hiroshi Nishiura
Hokkaido University, Japan
Hyojung Lee

The end of an epidemic has a substantial impact on the healthcare system and control strategies as well as economics associated with travel industry. We investigate the objective determination of the end of outbreak using mathematical models. We identify the source of infection for the last cases right before the declaration of Ebola-free. To compensate the shortage of objectiveness for the declaration of Ebola-free, we employ the stochastic dependence model which can describe and the impact of sexual transmission and undiagnosed cases. The probability of observing additional cases gives the objective standards to decide the end of an epidemic considered as probability of extinction. Then, we suggest the statistical determination of the end of Ebola outbreak with significant criteria. The declaration date should be delayed as the duration of persistence of Ebola virus or proportion of undiagnosed cases increase.

Dynamics of Mosquito Populations Incorporating Pulse Release of Genetically Modified Mosquitoes

Chengjun Sun
Kunming University of Science and Technology, Peoples Rep of China
Wei Yang

In this talk, pulse ODE models for the interactions between wild and genetically modified mosquitoes are proposed. Global dynamics of the models are presented. Optimal control strategies for pulse release are provided. Numerical examples are given to illustrate the rich dynamical features of models.

Special Session 54: Application of Ordinary Differential Equations in Medicine and Biology

Beata Jackowska-Zduniak, Warsaw University of Life Sciences, Poland
Urszula Forys, Warsaw University, Poland

This session will be focused on recent developments in applied mathematics in the field of tumor biology, conductivity system pathologies, mathematical models of other diseases and eco-epidemiological models. Topics will include modelling using ordinary differential equations, as well as novel protocols of treatment.

Modeling of Heterogeneous Tumor Growth - How to Avoid ACR

Urszula Forys
University of Warsaw, Poland
Piotr Bajger, Mariusz Bodzioch

We describe the competition between senstitve and resistant cells. This is done in the framework of competition between two populations of cells - one sensitive and the other resistant to applied drug. On this simple population-based approach we would like to propose optimal treatment which should keep resistant cells unders control.

A Comparison of Deterministic Vs Stochastic Simulation Models for Early Stage Cancer Under the Influence of Therapy

Beata Jackowska-Zduniak
Warsaw University of Life Sciences, Poland
Beata Jackowska-Zduniak

Two mathematical models: deterministic and stochastic describing the growth of early stage cancer were proposed. To develop these models for tumor growth we assume that cancer tumors are heterogenous populations consisting of six sub-populations of tumor cells: stem cells, tumor stem cells, healthy cells, cancer cells, quiescent cancer cells and quiescent cancer stem cells. Also we consider a situation when the heterogenous population of cells with different growth capabilities compete for resources in order to obtain more effective proliferation. The influence of the treatments for cancer cells is studied. Selected results of an extensive numerical analysis are presented. The advantages and disadvantages of each approach are discussed.

The Artificial Hamiltonian, First Integrals, and Closed-Form Solutions of Dynamical Systems for Epidemics

Imran Naeem
Lahore University of Management Sciences, Pakistan
Rehana Naz, Imran Naeem

The non-standard Hamiltonian system, also referred to as a partial Hamiltonian system in the literature appears widely in economics, physics, mechanics, and other fields. The non-standard (partial) Hamiltonian systems arise from physical Hamiltonian structures as well as from artificial Hamiltonian structures. We introduce the term artificial Hamiltonian for the Hamiltonian of a model having no physical structure. We provide here explicitly the notion of an artificial Hamiltonian for dynamical systems of ordinary differential equations (ODEs). Also, we show that every system of second-order ODEs can be expressed as a non-standard (partial) Hamiltonian system of first-order ODEs by introducing an artificial Hamiltonian. This notion of an artificial Hamiltonian gives a new way to solve dynamical systems of first-order ODEs and systems of second-order ODEs that can be expressed as a non-standard (partial) Hamiltonian system by using the known techniques applicable to the non-standard Hamiltonian systems. We employ the proposed notion to solve dynamical systems of first-order ODEs arising in epidemics.

Modelling of Inter-Hospital Transmission of Multidrug-Resistant Pathogens

Monika Piotrowska
University of Warsaw, Poland
Konrad Sakowski

Nowadays multidrug-resistant Enterobacteriaceae (MDR-E) is a significant public health threat in many European countries. While traditional infection control strategies primarily target the containment of intra-hospital transmission, there is growing evidence highlighting the importance of inter-hospital patient traffic for the spread of MDR-E within healthcare systems. We propose a network based model, which reflects a patient traffic in healthcare system and thus will provide the framework to systematic study of transmission dynamics of MDR-E and the effectiveness of infection control strategies to contain their spread within healthcare systems. Although model dynamics is based on the network structure, the spread of bacteria in the healthcare facilities is directly modelled by systems of ordinary equations. Our study is based on real anonymized patients' hospitalization data. However, due to the a strong emphasis on the privacy of patients, the hospitalization records available for our research are therefore limited. We would like to demonstrate what are the problems and limitations with derivation of a patient transmission network from these data and how to overcome them by presenting example(s) of such networks. Next we show preliminary results concerning simulations of infection spread within the network.

Understanding Inter-Patient Heterogeneity in Acute Leukemias - Insights from Mathematical Modeling

Thomas Stiehl
Institute for Applied Mathematics, Heidelberg University, Germany
Anthony D. Ho, Christoph Lutz, Anna Marciniak-Czochra

Acute leukemias are cancerous diseases of the blood forming (hematopoietic) system. A hallmark of acute leukemias is their clinical heterogeneity. Leukemias are derived from a leukemic stem cell (LSC) population that out-competes hematopoietic stem cells (HSC) which are required for blood cell formation. Experiments suggest that differences in the interaction between healthy and malignant cells contribute to the inter-patient heterogeneity. These interactions include leukemic cell response to endogenous growth factors and competition of stem cells for spaces in a supportive niche. We develop nonlinear ordinary differential equation models to study the impact of these interactions on inter-patient heterogeneity. Combining analytical results, computer simulations and patient data analysis we provide insights into the following clinically relevant questions:

- What can we learn about leukemic cell regulation based on clinical data? How does it differ between patients? What is the clinical impact of the inter-individual differences?
- Do leukemic cells require endogenous growth factors to expand or do they grow independently of regulatory signals? Does this make a difference for the clinical course and patient survival?
- How does competition inside the stem cell niche affect disease dynamics and treatment response?
- How can we use mathematical models to assess patient prognosis?

Systems of Second Order Differential Equations with Nonlocal Nonlinear Boundary Conditions

Katarzyna Szymanska-Debowska
Lodz University of Technology, Poland
J. Mawhin, B. Przeradzki

We study the nonlocal nonlinear Neumann boundary value problem of the following form

$$x'' = f(t, x, x'), \qquad x(0) = a, \qquad x'(1) = N(x'),$$

where $a \in \mathbb{R}^n$ is fixed, $t \in [0, 1]$, $f : [0, 1] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous, $N : C([0, 1], \mathbb{R}^n) \to \mathbb{R}^n$ is a continuous and not necessarily linear application. The case of linear boundary conditions are also considered, i.e.

$$N(y) := \int_0^1 y(s)\, dg(s),$$

where $g = \operatorname{diag}(g_1, \ldots, g_n)$ with $g_i : [0, 1] \to \mathbb{R}$, $i = 1, \ldots, n$. Using different topological methods, we show that the problem under consideration has at least one solution.

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Optimal Control of Time-Fractional Keller-Segel Equations

Qing Tang
China University of Geosciences(Wuhan), Peoples Rep of China
Qing Tang

The Keller-Segel equations are a model of chemotaxis, they have been used for modelling of tissue invasion by solid tumors. We discuss the optimal control of a Keller-Segel equation with time fractional derivative(Caputo derivative). We will forlmulate the first order necessary optimality conditions and discuss existence of optimal control. This model will be used to study collective motion of cells under subdiffusion.

Special Session 55: Advances in Analysis and Geometry of Nonlinear Waves and Integrable Systems

Changzheng Qu, Ningbo University, Peoples Rep of China
Ming Chen, University of Pittsburgh, USA

This session will focus on a number of recent developments in the very active area of nonlinear waves and integrable systems. The topics include Hamiltonian structures; derivation of physical model equations; well-posedness; formation of singularities; coherent structures e.g. solitons, rogue waves, peakons, compactons, etc.; stability analysis and geometric aspects. Directions of work related to new methods and their applications to integrable systems and nonlinear PDEs will be emphasized, with the aim of bringing together a number of researchers at all career stages working in on these topics.

Wave-Breaking Equations, Generalized Peakons, and Their Properties

Stephen Anco
Brock University, Canada
Elena Recio

In the past few decades in water wave theory, there has been considerable interest in nonlinear dispersive equations that model breaking waves. The best known example is the Camassa-Holm equation, which exhibits wave-breaking behaviour for a large class of initial data and also possesses exact travelling wave solutions having a peak. These solutions, called peakons, are not classical solutions but instead are weak or distributional solutions. This talk discusses recent work on a large family of nonlinear dispersive equations that generalize the Camassa-Holm equation and possess single and multi peakon solutions. The nonlinearities in these equations can be much stronger than the nonlinearity in the Camassa-Holm equation, and this has led to finding novel behaviour for 2-peakon interactions (such as bound pairs) as well as generalized peakon solutions (exhibiting acceleration, blow-up, decay, and other behaviour).

On Inverse Spectral Problems Related to Peakon Equations

Xiangke Chang
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Peoples Rep of China

A class of nonlinear integrable PDEs, such as the celebrated Camassa-Holm (CH) equation, Degasperis-Procesi (DP) equation, admit some special weak solutions called "peakons". Their N-peakons may be explicitly constructed by analyzing the regularizations of the smooth Lax pairs in peakon sectors and solving the associated inverse boundary value problems. In this talk, I will introduce some recent results on explicit constructions of peakons for some modifications of the CH equation, such as mCH/FORQ, 2mCH/SQQ, NLSP and HP equations. This talk is based on my several works with Stephen Anco, Xing-Biao Hu and Jacek Szmigielski.

Localized Excitations and Interactional Solutions for the Reduced Maxwell-Bloch Equations

Yong Chen
East China Normal University, Peoples Rep of China
Yong Chen

Based on nonlocal symmetry method, localized excitations and interactional solutions are investigated for the reduced Maxwell-Bloch equations. The nonlocal symmetries of the reduced Maxwell-Bloch equations are obtained by the truncated Painlevé expansion approach and the Möbious invariant property. The nonlocal symmetries are localized to a prolonged system by introducing suitable auxiliary dependent variables. The extended system can be closed and a novel Lie point symmetry system is constructed. By solving the initial value problems, a new type of finite symmetry transformations is obtained to derive periodic waves, Ma breathers and breathers travelling on the background of periodic line waves. Then rich exact interactional solutions are derived between solitary waves and other waves including cnoidal waves, rational waves, Painlevé waves, and periodic waves through similarity reductions. In particular, several new types of localized excitations including rogue waves are found, which stem from the arbitrary function generated in the process of similarity reduction. By computer numerical simulation, the dynamics of these localized excitations and interactional solutions are discussed, which exhibit meaningful structures.

Mixed Soliton Solution to a Nonlocal Coupled Nonlinear Schrödinger Equation with PT-Symmetry

Baofeng Feng
University of Texas Rio Grande Valley, USA
Mark J. Ablowitz, Xudan Luo, Ziad Musslimani

In this talk, we are concerned with a nonlocal coupled nonlinear Schrödinger (NLCNLS) equation with PT-symmetry for mixed zero and nonzero boundary conditions. By combining the Hirota's bilinear method and the Kadomtsev-Petviashvili (KP) hierarchy reduction method, general mixed-type soliton solution to the NLCNLS equation with PT-symmetry is constructed. To be specific, we start with a set of bilinear equations satisfied by the tau functions of the two-component KP hierarchy with shifted singular

points, N-bright-dark soliton solution is derived from a series of reductions such as complex conjugate and PT-symmetry reductions. It is quite interesting that, quite different from the nonlocal NLS equation with PT-symmetry, a nonsingular moving bright soliton solution exists in one-component while the the other component possessing a dark soliton solution.

Toda Systems and Their Symmetries

Chuanzhong Li
Ningbo University, Peoples Rep of China
Na Wang, Chuanzhong Li

In this talk, we will introduce all kinds of Toda systems including original Toda, triangular Toda, Full-Kostant Toda, bigraded Toda, extended bigraded Toda, B and C type Toda and so on. The tau functions of these systems have a lot of interesting applications in physics. These Toda systems have some interesting symmetries and these symmetries have nice algebraic structures including Block algebra, quantum torus algebra and so on.

Exact Solutions of Few-Cycle Pulses in Nonlinear Media

Ji Lin
Zhejiang Normal University, Peoples Rep of China

We investigate analytical solutions of few-cycle pulses in nonlinear mediums. Using the CTE method and the comformal transformation method, many exact soliton solutions are obtained.

On $N = 1$ Supersymmetric Evolutionary Integrable Equations of 0-Homogeneous Type

Kai Tian
China University of Mining & Technology, Beijing, Peoples Rep of China
Binfang Gao, Q. P. Liu and Jing Ping Wang

Regarding $N = 1$ supersymmetric integrable equations, various supersymmetric generalizations of the Harry Dym equation, as well as the supersymmetric Schwarzian Korteweg-de Vries equation, fall into the 0-homogeneous category. To comprehensively understand such equations, we classify 0-homogeneous equations of the form

$$U_t = U_{nx}F(U)+G\Big(U,(+\mathcal{D}U),U_x,\cdots,U_{(n-1)x},(\mathcal{D}U_{(n-1)x})\Big),$$

where the super field $U = U(x,\theta,t)$ is bosonic, $U_{kx}(k \in \mathbb{Z}_+)$ stands for the kth order derivative with respect to x (U_{1x} is abbreviated to U_x as usual), and the super derivative $\mathcal{D}$ is defined as $\mathcal{D} = \partial_\theta + \theta\partial_x$. Eight $N = 1$ supersymmetric integrable equations of such type are figured out by the symmetry approach, and most of them are shown to be connected with known ones by introducing appropriate changes of variables.

The Riemann-Hilbert Approach to the Second-Order Flow of Three-Wave Hierarchy

Deng-Shan Wang
Beijing Information Science and Technology University, Peoples Rep of China
Xiao-Yong Wen

The Lax pair of a generalized second-order flow of three-wave Hierarchy is constructed by prolongation technique. Then the Riemann-Hilbert approach is applied to study the initial value problem and initial-boundary value problem of this flow, respectively. As a result, the soliton solutions of the initial value problem of this flow are derived explicitly and the solution of the initial-boundary value problem is expressed in terms of the solution of a matrix-value Riemann-Hilbert problem.

Modulational Instability and Higher Order-Rogue Wave Solutions for the Generalized Discrete Hirota Equation

Xiao-Yong Wen
Beijing Information Science and Technology University, Peoples Rep of China
Deng-Shan Wang

This paper investigates the modulational instability and higher order-rogue waves in the generalized discrete Hirota system. The Lax pair and conservation laws for this system are constructed. Then the existent conditions for its modulational instability to form the rogue waves are given starting from the plane wave solution. Furthermore, the higher-order discrete rogue waves of this system are reported using the novel discrete version of generalized perturbation (n,N-n)-fold Darboux transformation. Finally, the dynamical behaviors of the strong and weak interactions of these higher-order discrete rogue waves are discussed analytically and numerically, which exhibits abundant nonlinear wave structures. These results may be useful for understanding some physical phenomena in optical fibers and relevant fields.

Drinfeld–Sokolov Hierarchies of Twisted Type, Revisited

Chaozhong Wu
Sun Yat-Sen University, Peoples Rep of China
Si-Qi Liu, Youjin Zhang, Xu Zhou

It is proved by Liu, Ruan and Zhang that part of the flows in an ADE Drinfeld–Sokolov hierarchy can be reduced to a BCFG Drinfeld–Sokolov hierarchy. Here we extend this result to a more general theorem, by which a Drinfeld–Sokolov hierarchy of twisted type can also be reduced from an ADE (or other type) Drinfeld–Sokolov hierarchy. For these hierarchies, we also study their solutions characterized by some ODEs of Painleve type.

Geometric Curve Flows and KdV Type Equations Associated to Affine Kac-Moody Algebra

Zhiwei Wu
Sun Yat-Sen University, Peoples Rep of China
Chuu-Lian Terng

In this talk, we will discuss curve flows in homogeneous spaces. The local differential invariant of these flows are solutions to certain KdV-type equations associated to affine Kac-Moody algebra. The periodic Cauchy problem of these curves can be solved and we will give an algorithm to compute the Bi-Hamiltonian structures of curve flows.

The Lump Solution and the Bilinear Backlund Transformation for the (4+1)-Dimensional Fokas Equation

Ruoxia Yao
Shaanxi Normal University, Peoples Rep of China
Yali Shen, Ruoxia Yao

In this paper, we first obtain a bilinear form with small perturbation u_0 for the (4+1)-dimensional Fokas equation by an appropriate transformation. By using the Hirota bilinear operator, we construct the bilinear Bäcklund transformation which consists of three bilinear equations and involves four arbitrary parameters and then obtain a new class of lump solution including some free parameters for the (4+1)-dimensional Fokas equation based on the bilinear forms. By choosing different values of the parameters, the dynamic properties of six different cases of the lump solution are shown graphically. From the graphs, we can see some interesting nonlinear phenomena which might provide us with useful information on the dynamics of relevant fields in nonlinear science. Finally, we analyze the mathematical reasons of the lump solution by using the extreme value theory under the above six cases.

On the Modified Coupled Dispersionless Equations and the Modified Short Pulse Equation

Guofu Yu
Shanghai Jiao Tong University, Peoples Rep of China
Guofu Yu, Jialiang Ji

In this talk, we study the real and complex modified coupled dispersionless (CD) equations, the real and complex modified short pulse (SP) equations geometrically and algebraically. From the geometric point of view, we first establish the link of the motions of space curves to the real and complex modified CD equations, then to the real and complex modified SP equations via hodograph transformations. The integrability of these equations are confirmed by constructing their Lax pairs geometrically. The N-soliton solutions in the form of determinants for the modified SP equation and two-component modified SP equation are provided.

Inverse Scattering Transformation for the PT-Symmetric Gross-Pitaevskii Equations

Dun Zhao
Lanzhou University, Peoples Rep of China
Yu-Juan Zhang, Wen-Xiu Ma

We present the inverse scattering transformation for a kind of Gross-Pitaevskii equations which possess the parity-time (PT) symmetric invariance. We find that unlike the local case, the PT -symmetry of the nonlocal Gross-Pitaevskii equation allows two different choices of the symmetry relations of the eigenfunctions which guarantee two different kinds of inverse scattering solutions.

Special Session 56: Analysis of Chemotaxis Models

Johannes Lankeit, Paderborn University, Germany
Tian Xiang, Renmin University of China, Beijing, Peoples Rep of China

Chemotaxis models are systems of PDEs arising from mathematical biology where they are used to describe simple organisms that are able to direct their movement according to the concentration gradient of a chemical substance. They play a role in so different contexts as aggregation patterns in bacteria, formation of slime moulds, skin pigmentation patterns, angiogenesis in tumor progression and wound healing. The most important examples of these systems are the Keller-Segel model and relatives thereof. Mainly due to the presence of cross-diffusive terms, they pose challenging analytical questions and lead to insights on e.g. structure formation, in its most extreme form signified by blow-up of solutions in finite or infinite time. Over the last decades, they have brought forth an active area of research and a large amount of literature. This special session will focus on recent analytical results concerned with the qualitative behavior of solutions to these systems, in accordance with current trends possibly including effects of coupling to other processes, like population growth, the interplay with a surrounding fluid, haptotaxis models, or interactions of several chemo-tactically active species. Examples of qualitative properties of interest are the question of boundedness versus blow-up, large-time behavior or the emergence of patterns. An important goal of this special session is to bring together young researchers and experts working in these areas, in order to stimulate discussion, facilitate the emergence of new ideas, and inspire new research.

Global Very Weak Solutions to 3D Chemotaxis-Fluid Systems with Nonlinear Diffusion

Tobias Black
Paderborn University, Germany

We discuss global (very) weak solutions to chemotaxis-fluid systems of the form

$$\begin{cases} n_t + u\cdot\nabla n = \nabla\cdot\left(D(n)\nabla n - S(x,n,c)\nabla c\right), & x\in\Omega, \quad t>0,\\ c_t + u\cdot\nabla c = \Delta c - c + n, & x\in\Omega, \quad t>0,\\ u_t + (u\cdot\nabla)u = \Delta u + \nabla P + n\nabla\phi, & x\in\Omega, \quad t>0,\\ \nabla\cdot u = 0, & x\in\Omega, \quad t>0, \end{cases}$$

in a bounded domain $\Omega\subset\mathbb{R}^3$ with smooth boundary. The prototypical choices involve linear diffusion or porous medium type diffusion, i.e. $D(n)=mn^{m-1}$ with $m\geq 1$ and tensor-valued sensitivities $S\in C^2\left(\overline{\Omega}\times[0,\infty)^2;\mathbb{R}^{3\times 3}\right)$ satisfying $|S(x,n,c)|\leq\frac{C_S}{(n+1)^\alpha}$ with $\alpha\geq 0$ and constant $C_S>0$.

A Generalized Global Weak Solution to a 3D Chemotaxis-Navier-Stokes System with Nonlinear Diffusion and Rotatioanl Flux

Li Feng
Southeast University, Peoples Rep of China

In this paper, we consider the chemotaxis-Navier-Stokes system with nonlinear diffusion Δn^m $(m>0)$ and rotational flux given by

$$\begin{cases} n_t + u\cdot\nabla n = \Delta n^m - \nabla\cdot(uS(x,n,c)\cdot\nabla c), & x\in\Omega, \quad t>0,\\ c_t + u\cdot\nabla c = \Delta c + n - c, & x\in\Omega, \quad t>0,\\ u_t + k(u\cdot\nabla)u = \Delta u + \nabla P + n\nabla\phi, & x\in\Omega, \quad t>0\\ \nabla u_\varepsilon = 0, & x\in\Omega, \quad t>0 \end{cases}$$

under homogenous Neumann boundary conditions in a bounded domain $\Omega\subset\mathbb{R}^3$, where $k\in\mathbb{R}$, $\phi\in W^{1,\infty}(\Omega)$, the given tensor-valued function $S:\overline{\Omega}\times[0,\infty)^2\to\mathbb{R}^{3\times 3}$ satisfying

$|S(x,n,c)|\leq S_0(n+1)^{-\alpha}$ for all $x\in\mathbb{R}^3$, $n\geq 0$, $c\geq 0$.

Resorting to some yet weaker solution concepts and imposing no restrictions on the size of the initial data, we establish the global existence of a weak solution while assuming $m+\alpha>\frac{4}{3}$ and $m>\frac{1}{3}$, which includes both non-degenerate $(m>1)$ and the degenerate $(m<1)$ cases.

Global Solutions to a Higher-Dimensional Chemotaxis System Related to Crime Modelling

Marcel Freitag
Universitaet Paderborn, Germany

We are concerned with a problem that differs from the usual setting both with respect to the system as well as the modelled process.

Given a bounded domain $\Omega \subset \mathbb{R}^n$ for $n \geq 2$, a positive constant χ and two nonnegative functions B_1 and B_2 in $L^\infty(\Omega)$ (as well as Neumann boundary conditions and suitable initial data), we consider the set of coupled PDEs

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot \left\{ \frac{u}{v} \nabla v \right\} - uv + B_1 & \text{in } \Omega \times (0, \infty), \\ v_t = \Delta v - v + uv + B_2 & \text{in } \Omega \times (0, \infty) \end{cases}$$

which several works ([1],[2]) have used to explain an aspect of the decision-making process of burglars. After Rodríguez and Winkler have proven the existence and uniqueness of global solutions for $n = 1$ in [3], it is our aim to add correspondent results for higher dimensions. The crucial question herein is how large we can choose χ while retaining our ability to detect the existence of global solutions.

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New Lyapunov-Like Functional of 1D Quasilinear Parabolic Chemotaxis System and Its Application

Kentarou Fujie
Tokyo University of Science, Japan
Tomasz Cieślak

We consider the fully parabolic 1d chemotaxis system with diffusion $1/(1+u)$. We prove that the above mentioned nonlinearity, despite being a natural candidate, is not critical. It means that for such a diffusion any initial condition, independently on the magnitude of mass, generates global-in-time solution. In order to prove our theorem we establish a new Lyapunov-like functional associated to the system. Moreover we will discuss boundedness of solutions. This talk is based on a joint work with Tomasz Cieślak.

On a Parabolic-Hyperbolic Keller-Segel System

Rafael Granero Belinchon
Universidad de Cantabria, Spain

In this talk we will present some new results regarding a parabolic-hyperbolic Keller-Segel system. This system was proposed by Othmers & Stevens based on biological considerations as a model of tumor angiogenesis and also appears as a singular limit of the classical parabolic-slliptic Keller-Segel model of aggregation of the slime mold *Dictyostelium discoideum.* In particular, we will present some global existence results in certain dissipation regimes.

Some Results on Reaction-Diffusion Systems with Density-Suppressed Motility

Hai-Yang Jin
South China University of Technology, Peoples Rep of China

In this talk, I will present some recent results on reaction-diffusion systems with density-suppressed motility including the boundedness, stabilization and pattern formation.

Recent Results on Consumptive Chemotaxis Models with Singular Sensitivity

Johannes Lankeit
Paderborn University, Germany

We will review a class of chemotaxis models combining consumption of the chemoattractant with a singular sensitivity function and discuss recent results.

Global Boundedness and Decay Property in a Three-Dimensional Keller-Segel-Stokes System Modeling Coral Fertilization

Jing Li
Minzu University of China, Peoples Rep of China

In this talk we introduce the mathematical models representing the fertilization process of corals and consider the following full model

$$\begin{cases} \rho_t + u \cdot \nabla \rho = \Delta \rho - \nabla \cdot (\rho S(x, \rho, c) \nabla c) - \rho e, \\ e_t + u \cdot \nabla e = \Delta e - \rho e, \\ c_t + u \cdot \nabla c = \Delta c - c + e, \\ u_t = \Delta u - \nabla P + (\rho + e) \nabla \phi, \quad \nabla \cdot u = 0, \end{cases}$$

in a bounded domain $\Omega \in \mathbb{R}^3$ with zero-flux boundary for ρ, e, c and no-slip boundary for u. The chemotactic sensitivity S is not a scalar function but rather attains values in $\mathbb{R}^{3\times 3}$, and satisfies $|S(x, \rho, c)| \leq C_S(1+\rho)^{-\alpha}$ with some $C_S > 0$ and $\alpha > 0$. In this talk, we will show the existence and boundedness of global classical solutions for arbitrarily large initial data under the assumption $\alpha > \frac{1}{3}$. Moreover, all the solutions are shown to decay to a spatially homogeneous equilibrium exponentially as time goes to infinity.

The Fast Signal Diffusion Limit in a Keller–Segel System Under Smallness Conditions for Initial Data

Masaaki Mizukami
Tokyo University of Science, Japan

The subject of this work is to construct a new approach to a parabolic-elliptic Keller–Segel system from its parabolic-parabolic case, and to use the parabolic-parabolic case as a step to establish new results in the parabolic-elliptic case. Our aim is, by considering that the parabolic-elliptic case is as a limit of its parabolic-parabolic system, to establish a result such that, only dealing with the parabolic-parabolic Keller–Segel system is enough to obtain the new properties for solutions of its parabolic-elliptic case. In this talk we consider fast signal diffusion limit in a Keller–Segel system, which namely is convergence of a solution for the parabolic-parabolic Keller–Segel system to that for its parabolic-elliptic version.

Global Existence and Boundedness of Solutions to a Chemotaxis Model with Singular Sensitivity

Giuseppe Viglialoro
University of Cagliari, Italy
Johannes Lankeit

W study a zero-flux chemotaxis system with singular sensitivity in a smooth and bounded domain Ω of $\mathbb{R}^2$. We show the existence of global classical solutions emanating from any regular initial data $u(x, 0)$. If additionally Ω is *convex* and $m := \int_\Omega u(x, 0)$ is sufficiently small, also their boundedness is achieved.

Uniqueness and Convergence of Equilibrium of the Subcritial Keller-Segel System

Zhian Wang
Hong Kong Polytechnic University, Hong Kong
Jun Wang, Wen Yang

It is well known that the classical Keller-Segel system has blow-up solutions in two dimensions when the initial mass is greater than the critical mass 8π (supercritical case), while has globally bounded solution if the initial mass is less than the critical mass 8π (sub-critical case). However the asymptotic behavior of solutions in the sub-critical case still remains unknown. It this talk, I will report a progress made recently. We prove that in the sub-critical case, the radially symmetric solutions will converge to constant equilibrium as time tends to infinity if the domain is a disk.

Chemotactic Aggregation Vs Logistic Damping in the Minimal Keller-Segel Model

Tian Xiang
Renmin University of China, Peoples Rep of China

We study chemotaxis effect (χ) vs logistic damping (μ) on boundedness (and large time behavior) for the minimal Keller-Segel model with logistic source in 2- and 3-D smooth and bounded domains. We obtain qualitative boundedness on χ and μ : up to a scaling constant depending only on initial data and the underlying domain, we provide explicit upper bounds for the solution components of the corresponding initial-boundary value problem. These bounds are increasing in χ and decreasing in μ.
In 2-D, the corresponding upper bounds have only one singularity in μ at $\mu = 0$. In contrast, in 3-D, the upper bounds, holding under a critical explicit relation between χ and μ (which has been shown to guarantee boundedness), are defined for all χ and $\mu > \text{const} \cdot \chi$, and have two singularities in μ at $\mu = 0$ and $\mu = \text{const} \cdot \chi$.
It is worthwhile to mention that, in the absence of logistic source, the corresponding classical KS model is well-known to possess blow-ups for even small initial data. We hope that these qualitative findings presented here would produce some new principles on finite-time blow-up to chemotaxis systems with weak logistic damping sources.

Long Time Behavior of Solutions to the 2D Keller-Segel Equation with Degenerate Diffusion

Yao Yao
Georgia Institute of Technology, USA
Jose Carrillo, Sabine Hittmeir, Bruno Volzone

The Keller-Segel equation is a nonlocal PDE modeling the collective motion of cells attracted by a self-emitted chemical substance. When this equation is set up in 2D with a degenerate diffusion term, it is known that solutions exist globally in time, but their long-time behavior remains unclear. In a joint work with J. Carrillo, S. Hittmeir and B. Volzone, we prove that all stationary solutions must be radially symmetric up to a translation, and use this to show convergence towards the stationary solution as the time goes to infinity.

Boundedness in the Keller-Segel System with Signal-Dependent Sensitivity

Tomomi Yokota
Tokyo University of Science, Japan
Masaaki Mizukami

In this talk we prove boundedness of classical solutions to the fully parabolic Keller-Segel system with signal-dependent sensitivity under the smallness condition for the coefficient of the sensitivity function. The proof is based on the construction of the ordinary differential inequality for the integral of the p-th power of the first solution component multiplied by a function depending on the second solution component.

Global Dynamics in a Two-Competing-Species Chemotaxis-Fluid System with Two Chemicals

Pan Zheng
Chongqing University of Posts and Telecommunications, Peoples Rep of China

In this talk, we consider a two-competing-species chemotaxis-fluid system with two different signals under homogeneous Neumann boundary conditions in a smooth bounded domain. This system describes the evolution of two-competing species which react on two different chemical signals in a liquid surrounding environment. The cells and chemical substances are transported by a viscous incompressible fluid under the influence of a force due to the aggregation of cells. Firstly, when $N = 2$ and $\kappa = 1$, based on the standard heat-semigroup argument, it is proved that for all appropriately regular nonnegative initial data and any positive parameters, this system possesses a unique global bounded solution. Secondly, when $N = 3$ and $\kappa = 0$, by using the maximal Sobolev regularity and semigroup technique, it is proved that the system admits a unique globally bounded classical solution under some suitable conditions. Finally, by means of energy functionals and comparison arguments, it is shown that the global bounded solution of the system converges to different constant steady states, according to the different values of a_1 and a_2. Furthermore, we give the precise convergence rates of global solutions.

Special Session 57: Parabolic-Hyperbolic Coupled Partial Differential Equations

Yachun Li, Shanghai Jiao Tong University, Peoples Rep of China
Weike Wang, Shanghai Jiao Tong University, Peoples Rep of China
Xiongfeng Yang, Shanghai Jiao Tong University, Peoples Rep of China

This session is concerned with parabolic-parabolic coupled partial differential equations. It could be the coupling of parabolic domain and hyperbolic domain, like degenerated convection-diffusion equations, or the coupling of parabolic equation and hyperbolic equation in a system of PDEs, like Navier-Stokes equations. There are a lot of applications of this class of equations in physics, fluid dynamics, bio-mathematics, engineering mechanics, material sciences, financial mathematics, etc. There are very rich phenomena caused by the coupling of parabolicity and hyperbolicity, This session is aimed at bring together researchers from this field or even a wider range and make deep discussions.

Stability of Rarefaction Wave to the 1-D Piston Problem for the Compressible Euler Equations

Min Ding
Wuhan University of Technology, Peoples Rep of China
Yongqian Zhang

We study the 1-D piston problem for compressible full Euler equations under the assumptions that the total variation of the initial data and the perturbation of the piston velocity are sufficiently small. By a modified wave front tracking method, we establish the global existence of entropy solutions including a strong rarefaction wave without restriction on the strength. Moreover, we establish the asymptotic behavior of the solutions.

Some Relaxation Models in Hydrodynamics

Yuxi Hu
China University of Mining and Technology, Beijing, Peoples Rep of China
Reinhard Racke, Na Wang

The compressible Navier-Stokes equations are to describe the dynamics of compressible fluid. The system are composed of mass equation, momentum equation and energy equation. Mathematically, the system are underdetermined and one need to give constitutive equations to close the system. Newtonian law in describing the relation of stress tensor and velocity, and Fourier law in describing the heat conductivity are the two main constitutive relations in fluid dynamics. In this talk, we shall investigate some other constitutive relations, namely, Cattaneo's law for heat conductivity and Maxwell's law for the stress tensor and velocity, which can be considered as some kind of relaxation of the classical system. The importance of these two constitutive relations are demonstrated by many physicists and biologists recently, in the field such as skin burns, nano fluids , biological materials, nanoscale mechanical devices vibrating in simple fluid, etc. However, the corresponding mathematical results are still very few. We shall present some related mathematical results and give some new results.

Well-Posedness of Radiation Hydrodynamics Equations

Yachun Li
Shanghai Jiao Tong University, Peoples Rep of China
Has Li, Shuai Xi, Shengguo Zhu

In this talk I will present recent progress on viscous or inviscid radiation hydrodynamics equations for compressible fluids (Navier-Stokes-Boltzmann or Euler-Boltzman equations). The results include the local existence of classical solutions with vacuum, some blow-up results of classical solutions, and some regularity criteria.

Incompressible Limit of the Degenerate Quantum Compressible Navier-Stokes Equations

Fucai Li
Nanjing University, Peoples Rep of China

In this paper we study the incompressible limit of the degenerate quantum compressible Navier-Stokes equations in a periodic domain $\mathbb{T}^3$ and the whole space $\mathbb{R}^3$ with general initial data. In the periodic case, by applying the refined related entropy method and carrying out the detailed analysis on the oscillations of velocity, we prove rigorously that the gradient part of the weak solutions (velocity) of the degenerate quantum compressible Navier-Stokes equations converge to the strong solution of the incompressible Navier-Stokes equations. While for the whole space case, thanks to the Strichartz's estimates of linear wave equations, we can obtain the convergence of the weak solutions of the degenerate quantum compressible Navier-Stokes equations to the strong solution of the incompressible Navier-Stokes/Euler equations with a linear damping term. Moreover, the convergence rates are also given.

Isentropic Approximation

Ronghua Pan
Georgia Institute of Technology, USA

In the study of compressible flows, the isentropic model was often used to replace the more complicated full system when the entropy is near a constant. This is based on the expectation that the corresponding isentropic model is a good approximation to the full system when the entropy is sufficiently close to the constant. We will discuss the mathematical justification of isentropic approximation in Euler flows and in Navier-Stokes-Fourier flows. This is based on the joint work with Y. Chen, J. Jia, and L. Tong.

Nonlinear Stability of Large Perturbation Around Viscous Shock Wave for 2-D Scalar Viscous Conservation Law

Renkun Shi
Hohai University, Peoples Rep of China
Weike Wang

We are interested in nonlinear stability of large perturbation around the planar shock wave for scalar viscous conservation law in two dimensions. We prove that the problem always admits a global classical solution if the initial perturbation is in $L^1 \cap H^4(\mathbb{R}^2)$. The proof is carried out by classical energy estimates based on the maximal principle and the contraction property. More important, for large perturbation problems of certain types of the 2D scalar viscous conservation law, we obtain nonlinear stability of the shock profile for weak shock, and establish the L^2 decay rate $t^{-\frac{1}{4}}$ and L^∞ decay rate $t^{-\frac{1}{2}}$ of solutions toward the planar shock wave. The idea of the proof uses a technique combining the semigroup approach and the energy method to get some smallness estimates, and then to obtain the asymptotic behavior.

Global Existence of Large Solutions to Conservation Laws with Nonlocal Dissipation-Type Terms

Lijuan Wang
Shanghai University of International Business and Economics, Peoples Rep of China
Weike Wang, Xin Xu

We are concerned with the Cauchy problem of a scalar conservation law with a nonlocal dissipation term. By the Green function and nonlinear maximum principle method, global classical solution and its long-time behavior are established in present paper for arbitrary large initial data.

Large Time Behavior of Solutions to a Viscous Liquid-Gas Two-Phase Flow Model

Wenjun Wang
University of Shanghai for Science and Technology, Peoples Rep of China
Weike Wang

In this talk, the Cauchy problem of a three-dimensional compressible viscous liquid-gas two-phase flow model is considered. We obtain the global solution and L^2-L^2 decay rates (in time) with some smallness assumptions.

Green's Function and Large Time Behavior of Solution for Nonlinear Evolution Systems

Weike Wang
Shanghail Jiao Tong University, Peoples Rep of China

In this talk, we will show the pointwise estimates of solutions to initial value problems of large time behavior of nonlinear evolution equations in small perturbation cases by using the Green function method. Introducing the basic idea of Green function, this paper will exemplify under hyperbolic, parabolic and nonlinear mechanism, how Green function helps the solution to initial value problems of dissipative and hyperbolic structured nonlinear evolution equations sort out various mechanisms, find out fine structures, and figure out some amazing phenomenon with which other methods could not deliver a clear explanation.

Dirichlet Problems for Degenarate Parabolic-Hyperbolic Equations

Qin Wang
Yunnan University, Peoples Rep of China

In this talk we will discuss the well-posedness (existence and uniqueness) of entropy solutions to the anisotropic degenerate parabolic-hyperbolic equations with L^∞ initial data and nonhomogeneous Dirichlet boundary condition. We will also consider the homogeneous Dirichlet boundary problem for weakly coupled degenerate parabolic-hyperbolic system. We will use doubling variables device and regularization method to prove the uniqueness and existence result, respectively.

Stability of Stationary Solution for the Compressible Viscous Magnetohydrodynamic Equations with Large Potential Force in Bounded Domain

Xiongfeng Yang
Shanghai Jiao Tong University, Peoples Rep of China
Yeping Li

In this paper, we consider the 3-D compressible viscous magnetohydrodynamic (MHD) equations with some large potential force in bounded rigid vessel. We firstly construct the non-constant stationary solutions of the compressible viscous MHD equations under suitable constitutive assumptions. Next, a critical energy identity is established to achieve a universal stability criterion of the stationary solution. In this case, the stationary solution is exponential stable for any large external potential force. Finally, we show the well-posedness of the initial boundary value problem for the compressible viscous MHD equations with the large potential force, provided that the prescribed initial data is close to the stationary solution. It implies that the set satisfying the stability criterion is not empty.

Exact Boundary Controllability of Entropy Solutions to Hyperbolic Systems of Balance Laws

Lei Yu
Tongji University, Peoples Rep of China
Tatsien Li

Compared with the study of boundary controllability of classical solutions to quasilinear hyperbolic systems of balance laws, only few corresponding results have been obtained in the framework of entropy solutions. In this talk, I will give a survey on recent development in this field and introduce our works on the exact boundary controllability of entropy solutions to a class of hyperbolic systems of balance laws. These works are based on the well-posdedness of semi-global solutions as the limits of (ϵ, h)-approximate front tracking solutions (resp. (h, ϵ)-approximate front tracking solutions) to the forward (resp. rightward) mixed initial-boundary value problem with nonlinear boundary conditions. Some applications and possible improvement of these works will also be discussed.

Special Session 58: Geometric and Nonlinear PDEs

Frederic Robert, Université de Lorraine, France
Jerome Vetois, McGill University, Canada

Our session will bring together leading experts from different areas of nonlinear PDEs with geometric backgrounds. Important discoveries have recently emerged at the crossroads between geometry and nonlinear PDEs. This includes for instance spectacular breakthroughs on the De Giorgi problem, the formation of singularities in conformal geometry, and the analysis of nonlocal operators in Euclidean and geometric settings. These topics and more will be addressed in our session by leading experts in the field. This will be a great opportunity for mathematicians in these areas to learn about the achievements of others, foster collaboration, and enable new breakthroughs.

Uniform Regularity Results for Critical and Subcritical Surface Energies

Yann Bernard
Monash University, Australia
Tristan Riviere

We establish regularity results for critical points to energies of immersed surfaces depending on the first and the second fundamental form exclusively. These results hold for a large class of intrinsic elliptic Lagrangians which are subcritical or critical. They are derived using uniform ϵ-regularity estimates which do not degenerate as the Lagrangians approach the critical regime given by the Willmore integrand.

Radial Solutions to the Keller-Segel Equation

Jean-Baptiste Casteras
Université Libre de Bruxelles, Belgium

In this talk, we will be interested in the Keller-Segel equation

$$\begin{cases} -\Delta u + u = \lambda e^u, \ u > 0, \ \text{in } \Omega, \\ \partial u = 0, \ \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$ and $\lambda > 0$. This equation arises when looking for steady states to the Keller-Segel system which describes chemiotaxis phenomena. We will make a radial bifurcation analysis of this equation with respect to the parameter λ and describe the solutions when $\lambda \to 0^+$. Joint work with Denis Bonheure, Juraj Fóldes, Benedetta Noris and Carlos Román.

Limit of Sobolev Inequalities

Sun-Yung Alice Chang
Princeton University, USA
Fang Wang

We derive the Moser-Trudinger-Onofri inequalities on the 2-sphere and the 4-sphere as the limiting cases of the fractional power Sobolev inequalities on the same spaces, and justify our approach as the dimensional continuation argument initiated by Thomas P. Branson.

Higher Dimensional Willmore Energies and Invariants

Rod Gover
University of Auckland, New Zealand

The Willmore energy of a surface is a conformal measure of its failure to be conformally spherical. In physics the energy is variously called the bending energy, or rigid string action. In both geometric analysis and physics it has been the subject of great recent interest. Its Euler-Lagrange equation is an extremely interesting equation in conformal geometry: the energy gradient is a fundamental curvature that is a scalar-valued hypersurface analogue of the Bach tensor (of dimension 4) of intrinsic conformal geometry. We next show that that these surface conformal invariants, i.e. the Willmore energy and its gradient (the Willmore invariant), are the lowest dimensional examples in a family of similar invariants in higher dimensions. A generalising analogue of the Willmore invariant arises directly in the asymptotics associated with a singular Yamabe problem on conformally compact manifolds. It was shown by Graham that an energy giving this (as gradient with respect to variation of hypersurface embedding) arises as a so-called anomaly term in a related renormalised volume expansion. We show that this anomaly term is, in turn, the integral of a local Q-curvature quantity for hypersurfaces that generalises Branson's Q-curvature by including coupling to the (extrinsic curvature) data of the embedding. This is joint work with Andrew Waldron arXiv:1506.02723, CMP 2017 (arXiv:1603.07367), Communications Contemp. Math. 2018 (arXiv:1611.08345).

On Least-Energy Solution of Elliptic Equation Involving the Hardy-Sobolev Critical Exponent

Masato Hashizume
Ehime University, Japan

We consider an elliptic equation involving the Hardy-Sobolev critical exponent. Concerning this problem, position of the singularity on the bounded domain plays a crucial role. We study existence and nonexistence of least-energy solution and effect of the mean curvature at the singularity. In order to prove existence and nonexistence, we investigate asymptotic behavior of least-energy solution. By investigating asymptotic behavior, we can find the scale of the domain also plays an important role on existence and nonexistence in addition to the mean curvature.

Moser-Trudinger and Adams Inequalities

Sandeep Kunnath
TIFR-CAM Bangalore, India

In this talk we will discuss the Moser-Trudinger and Adams inequalities in various contexts. We will recall these inequalities in their original form and their various extensions especially to the case of infinite measure like Hyperbolic space. We will also present some of the recent results obtained in this direction.

Analysis of Conformally Invariant Problem

Paul Laurain
Université Paris 7, France
L. Lin, R. Petrides, T. Riviére

Preliminary I will expose a technique developed with T. Riviére to prove energy identities for limits of sequences of solutions of conformally invariant problem in dimension 2. I will explain in the following how with L. Lin and R. Petrides, we transpose it to open problems in the higher dimension or with free boundary and how it permits to prove the convexity of certain functional around their critical point of low energy.

Blow-Up Analysis for a Nonlinear Elliptic Equation with Critical Growth and Hardy Weight

Saikat Mazumdar
University of British Columbia, Canada

In this talk we will study the asymptotic behaviour of a sequence of solutions to a family of elliptic PDEs with Hardy weight and Sobolev critical growth. The blow-up analysis will give us conditions under which compactness holds for this sequence and this in turn will help establish the existence of solutions.
The location of the singularity, be it in the interior of the domain or on its boundary, affects the analytical properties of the equation and makes the two situations quite different. When the singularity is in the interior then a lower order perturbation suffices for high dimensions, while the curvature of the boundary plays a crucial role if the singularity is on the boundary. This is a joint work with Nassif Ghoussoub(UBC) and Frederic Robert(Université de Lorraine).

Some Regularity Results for the Equation $U_{11}U_{22} = 1$

Connor Mooney
UC Irvine, USA
Ovidiu Savin

The equation $(u_{11}u_{22})^{1/2} = 1$ in $\mathbb{R}^2$ is a simple example of a concave, fully nonlinear elliptic PDE that is not a Hessian equation. It shares interesting features with the complex Monge-Ampere equation, such as non-convexity of solutions and invariance under adding certain quadratics. We will present some regularity results for this equation and its higher-dimensional analogue, as well as open problems. This is joint work with O. Savin.

Blow-Up Over the Threshold of the Scalar Curvature in Small Dimensions

Bruno Premoselli
Université Libre de Bruxelles ULB, Belgium
P.D. Thizy

On a closed manifold (M^n, g) of dimension $n \in \{4, 5\}$ we construct new blow-up configurations for perturbations of a purely critical stationary Schrödinger equation. We construct positive solutions which blow-up as the sum of two bubbles, where the highest one concentrates at a point ξ where the potential k of the equation lies above the geometric threshold of the scalar curvature. This condition requires the bubbles to blow-up at very different speeds. To take care of this we perform a construction which combines a priori asymptotic analysis methods with a Lyapounov-Schmidt reduction.

Recent Progress on the Moser-Trudinger Equation

Pierre-Damien Thizy
University of Lyon, France
Olivier Druet, Gabriele Mancini.

We will explain how recent results on the blow-up analysis of the critical Moser-Trudinger equation in dimension 2 allow 1) to prove that the Adimurthi-Druet inequality may have no extremals, 2) to understand almost thoroughly the existence (or not) of extremals for a general class of perturbed Moser-Trudinger inequalities, 3) to get general existence results of solutions for the Moser-Trudinger equation itself, even at supercritical energy levels.

Elliptic Equations Related to Lane-Emden and Hardy-Littlewood-Sobolev Conjectures

John Villavert
University of Texas, Rio Grande Valley, USA
Congming Li

In this talk, we will examine a family of nonlinear elliptic PDEs and closely related integral equations. The equations we consider play key roles in problems from analysis and geometry, and they also serve as basic models in mathematical physics. We will discuss several key properties of positive solutions to these problems, e.g., the optimal conditions for their existence as well as their asymptotic behavior and classification. We will also discuss the importance of these properties, especially their relationship to some open questions in nonlinear elliptic PDEs.

Special Session 59: Efficient Algorithms for Flow and Transport in Porous Media

Shuyu Sun, King Abdullah University of Science & Technology, Saudi Arabia
Yanping Chen, South China Normal University, Peoples Rep of China

Flow and transport in porous media are common phenomena encountered in daily life and in engineering practice. Natural natural porous media, such as aquifers, petroleum reservoirs, or even biological tissues, provide energy and resource to human being. Man-made or engineered porous materials, including cements, ceramics, and many others, serve a key role in many engineered systems. Flow and transport phenomena within porous media are especially important in petroleum engineering, bio-remediation techniques, filtration, and hydrogeology. In this workshop, we invite speakers from diverse scientific groups of applied mathematicians, modelers, computational scientists, software engineers, chemists, physicists, and geoscientists who share an interest in developing mathematical formulation and efficient algorithms for better understanding the flow and transport in porous media. The topics of the workshop include, but are not limited to: 1) novel and advanced numerical methods for the simulation of subsurface flow and transport, and associated aspects such as discretization, gridding, up-scaling, multi-scale algorithms, optimization, data assimilation, uncertainty assessment, and high performance parallel and grid computing; 2) modeling and simulation of single-phase and multi-phase flow in porous media, and its applications to earth sciences and engineering; 3) computational thermodynamics of fluids, especially hydrocarbon and other oil reservoir fluids, and its interaction with flow and transport; 4) pore-network modeling of multi-phase flow in porous media and other pore-scale modeling of porous media flow, as well as related algorithms; 5) molecular dynamics simulation and Monte Carlo molecular simulation of fluid in a single pore or pore network, as well as related algorithms.

A Conforming Enriched Finite Element Method for Stokes Interface Problems

Jinru Chen
Jiangsu Second Normal University, Peoples Rep of China
Hua Wang, Jinru Chen, Pengtao Sun, Nan Wang

A new conforming enriched finite element method is presented for Stokes interface problems with interface-unfitted meshes. The conforming enriched finite element pair is constructed based on the Mini element pair. A ghost penalty term is used in the standard discretization form as a stabilization term. An inf-sup stability result is derived, which is uniform with respect to the mesh size. Our method does not limit the viscosity coefficient to be a piecewise constant. Finite element errors are proved to be optimal. Numerical experiments are carried out to validate theoretical results.

A Posteriori Error Estimate for the Stokes Darcy System

Ming Cui
Beijing Univerisity of Technology, Peoples Rep of China
Ningning Yan

We consider the a posteriori error estimates for finite element approximations of the Stokes Darcy system. The finite element spaces adopted are the Hood-Taylor element for the velocity and the pressure in fluid region and conforming piecewise quadratic element for the pressure in porous media region. The a posteriori error estimate is based on a suitable evaluation on the residual of the finite element solution. It is proven that the a posteriori error estimate provided in this paper is both reliable and efficient.

A High-Order Conservative Flux Optimization Finite Element Scheme for Fluid Flow Models in Porous Media

Yujie Liu
Sun Yat-Sen University, Peoples Rep of China
Junping Wang

A high order flux-conservative finite element scheme is proposed and analyzed for the fluid flow models in porous media. This scheme is based on the classical Galerkin finite element method enhanced by a flux approximation on the boundary of a prescribed set of control volumes. The numerical approximations can be characterized as the solution of a constrained-minimization problem with constraints given by the flux conservation equations for each control element. It is shown that both the finite element solution and the numerical flux converge to the exact solutions with optimal orders. The theoretical results are verified by numerical experiments on test problems for which exact solutions are known. A simplified two-phase fluid flow model in porous media has been simulated to illustrate the motivation and the efficiency of this work. The numerical results match the underlying physics and hence show that the high-order conservative fluxes obtained by the scheme perform well.

Finite Element Methods for Wave Propagation with Debye Polarization in Nonlinear Dielectric Materials

Changhui Yao
Zhengzhou University, Peoples Rep of China
Qiumei Huang, Shanghui Jia, Fei Xu, Zhongwen Xu

In this paper, we consider the wave propagation with Debye polarization in nonlinear dielectric materials. For this model, the Rother's method is employed to derive the well-posedness of the electric fields and the existence of the polarized fields by monotonicity theorem as well as the boundedness of the two fields are established. Then, decoupled the full-discrete scheme of the Euler in time and Raviart-Thomas-N*édé*lec element $k \geq 2$ in spatial is established. Based on the truncated error, we present the convergent analysis with the order $O(\Delta t + h^s)$ under the technique of a-prior L^∞ assumption. For the $k = 1$, we employ the superconvergence technique to ensure the a-prior L^∞ assumption. In the end, we give some numerical examples to demonstrate our theories.

Convergence Analysis of Algorithm for Distributed Optimal Control Problems Governed by Stokes Equations with Spectral Approximations

Jianwei Zhou
Beijing Computational Science Research Center, Peoples Rep of China
Jianwei Zhou

In this work, spectral approximations for distributed optimal control problems governed by the Stokes equation are considered. And the constraint set on velocity is stated with L^2-norm. The optimality conditions of the continuous and discretized systems are deduced with the Karush-Kuhn-Tucker conditions. The Lagrange multiplier is dependent on the constraint. Galerkin spectral approximations are employed to solve the optimal control systems. Meanwhile, adopted the first inf-sup condition, we study *a priori* error estimates for the velocity and pressure. Specially, an efficient algorithm based on the Uzawa algorithm is proposed and its convergence analysis is proved in detail. Numerical experiments are performed to confirm the theoretical results.

Special Session 60: Recent Trends in Nonlocal Nonlinear PDEs

K. Sreenadh, Indian Institute of Technology (IIT), Delhi, India
Mousomi Bhakta, Indian Institute of Science Education and Research (IISER), Pune, India
Phuoc Tai Nguyen, Masaryk University, Czech Rep

This session will focus on some recent developments in the theory of nonlocal type nonlinear PDEs which has been of great interest lately. Questions like existence, uniqueness, multiplicity, qualitative properties of solutions will be central topics of discussion. The goal of this minisymposium is to provide a forum to discuss the recent progress and promising directions in PDEs, calculus of variations, conformal geometry, minimal surface and their applications to physics, engineering, optimization and finance, obstacle problem and many more.

New Distributional Sense of Isolated Singularities on Hardy Equations

Huyuan Chen
Jiangxi Normal University, Peoples Rep of China
Feng Zhou, Alexander Quaas

In this talk, we present some suitable distributional identities of the solutions for non-homogeneous elliptic equations involving the Hardy potentials and the fractional Laplacian. Qualitative properties of the solutions to the corresponding non-homogeneous problems are investigated by the distributional identities. We address some applications on the non-existence of some non-homogeneous problems with the Hardy potentials.

Existence Results for P–Fractional Hardy–Schrödinger–Kirchhoff Systems Involving Critical Nonlinearities

Alessio Fiscella
Universidade Estadual de Campinas, Brazil

In this talk, we present recent existence results for Hardy–Schrödinger–Kirchhoff systems driven by p–fractional operators and involving critical Sobolev nonlinearities. The proof techniques are based on different variational methods. In particular, to overcome the lack of compactness arising from various reasons, we exploit a tricky qualitative analysis, which we introduced successfully for scalar Kirchhoff equations. Finally, we discuss about some appealing open questions, concerning the study of our problems.

Fractional Singular Problems

Jacques Giacomoni
Université de Pau et des Pays de l Adour, France
Adimurthi, S. Santra

In this talk, we study the positive solutions to the following singular and non local elliptic problem posed in a bounded and smooth domain $\Omega \subset \mathbb{R}^N$, $N > 2s$:

$$(P_\lambda)\begin{cases} (-\Delta)^s u = \lambda(K(x)u^{-\delta} + f(u)) \text{ in } \Omega \\ u > 0 \text{ in } \Omega \\ u \equiv 0 \text{ in } \mathbb{R}^N \backslash \Omega. \end{cases}$$

Here 00 and $f : \mathbb{R}^+ \to \mathbb{R}^+$ is a positive C^2 function. $K : \Omega \to \mathbb{R}^+$ is a Hölder continuous function in Ω which behave as $\text{dist}(x, \partial\Omega)^{-\beta}$ near the boundary with $0 \leq \beta$ and for $\lambda >$ small enough, we prove the existence of solutions to (P_λ).
Next, for a suitable range of values of δ, we show the existence of an unbounded connected branch of solutions to (P_λ) emanating from the trivial solution at $\lambda = 0$. For a certain class of nonlinearities f, we derive a global multiplicity result that extends results proved in Peral-Al. To establish the results, we prove new properties which are of independent interest and deal with the behavior and Hölder regularity of solutions to (P_λ).

Existence of Infinitely Many Solutions for Equation with Fractional Laplacian in the Zero Mass Case

Norihisa Ikoma
Kanazawa University, Japan

This talk is concerned with the existence of infinitely many solutions of equation with fractional Laplacian in the zero mass case. The equation we consider here is a fractional counterpart of the classical scalar field equation studied by Berestycki–Lions ('83) and Struwe ('82), and we generalize the recent result by Ambrosio ('18). The proof is based on a symmetric mountain pass type argument which is slightly different from that of Ambrosio and gives another proof to the results by Berestycki–Lions and Struwe.

Solutions to Fractional Lane-Emden Equations and Systems

Seunghyeok Kim
Hanyang University, Korea
Woocheol Choi, Ki-Ahm Lee, Wenjing Chen, Shengbing Deng, Fethi Mahmoudi

The goals of this talk are two-folded. Firstly, we describe asymptotic behavior of least energy solutions to the fractional Lane-Emden equations and systems as the exponents of the nonlinearities tend to certain critical exponents. Secondly, we construct a variety of solutions such as multi-bubbles, bubble-towers and higher-dimensional concentrating solutions. The existence of such solutions tells us that the conformal covariance property of the equations "prevails" their nonlocal property.

Nonlinear Choquard Equations

Sreenadh Konijeti
Indian Institute of Technology Delhi, India
Tuhina Mukherjee

In this talk we will discuss the recent developments on Nonlocal problems motivated by Choquard type convolution nonlinearity of critical growth. The main focus will be on some recent results on doubly nonlocal problems involving fractional powers of Laplacian and the corresponding critical growth nonlinearities

The Moving Plane Method and Qualitative Properties in a Non Local Setting

Luigi Montoro
UNICAL, Italy

We will present some recent results about qualitative problems of some non local problems involving the Hardy potential.

Groundstates and Radial Solutions to Schrödinger-Poisson-Slater Equations at the Critical Frequency

Vitaly Moroz
Swansea University, Wales
Jacopo Bellazzini, Marco Ghimenti, Carlo Mercuri, Jean Van Schaftingen

Schrödinger-Poisson-Slater (SPS) equation is derived as a mean-filed limit of the linear N-body Schrödinger problem for fermionic particles. We discuss the fractional Coulomb-Sobolev function spaces, which appear as the natural domain for the energy functional of a class of equations of SPS type, and establish a family of optimal interpolation inequalities associated with the Coulomb-Sobolev spaces. We also prove the existence of optimizers for the inequalities, which implies the existence of ground-states to SPS equations for a certain range of the parameters. Finally, we discuss radial Strauss type estimates and use them to prove the existence of radial solutions to SPS equations in a range of parameters which is in general wider than the range of existence parameters obtained via interpolation inequalities. The latter suggests a striking radial symmetry breakup conjecture.

Nonlocal Fractional Problems: New Results and Open Questions

Patrizia Pucci
University of Perugia, Italy

A great attention has been drawn to the study of fractional and nonlocal problems of Kirchhoff type, since they arise in a quite natural way in many different applications. The talk is based on very recent results contained in a series of papers. In particular, we present wave Kirchhoff problems driven by a nonlocal integro-differential operator and produce global existence (even under critical initial conditions), vacuum isolating and blow up of solutions. The proof arguments combine the Galerkin method with the potential well theory. The results also raise, and leave open, a number of other intriguing questions.

On Free Boundary Problems for Conformally Invariant Variational Functions

Armin Schikorra
U Pittsburgh, USA

I will present a regularity result at the free boundary for critical points of a large class of conformally invariant variational functionals. The main argument is that the Euler-Lagrange equation can be interpreted as a coupled system, one of local nature and one of nonlocal nature, and that both systems (and their coupling) exhibit an antisymmetric structure which leads to regularity estimates.

On the Moving Plane Method for Nonlocal Problems

Berardino Sciunzi
UNICAZ, Italy

I will present some results on the study of symmetry and monotonicity properties of singular solutions to nonlocal problems.

Fractional Gradient Partial Differential Equations

Daniel Spector
National Chiao Tung University, Taiwan
Tien-Tsan Shieh, Armin Schikorra

In this talk we will survey some results concerning fractional PDE written in terms of the Riesz fractional gradient. This is based on joint work with Tien-Tsan Shieh and Armin Schikorra.

Critical and Subcritical Fractional Trudinger-Moser Type Inequalities on $\mathbb{R}$

Futoshi Takahashi
Osaka City University, Japan

In this talk, we are concerned with the critical and subcritical Trudinger-Moser type inequalities for functions in a fractional Sobolev space $H^{1/2,2}$ on the whole real line. We prove the relation between two inequalities and discuss the attainability of the suprema.

Symmetric Property to Semi-Linear Nonlocal Equations

Ying Wang
Jiangxi Normal University, Peoples Rep of China

In this paper, we consider the symmetry results of positive solutions for some nonlocal problems, such as the fractional equations, the mixed integro-differential equations and so on. We prove our results through the method of moving planes, using the ABP estimates for the fractional Laplacian to obtain the Maximum Principle for small domain. We use a truncation technique to overcome the difficulty caused by the nonlocal character of the differential operator.

A Unified Approach to Nirenberg Problem and New Results

Jingang Xiong
Beijing Normal University, Peoples Rep of China
Tianling Jin, Yanyan Li

I will talk about a unified approach to Nirenberg problem of different orders, which also yields new results. A blow up analysis procedure by potential representation will be sketched.

The Brezis-Nirenberg Problem for the Fractional P-Laplacian

Yang Yang
Jiangnan University, Peoples Rep of China
Sunra Mosconi, Kanishka Perera, Marco Squassina

We obtain nontrivial solutions to the Brezis-Nirenberg problem for the fractional p-Laplacian operator, extending some results in the literature for the fractional Laplacian. The quasilinear case presents two serious new difficulties. First an explicit formula for a minimizer in the fractional Sobolev inequality is not available when $p \neq 2$. We get around this difficulty by working with certain asymptotic estimates for minimizers. The second difficulty is the lack of a direct sum decomposition suitable for applying the classical linking theorem. We use an abstract linking theorem based on the cohomological index to overcome this difficulty.

Special Session 61: Stochastic Filtering, Optimal Control, and their Applications

Guangchen Wang, Shandong University, Peoples Rep of China
Jie Xiong, Southern University of Science and Technology, Peoples Rep of China

Optimal control problems are of important theoretical and practical significance with many application fields such as mathematical finance. A standard assumption in the literature is that the stochastic noises in the model are completely observed. However, this is rarely the case in real world situations. The optimal control problems under complete information are studied extensively. Nevertheless, very little is known about these problems when the information is not complete.
Stochastic filtering is a classical topic with revitalized interest. The aforementioned optimal control problems under partial information calls for the combined study of optimal control and stochastic filtering. The goal of this session is to bring experts from both fields to exchange ideas and to encourage collaborations.

Optimal Control for a Class of Queueing Models at the Law-Of-Large-Numbers Scale

Rami Atar
Technion, Israel
Anup Biswas, Haya Kaspi, Kavita Ramanan

Motivated by queueing applications, we present dynamics on measures on the real line associated with an optimal control problem. The dynamics are described by a newly introduced tool that we refer to as a Skorohod map on measure-valued paths. This tool is used, in addition, to prove scaling limits at the law of large numbers scale.

Nonlinear Filtering and Averaging in Two Time Scale Systems

Vivek Borkar
Indian Institute of Technology Bombay, India

This talk will outline recent applications of nonlinear filtering in deriving averaging results for two time scale stochastic differential systems as the time scale separation diverges and the slower time scale “sees” its dynamics averaged with respect to the stationary distribution of the faster dynamics.

Large Deviations from the Hydrodynamic Limit for a System with Nearest Neighbor Interactions

Amarjit Budhiraja
University of North Carolina at Chapel Hill, USA
Sayan Banerjee, Michael Perlmutter

We give a new proof of the large deviation principle from the hydrodynamic limit for the Ginzberg-Landau model studied in Donsker-Varadhan(1989) using techniques from the theory of stochastic control and weak convergence methods. The proof is based on characterizing subsequential hydrodynamic limits of controlled diffusions with nearest neighbor interaction that arise from a variational representation of certain Laplace functionals. The approach taken here does not require superexponential probability estimates or estimation of exponential moments that are central ingredients in previous proofs. Instead, proof techniques are very similar to those used for the law of large number analysis, namely in the proof of convergence to the hydrodynamic limit (cf. Guo-Papanicolaou-Varadhan(1988)). Specifically, the key step in the proof is establishing suitable bounds on relative entopies and Dirichlet forms associated with certain controlled laws. This general approach has the promise to be applicable to other interacting particle systems as well and to the case of non-equilibrium starting configurations, and to infinite volume systems. This is joint work with S. Banerjee and M. Perlmutter.

Time Discretizations of the Solution of the Non-Linear Filtering Problem

Dan Crisan
Imperial College London, England
Salvator Ortiz-Latorre

The solution of the continuous time filtering problem can be represented as a ratio of two expectations of certain functionals of the signal process that are parametrized by the observation path. We introduce a class of discretization schemes of these functionals of arbitrary order. The result generalizes the classical work of Picard, who introduced first order discretizations to the filtering functionals. For a given time interval partition, we construct discretization schemes with convergence rates that are proportional with the m-power of the mesh of the partition for arbitrary integers m. The result paves the way for constructing high order numerical approximation for the solution of the filtering problem.

Nonlinear Filtering with Levy Processes

Erika Hausenblas
Montanuniversitaet Leoben, Austria
Pani Fernando

The objective in stochastic filtering is to reconstruct the information about an unobserved (random) process, called the signal process, given the current available observations of a certain noisy transformation of that process, called observation process. Usually X and Y are modeled by stochastic differential equations driven by a Brownian motion or a jump (or Lévy) process. We are interested in the situation where both the state process X and the observation process Y are perturbed by coupled Levy processes. In the talk we first present some theoretical results. More precisely, we consider the situation where $L = (L_1, L_2)$ is a 2–dimensional Levy process in which the structure of dependence is described by a Levy copula. L_1 appears in the signal process, L_2 appears in the observation process. Secondly, we introduce the approximation of the density by a particle system.

Mean-Variance Problems for Finite Horizon Continuous-Time Markov Decision Processes

Yonghui Huang
Sun Yat-Sen University, Peoples Rep of China

This talk discusses mean-variance problems in the context of finite horizon continuous-time Markov decision processes. The state and action spaces are assumed to be general spaces, while reward functions and transition rates are allowed to be unbounded. Using the first-jump analysis, we succeed in converting the variance of the finite-horizon reward to a mean of a finite-horizon reward with new reward functions, and then, we design a new method called successive approximation, via which we prove the existence of a solution to the Hamilton-Jocobi-Bellman equation and of an optimal policy under some growth and compact-continuity conditions.

Multilevel Smoothing and Filtering

Ajay Jasra
National University of Singapore, Singapore

I present several techniques for performing filtering and smoothing for partially observed stochastic differential equations. The methodology focuses upon utilizing the multilevel Monte Carlo method. I show how optimal transport and/particle filtering methods can be used in this context as well as associated complexity theorems, giving a justification of their practical application.

Multilevel Particle Filtering

Kody Law
Oak Ridge National Laboratory, USA
Ajay Jasra, Kengo Kamatani, Yan Zhou, Prince Osei

I will discuss the filtering of partially observed diffusions, with discrete-time observations. It is assumed that only biased approximations of the diffusion can be obtained, for choice of an accuracy parameter. A multilevel estimator is proposed, consisting of a telescopic sum of increment estimators associated to the successive levels. The work associated to a given mean-square error between the multilevel estimator and average with respect to the filtering distribution is shown to scale optimally, for optimal rates of convergence of the underlying diffusion approximation. The method is illustrated on some examples.

Stochastic Maximum Principle on a Continuous-Time Behavioral Portfolio Model

Qizhu Liang
University of Macau, Peoples Rep of China
Jie Xiong

Within the framework of Kahneman and Tversky's cumulative prospective theory, this paper considers a continuous-time behavioral portfolio selection model, which includes both running and terminal terms in the objective functional. Despite the existence of S-shaped utility functions and probability distortions, a necessary condition for optimality is derived by stochastic maximum principle.

Market-Reaction-Adjusted Optimal Central Bank Intervention Policy in a Forex Market with Jumps

Hongwei Long
Florida Atlantic University, USA
Sandun Perera, Winston Buckley

Impulse control with random reaction periods (ICRRP) is used to derive a country's optimal foreign exchange (forex) rate intervention policy when the forex market reacts to the interventions. We extend the previous work on ICRRP by incorporating a multi-dimensional jump diffusion process to model the state dynamics. Furthermore, we employ a novel minimum cost operator that simplifies the computations of the optimal solutions. Finally, we demonstrate the efficacy of our framework by finding a market-reaction-adjusted optimal central bank intervention (CBI) policy for a country. Our numerical results suggest that market reactions and the jumps in the forex market are complements when the reactions increase the forex rate volatility; otherwise, they are substitutes.

Sequential Empirical Bayes Method for Filtering Dynamic Spatiotemporal Processes

Vasileios Maroulas
University of Tennessee, USA

We consider online prediction of a latent dynamic spatiotemporal process and estimation of the associated model parameters based on noisy data. The problem is motivated by the analysis of spatial data arriving in real-time and the current parameter estimates and predictions are updated using the new data at a fixed computational cost. Estimation and prediction is performed within an empirical Bayes framework with the aid of Markov chain Monte Carlo samples. Samples for the latent spatial field are generated using a sampling importance resampling algorithm with a skewed-normal proposal and for the temporal parameters using Gibbs sampling with their full conditionals written in terms of sufficient quantities which are updated online. The spatial range parameter is estimated by a novel online implementation of an empirical Bayes method, called herein sequential empirical Bayes method. A simulation study shows that our method gives similar results as an offline Bayesian method. We also find that the skewed-normal proposal improves over the traditional Gaussian proposal. The application of our method is demonstrated for online monitoring of radiation after the Fukushima nuclear accident.

Large Deviations for the Nonlinear Filter with Levy Noise

Xiaoyang Pan
University of Tennessee, USA
Vasileios Maroulas, Jie Xiong

We focus on the asymptotic behavior of the optimal filter where both signal and observation processes are driven by Lévy noises. Indeed, we study large deviations for the case where the signal-to-noise ratio is small by considering weak convergence arguments. To that end, we first prove the uniqueness of the solution of the controlled Zakai and Kushner-Stratonovich equations, using a method which transforms the associated equations into SDEs in an appropriate Hilbert space. Taking into account the controlled analogue of Zakai and Kushner-Stratonovich equations, respectively, the large deviation principle follows by employing the existence, uniqueness and tightness of the solutions.

Stochastic Control Under Partial Observation

Huyen Pham
University Paris Diderot, France
E. Bandini, A. Cosso, M. Fuhrman

We study and revisit the optimal control problem of partially observed stochastic systems. By using a control randomization method, we provide a backward stochastic differential equation (BSDE) representation for the value function in a general framework including path-dependence in the coefficients (both on the state and control) and without any non degeneracy condition on the diffusion coefficient. In the standard Markovian case, this BSDE representation has important implications: it allows us to obtain a corresponding randomized dynamic programming principle (DPP) for the value function, which is obtained from a flow property of an associated filter process. This DPP is the key step towards our main result: a characterization of the value function of the partial observation control problem as the unique viscosity solution to the corresponding dynamic programming Hamilton-Jacobi-Bellman (HJB) equation. The latter is formulated as a new, fully non linear partial differential equation on the Wasserstein space of probability measures, and is derived by means of the recent notion of differentiability with respect to probability measures introduced by P.L. Lions in his lectures on mean-field games at the Collège de France. An important feature of our approach is that it does not require any condition to guarantee existence of a density for the filter process solution to the controlled Zakai equation, as usually done for the separated problem. We give an explicit solution to our HJB equation in the case of a partially observed non Gaussian linear quadratic model.

An Optimal Consumption and Investment Problem with Partial Information

Shuenn-Jyi Sheu
National Central University, Taiwan
Hiroaki Hata

We consider optimal consumption problem on finite time or on infinite time horizon. An investor wants to maximize the expected HARA utility of consumption. We treat a stochastic factor model that the mean returns of risky assets depend linearly on underlying economic factors formulated as the solutions of linear stochastic differential equations. We discuss the partial information case that the investor can not observe the factor process and use only past information of risky assets. Then our problem is formulated as a stochastic control problem with partial information. Using filtering equation, we can reformulate the problem with complete information. We derive the HJB equation. The equation can be solved to obtain an explicit form of the value function and the optimal strategy for this problem. Moreover, we also introduce martingale method and obtain solution without solving PDE. We compare the solution using dynamic programming approach and martingale method. The discussion is based on a joint work with Hiroaki Hata.

A Kind of Stochastic Linear-Quadratic Stackelberg Differential Game with Overlapping Information

Jingtao Shi
Shandong University, Peoples Rep of China
Guangchen Wang, Jie Xiong

This talk is concerned with a kind of stochastic linear-quadratic Stackelberg differential game with overlapping information. Here the term overlapping means that the follower's and the leader's information have some joint part, while they have no inclusion relation. Optimal controls of the follower and the leader's are proved by the stochastic maximum principle, the direct calculation of the derivative of the cost functional and stochastic filtering. A new system of Riccati equations is introduced to represent the state feedback of the Stackelberg equilibrium strategy.

Ensemble Kalman Filter with a Small Effective Dimension

Xin Tong
National University of Singapore, Singapore
A. J. Majda

The successful EnKF prediction skill with an ensemble size K much smaller than the dimension d is an intriguing mystery. The practitioners often attribute this success to a low effective dimension p, of which the formal definition is unclear. The first part of our framework proposes a natural definition for the effective dimension, using the covariance spectrum of an associated Kalman filter. The second component employs the Mahalanobis norm to quantify the EnKF performance, which is intrinsically dissipative for Kalman type of filter updates. This dissipative mechanism is stable enough to wither the noisy perturbation from model or small sampling error. The low effective dimension plays a vital role here, since when $K > Cp$ for a constant C, the sample forecast covariance matrix can concentrate around its expected value, using a new random matrix theory result. Practical covariance inflation and spectral projection are employed to our EnKF. The fact that these augmentations are necessary for our proof, indicates the theoretical significance of these augmentations, while their practical significance has already been observed and well documented.

Linear-Quadratic Stochastic Optimal Control with Fast-Slow Dynamics

James Yang
University of Sydney, Australia
Beniaman Goldys, Gianmario Tessitore

In this talk, we discuss the convergence of the value function of a linear-quadratic stochastic optimal control problem as the ratio between the speed of the two-scale state dynamics diverge. More specifically, we represent the value function in terms of a Riccati representation and show that it converges towards a reduced backwards equation. In the simplified one dimensional case, we show that the reduced backwards equation is the Riccati equation of a classical linear-quadratic stochastic control problem. Connections to ergodic control will also be discussed. This work is inspired by the study of the nonlinear case done by Alvarez-Bardi [1] and Guatteri-Tessitore [2], via HJB and BSDE methods respectively. It should be noted that both papers adopt Lipschitz and boundedness assumptions which do not cover the linear-quadratic case. This is a joint work Beniamin Goldys and Gianmario Tessitore.

References

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Estimation and Filtering in Environmental Pollution

Xingqiu Zhao
The Hong Kong Polytechnic University ShenZhen Research Institute, Peoples Rep of China
Qizhu Liang, Jie Xiong

We consider the environmental pollution model. In the model, the undesired chemical is released from an unknown location by a Poisson process with unknown intensity. Actually, in many cases, it is not possible to directly observe the concentration of undesired chemical. The problem is to find consistent estimators of unknown source of the pollution and unknown intensity and predict the pollution distribution based on the observed data. For the problem, we develop an estimation procedure, construct an approximating filter and establish its asymptotic properties.

Special Session 62: Asymptotic Behavior in Nonlinear Elliptic and Parabolic Problems

Yoshitsugu Kabeya, Osaka Prefecture University, Japan
Vitaly Moroz, Swansea University, Wales
Dmitri Finkelshtein, Swansea University, Wales

Understanding the asymptotic behavior of solutions to stationary and time dependent nonlinear PDEs is crucial for the analysis of the existence and qualitative properties of the solutions such as regularity, symmetry, stability, front propagation. It is also fundamental for the apriori nonexistence results such as Liouville type theorems in elliptic and finite time blowup in parabolic problems. The aim of this special session is to bring together leading experts on linear and nonlinear PDEs for the exchange of ideas and most recent advances in these areas. Relevant topics from the linear theory will include Green's functions and heat kernels estimates; maximum principles and regularity theory; involving probabilistic methods. The focus in nonlinear PDEs will be on elliptic and parabolic problems including equations on manifolds and problems with nonlocal interactions, as well as travelling waves solutions.

Domain Variations and Applications to Elliptic Problems with Robin Boundary Conditions

Catherine Bandle
University of Basel, Switzerland

In the calculus of variations symmetrization techniques are useful for estimating the energy or the Rayleigh quotient provided the solutions of the underlying problems satisfy a Dirichlet boundary condition. The ball is then the optimal domain. In the case of Robin boundary conditions symmetrization is not applicable in general, and the question whether or not the ball is optimal is not clear. We focus on nearly spherical domains and study how much their solutions differ from the ones in the ball. This talk is based on a study in collaboration with Alfred Wagner.

Structure of Singular Solutions for Nonlinear Elliptic Equations with the Critical Potential

Jann-Long Chern
National Central University, Taiwan
Chih-Her Chen, Eiji Yanagida

We consider the structure of singular solutions for elliptic equations with the Hardy potential and critical nonlinearity under quite general conditions on the potential terms. It is shown that there exists a unique special singular solution, and other infinitely many singular solutions are oscillatory around the special singular solution. We also study the asymptotic behavior of the solutions around the singular point. Our results can be applied to various problems such as the scalar field equation, a self-replication model and the Cafarelli-Kohn-Nirenberg inequality.

Doubly Nonlocal Fisher-KPP Equation: Minimal Speed of Travelling Waves

Dmitri Finkelshtein
Swansea University, Wales

We consider a reaction-diffusion equation with nonlocal anisotropic diffusion and a linear combination of local and nonlocal monostable-type reactions in a space of bounded functions on $\mathbb{R}^d$. We study asymptotic properties of the travelling waves profiles, find explicit formula for the minimal travelling wave speed and prove the uniqueness of the profiles up to shifts, including the travelling wave with the minimal speed. We cover the special case when the abscissa of the travelling wave with the minimal speed (in a direction) coincides with the abscissa of the diffusion kernel in this direction.

Gelfand Type Problems for Reactive Jets: Autoignition of Turbulent Jets

Peter Gordon
Kent State University, USA
Uday G. Hegde and Michael C. Hicks

Gelfand problem is one of the canonical problems in the theory of non-linear parabolic and elliptic PDEs. This problem naturally arises in the Frank-Kamenetskii theory of thermal explosion (autoignition) and describes an initial stage of evolution of a temperature field in reactive materials and mixtures. In this talk I will present a generalization of Gelfand problem for the analysis of autoignition of reactive turbulent jets. I will present both the derivation of this new model and its analysis. The latter is performed using a combination of rigorous, formal asymptotic and numerical techniques. It will be shown that similar to the classical Gelfand problem an autoignition in jets occur exclusively owing to the absence of self-similar temperature distribution which, in mathematical terms, leads to loss of regularity (blow-up) of underlying PDE. The detailed analysis of self-similar temperature profiles will be presented and a sharp characterization of an autoignition event in terms of principal geometric and physical parameters of the problem will be given. This a joint work with U.G. Hegde and M.C. Hicks of NASA Glenn Research Center.

Bifurcation Diagrams of a Nonlinear Elliptic Equation on a Spherical Cap

Yoshitsugu Kabeya
Osaka Prefecture University, Japan

We consider a nonlinear elliptic equation on a spherical cap and discuss the existence of non-azimuthal solutions. This talk is based on the joint work with Professors C. Bandle (Basel, Switzerland) and H. Ninomiya (Meiji, Japan).

Reaction-Diffusion Equations on a Singularly Perturbed Domain

Toru Kan
Osaka Prefecture University, Japan

On a domain given as a tubular neighborhood of a graph, we consider a reaction-diffusion equation with the Neumann boundary condition. The domain converges to a line segment on each edge of the graph, while it shrinks faster to a point at each node. Then the equation is expected to be approximated by some one-dimensional limiting equation on the graph with appropriate matching conditions at the nodes. We formally derive the limiting equation and then show that it indeed approximates the original equation.

Concentration Phenomena of a Least Energy Solution to Semilinear a Neumann Problem with a Non-Smooth Boundary

Atsushi Kosaka
Bukkyo University, Japan

We consider the condensation phenomena of a positive solution to semilinear Neumann problems. By singular perturbation some positive solution tends to zero except for some points, and its energy concentrates around those points. That is said to be concentration phenomena. In precedent studies if the domain of a Neumann problem has a smooth boundary, then the mean curvature plays an important role in concentration phenomena. Especially if a solution has the least positive critical value, that is, a least-energy solution, then the solution concentrates at the point at which the mean curvature attains its maximum. In our talk we assume that a domain of our problem has a piecewise smooth boundary. Especially we consider the case that the profile of the boundary is like a cone around a non-smooth point. Then we expect that the angle of the cone plays a similar role to the mean curvature. In fact, for a least-energy solution, we can prove that when the dimension of the domain is 2 or 3. Namely the least-energy solution concentrates at the peak having the least angle. Moreover we can investigate the profile of the least-energy solution around the peak.

The Choquard Equation Under a Logarithmic Potential

Jean van Schaftingen
Université Catholique de Louvain, Belgium
Luca Battaglia, Silvia Cingolani, Denis Bonheure

The coupling of the Schrödinger equation with a Newtonian self-interaction field leads in the two-dimensional case to a Choquard equation with a logarithmic internal potential

$$-\Delta u + u = \Bigl(\log \frac{1}{|\cdot|} * |u|^2\Bigr) u$$

The divergence of latter logarithmic potential at infinity prevents the natural energy functional to be well-defined and smooth on the natural Sobolev space induced by the linear part of the equation. I will present a new relaxation scheme that Luca Battaglia and I have developed to construct solutions to this Choquard problem and some nondegeneracy result obtained in collaboration with Denis Bonheure and Silvia Cingolani.

On the Maximizing Problem Associated with Sobolev Type Embeddings Under Homogeneous Constraints

Hidemitsu Wadade
Kanazawa University / Institute of Science and Engineering, Japan
Michinori Ishiwata

We consider the attainability of a maximizing problem associated with the Sobolev embedding. The existence and non-existence of a maximizer are related to the exponents appearing in the problem. Indeed, for some range of the exponents, we have a non-existence result in spite of the sub-critical setting and the radial nature of the problem. In fact, we characterize the value which is the threshold in terms of the attainability of the problem. We also consider the attainability of the problem when the exponent becomes the threshold number with its computation of the exact value for some range of the exponents. This content is joint work with Prof. Michinori Ishiwata in Osaka University.

Special Session 63: Theoretical and Numerical Aspects of Mathematical Geophysical Dynamics

Olga Rozanova, Moscow State University, Russia
Alina Chertock, North Carolina State University, USA
Alexander Kurganov, Southern University of Science and Technology, China and Tulane University, USA, Peoples Rep of China
Jui-Ling Yu, Providence University, Taiwan

This special session is devoted to various aspects of the nonlinear dynamics of fluid or gas over plane or curved surface in the presence of the Coriolis force. In particular, we are interested in a vortex motion of different scale: nonlinear stability of atmospheric and oceanic eddies, their trajectories, the interaction with the land, the conditions of formation in the context of the study of tropical cyclones. Special attention is paid to the problems connected with the shallow water equations over the bottom topography of a complex configuration, multi-layer media, and modeling of interfaces of water and air. We focus both on the theoretical results in the field and on numerical simulations

A Mathematicam Model of the Evolution of a Typhoon Based on the Trajectories of the Wind

Hisao Fujita Yashima
Ecole Normale Supérieure Assia Djebar de Constantine, Algerie
Hisao Fujita Yashima, Meriem Aouaouda

In this talk a model of the evolution of a typhoon is considered and studied numerically. We consider the motion of the air in a cylindrical domain with axial symmetry. Neglecting the viscosity and the heat-conductivity, we transform the equations in those on the trajectories of the air. On each trajectory the process of condensation of water vapor in ascending flow of wet air and the consequent heating by the latent heat are described. The force pushing upwards the air is determined directly by the relation with the pressure in the external region. The result of numerical computation shows the increase of the wind due to the latent heat of the condensation and the stabilization due to the accumulation of liquid or solid water in the air, which brakes down the ascending motion of the air.

Well-Balanced Schemes Via Conservative Formulation Using Global Fluxes

Alexander Kurganov
Southern University of Science and Technology, China and Tulane University, USA, Peoples Rep of China

I will present a new way of designing well-balanced schemes for hyperbolic systems of balance laws. Our approach is based on a conservative formulation of the system using global fluxes obtained by introducing nonlocal equilibrium variables. A crucial step in the construction of well-balanced second- and higher-order schemes is a well-balanced piecewise polynomial reconstruction of equilibrium variables combined with a well-balanced evolution in time.
A difficulty one may face while implementing our approach is a necessity of (approximately) solving Riemann problems for hyperbolic systems with global fluxes. This may be hard or even impossible. We therefore use Riemann-problem-solver-free central schemes. Our particular choice is central-upwind schemes, which may need to be adapted to reduce the amount of numerical viscosity when the flow is at (near) steady-state regime.
The new method can be applied to a wide variety of hyperbolic systems of balance laws. I will present the new well-balanced central-upwind schemes for shallow water equations and compressible Euler equations with gravitation.

Nonlinear Stability of Gas Cloud in the Earth Atmosphere

Olga Rozanova
Moscow State University, Russia

We consider a model of compressible rotating stratified gas cloud in the field of Coriolis force. Our main goal is to study a possibility of existence of stable structure, corresponding to ling-living atmospheric vortex. We study the influence of surface friction and processes of mass inflow/outflow on the stability of gas cloud.

Qualitative Precipitation Prediction: Past, Present, and Future

Shih-Hao Su
Chinese Culture University, Taiwan
Ting-Shuo Yo, Jung-Lien Chu

Extreme rainfall is one of the major natural disasters in Taiwan and it made significant environmental and societal impacts. The occurrence of rainfall extremes involves complicated multiscale interactions and hence the ability in Quantitative Precipitation Estimations(QPEs) / Quantitative Precipitation Forecasts (QPFs) is limited. In the past, we used the numerical model which based by the physical concepts to predict the rainfall amount. Limited by model resolution and traditional theoretical architectures, many simplifying assumptions and uncertainty in convection simulation lead to large prediction errors. By incorporating machine learning(ML) techniques, a new approach of multi-time scales QPEs and QPFs is proposed. The designed framework is based on our prior work on classifying weather events and was proved to outperform traditional objective analysis. For QPEs/QPFs tasks, we used objective weather events data, real-time surface/radar observations, and numerical model output as dependent variables. By combining the machine learning techniques and multiple data sources, we developed an extreme rainfall warning system. The preliminary result of using ML method to detect the heavy rainfall events in Taiwan shows the hit rate is about 79%-82% for different classifiers of all positive events.

Understanding Dynamics of Equatorial Atmosphere with Moist-Convective Rotating Shallow Water Model

Vladimir Zeitlin
Sorbonne University, France
M. Rostami

One- (or multiple-) layer rotating shallow water (RSW) models are of routine use in geophysical fluid dynamics (V. Zeitlin 2018, "Geophysical Fluid Dynamics: understanding (almost) everything with rotating shallow-water models", OUP) . The models allow for efficient high-resolution finite-volume numerical methods, and straightforward inclusion of bottom topography. Still, traditional RSW describes "pure" dynamics, and such crucial atmospheric phenomena as water-vapor condensation and evaporation, and related moist convection are not captured. The recently proposed "moist- convective" mcRSW models (J. Lambaerts *et al* 2011, Phys. Fluids, **23**, 046603) incorporate thermodynamics of the moist air in the shallow-water framework in a self-consistent way, and were shown to correctly reproduce dynamics of the moist atmosphere, including evolution of tropical storms. We demonstrate how the mcRSW models can be further improved to include precipitable water. The models, thus, become cloud-resolving, but keep minimal complexity, and are computationally friendly. A variant of the RSW allowing for variable temperature/density of the fluid is used to render the model even more realistic and couple it to sea-surface temperature. We illustrate the capacities of the new models by simulating interactions of equatorial waves with maritime continent and Western-Pacific warm-pool in the context of Madden-Julian oscillation and tropical cyclogenesis in South-East Asia.

Special Session 64: Delay Equations in Population Dynamics

Gergely Rost, University of Szeged, Hungary
Philipp Getto, Center for Dynamics, TU Dresden, Germany
Yukihiko Nakata, Shimane University, Japan

Delay equations and functional differential equations are suitable to model biological systems where the evolution of the system depends also on the past states. In population dynamics, time delays arise naturally due to maturation periods, times needed for spatial movement, times to initiate population control and so on. Modeling complex biological phenomena often requires the use of multiple, variable, distributed, infinite delays, delays in terms with a derivative, or state dependent delays that depend on the solution itself. In this session we propose to summarize recent progress in this field from theoretical, numerical and application points of view.

Periodic Solutions of a Stem Cell Population Model with State-Dependent Delay

Istvan Balazs
Bolyai Institute, University of Szeged, Hungary
Philipp Getto

We consider a system of differential equations with state-dependent delay, describing the size of a population of stem cells. We show that, for some initial functions, the solution is slowly oscillating. Using fixed point index theory developed by Roger Nussbaum, we prove existence of nontrivial slowly oscillating periodic solutions.

Dynamics of a Stage Structured Intraguild Predation Model

Juancho Collera
University of the Philippines Baguio, Philippines
Felicia Maria G. Magpantay

We consider a three-species intraguild predation (IGP) model which includes a predator (IG predator) and its prey (IG prey) that share a common resource, and where the IG prey population is partitioned into juvenile and adult stages. The juvenile IG preys are assumed to have little ability of predation and are able to avoid the IG predators by taking refuge. The maturation age of the IG prey population is reflected by a time delay. Conditions for the existence and local stability of all non-negative equilibria are given using the delay as the main parameter. In particular, we show that the positive equilibrium may switch stability at some critical delay value where Hopf bifurcation occurs. Numerical continuation in DDE-Biftool is used to illustrate our results.

A Prey-Predator Model with Gestation Delay

Yoichi Enatsu
Tokyo University of Science, Japan
Yoichi Enatsu

In this talk, we consider the dynamics of a Lotka-Volterra prey-predator model by a class of delay differential equations. The number of prey varies due to a general nonlinear predators‘ consumption rate with delays. Under the assumption that the consumption rate is monotonically increasing with respect to the number of prey, we theoretically explain that the less effective the predator consume prey or the longer gestation time lag becomes, the numbers of prey and predator tend to oscillate. For the case where the consumption rate is described not only as Holling type I,II but also as Holling type III, we present some ongoing studies. We also mention the difference of the assumptions on the incidence rates appearing in prey-predator models with those in epidemiological models.

Stability Analysis for a Differential Equation with State-Dependent Delay

Philipp Getto
TU Dresden, Germany

The presentation will be on the analysis of a differential equation that describes the regulated maturation of a stem cell population. The equation features distributed and state dependent delays. Recent results on well-posedness and a linearized stability theorem will be presented. Based on the latter is current research on the analytical and numerical computation of maximal regions of (in)stability.

ODE-Reducibility of Delay Equations Describing Structured Populations

Mats Gyllenberg
University of Helsinki, Finland

I show how models of physiologically structured populations could (and should) be formulated as systems of nonlinear delay equations and consider the question of when such a system of delay equations can be faithfully represented by a system of ordinary differential equations.

Mathematical and Numerical Study of Blow-Up Problem for Some Oscillation Model with a Delay

Tetsuya Ishiwata
Shibaura Institute of Technology, Japan
Emiko Ishiwata, Yukihiko Nakata

It is well-known that time-lags or histories sometimes play a essential role in the phenomena and thus many mathematical model with delay-effects are studied mathematically and numerically. It is also well-known that the delay-effects cause an instability or an oscillatory.
In this talk we consider the effects of time-delay for such instabilities from the viewpoint of a finite time blow-up of the solutions and show delay-induced blow-up phenomena for a very simple oscillation model with a constant delay. We also show the emergence of infinity many unstable periodic solutions, while the non-delay system has only one limit cycle. Finaly we show some numerical examples and give our observations.

Controlling Mackey–Glass Chaos

Gabor Kiss
University of Szeged, Hungary
Gergely Rost

The Mackey–Glass equation is the representative example of delay induced chaotic behavior. Here we propose various control mechanisms so that otherwise erratic solutions are forced to converge to the positive equilibrium or to a periodic orbit oscillating around that equilibrium. We take advantage of some recent results of the delay differential literature, when a sufficiently large domain of the phase space has been shown to be attractive and invariant, where the system is governed by monotone delayed feedback and chaos is not possible due to some Poincaré–Bendixson type results. We systematically investigate what control mechanisms are suitable to drive the system into such a situation, and prove that constant perturbation, proportional feedback control, Pyragas control and state dependent delay control can all be efficient to control Mackey–Glass chaos with properly chosen control parameters.

Reduction Theory Based on the Floquet Theorem for Delay Differential Equations

Kiyoshi Kotani
The University of Tokyo, Japan
Yutaro Ogawa, Sho Shirasaka, Akihiko Akao, Hiroya Nakao, Yasuhiko Jimbo

Oscillatory dynamics induced by time delay are frequently observed in natural and artificial systems. However, analytical frameworks for delay induced oscillations are not fully developed. Here, we propose a framework of model reduction by using the Floquet theorem for general delay-differential equations (DDEs) exhibiting limit-cycle oscillations. Especially, we illustrate that the adjoint eigenfunction corresponding to the Floquet zero eigenvalue provides analytical insights of synchronization properties. We further propose a practical numerical method to derive the adjoint eigenfunction. By using this function, original DDEs can be reduced to a simple phase equation without time-delay. We then demonstrate that the analyses of the phase equations provide analytical insights into synchronization properties of biological systems modelled by DDEs.

Dynamics of a Mathematical Model for Hematopoietic Stem Cells with Diffusion and Time Delay

Toshikazu Kuniya
Kobe University, Japan
Mostafa Adimy, Abdennasser Chekroun

In this study, we investigate a mathematical model for hematopoietic stem cells. The model is described by a system of partial differential equations, which depend on space and age. By applying the method of characteristics, we reformulate the model into a reaction-diffusion equation with a nonlocal spatial term and time delay. We prove the existence, uniqueness and positivity of the solution, and obtain a threshold condition for the global asymptotic stability of the trivial equilibrium. Moreover, we obtain sufficient conditions for the existence of nontrivial equilibrium and the uniform persistence of the system.

An Alternative Formulation for a Distributed Delayed Logistic Equation

Chiu-Ju Lin
McMaster University, Canada
Lin Wang, Gail S.K. Wolkowicz

We study the alternative single species logistic distributed delay differential equation (DDE) with decay-consistent delay in growth. Population oscillation is rarely observed in nature, in contrast to the outcomes of the classical logistic DDE. In the alternative discrete delay model proposed by Arino et al. [J. Theor. Biol. 2006, pp109–119], oscillating behavior is excluded. This study adapts their idea of the decay-consistent delay and generalizes their model. We establish a threshold for survival and extinction: in the former case, it is confirmed using Lyapunov functionals that the population approaches the delay modified carrying capacity; in the later case the extinction is proved by the fluctuation lemma. We further use the adaptive dynamics to conclude that the evolutionary trend is to make the mean delay in growth as short as possible. This confirms Hutchinson's conjecture and fits biological evidence. This is a joint work with Lin Wang and Gail S.K. Wolkowicz.

Monotone Dynamics Or Not? Dynamical Consequences of Various Mechanisms for Delayed Logistic Growth

Torsten Lindström
Linnaeus University, Sweden

In this paper we interpret the global stability properties of the delayed single species chemostat in terms of monotone dynamics on an asymptotically invariant hyperplane in the state space. The consequence is a translation of advanced analysis and delay differential equations into sign checks and ordinary differential equations for an important single species model with explicit resource dynamics. Complete proofs are included, since the limiting behavior at asymptotically invariant sets may not agree with the limiting behavior of the original system even in the finite dimensional case (Thieme (1992)).
A delayed logistic equation based on explicit resource dynamics falls out as a limiting case of the chemostat and we claim this to be a new mechanistic interpretation of delayed logistic models. We continue by comparing these results to several other delayed logistic models that has been mechanistically justified in the literature. We conclude that monotone dynamics apply in several cases. We improve one global stability result that cannot be obtained with by the use of monotone dynamics and end up by pointing out the dynamical differences between Hutchinson's (1948) delayed logistic equation and those with mechanistic interpretations.

An Age-Structured Population Model with State-Dependent Time Delay

Felicia Magpantay
Queen's University, Canada
N. Kosovalic, J. Wu

We present an age-structured population model that accounts for the following aspects of complex life cycles: (i) There are juvenile and adult stages, (ii) only the adult stage is capable of reproducing, (iii) cohorts of juveniles can transition to the adult stage when they have consumed enough nutrition and (iv) the juvenile and adult populations consume different limited food sources. Taking all of these into account leads to a new mathematical model that cannot be directly analyzed using the established framework of functional differential equations or simulated by standard numerical schemes for age-structured populations. Here we present the model derivation, properties and a scheme to numerically integrate the equations.

Periodic Solutions of a Delay Differential Equation

Yukihiko Nakata
Shimane University, Japan

We would like to discuss periodic solutions of a delay differential equation, which models disease transmission dynamics in a population. We show that for a special case the model has a periodic solution, expressed in terms of the Jacobi elliptic functions. Implications will be discussed.

A Unification of Theory of Well-Posedness for Delay Differential Equations

Junya Nishiguchi
Tohoku University, Japan

In this talk, I will develop a unified theory of well-posedness for delay differential equations (DDEs) by "history spaces" and "prolongations." One of the main results is the following: under the assumption that the history space H is C^0-prolongable and C^0-regulated, the trivial equation $\dot{x} = 0$ generates a continuous semiflow on H is necessary and sufficient for the property that the initial value problem of $\dot{x}(t) = F(t, x_t)$ is well-posed for any history functional F which is continuous and uniformly locally Lipschitzian about C^0-prolongations. I will also generate this result to the setting that H is C^1-prolongable and C^1-regulated in order to cover DDEs with general state-dependent delays.

Stability Properties of Solutions to Nonlinear PDEs and ODEs with State-Dependent Delays

Alexander Rezunenko
Kharkiv University & AVCR, Ukraine

The well-posedness in the sense of Hadamard for systems of PDEs and ODEs is investigated. Particularly, we are interested in reaction-diffusion equations and systems in bounded domains with delays in reaction terms. Different types of solutions (different phase spaces) are discussed. A recent study of viral in-host infection models with state-dependent delay is also described. It includes PDE and/or ODE infection models with /without CTL and antibody immune responses. A delay in such systems is a natural parameter (functional of the state) describing the biological delay between the time a virus particle contacts a target cell and the time the cell becomes actively infected (start producing new virions).

Sharp Conditions on Global Stability and Limit Cycles for Biological Control Systems

Yasuhisa Saito
Shimane University, Japan

To control or eradicate pest, providing their predators with additional food is known as a biological control method using the scenario of apparent competition; additional food increasing predators and also their effect on target prey (i.e., pest), thereby helping to decrease the abundance of the pest. In the context, this talk presents some predator-prey differential equations constructed by taking into account constant additional food on a general Holling-typed predation to discuss limit cycles and the global asymptotic stability of a positive equilibrium. These results suggest that the scenario of apparent competition mentioned above does not always hold, where increasing additional food may cause their large fluctuations, depending on the choice of additional food.

Numerical Bifurcation Analysis of Delay Equations in Biology

Francesca Scarabel
University of Helsinki, Finland
Dimitri Breda, Odo Diekmann, Mats Gyllenberg, Francesca Scarabel, Rossana Vermiglio

Delay equations are increasingly used in mathematical models for biological systems. Populations with age or size structure, for instance, can be described with a renewal equation for the population birth rate, possibly coupled with a delay differential equation for the environmental variable. But no software is capable of studying numerically the bifurcation properties of this kind of nonlinear systems. To address this gap, we propose the pseudospectral discretization technique as a way to approximate a general nonlinear delay equation with a low-dimensional system of ordinary differential equations, whose properties can be studied with existing software. The technique can be applied to systems coupling both renewal and delay differential equations, and involving bounded or unbounded delays. Using some numerical examples, we explore the effectiveness and flexibility of the method.

Special Session 65: Propagation Dynamics in Nonlinear Evolution Systems

Jian Fang, Harbin Institute of Technology, Peoples Rep of China
Xing Liang, University of Science and Technology of China, Peoples Rep of China
Xiao-Qiang Zhao, Memorial University of Newfoundland, Canada

This special session is focused on the theory, methods and applications of propagation dynamics in various nonlinear evolution systems. The topics include travelling waves, spreading speeds, asymptotic patterns, linear and nonlinear selection, temporal and spatial dynamics, local and nonlocal dispersal, and biological invasions.

Domain Decomposition Methods for a Spatial Heterogeneous Delay R-D Equation

Yuming Chen
Wilfrid Laurier University, Canada
Taishan Yi

We derive and study a spatial heterogeneous delayed reaction-diffusion equation, which models a single species with different mature-immature habitats accounting for a scenario where the boundaries are hostile to the species. We introduce new solid cones, analyze spectral bounds of several spatial heterogeneous operators, and establish limiting non-negativeness property for the whole space and the eventual comparison principle for bounded domains. As a result, we develop new domain decomposition methods so that one can compare solutions with those to associated equations from a suitable bounded spatial domain to the whole space. Then by employing domain decomposition methods and dynamical system approaches, we obtain threshold results under the supremum norm, which is greatly different from the existing results of other evolution equation in unbounded or all space. The main results are applied to two examples with the Ricker birth function and with the Mackey-Glass birth function. It reveals that the size of the immature habitat can affect the reproduction and spread of the population. This is a joint work with Dr. Taishan Yi.

Dynamics of Time-Periodic Reaction-Diffusion Equations with Compact Initial Support on R

Weiwei Ding
Meiji University, Japan
Hiroshi Matano

This work is concerned with the asymptotic behavior of bounded solutions of the Cauchy problem

$$\begin{cases} u_t = u_{xx} + f(t,u), & x \in \mathbb{R},\ t > 0, \\ u(x,0) = u_0, & x \in \mathbb{R}, \end{cases}$$

where u_0 is a nonnegative bounded function with compact support and f is periodic in t and satisfies $f(\cdot,0) = 0$. We first prove that the ω-limit set of any bounded solution either consists of a single time-periodic solution or it consists of time-periodic solutions as well as heteroclinic solutions connecting them. Furthermore, under a minor nondegenerate assumption on time-periodic solutions of the corresponding ODE, the convergence to a time-periodic solution is proved. Lastly, we apply these results to equations with bistable nonlinearity and combustion nonlinearity, and specify more precisely which time-periodic solutions can possibly be selected as the limit.

Accelerated Nonlocal Nonsymmetric Dispersion for Monostable Equations on the Real Line

Dmitri Finkelshtein
Swansea University, Wales

We consider the accelerated propagation of solutions to equations with a nonlocal linear dispersion on the real line and monostable nonlinearities (both local or nonlocal, however, not degenerated at 0), in the case when either of the dispersion kernel or the initial condition has regularly heavy tails at both $\pm\infty$, perhaps different. We show that, in such case, the propagation to the right direction is fully determined by the right tails of either the kernel or the initial condition. We describe both cases of integrable and monotone initial conditions which may give different orders of the acceleration. Our approach is based, in particular, on the extension of the theory of sub-exponential distributions, which we introduced early.

Genetic and Demographic Consequences of Fast Propagation

Jimmy Garnier
CNRS – University Savoie Mont-Blanc, France
E. Bouin, O. Bonnefon, J. Coville, C. Henderson, F. Patout, F. Hamel, L. Roques

Dispersal is one of the critical process during the spreading stage of biological invasions. In particular, long distance dispersal events may increase dispersal ability of invasive species and thus accelerate the spread of the biological invasion. These events might also have huge consequences on the genetic diversity of the species. During my talk, I will present nonlocal equations model which can take into account long distance dispersal events. I will first focus on the propagation properties of the solutions of these nonlocal equations. I will show that the tail of the dispersal kernel play a critical role in the spreading speed of the invasion. Then, I will present some results on the inside dynamics of these solutions.

Bistable and Multistable Pulsating Fronts in High Dimensions

Thomas Giletti
University of Lorraine, France

Travelling fronts typically connect two stationary states of a reaction-diffusion equation and often accurately describe the large-time behavior of solutions. However, when many such steady states exist, a more complicated dynamics may appear involving a layer of several fronts, or propagating terrace. In this talk, we will consider a spatially periodic problem of the multistable type, where the equation admits a finite sequence of ordered stable steady states. In this rather general context, we will show that a propagating terrace exists in any direction. Our approach relies on an abstract discrete framework together with an iterative argument. Surprisingly, due to the loss of symmetry induced by the heterogeneity, the shape of the propagating terrace may differ depending on the direction. In particular, travelling waves may exist only in some directions. This is a joint work with Luca Rossi from EHESS.

Population Persistence in a Benthic-Drift River Environment

Yu Jin
University of Nebraska-Lincoln, USA
F.-B. Wang

We consider a river environment where species grow on the benthos, drift in the water column and transfer between the water column and the benthos. We use reaction-diffusion-advection equations coupled with ordinary differential equations to describe the dynamics of a single species and of two competitive species. We study the population persistence criteria, based on persistence measures, including the net reproductive rate and eigenvalues of corresponding eigenvalue problems. We then use these measures to numerically investigate the influences of factors, such as the birth rate, various flow regimes, diffusion rates, competition rates, transfer rates, and spatial heterogeneity on population persistence. The theory developed here provides the basis for effective decision-making tools for water managers.

Invasion Speeds in Microbial Systems with Toxin Production and Quorum Sensing

Bingtuan Li
University of Louisville, USA
Sharon Bewick, Phillip P.A. Staniczenko, David K. Karig, William F. Fagan

The theory of invasions has traditionally been studied in macroscopic systems. Surprisingly, microbial invasions have received less attention. Although microbes share many of the features associated with competition between larger-bodied organisms, they also exhibit distinctive behaviors that require new mathematical treatments to fully understand invasions in microbial systems. We model bacterial invasion using a system of reaction-diffusion equations. Our model considers a competitive system with diffusible toxins that, in some cases, are expressed in response to quorum sensing. We derive analytical approximations for invasion speeds in the limits of fast and slow toxin diffusion. Interestingly, we find that toxins should diffuse quickly when used offensively, but that there are two optimal strategies when toxin is used as a defense mechanism. Specifically, toxins should diffuse quickly when their killing efficacy is high, but should diffuse slowly when their killing efficacy is low. Our approach permits an explicit investigation of the properties of diffusible compounds used in non-local competition, and is relevant for microbial systems and select macroscopic taxa, such as plants and corals, that can interact through biochemicals.

Entire Solutions of the Fisher-KPP Equation on the Half Line

Bendong Lou
Shanghai Normal University, Peoples Rep of China
Junfan Lu, Yoshihisa Morita

I will talk about entire solutions of the Fisher-KPP equation $u_t = u_{xx} + f(u)$ on the half line $[0, \infty)$ with Dirichlet boundary condition at $x = 0$. (1). For any $c \geq 2\sqrt{f'(0)}$, we show the existence of an entire solution $\mathcal{U}^c(x,t)$ which connects the traveling wave solution $\phi^c(x + ct)$ at $t = -\infty$ and the unique positive stationary solution $V(x)$ at $t = +\infty$; (2). We also construct an entire solution $\mathcal{U}(x,t)$ which connects the solution of $\eta_t = f(\eta)$ at $t = -\infty$ and $V(x)$ at $t = +\infty$. Our result presents a rather complete description on the relationship among the entire solutions.

Propagation Enhancement for Fisher-KPP Problems with Diffusion and Reaction Heterogeneities in Adjacent Domains

Andrea Tellini
Universidad Autonoma de Madrid, Spain
H. Berestycki and L. Rossi (EHESS, Paris)

I will consider a system of two Fisher-KPP type equations posed on adjacent domains, where the diffusion coefficients and the reaction terms are different. I will present the qualitative properties of the asymptotic speed of propagation of the solutions starting from compactly supported initial data. In particular, I will show the differences with respect to the case of a homogeneous environment, as well as the relations with the corresponding “road-field” model.

Propagation Dynamics of a Time Periodic and Delayed Reaction-Diffusion Model Without Quasi-Monotonicity

Zhi-Cheng Wang
Lanzhou University, Peoples Rep of China
Liang Zhang, Xiao-Qiang Zhao

In this talk we consider a time periodic non-monotone and nonlocal delayed reaction-diffusion population model with stage structure. We first prove the existence of the asymptotic speed c^* of spread by virtue of two auxiliary equations and comparison arguments. We then establish the existence of time periodic traveling wave solutions with speed $c \geq c^*$.

The Stability of Travelling Waves for Autocatalytic Reaction Systems with Or Without Decay

Yaping Wu
Capital Normal University, Peoples Rep of China
Yi Li, Niannian Yan

Consider the following general autocatalytic reaction system without or with decay

$$\begin{cases} u_t = d\triangle u - uf(v) \\ v_t = \triangle v + uf(v) - Kv^q. \end{cases}$$

In this talk we shall talk about our recent progress on the asymptotic stability of traveling fronts for the system without decay ($K = 0$) and the spectral stability of the wave solution (Front, Pulse) for the system with strong decay ($K > 0$ and $q > 1$). Our arguments are based on Evan's function method, spectral analysis, semigroup estimates and numerical simulation on Evans function method.

Logarithmic Corrections in Fisher-KPP Problems for the Porous Medium Equation

Maolin Zhou
University of New England, Australia
Yihong Du, Fernando Quirós

We consider the large time behaviour of solutions to the porous medium equation with a Fisher-KPP type reaction term and nonnegative, compactly supported initial function in $L^\infty(\mathbb{R}^N) \setminus \{0\}$:

$$(*) \quad u_t = \Delta u^m + u - u^2 \quad \text{in } Q := \mathbb{R}^N \times \mathbb{R}_+,$$

with $u(\cdot, 0) = u_0$ in $\mathbb{R}^N$. It is well known that the spatial support of the solution $u(\cdot, t)$ to this problem remains bounded for all time $t > 0$. In spatial dimension one it is known that there is a minimal speed $c_* > 0$ for which the equation admits a traveling wave solution Φ_{c_*} with a finite front, and this traveling wave solution is asymptotically stable in the sense that if the initial function $u_0 \in L^\infty(\mathbb{R})$ satisfies $\liminf_{x\to-\infty} u_0(x) > 0$ and $u_0(x) = 0$ for all large x, then $\lim_{t\to\infty} \left\{\sup_{x\in\mathbb{R}} |u(x,t) - \Phi_{c_*}(x - c_*t - x_0)|\right\} = 0$ for some $x_0 \in \mathbb{R}$. In dimension one we obtain an analogous stability result for the case of compactly supported initial data, not necessarily symmetric. In higher dimensions we show that Φ_{c_*} is still attractive, albeit that a logarithmic shifting occurs. More precisely, if the initial function in $(*)$ is additionally assumed to be radially symmetric, then there exists a second constant $c^* > 0$ independent of the dimension N and the initial function u_0, such that

$$\lim_{t\to\infty} \left\{ \sup_{x\in\mathbb{R}^N} |u(x,t) - \Phi_{c_*}(|x| - c_*t + (N-1)c^* \log t - r_0)| \right\} = 0$$

for some $r_0 \in \mathbb{R}$ (depending on u_0). If the initial function is not radially symmetric, then there exist $r_1, r_2 \in \mathbb{R}$ such that the boundary of the spatial support of the solution $u(\cdot, t)$ for all large time t is contained in the spherical shell $\{x \in \mathbb{R}^N : r_1 \leq |x| - c_*t + (N-1)c^* \log t \leq r_2\}$, and for any $c \in (0, c^*)$, $\lim_{t\to\infty} u(x,t) = 1$ uniformly in $\left\{|x| \leq c_*t - (N-1)c \log t\right\}$.

Special Session 66: Nonlinear and Nonlocal Evolution PDEs

Hantaek Bae, Ulsan National Institute of Science and Technology, Korea
Rafael Granero-Belinchon, Universidad de Cantabria, Spain

Nonlocal and nonlinear effects appear in many applications in natural processes as incompressible flows (Euler or Navier-Stokes equations), free boundary problems (water waves or Muskat/Hele-Shaw problem) or aggregation equations (Keller-Segel system and other chemotaxis problems), etc. This session is devoted to the mathematical analysis of nonlinear and nonlocal partial differential equations with a particular emphasis in equations arising in fluid dynamics and mathematical biology. This session will bring together experts in this research area to exchage ideas and to have in-depth discussion that will benefit each other.

Convergence of a Boundary Integral Method for 3D Interfacial Flow with Surface Tension

David Ambrose
Drexel University, USA

We present a boundary integral method for 3D interfacial Darcy flow with surface tension. We make several choices inspired by analytical work on well-posedness, including use of an isothermal parameterization of the free surface. The method uses a small-scale decomposition to remove stiffness, in the spirit of prior work by Hou, Lowengrub, and Shelley. In addition to showing results of the method as implemented, we will show some details of our proof of convergence of a version of the method. This includes joint work with Yang Liu, Nader Masmoudi, Michael Siegel, and Svetlana Tlupova.

A Nonlocal Approach for Waves of Maximal Height for the Reduced Ostrovsky Equation

Gabriele Bruell
Karlsruhe Institute of Technology, Germany
Raj Narayan Dhara

We discuss periodic traveling wave solutions of the reduced Ostrovsky equation, which arises in the context of long surface and internal gravity waves in a rotating fluid. Our aim is to reformulate this local equation into a nonlocal dispersive equation, and study the properties of their traveling waves by analyzing the corresponding convolution kernel. Of particular interest is the existence of a highest, peaked, traveling wave solution, which we obtain as a limiting case at the end of a global bifurcation branch. We show that the regularity at the crest of a highest wave is precisely Lipschitz. While the reduced Ostrovsky equation is of order -2, similar investigations have been done for equations of different orders. This work is part of a longer study with the aim of understanding the interaction of nonlinearities and dispersion, which leads to families of ever-higher waves, ending in a typically singular and highest one.

Fractional Patlak-Keller-Segel Equation

Jan Burczak
University of Oxford, Poland
Rafael Granero-Belinchón

We will present recent regularity results for the Patlak-Keller-Segel system with fractional diffusion, including

- Disproof of the finite-time blowup conjecture in the critical, 1d case.
- Global-in-time smoothness results in 1d and 2d for the logistically damped case, but with both supercritical diffusion and weak damping.
- Criteria for homogenous asymptotics.

Degraded Mixing Solutions for the Incompressible Porous Media

Angel Castro
ICMAT, Spain
Diego Cordoba, Daniel Faraco, Francisco Mengual

I will present the construction of degraded mixing solutions for the IPM system. This system models the dynamics of an incompressible and viscous fluid in a porous media and under the gravitational force. When the initial density of the fluid just takes two values the existence of solutions for IPM is known as the Muskat problem. In a previous work, together with D. Cordoba and D. Faraco, we showed the existence of solutions in the unstable regime which consist of the mixing of the two densities. In this talk I will skech a new construction in which we show that the solutions, in average, mix in a linear way. This is a work in collaboration with D. Faraco and F. Mengual Breton.

Stokes Expansions and Asymptotic Models of Water Waves

Ching-Hsiao Cheng
National Central University, Taiwan
Rafael Granero-Belinchon, Steve Shkoller

The study of irrotational incompressible Euler equations has been a long tradition in the fluid community. When the free surface was taken into account, both the theoretical study and robust numerical schemes become very challenging, especially for the case of deep water (Euler equations on a fluid domain with infinite depth). In the numerical side, various methods used to compute the Dirichlet-to-Neumann map (which is highly related to the water wave equations) proposed by W. Craig et al (1993) and M.J. Ablowitz et al (2006, 2008) involve highly ill-conditioned intermediate calculations (while the difficulties can be overcome by implementing multiple-precision arithmetic). The boundary integral collocation method and the transformed field expansion method are then introduced to avoid catastrophic cancellation of digits in the intermediate results; however, carrying out those methods in the three-dimensional case seems difficult. Therefore, the search for good asymptotic models for water waves become appealing for it might provide models that can be easily implemented and at the same time provide accurate enough evolution of the free surface. In this talk, I will present how the Stokes expansions can be used to derive asymptotic models up to any order.

A Full Dispersion Model for the Propagation of Gravity Waves in the Shallow Water Regime

Vincent Duchene
Univ. Rennes and CNRS, France
Samer Israwi, Raafat Talhouk, Dag Nilsson, Erik Wahlèn

We will present a model for the propagation of gravity water waves, which can be seen as a modification of the so-called Green-Naghdi system, where nonlocal operators (Fourier multipliers) have been inserted. In some sense, the model can be viewed as a fully nonlinear and bidirectional Whitham equation. We will then discuss some basic properties shared by a class of systems including the original Green-Naghdi system and the modified one, namely the well-posedness of the Cauchy problem, and the existence of solitary waves.

Propagation of Geometric Structures for Non-Homogeneous Fluids and Applications

Francesco Fanelli
Université de Lyon, France

The study of propagation of geometric structures for incompressible flows plays a fundamental role in understanding well-posedness and regularity issues for the homogeneous Euler equations (e.g. in the investigation of the regularity persistence of vortex patches). Some results in the last years allowed to extend the framework also to the case of (both viscous and inviscid) density-dependent incompressible fluids. In the present talk, we show how propagation of tangential regularity, combined with a maximal regularity approach, can be used to establish a well-posedness result for viscous compressible flows with only bounded density, in any space dimesion. This talk is based on a joint work with *Raphael Danchin* and *Marius Paicu*.

Regularity Vs Singularity for Immiscible Incompressible Navier-Stokes Fluids

Francisco Gancedo
University of Seville, Spain
Eduardo Garcia-Juarez

The mathematical analysis of fluid mechanics models in PDEs is a classical topic of research since Euler's 1757 paper, where the equation of an ideal flow was first derived. For the well established models, such us Navier-Stokes and Euler, the incompressible case presents basic and important open questions such as regularity and finite time singularity formation of the solutions. In this talk we consider several scenarios involving the interaction among incompressible fluids of different nature. The main concern is the dynamics of the free boundary separating the fluids, which evolves with the velocity flow. The important questions to address is whether the regularity is preserved in time or, on the other hand, the system develops singularities. We focus on Navier-Stokes models, where the viscosity of the fluids play a crucial role. At first showing results of finite time blow-up for the case of vacuum-fluid interaction. Later discussing new recent results on global existence for 1996 P.L. Lions' conjecture for density patches evolving by inhomogeneous Navier-Stokes equations.

Stability Shift for the Muskat Problem

Javier Gomez-Serrano
Princeton University, Spain
Diego Cordoba, Andrej Zlatos

The Muskat problem models the evolution of incompressible fluids of different nature in porous media. It is known that if the denser fluid is below the lighter fluid, the system is stable, and unstable otherwise. In this talk we will show the existence of solutions that transition between stable and unstable regimes several times, without breaking down. Joint work with Diego Cordoba and Andrej Zlatos.

Breaking and Disintegration in Shallow Water Models

Vera Mikyoung Hur
University of Illinois at Urbana-Champaign, USA

In the 1960s, Benjamin and Feir, and Whitham, discovered that a Stokes wave is unstable to long wavelength perturbations, provided that (the carrier wave number)$\times$(the undisturbed water depth)$> 1.363\dots$. In the 1990s, Bridges and Mielke studied the corresponding spectral instability in a rigorous fashion. But it leaves some important issues open, such as the spectrum away from the origin. The governing equations of the water wave problem are very complicated. One may resort to simple approximate models to gain insight. I will begin with wave breaking and ill-posedness in Whitham's shallow water model and Korteweg-de Vries equations with fractional dispersion. I will then discuss modulational instability of a small amplitude wave of the Whitham equation and an extension to large amplitude waves for a class of Hamiltonian systems, permitting nonlocal dispersion.

Axisymmetric Flow of Ideal Fluid Moving in a Narrow Domain

Tak Kwong Wong
The University of Hong Kong, Hong Kong
Robert M. Strain

In applications in blood flow and pipeline transport, the radial length scale of the underlying flow is usually small compared to the horizontal length scale. In this talk, we will introduce a new model called the axisymmetric hydrostatic Euler equations, which describe the leading order behavior of an ideal and axisymmetric fluid moving in such narrow channel. After providing the formal derivation, we will discuss the mathematical analysis of this model under a new sign condition. This is a joint work with Robert M. Strain.

Instantaneous Vortex-Stretching and Anomalous Dissipation on the 3D Euler Equations

Tsuyoshi Yoneda
University of Tokyo, Japan

By DNS of Navier-Stokes turbulence, Goto-Saito-Kawahara (2017) showed that the turbulence consists of a self-similar hierarchy of anti-parallel pairs of vortex tubes, in particular, stretching in larger-scale strain fields create smaller-scale vortices. Inspired by their numerical result, in this talk, we give a typical example of instantaneous vortex-stretching Euler flow, and suggest that it may be related to "anomalous dissipation" which is closely related to the famous Onsager conjecture.

Global Solutions to the Simplified Ericksen Leslie System

Yong Yu
The Chinese University of Hong Kong, Hong Kong
Yuan Chen, Soojung Kim

The Ericksen Leslie system is a hydrodynamical system in the theory of liquid crystal. It is a coupled system between Navier-Stokes equation and the transported harmonic map heat flow. In this talk we will present three global solutions to the Ericksen-Leslie system. They correspond to three global instability phenomena in nature. More precisely we will focus on oscillation instability, concentration instability and Freedericksz transition. They are caused by nontrivial circulation Reynolds number, non-trivial twist rate and high external magnetic fields, respectively.

Special Session 68: Viscosity Solutions: Beyond the Well-Posedness Theory

Hung V. Tran, University of Wisconsin Madison, USA
Hiroyoshi Mitake, Hiroshima University, Japan
Yifeng Yu, University of California Irvine, USA

The session focuses on some recent developments in the theory of viscosity solutions and related topics. The well-posedness theory of viscosity solutions has been developed since 1980s and has various connections to calculus of variations, differential games, dynamical systems, front propagations, homogenization theory, mean field games, etc. It is extremely important to go beyond the well-posedness theory and to study deep properties of the solutions and other quantities. We plan to invite people working on the aforementioned directions coming to discuss new aspects and developments, and to find common grounds to set up a long term research theme.

On and Beyond Propagation of Singularities

Wei Cheng
Nanjing University, Peoples Rep of China
Piermarco Cannarsa, Albert Fathi

In this talk, we will discuss the intrinsic approach of the propagation of singularities of viscosity solutions of the first order Hamilton-Jacobi equations, mainly based on the joint work with Piermarco Cannarsa and Albert Fathi. Let H be a Tonelli Hamiltonian. We consider the propagation of singularities along generalized characteristics by an intrinsic method. We will show that, for a prescribed solution u which has the representation in the form of inf-convolution, the relevant precess of sup-convolution determines the propagation of singulars and generalized characteristics for singular initial data. This method leads to the global result under mild Tonelli conditions. Based on this global result, we can discuss the associated singular dynamics in both topological and differential sense. We obtained the homotopy equivalence result between the complement of Aubry set and the cut locus of u, and the local path-connected result of the cut locus. We also studied the ω-limit set of the semi-flows defined by generalized characteristics, and their connections to the regular dynamics. We shall also discuss this problem under different type of initial/boundary conditions.

Weak Solutions to Mean-Field Games

Rita Ferreira
KAUST, Saudi Arabia
Diogo A. Gomes

In this seminar, we will address the problem of existence of weak solutions to monotone mean-field games. We introduce a notion of weak solution and prove the existence of such weak solutions under a minimal set of assumptions. We will also examine the properties of these solutions in several examples. Our approach is based on monotone operator methods and provides a general framework to construct weak solutions to MFGs with local, nonlocal, or congestion terms. This is a joint work with Diogo A. Gomes.

First-Order, Stationary Mean-Field Games with Congestion

Diogo Gomes
KAUST, Saudi Arabia
D. Evangelista, R. Ferreira, L. Nurbekyan, V. Voskanyan

Mean-field games (MFGs) are models for large populations of competing rational agents that seek to optimize a suitable functional. In the case of congestion, this functional takes into account the difficulty of moving in high-density areas. Here, we study stationary MFGs with congestion with quadratic or power-like Hamiltonians. First, using explicit examples, we illustrate two main difficulties: the lack of classical solutions and the existence of areas with vanishing densities. Our main contribution is a new variational formulation for MFGs with congestion. With this formulation, we prove the existence and uniqueness of solutions. Finally, we consider applications to numerical methods.

The Vanishing Discount Problem for Fully Nonlinear Degenerate Elliptic PDEs

Hitoshi Ishii
Tsuda University, Japan
Hiroyoshi Mitake, Hung V. Tran

I discuss an approach, based on generalized Mather measures, to the vanishing discount problem for fully nonlinear, degenerate elliptic, partial differential equations. Under mild assumptions, we introduce viscosity Mather measures for such PDEs, which are natural extensions of Mather measures, originally due to J. Mather. Using the viscosity Mather measures, one can show that the whole family of solutions v^λ of the discounted problem, with the discount factor λ, converges to a solution of the ergodic problem as λ goes to 0. This is based on joint work with Hiroyoshi Mitake (Hiroshima University) and Hung V. Tran (University of Wisconsin, Madison).

Holder Gradient Estimates for a Class of Singular Or Degenerate Parabolic Equations

Tianling Jin
Hong Kong University of Science and Technology, Hong Kong
Cyril Imbert, Luis Silvestre

We prove interior Holder estimates for the spatial gradients of viscosity solutions to a class of singular or degenerate parabolic equations, which unifies the proof of the "from L^∞ to Holder" regularity estimates for the parabolic p-Laplacian equations in both divergence form and non-divergence form.

Improved Regularity in the Periodic Homogenization of Hamilton-Jacobi Equations

Wenjia Jing
Tsinghua University, Peoples Rep of China
Hung V. Tran

We consider Hamilton-Jacobi equations with highly oscillatory Hamiltonians in space-time. In situations where homogenization theory is proved, we use the compactness method due to Avellaneda and Lin to show that the solutions to the HJ equations with oscillatory Hamiltonians have better regularity that is uniform with respect to the scale of oscillations in the environment.

Hamilton-Jacobi Equations with Large Hamiltonian Drift Terms

Taiga Kumagai
Waseda University, Japan

In this talk, we consider the asymptotic behavior of solutions of Hamilton-Jacobi equations with large Hamiltonian drift terms in an open subset of two dimensional Euclidean space. When the Hamiltonian admits degenerate critical points, we establish the convergence of solutions of the Hamilton-Jacobi equations and identify the limit of the solutions as the solution of systems of ordinary differential equations on a graph. The graph has one node and arbitrarily many edges depending on the Hamiltonian.

Large Exponent Behavior of Power-Type Evolution Equations and Applications

Qing Liu
Fukuoka University, Japan
Naoki Yamada

More precisely, we consider the level set formulation of the power curvature flow with a Lipschitz initial value. It is well known that for any given exponent there exists a unique viscosity solution. In this talk, we are interested in the limit behavior of such a solution as the exponent tends to infinity, which has applications in image denoising. When the initial value satisfies a convexity assumption, we show that the limit equation can be characterized as the following stationary obstacle problem involving 1-Laplacian. The large exponent behavior is more complicated when the convexity assumption of the initial value is dropped. In this case, we will mainly discuss a simplified problem with another application related to a math model describing unstable sandpiles. Part of this talk is based on joint work with Naoki Yamada at Fukuoka University.

Stability of Feedback Solutions for Infinite Horizon Noncooperative Differential Games

Tien Khai Nguyen
North Carolina State University, USA
Alberto Bressan

Consider a non-cooperative game in infinite time horizon, with linear dynamics and exponentially discounted quadratic costs. Assuming that the state space is one-dimensional, we prove that the Nash equilibrium solution in feedback form is stable under nonlinear perturbations. The analysis shows that, in a generic setting, the linear-quadratic game can have either one or infinitely many feedback equilibrium solutions. For each of these, a nearby solution of the perturbed nonlinear game can be constructed.

Regularity of Interfaces for a Pucci-Type Segregation Problem

Stefania Patrizi
UT Austin, USA
L. Caffarelli, V. Quitalo, M. Torres

Motivated by a model studied by V. Quitalo and describing population segregation, we consider a free boundary problem involving Pucci-type operators. We show the existence of a Lipschitz viscosity solution and prove that the set of regular points of the free boundary, i.e. the boundary of the positivity set of the solution, is relatively open and locally of class $C^{1,\alpha}$.

Incompressible Limit of the Porous Medium Equation with a Drift

Norbert Pozar
Kanazawa University, Japan

In this talk, I will present results concerning the singular limit of the solutions of the porous medium equations with a drift. Under the assumption that the drift field is compressive, we show that, in the incompressible limit, the solutions converge to the solution of a constrained transport equation with no diffusion. The limit solution reaches the constraint in a so-called congested set, which can be characterized by a certain Hele-Shaw type problem. This convergence result establishes the relationship of the diffuse interface models and sharp interface models of crowd motion and tumor growth. This is joint work with Inwon Kim and Brent Woodhouse.

A Finite Difference Method in Hamilton-Jacobi Equations and Weak KAM Theory

Kohei Soga
Keio University, Japan

I show that some finite difference scheme applied to Hamilton-Jacobi equations with Tonelli Hamiltonians provides convergent approximation of not only viscosity solutions but also their derivatives and characteristic curves at the same time. Then, I apply this approximation technique to weak KAM theory. These results are reminiscent of the approximation theories based on the vanishing viscosity/discount method.

Modulus of Continuity of Weak KAM Solutions

Jinxin Xue
Tsinghua University, Peoples Rep of China
Chong-Qing Cheng

It is an important problem to decide how regular is dependence of the weak KAM solutions on the cohomology classes. In this talk, we will explain a work in which we establish 1/3-Holder regularity dependence of the weak KAM solutions on an external parameter if the configuration space is the two-torus. We will explain how this is applied to our work on the problem of Arnold diffusion.

Special Session 69: Global or/and Blowup Solutions for Nonlinear Evolution Equations and Their Applications

Shaohua Chen, Cape Breton University, Canada
Ming Mei, McGill University, Canada
Runzhang Xu, Harbin Engineering University, Peoples Rep of China

This session is devoted to the recent developments in global or/and blowup solutions for nonlinear evolution equations and their applications, including reaction diffusion equations, fluid dynamics, delay, localized, non-local, degenerate evolution equations, traveling waves, steady states and their properties.

Initial Boundary Problem for a Class of Quasilinear Pseudo-Parabolic Equation with Nonlinear Source

Jianqing Chen
Fujian Normal University, Peoples Rep of China

In this paper, we discuss the Cauchy problem of pseudo-parabolic equation

$$u_t - \Delta u_t - \Delta u - \nabla \cdot \left(|\nabla u|^{2q} \nabla u\right) = u^p,$$

where $p \geq 2q+1$. Firstly, under $p < \frac{n+2}{n-2}$, we prove the local existence of weal solutions by the Galerkin method. Secondly, under $p > 2q+1$ and certain initial datum, we establish the blow-up in finite time and the global existence of solutions by the potential well method and the Poincaré inequality. Thirdly, we discuss the blow-up in finite time of solutions under $p = 2q+1$ and the non-positive initial energy. Fourthly, we study the blow-up in finite time of solutions under $p = 2q+1$, Δu vanishing and the negative initial energy. Finally, we determine a better lower bound for blow-up time if blow-up does occur.

Global Existence and Blow-Up of Positive Solutions for a Singular Gierer-Meinhardt System

Shaohua Chen
Cape Breton University, Canada

We discuss global existence and blow-up of positive solutions for a singular Gierer-Meinhardt system subject to zero Dirichlet boundary conditions. We first prove the existence of a local solution and then obtain a global solution and several blow-up solutions under various conditions for parameters.

Finite Time Blow Up for Two Dimensional Generalized Boussinesq Equations

Siyan Guo
Harbin Engineering University, Peoples Rep of China

In this talk, we consider the Cauchy problem of two dimensional generalized Boussinesq type equation. We prove the finite time blow up of the solution to the Boussinesq equation with arbitrarily positive initial energy for solutions. There are very few works on Boussinesq equation with nonlinear exponential growth term by potential well method. This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation oriented Talents Cultivation.

Entropy-Bounded Solutions of the Compressible Navier-Stokes Equations with Vacuum

Jinkai Li
The Chinese University of Hong Kong, Hong Kong
Zhouping Xin

The entropy is one of the fundamental states of a fluid and, in the viscous case, the equation that it satisfies is both degenerate and singular in the region close to the vacuum. In spite of its importance in the gas dynamics the mathematical analyses on the behavior of the entropy near the vacuum region, were rarely carried out; in particular, in the presence of vacuum, either at the far field or on the physical boundaries, it was unknown if the entropy retains its boundedness. It will be shown in this talk that the ideal gases retain their uniform boundedness of the entropy, locally or globally in time, for both the Cauchy problem and the initial-boundary value problems, if the vacuum occurs only at the far field or on the physical boundary, as long as the initial density behaves well at the far field or near the boundary.

Behaviour in Time of Solutions to a Class to Hyperbolic and Parabolic Systems

Monica Marras
University of Cagliari, Italy
Stella Vernier Piro, Giuseppe Viglialoro

We discuss blow-up phenomena to solution of some classes of parabolic systems under Neumann boundary conditions and nonlinear hyperbolic coupled systems of fourth order under Dirichlet or Navier boundary conditions. The solutions may blow up in finite time t^* and under appropriate assumptions on data, a safe interval of existence of the solution is derived with a lower bound of the lifespan. The proofs are based on some inequalities and coupled estimates techniques.

Decay in a Nonlinearly Damped Wave Equation with Variable Exponents of Nonlinearity

Salim Messaoudi
King Fahd University of Petroleum and Minerals, Saudi Arabia
A. Talahmeh

In this paper, we consider the following nonlinear wave equation with variable exponents:

$$u_{tt} - \Delta u + |u_t|^{m(\cdot)-2} u_t = 0.$$

By using a lemma by Komornik, we prove the decay estimates for the solution under suitable assumptions on the variable exponents m and the initial data.

On the Wave-Breaking Phenomena and Global Existence for the Periodic Rotation-Two-Component Camassa-Holm System

Byungsoo Moon
Incheon National University, Korea

Considered herein is the periodic rotation-two-component Camassa-Holm system, which can be derived from the f-plane governing equations for the geophysical water waves with a constant underlying current. The nonlocal nonlinearities on blow-up criteria and wave-breaking phenomena are established. Finally, a sufficient condition for global solutions is obtained by using a method of the Lyapunov function.

Global Well-Posedness of Damped Multidimensional Generalized Boussinesq Equations

Yi Niu
Shandong Normal University, Peoples Rep of China
Peng Xiuyan, Zhang Mingyou

We study the Cauchy problem for a class of sixth order Boussinesq equations with the generalized source term and damping term. By using Galerkin approximations and potential well methods, we prove the existence of global weak solution. Furthermore, we talk about the condition of damped coefficient k and obtain the finite time blow up solution.

Blow-Up Phenomena in Hyperbolic 4th Order Equations

Stella Maria Piro (Vernier)
University of Cagliari, Italy
Gerard Philippin

We consider some initial-boundary value problems for a class of nonlinear hyperbolic equations of the fourth order with time dependent coefficients, whose solution $u(x,t)$ may or may not blow up in finite time. Under suitable conditions on data, a lower bound for t^* is derived, where $[0, t^*)$ is the time interval of existence of $u(x,t)$. Moreover we discuss some extensions for some classes of nonlinear fourth order hyperbolic systems.

Global Solutions to a Class of Nonlinear Parabolic Equations

Maria Michaela Porzio
Sapienza University of Rome, Italy

We will prove the existence of global solutions to a class of nonlinear parabolic equations appearing in many physical applications being a nonlinear version of the heat equation. We will describe the regularity properties and the behavior in time of the solutions with special attention to the autonomous case. Finally, we will show that in the non autonomous case it is possible to estimate the difference between the solutions of the evolution problem and the solutions to suitable stationary problems.

Stability Results of Semi-Wavefronts in Reaction-Diffusion Equations with Delay

Abraham Solar
Pontifical Catholic University of Chile, Chile

This talk is about some (global and local) stability results of semi-wavefronts for the equation $u_t(t,x) = u_{xx}(t,x) - u(t,x) + \int_{\mathbb{R}} k(x-y)g(u(t-h,y))dy$ for $t > 0, x \in \mathbb{R}$ where $h \geq 0$ and the kernel $k \in L^1(\mathbb{R})$ is not assumed even. The monotonicity of g (monostable type) is not assumed and non monotone semi-wavefronts are considered. A typical example of this is the Nicholson's blowflies model where g is an unimodal function.

Cauchy Problem for Two-Dimensional Singularly Perturbed Boussinesq-Type Equation

Changming Song
Zhongyuan University of Technology, Peoples Rep of China
Jianlin Zhang, Li Chen

In this talk, we discuss the existence and uniqueness of the local classical solution to Cauchy problem for the two-dimensional singularly perturbed Boussinesq-type equation, and give the sufficient conditions of blow-up of the solution in finite time.

A Class of Nonlocal Parabolic PDEs and Nonlocal Curvature Flows

Xiaoliu Wang
Southeast University, Peoples Rep of China
Li Huiling, Tao Weirun, Tsai Dong-Ho, Wang Hengling

In this talk, we introduce a class of nonlocal parabolic PDEs with integral constraint, for which the maximum principle does not always hold. The sufficent and necessary conditions for the existence of global solutions are established, together with the behaviour of global solutions and nonglobal solutions. Moreover, a relative study on the evolution of nonlocal curvature flows with area-constraint or length-constraint will also be investigated.

Nonlinear Behavior of Impact System with Symmetry Barriers

Ligang Wang
Harbin Engineering University, Peoples Rep of China
Xueying Bai, Fuwang Dong, Hongyan Jing

In this talk, I will report our recent results on the nonlinear behavior of a kind of impact system subjected to harmonic loading. The system is represented by a cantilever beam which hits and rebounds from two symmetry rigid barriers during vibration. The periodic solution to the motion equations is formulated based on the ordinary differential equation theory. The numerical simulation is employed to analysis the nonlinear behavior. The grazing bifurcation and double-periodic bifurcation are discussed.

Boundedness, Stabilization and Pattern Formation Driven by Density-Suppressed Motility

Zhian Wang
Hong Kong Polytechnic University, Hong Kong
Haiyang Jin, Yong-Jung Kim

In two papers by C. Liu et al. (Science, 2011) and Fu et al. (Phys. Rev. Lett. 2012), a new argument driving spatio-temporal pattern formation by density suppressed motility through self-trapping was proposed and verified by a mathematical model. In this talk, we shall report some results on the density-suppressed motility model proposed therein. The main challenge arising in this model is the possible degeneracy of diffusion. In our work, we shall use the motility function as a weight function and employ the weighted energy estimates to get the boundedness of solutions and hence rule out the possible degeneracy to obtain the existence of global classical solutions. We also discuss the large-time behavior of solutions and pattern formations with numerical simulations. This is a joint work with Haiyang Jin (South China University of Technology) and Yong-Jung Kim (KAIST).

Global Attractor for a Strongly Damped Wave Equation with Fully Supercritical Nonlinearities

Zhijian Yang
Zhengzhou University, Peoples Rep of China
Zhiming Liu

In this talk, we investigate the existence of global attractor for a strongly damped wave equation with fully supercritical nonlinearities: $u_{tt} - \Delta u - \Delta u_t + h(u_t) + g(u) = f(x)$. In the case when the nonlinearities $h(u_t)$ and $g(u)$ are of fully supercritical growth, which leads to that the weak solutions of the equation lose their uniqueness, by introducing the notion of multi-valued operators, we establish some abstract criteria and use them to obtain the existence of global attractor of the equation in natural energy space in the sense of strong topology. Moreover, the geometrical structure of the global attractors of the corresponding multi-valued operators is shown.

Blow Up to the Initial-Boundary Value Problem for a System of M-Laplace Nonlinear Heat and Wave Equations

Linlin Zhai
Harbin Engineering University, Peoples Rep of China

In this talk, we consider the initial boundary value problem for a system of m-Laplace nonlinear heat and wave equations on a bounded domain. In the framework of potential well, we discuss the blow up of solutions with low and critical initial energy, the global existence and blow up of solutions with the high initial energy.
This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

Cooperative Output Regulation of Heterogenous Multi-Agent Systems Based on Passivity

Xuxi Zhang
Harbin Engineering University, Peoples Rep of China

This talk investigates the problem of cooperative output regulation of heterogenous linear multi-agent systems where the agent dynamics may be different. A passive framework is presented for the stabilization analysis of cooperative output regulation, which can overcome the difficulty caused by the fact that the global dynamics of heterogeneous multi-agent systems depends on the global communication structure. Based on passivity design, both a distributed state feedback protocol and a distributed output feedback protocol are designed for output synchronization of heterogenous multi-agent systems. The gain matrices of the distributed protocols and observers are obtained by a Riccati equation design approach. Furthermore, sufficient local conditions for solving the problem of cooperative output regulation of heterogenous multi-agent systems are presented. Finally, numerical simulation results are given to illustrate the effectiveness of the proposed distributed control schemes.

Finite Time Blowup for Kirchhoff-Type Equation Involving the Fractional Laplacian at High Energy Level

Meina Zhang
Harbin Engineering University, Peoples Rep of China

In this talk, we study the finite time blow up of the solution for degenerate Kirchhoff type hyperbolic equation involving the fractional Laplacian. By introducing a new auxiliary function and an adapted concavity method we establish some sufficient conditions on initial data such that the solution blow up in finite time with high initial energy level. This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.

Local Existence and Blow-Up of Solutions for Nonlocal Kirchhoff Diffusion Problems

Binlin Zhang
Heilongjiang Institute of Technology, Peoples Rep of China
Mingqi Xiang, Vicentiu D. Radulescu

In this talk, we discuss a diffusion model of Kirchhoff-type driven by a nonlocal integro-differential operator. Under some appropriate conditions, the local existence of nonnegative solutions is obtained by employing the Galerkin method. Then, by virtue of a differential inequality technique, we prove that the local nonnegative solutions blow-up in finite time with arbitrary negative initial energy and suitable initial values. Moreover, we give an estimate for the lower and upper bounds of the blow-up time. The main novelty is that the Kirchhoff term could be zero at the origin.

Special Session 70: Lie Symmetries, Conservation Laws and Other Approaches in Solving Nonlinear Differential Equations

Chaudry Masood Khalique, North-West University, So Africa
Muhammad Usman, University of Dayton, USA
Maria Luz Gandarias, University of Cadiz, Spain

This session is devoted to research areas that are related to nonlinear differential equations and their applications in science and engineering. The main focus of this special session is on the Lie symmetry analysis, conservation laws and their applications to ordinary and partial differential equations. Other approaches in finding exact solutions to nonlinear differential equations will also be discussed. This includes, but not limited to, asymptotic analysis methodologies, the simplest equation method, the multiple exp-function method, inverse scattering transform techniques, the upper-lower solutions method, the Hirota method, and others.

On the Solutions and Conservation Laws of a Two-Dimensional Korteweg de Vries Model: Multiple Exp-Function Method

Abdullahi Adem
North-West University, So Africa

Under investigation in this talk is a two-dimensional Korteweg de Vries model, which is a spacial extention of the Korteweg de Vries model. An infinite number of nonlocal conservation laws are given, which indicate the integrability of this model. Exact soliton solutions are then respectively derived by means of multiple exp-function method.

Common Errors in Finding Exact Solutions and Conservation Laws of Differential Equations

Stephen Anco
Brock University, Canada

Several years ago, two important papers [1,2] were published on common errors that are often made in investigations of finding exact solutions to nonlinear partial differential equations. These papers were motivated by a large (and still growing) number of publications appearing in the literature in which basic errors were repeated concerning correctness, generality, and novelty of exact solutions, as well as methods for their construction, and proper citation of previous work.
In the past decade, a similar situation has arisen with investigations of finding conservation laws of nonlinear partial differential equations. I will point out several common errors that are made in the literature on this topic. The errors deal with generality and novelty of conservation laws, as well as methods for their construction [3] and proper citation of previous work [4].

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Integrability Analysis of the Partial Differential Equation Describing the Classical Bond-Pricing Model of Mathematical Finance

Taha Aziz
North West University, Potchefstroom Campus, So Africa
Aeeman Fatima, C.M. Khalique

The invariant approach is employed to solve the Cauchy value problem for the bond-pricing partial differential equation (PDE) of mathematics of finance. We first briefly review the invariant criteria for scalar second-order parabolic partial differential equation in two independent variables. The criteria is then utilized to reduce the bond-pricing equation to different Lie canonical forms. We find that the invariant approach aids in transforming the bond-pricing equation to the second Lie canonical form and with a proper parametric selection, the bond-pricing PDE can be converted to the first Lie canonical form

which is the classical heat equation. Different cases have been deduced for which the original equation can be reduced to the first and the second Lie canonical forms. For each of the cases, we also work out the transformations which map the bond-pricing equation into the heat equation and also to the second Lie canonical form. We construct the fundamental solutions for the bond-pricing model via these transformations by utilizing the fundamental solutions of the classical heat equation as well as solution to the second Lie canonical form. Finally, the closed-form analytical solutions of the Cauchy initial value problems of the bond-pricing model with proper choice of terminal boundary conditions are also obtained.

Exact Solutions and Conservation Laws of a Generalized Variable-Coefficient Gardner Equation

Rafael de la Rosa
Universidad de Cádiz, Spain

In this talk, we study a generalized variable-coefficient Gardner equation from the point of view of symmetries. We determine the group-invariant solutions of this equation. Moreover, we present a complete classification of all low-order conservation laws.

Conservation Laws for a Double Dispersion Equation

Maria Luz Gandarias
University of Cádiz, Spain

In this talk we will present all the low-order conservation laws for a double dispersion equation which has many applications in mathematical physics.

A Model of Transmission Dynamics of Zika Fever with Horizontal, Vertical Transmission of Disease and Isolation

Mudassar Imran
Gulf University for science and technology, Kuwait

In this paper, a deterministic model is proposed to perform a thorough investigation of the transmission dynamics of Zika fever. Our model, in particular, takes into account the effects of horizontal as well as vertical disease transmission of both humans and vectors. The expression for the basic reproductive number R0 is determined in terms of horizontal as well as vertical disease transmission rates. An in-depth stability analysis of the model is performed, and it is consequently shown, that model has a globally asymptotically stable disease-free equilibrium when the basic reproduction number $R0 < 1$. It is also shown that the disease strongly uniformly persists when $R0 > 1$ and in this case there exists a unique endemic equilibrium which is locally asymptotically stable when $R0 > 1$. Assuming infection prevalence in the population under the constant control, optimal control theory is used to devise an optimal isolation control strategy for the model. The impact of isolation on the number of infected individuals and the accumulated cost is assessed and compared with the constant control case.

A Study of a (3+1)-Dimensional Modified Benjamin-Bona-Mahoney Equation

Chaudry Khalique
North-West University, So Africa

In this talk we study a physical space-time (3+1)-dimensional modified Benjamin-Bona-Mahoney model, which has applications in many scientific fields. We compute conservation laws for the underlying equation and present exact solutions of this equation.

A Novel Optimal Control Technique with Applications to Radiobiology

Adnan Khan
Lahore University of Management Sciences, Pakistan
Sultan Sial

Radiotherapy is an effective tool in the treatment of cancerous tumors. Dosage strategies currently employed vary form a large single dose or a few large doses (hypofractionation) to many smaller doses (hyperfractionation). These can be derived from a time-dose relationship widely used in radiotherapy, the Linear Quadratic (LQ) model. In this study we determine the optimal radiation dosing strategy, using a novel optimal control technique using mechanistic radiobiological models.

On a Diffusive Competing Lotka-Volterra Type System with an Advection Along Resources Gradient Under Lethal Boundary Environment

Kwangjoong Kim
Kookmin University, Korea
Wonhyung Choi, Inkyung Ahn

We study the effects of advection along environmental gradients on a diffusive competition model of the Lotka-Volterra type in a spatially heterogeneous environment. It is assumed that while the two species are ecologically equivalent, they adopt different dispersal strategies. One species disperses randomly, and the other species follows a directed path along the environmental gradient in addition to the random diffusion. It is known that the species with advection survives against the competing species with random dispersal when the boundary of the habitat is assumed to be a reflecting barrier to the population. In particular, sufficiently rapid movement in the direction of a source is always beneficial. In this study, a lethal environment is assumed at the boundary of a habitat.

We show that under the lethal boundary condition, movement up the gradient of a resource may be either beneficial or harmful to a given species depending on the properties of the resource function. Competitive exclusion and coexistence can occur depending on the situation of the environmental gradient and strength of advection. Simulations are presented based on our results in the 1-dimension case.

Characterization of Approximate Partial Hamiltonian Operators and Related First Integrals

Rehana Naz
Lahore School of Economics, Pakistan

The approximate partial Hamiltonian systems for the characterization of associated approximate operators and first integrals are investigated. It is proved that if an approximate operator is an approximate partial Hamiltonian operator which provides a first integral, then its evolutionary representative is an approximate partial Hamiltonian operator providing a first integral. The extra operator conditions are provided for an approximate partial Hamiltonian operator in evolutionary form to yield an approximate first integral. Moreover, characterization of approximate partial Hamiltonian operators and associated first integral conditions are explicitly provided for the approximate Hamiltonian system. Several examples are constructed to explain the theory.

Thermo-Chemo-Mechanical Coupling in Single Fluid Flows Through Porous Media

Lehlohonolo Phali
North-West University, So Africa

We consider a system that captures thermal, chemical and mechanical effects in the form of a solid, fluid and an interface, each having species and thermodynamic properties. Here we propose an approach which presents a more general setting for the system's entropy inequality. The approach we follow, with our slight variation was pioneered by Gray and Miller in their Thermodynamically Constrained Averaging Theory approach. Due to the dependence of fluid flow on the thermal conditions and flow conductivity, we modify Darcy's law to capture these effects. Depending on the temperature conditions and the flow forms we develop closed forms which are non-isothermal, locally isothermal and isothermal using the Darcy's law modifications. We present thermo-chemo-mechanical closed forms in the form of single-phase flow with species and thermo-mechanical closed forms in the form of single-phase flow without species.

Differential Invariants for a Class of Generalized Diffusion Equations

Muhammad Safdar
NUST Islamabad, Pakistan
Riaz A. Khan, Safia Taj

We consider a class of generalized diffusion equations to determine associated invariants. Equivalence group classification of this family of diffusion equations has already been done. We make use of the derived group of equivalence transformations to determine the associated invariants. We employ Lie infinitesimal method for deriving invariants. Using the obtained invariants we reduce the generalized diffusion equation to a much simpler class of this family.

Existence in Probability of Local and Global Strong Solution for Stochastic Magnetohydrodynamics Equations in Spaces of Besov Type

Tesfalem Tegegn
University of Pretoria, So Africa
Mamadou Sango

We established existence and uniqueness results for stochastic Magnetohydrodynamics equations in probabilistic evolution spaces of Besov type. This is done first by establishing regularity results for the stochastic heat equation with additive noise in the space $\mathcal{L}^{\sigma}_{\Omega}\mathcal{L}^{\infty}_{T}(\dot{B}^{\frac{s}{\sigma}}_{2,q}) \cap \mathcal{L}^{\sigma}_{\Omega}\mathcal{L}^{\sigma}_{T}(\dot{B}^{\frac{s+2}{\sigma}}_{2,q})$; where $s \in \mathbb{R}, \sigma \in [2, \infty)$ and $q \in [2, \infty]$. The regularity result is used to prove a global and local in time existence and uniqueness of solution to the stochastic Magnetohydrodynamics equation. The existence result holds with a positive probability which can be made arbitrarily close to one. The work is carried out by blending harmonic analysis tools such as Littlewood-Paley decomposition, Jean-Micheal bony paradifferential calculus and stochastic calculus. The law of large numbers is a key tool in our investigation.

An Efficient Space-Time Approximate Scheme for Nonlinear Burgers' Equation

Marjan Uddin
University of Engineeing and Technology Peshawar, Pakistan
Hazrat Ali

It is well known that the major error occurs in the time integration instead of the spatial approximation. In this work the anisotropic kernels are used for temporal as well as spatial approximation. We extended to construct a numerical scheme for solving nonlinear Burgers' equation. The time-dependent PDEs are collocated in both space and time dimension as contrary to spatial discretization first and then apply time steeping procedures for time integration. Physically one cannot in general expect that the spatial

and temporal features of the solution behaves on the same order. Hence one should have to incorporate anisotropic kernels. The nonlinear Burgers' equation are converted by nonlinear transformation to linear equation. The spatial discretizations are carried out to construct sparse differential differentiation matrices instead of dense full ill-conditioned differential matrices. Comparisons with most available numerical methods are made for solving the Burgers' equation.

Nonlinear and Stability Analysis for a Ship with a General Roll-Damping Model Using an Asymptotic Perturbation Method

Muhammad Usman
University of Dayton, USA

In this work, the response of a ship rolling in longitudinal waves is studied under a slow change in frequency. The model is one degree of freedom model for nonlinear ship dynamics. The model consists of the terms containing inertia, damping, restoring forces and external forces. The asymptotic perturbation method is used to study the primary resonance phenomena. The effects of various parameters are studied on the stability of steady states. It is shown that with the variation of bifurcation parameters affects the bending of bifurcation curve. The slope stability theorems are also presented.

Special Session 71: Qualitative Properties of Solutions to Local and Nonlocal Problems

Berardino Sciunzi, UNICAL, Italy
Luigi Montoro, UNICAL, Italy

The session is devoted to discuss recent trends in the study of qualitative properties of solutions to local and nonlocal problems. Elliptic or parabolic issues are addressed also for the case of degenerate operators.

Periodic Solutions for the One-Dimensional Fractional Laplacian

Begoña Barrios
Universidad de La Laguna, Spain

Our interest in the present talk is to deal with non-local problems posed in the whole real line $\mathbb{R}$

$$(-\Delta)^s u = f(u) \quad \text{in } \mathbb{R}. \tag{1}$$

Particular types of solutions have been obtained for problem (1) depending on the "shape" of the nonlinearity. For instance, layer solutions or ground states. However, at the best of our knowledge, the existence of periodic solutions to (1) has not been obtained so far except for a very particular f. We will introduce a suitable framework which allows to reduce the search for such periodic solutions to the resolution of a boundary value problem in a suitable Hilbert space, thereby making it possible to reach for the usual tools of nonlinear analysis, like bifurcation theory or variational methods.
We obtain some existence theorems which are lately enlightened with the analysis of some examples.

Sign-Changing Prescribed Gaussian Curvature

Francesca de Marchis
University of Rome Sapienza, Italy

I will consider the problem of prescribing the Gaussian curvature (under pointwise conformal change of the metric) on surfaces with conical singularities. This question has been first raised by Troyanov [TAMS,1991] and it is a generalization of the Kazdan-Warner problem for regular surfaces, known as the Nirenberg problem on the sphere. From the analytical point of view, this amounts to solve a singular Liouville-type equation on the surface. Initially, in the supercritical regime, only the case of positive prescribed Gaussian curvature has been attacked. In this talk I will present some new results (obtained in collaboration with T. D'Aprile, I. Ianni, S. Kallel, R. López-Soriano and D. Ruiz) concerning the sign-changing case.

On the Uniqueness of Positive Solutions to the Lane-Emden Problem in Convex Planar Domains

Isabella Ianni
Università degli Studi della Campania, Italy

This talk concerns the uniqueness of the solution of the semilinear elliptic problem

$$\begin{cases} -\Delta u = u^p & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \\ u > 0 & \text{in } \Omega \end{cases}$$

when $\Omega \subset \mathbb{R}^2$ is a convex smooth bounded domain and $p \in (1, +\infty)$. We give a partial answer to this longstanding open problem, proving the uniqueness for any finite energy solution when p is sufficiently large, where how large depends on the energy level considered.

Comparison Principle for Quasilinear Elliptic and Parabolic Equations

Tommaso Leonori
Sapienza, Università di Roma, Italy
M. Magliocca, A. Porretta, G. Riey

In this talk we discuss the comparison principle for quasilinear elliptic and parabolic equations with gradient terms. We focus our attention on the case in which the sub/supersolutions are smooth (say bounded) and irregular.

Maximum Principles at Infinity and the Ahlfors-Khas'Minskii Duality

Luciano Mari
Scuola Normale Superiore, Italy
B. Bianchini, M. Rigoli, P. Pucci, D. Valtorta and L. F. Pessoa

Maximum principles at infinity (or almost maximum principles), such as for instance Ekeland and Omori-Yau principles, are a powerful tool to investigate the geometry of Riemannian manifolds. Their validity is intimately related to the geometry of the underlying space, and exhibit deep relations with the theory of stochastic processes and to potential theory. In the first part of the talk, I will present a survey of a few geometric applications to motivate the study of these principles, and discuss their link with probability. Then, I will discuss a recent underlying duality with the existence of suitable exhaustion functions called Khasminskii potentials. Indeed, duality holds for a broad class of fully-nonlinear operators of geometric interest. This is based on joint works with B. Bianchini, M. Rigoli, P. Pucci, D. Valtorta and L.F. Pessoa.

Mixed Boundary Conditions in a Fractional Eigenvalue Problem

Maria Medina
Pontificia Universidad Catolica de Chile, Chile

In this talk we will analyze the non local problem

$$\begin{cases} (-\Delta)^s u &= \lambda_1 u & \text{in } \Omega, \\ u &= 0 & \text{in } D, \\ \mathcal{N}_s u &= 0 & \text{in } N, \end{cases}$$

where $s \in (0,1)$, λ_1 is the first eigenvalue associated to the problem, N and D are two open sets of $\mathbb{R}^N \setminus \Omega$ of positive measure satisfying

$$D \cap N = \emptyset, \left|\mathbb{R}^N \setminus (\Omega \cup D \cup N)\right| = 0,$$

and $\mathcal{N}_s u$ denotes the non local Neumann boundary condition of u. A natural question here is how the configuration of the sets D and N determines the behavior of u. Is it similar to the solution of the Dirichlet problem when N is small? Does it behave like the Neumann eigenfunction when N is large?
When $s = 1$ and the boundary conditions are prescribed on $\partial\Omega$ it is known that u behaves in very different ways depending on the size and the location of D and N. As we will see, the fact that in the fractional case the boundary is the whole $\mathbb{R}^N \setminus \Omega$ completely changes the possible configurations of the sets (one can even have both sets of unbounded measure). The purpose of this talk will be to translate the notion of "small" boundary set and to analyze wether D and N can be prescribed to recover the classical results.

This is a joint work with T. Leonori, I. Peral, A. Primo and F. Soria.

Nonlinear Helmholtz Equations: Infinite Energy Solutions

Benedetta Pellacci
Università della Campania "Luigi Vanvitelli", Italy
Eugenio Montefusco, Rainer Mandel

The aim of this talk is to present some existence results of radially symmetric oscillating solutions for a class of nonlinear autonomous Helmholtz equations and in particular to analyse their exact asymptotic behavior at infinity. Some generalizations to non-autonomous radial equations as well as existence results for non-radial solutions will be discussed. These results are linked with the existence of standing waves solutions of nonlinear wave equations with large frequencies.

End-To-End Construction of the Maximal Solutions of the Liouville Equation

Giusi Vaira
University of Campania Luigi Vanvitelli, Italy
M. Kowalczyk, A. Pistoia

We consider the Liouville equation $\Delta u + \lambda^2 e^u = 0$ with the Dirichlet boundary conditions in a two dimensional, doubly connected domain Ω. We show that there exists a simple, closed curve $\gamma \in \Omega$ such that for a sequence $\lambda_n \to 0$ there exist a sequence of solutions $u^{max}_{\lambda_n}$ such that $\frac{\lambda_n^2}{\log \frac{1}{\lambda_n}} \int_\Omega e^{u^{max}_{\lambda_n}}\, dx \to c_0|\gamma|$.

Special Session 72: Recent Developments in Problems of Fluid Mechanics

Chaudry Masood Khalique, North-West University, So Africa
Asim Aziz, National University of Sciences and Technology, Pakistan
Noreen S. Akbar, National University of Sciences and Technology, Pakistan

Fluid mechanics is the study of fluids either in motion or at rest. Liquids, gases and plasmas are all classified as fluids. The scope of fluid mechanics is vast and has countless applications in engineering and human activities. Researchers have extensively studied flow models for different geometries influenced by a number of factors including, fluid viscosity, bounding surface characteristics, external forces, boundary conditions, heat transfer analysis for Newtonian, non-Newtonian and nanofluids. The aim of this mini-symposium is to present and discuss recent advances in the physical understanding and applications of fluid flow using theoretical, numerical and experimental approaches.

Thermal Engineering Nano Model Study for Thermal Conductivity Knf/Kf of Shape Factor with Variable Fluid Properties

Noreen Akbar
National University of Sciences and Technology Pakistan, Pakistan

The present investigation is associated with the contemporary study of viscous flow with temperature dependent viscosity for different shape nanoparticles in a flexible tube. The main flow problem will be presented using cylindrical coordinates and flow equations with simplified to ordinary differential equations using longwave length and low Reynold's number approximation and exact solutions will be presented for velocity, pressure gradient and temperature. Results acquired will be discussed graphically for better understanding. It is will also be presented that velocity field is small for Platelets shape particles as compared to Bricks and cylinders shape particle.

Entropy Analysis for Slip Flow of Steady MHD Maxwell Nanofluid Over a Porous Stretching Surface with Variable Thermal Conductivity

Yasir Ali
National University of Sciences and Technology, Pakistan

In this article a mathematical model is developed to study the flow, heat transfer and entropy of the Maxwell nanofluid over a linearly stretching porous surface. The slip conditions are employed at the boundary and the thermal conductivity is considered as a function of temperature. Moreover a uniform magnetic field is applied in the transverse direction to the flow. The governing continuity, momentum, energy and entropy equations are first transformed using suitable similarity transformation technique and then solved numerically using shooting technique along with fourth order RK method. The graphs are plotted for non-dimensional velocity, temperature and entropy profiles for different values of governing parameters and are discussed from physical point of view.

The Shape Effects on Heat Transfer and Entropy of MHD Casson Nanofluid Over a Stretching Surface with Slip Condition, Thermal Radiation and Variable Thermal Conductivity

Asim Aziz
NUST, Pakistan

In the present research a simplified mathematical model is numerically studied for heat transfer and entropy generation analysis of non Newtonian nanofluids effected by the different shape of the nanoparticles. The flow is induced by the non-linear stretching of the porous horizontal surface with fluid occupying a space over the surface. Non Newtonain Casson fluid model is utilized for the nanofluids and thermal conductivity is to vary as a linear function of temperature. Moreover a uniform magnetic field is applied in the transverse direction to the flow with slip and convective boundary conditions are employed at the boundary. The mathematical formulation is carried out through boundary layer approach and the numerical computations are carried out using Keller box method. The numerical results are presented for five different nanoparticles shapes namely, sphere, hexahedron, tetrahedron, cylinder and lamina. The effect non dimensional physical parameters appearing in the system have been observed on temperature and entropy of the system, velocity gradient (skin friction coefficient) and the strength of convective heat exchange (Nusselt number).

Group Theoretical and Compatibility Analysis for Generalized Stokes Flow of Non-Newtonian Fluid in a Darcy Porous Medium with Suction/Blowing

Taha Aziz
North West University, Potchefstroom Campus, So Africa

In this study a generalized Stokes flow of a third grade non-Newtonian fluid model is investigated by the view point of group theoretical approach.

Heat Transfer and Entropy Analysis of Powell-Eyring Nanofluid Over a Stretching Surface Using Slip and Cattaneo-Christov Heat Flux Model

Wasim Jamshed
Capital University of Science and Technology, Pakistan
Asim Aziz

In this paper a simplified mathematical model is presented to study the flow, heat transfer and entropy generation analysis of Powell-Eyring nanofluid over an infinite horizontal surface. The flow is induced by the nonlinear stretching of the surface and partial slip conditions are assumed at the boundary. The Cattaneo-Christov heat flux model and thermal radiation effects are also considered in the present study. The governing system of partial differential equations are first transformed into a system of ordinary differential equation and then solved numerically using Keller box numerical scheme. Numerical computations are carried out for Copper-water and Titanium oxide-water nanofluids. Moreover five different types of nanoparticle shapes, namely, sphere, hexahedron, tetrahedron, cylinder and lamina are considered. The effect of non-dimensional physical parameters appearing in the system have been observed on the velocity, temperature and entropy profiles along with the velocity gradient and the rate of heat transfer at the boundary

Exact Solutions of the Modeling Equations of Natural Convection and Mass Transfer of MHD Fluid Flow Over an Oscillating and Translating Porous Plate

Arshad Alam Khan
National University of Sciences and Technology, Pakistan

The focus of this study is to analyze the slip flow of heat and mass transfer in an electrical conducting fluid over an oscillatory and translatory porous plate. This analysis is carried out for porous medium in presence of magnetic field. The governing system of PDEs, representing flow, energy and concentration of fluid, are transformed into a system of ODEs. These ODEs are solved analytically and effects of the various physical parameters like translation, magnetic parameter, heat source parameter, Grashof number for heat and mass transfer, permeability parameter, Schmidt number, Prandtl number and suction/injection parameter on the velocity, temperature and concentration profile are discussed in details.

Simulation of Mixed Convection Flow for Physiological Transport of Rabinowitsch Fluid Model with Convective Conditions

Hina Sadaf
NUST, Pakistan
S. Nadeem

The main objective of this paper is to discuss the simulation of mixed convection flow for physiological transport of Rabinowitsch fluid model with convective conditions. Peristaltic flow motion is considered in an inclined tube. Effects of mixed convection and convective boundary conditions are also considered for the Rabinowitsch fluid model. The present problem is modeled and exact solutions are managed under long wavelength and low Reynolds number approximation. We have used mathematica numerical simulation to calculate pressure rise. The effect of different parameters on temperature and velocity profile are displayed graphically. The characteristics of the peristaltic motion are inspected by plotting graphs and results for shear thinning and shear thickening fluids are discussed separately.

Rayleigh Surface Waves in Orthotropic Materials with Voids

Moniba Shams
National University of Sciences and Technology, NUST, Pakistan
Noureen Tayyab

The linear governing equation for the Rayleigh-type waves are formulated and solved with the appropriate boundary conditions at the free surface of an orthotropic half-space with voids. The frequency equation for Rayleigh waves is presented in its nondimensional form. It is found that these waves are dispersive and also dependent on the void parameters. For different orthotropic materials, graphs are presented to elaborate the effects of various void parameters and wave number on the wave speed.

Non-Newtonian Flow in Deformable Porous Media: Modeling and Simulations for Compression Molding Processes

Javed Siddique
Penn State University, USA

The aim of our study is to develop mathematical model which can assist in improving industrial manufacturing of composite materials by compression molding process. Modeling based on the unidirectional compression of preimpregnated layers of deformable media filled with non-Newtonian fluid. The main idea is to used Eulerian coordinates and then Lagrangian coordinates fixed on sold preimpregnated pile by considering the dynamics controlled by pressure applied on the piston or velocity of piston.

Semi Invariants for Linear Third Order Evolution Equation

Safia Taj
NUST Islamabad, Pakistan
M. Safdar, Riaz A. Khan

We obtain equivalence transformations for linear third order evolution equation by exploiting Lie infinitesimal approach. Invariants for this class of equations are calculated using the derived set of equivalence transformations. We find second order invariants for the third order evolution equation under transformations of the dependent and independent variables, separately. Further, these quantities are used to reduce the linear third order evolution equations with variable coefficients to a simpler equation of the same family with constant coefficients.

Effect of Magnetic Field on Liquid Thin Film Flow of Magnetic-Nanofluid Having Variable Viscosity and Thermal Conductivity Embedded with Graphene Nanoparticles

Iffat Zehra
Air University, PAF complex, E-9, Islamabad, Pakistan
E.N. Maraj

Nowadays graphene is emanating as one of the most encouraging nano-material due to its continuous electrical conducting behavior even at zero carrier concentration. Keeping this in mind, present article is a numerical investigation of flow and heat transfer of liquid film flow of magnetic nano fluid towards an extending surface under the influence of applied uniform magnetic field along with non-uniform internal heat generation/absorption and thermal radiation. Water and mixture of water and ethylene glycol are taken as the base fluids. Moreover, variable viscosity and variable thermal conductivity effects are incorporated for realistic analysis. By means of similarity analysis system of governing equations are simplified to nonlinear system of differential equations which is tackled numerically. Effect of pertinent parameters are examined through graphs and tables.

Special Session 73: Dynamics of Ordinary Differential Equations

Jifeng Chu, Shanghai Normal University, Peoples Rep of China
Juntao Sun, Shandong University of Technology, Peoples Rep of China
Zhaosheng Feng, University of Texas-Rio Grande Valley, USA

This session will be devoted to the study of the dynamical aspects of ordinary differential equations and related differential equations arising in physical and biological phenomena. The topics include (but not limited to) the existence of positive/periodic solutions, stability, bifurcation, persistence, oscillations, chaotic phenomena, homoclinic solutions, exponential dichotomy etc.

A Fast $O(N \log N)$ Second-Order Numerical Method for Space-Fractional Diffusion Equations

Treena Basu
Occidental College, USA
Greg Capra

Fractional diffusion equations have been proven to accurately model anomalous diffusion processes in nature. However, numerical schemes applied to space-fractional diffusion equations result in dense or full coefficient matrices with computational complexity and storage capacity of $O(N^3)$ per time step and $O(N^2)$ respectively, which is increasingly problematic for larger N. This talk seeks to provide a more efficient and robust algorithm for numerically approximating a second-order accurate numerical solution to the discretized one-dimensional two-sided space-fractional diffusion equation that requires only $O(N \log N)$ computational work per time step and $O(N)$ memory by utilizing the Crank-Nicolson scheme and studying the structure of the resulting coefficient matrix. A fast iterative scheme is used to solve the resulting system of equations. Numerical results are shown to illustrate the second-order accuracy and efficiency of the new method.

Group Actions on Monotone Skew-Product Semiflows with Applications

Feng Cao
Nanjing University of Aeronautics and Astronautics, Peoples Rep of China
Mats Gyllenberg, Yi Wang

We discuss a general framework of monotone skew-product semiflows under a connected group action and establish a theory concerning symmetry or monotonicity properties of uniformly stable 1-cover minimal sets. We then apply this theory to show rotational symmetry of certain stable entire solutions for a class of nonautonomous reaction-diffusion equations, as well as monotonicity of stable traveling waves of some nonlinear diffusion equations in time recurrent structures including almost periodicity and almost automorphy.

A Computable Criterion for the Existence of Silnikov Saddle-Focus Chaos

Brian Coomes
University of Miami, USA
Huseyin Kocak, Kenneth J. Palmer

Recently, we developed a general theory that guarantees the existence of an orbit connecting two hyperbolic equilibria of a parametrized autonomous differential equation in R^n near a suitable approximate connecting orbit generated using numerical methods. We applied this theorem to three systems in R^3, and, additionally, proved the existence of Silnikov saddle-focus chaos in each. The subject of this talk is recent work on proving the existence of Silnikov saddle-focus chaos in higher dimensions with an explicit example in R^4.

Traveling Wave Solutions of a Modified Vector-Disease Model

Zengji Du
Jaingsu Normal University, Peoples Rep of China

In this talk, we discuss the existence and asymptotic behavior of traveling wave fronts in a modified vector-disease model. We first establish the existence of traveling wave solutions for the modified vector-disease model without delay, then the existence of traveling fronts for the model with a special local delay convolution kernel are obtained by employing geometric singular perturbation theory and the linear chain trick. At last, we investigate the local stability of the steady states, the existence and the asymptotic behavior of traveling wave solutions for that model with a special non-local delay convolution kernel.

Approximate Analysis to the KdV-Burgers-Kuramoto Equation

Zhaosheng Feng
University of Texas-Rio Grande Valley, USA

In this talk, we consider the KdV-Burgers-Kuramoto equation, a partial differential equation that occupies a prominent position in describing some physical processes in motion of turbulence and other unstable process systems. We convert the problem into an equivalent 3D-dimensional system and analyze its local dynamical behaviors. By means of the Lie symmetry reduction method and the Preller-Singer procedure, we show that there exist nontrivial bounded wave solutions under certain parametric conditions. Numerical simulations of wave phenomena are illustrated, which provide us rich dynamical information and are in agreement with our theoretical analysis.

A Perron-Type Theorem for Nonautonomous Differential Equations with Different Growth Rates

Yongxin Jiang
Hohai University, Peoples Rep of China

We show that if the Lyapunov exponents associated to a linear equation $x' = A(t)x$ are equal to the given limits, then this asymptotic behavior can be reproduced by the solutions of the nonlinear equation $x' = A(t)x + f(t,x)$ for any sufficiently small perturbation f. We consider the linear equation with a very general nonuniform behavior which has different growth rates.

Homoclinic Solutions for a Higher Order Difference Equation

Lingju Kong
University of Tennessee at Chattanooga, USA

We study a higher order difference equation defined on the set of integers with p-Laplacian and containing both advance and retardation. By using the critical point theory, sufficient conditions are obtained for the existence of multiple homoclinic solutions of the equation.

Lifting Dynamics and Ergodicity in Stochastic Differential Equations

Xiliang Li
Shandong Technology and Business University, Peoples Rep of China
Xidong Sun, Baifeng Liu, Wei Liu

In this talk, we investigated the lifting dynamics in stochastic differential equations. Recurrent processes, recurrent measures and ergodicity are discussed. This is part of joint work with Prof. Xidong Sun, Baifeng Liu and Dr. Wei Liu

The Filtration of Nehari Manifold and Its Application in Some Nonlocal Problems

Juntao Sun
Shandong University of Technology, Peoples Rep of China
Tsung-fang Wu, Zhaosheng Feng

In this talk, by introducing a new set, which is regarded as a filtration of the Nehari manifold, and together with variational methods, we are concerned with the existence of positive solution for a class of nonautonomous nonlocal equations.

Non-Perturbative Positive Lyapunov Exponent of Schrödinger Operator with Skew-Shift Mapping

Kai Tao
Hohai University, Peoples Rep of China

We study the discrete analytic Schrödinger operator with a family of mappings. We first show that if the coupling number is large, then the Lyapunov exponent equals approximately to its logarithm. When applying it to the skew-shift mapping, we prove, that the Lyapunov exponent is positive and week Hölder continuous, and the spectrum satisfies Anderson Localization and contains large intervals. Moreover, all of these conclusions for the skew shift Schrödinger operator are non-perturbative.

Prevalence of Stable Periodic Solutions in the Forced Relativistic Pendulum Equation

Feng Wang
Nanjing Normal University, Peoples Rep of China
Jifeng Chu, Zaitao Liang

We study the prevalence of stable periodic solutions of the forced relativistic pendulum equation for external force which guarantees the existence of periodic solutions. We prove the results for a general planar system.

Lie Symmetry to Travelling Wave Solutions of Sharma-Tasso-Olver Equation

Rong Yin
Nantong University, Peoples Rep of China
Zhaosheng Feng, Jihui Zhang

Sharma-Tasso-Olver equation which includes a linear dispersive term and two nonlinear terms has the important status in scientific applications. In this talk we will study travelling wave solutions of Sharma-Tasso-Olver equation by Lie symmetry method. Firstly, we will get the infinitesimal generators through Lie symmetry method. Then according to the infinitesimal generators we can know that there exist the travelling wave solutions and get them by Lie symmetry method.

Quasi-Periodic Solutions of High Dimensional Schrödinger Equation with Liouvillean Basic Frequencies

Dongfeng Zhang
Southeast University, Peoples Rep of China
Junxiang Xu, Xindong Xu

In this paper we consider the high dimensional Schrödinger equation $-\frac{d^2y}{dt^2} + S(t)y = Ey, y \in \mathbb{R}^n$, where $E = \mathrm{diag}(\lambda_1^2, \ldots, \lambda_n^2)$ is a diagonal matrix, $S(t)$ is a real analytic quasi-periodic symmetric ma-

trix with basic frequencies $\omega = (1, \alpha)$, where α is irrational, it is proved that for most of sufficiently large $\lambda_j, j = 1, \ldots, n$, the Schrödinger equation has n pairs of conjugate quasi-periodic solutions, if the basic frequencies satisfy that $0 \leq \beta(\alpha) < r$, where $\beta(\alpha)$ measures how Liouvillean α is, r is the initial analytic radius.

Positive Homogeneity, Almost Periodicity, Rotation Number

Zhe Zhou
Academy of Mathematics and Systems Science, CAS, Peoples Rep of China

In this talk, we will establish the rotation number for the Schreodinger equation. The essential elements in the provement are positive homogeneity and almost periodicity. From this point of view, the concept of rotation numbers may be introduced in the case of discontinuous potentials.

Stability of Lyapunov Exponents, Weak Integral Separation and Nonuniform Dichotomy Spectrum

Hailong Zhu
Anhui University of Finance and Economics, Peoples Rep of China
Zhaoxiang Li, Xiuli He

A necessary and sufficient condition for the stability of Lyapunov exponents of linear differential system are proved in the sense that the equations satisfy the weaker form of integral separation instead of its classical one. Furthermore, the existence of full nonuniform exponential dichotomy spectrum under the condition of weak integral separateness is also presented.

Special Session 74: Perturbation Techniques in Stochastic Analysis and Its Applications

Stephane Menozzi, Université d'Evry-Val-d'Essonne, France
Arturo Kohatsu-Higa, Ritsumeikan, Japan
Valentin Konakov, HSE, Russia

Perturbation techniques have existed for a long time in the field of Functional Analysis and its near applications such as ordinary/partial differential equations. These techniques have proven very efficient in various fields in probability including stochastic analysis. They have been applied for example, in the study of resonance in engineering problems, in the oscillations problems in finance such as the Cox-Ingersoll-Ross model of interest rates. Still many challenging problems remain and the aim of this session is to bring distinguished researchers from different fields to promote a deeper discussion of the challenging problems that appear in applications and the techniques that researchers in different fields have developed. Therefore the goal of this session is to present the large scope of perturbative methods as applied in stochastic dynamical systems. Examples of these applications are: parametrix methods for the study of densities of linear and non-linear stochastic differential equations in finite and infinite dimensions with non-smooth coefficients, properties of optimal control problems, martingale problems, study of the heat equation in geometrical contexts, the role of noise in the construction of strong solutions for stochastic equations, Perturbation methods in order to obtain feasible numerical approximations with applications in biological or financial models, long time behavior of population dynamics through spectral analysis etc.

Well-Posedness for Some Non-Linear Diffusion Processes and Related PDE on the Wasserstein Space

Noufel Frikha
University Paris Diderot, France
Paul-Eric Chaudru de Raynal

During this talk, we present some results on the well-posedness of the martingale problem for non-linear stochastic differential equations in the sense of McKean-Vlasov under mild assumptions on the coefficients and a class of associated linear partial differential equations defined on $[0,T] \times \mathbb{R}^d \times \mathcal{P}_2(\mathbb{R}^d)$, for any $T > 0$, $\mathcal{P}_2(\mathbb{R}^d)$ being the Wasserstein space, that is, the space of probability measures on $\mathbb{R}^d$ with a finite second-order moment. The martingale problem is addressed by a perturbation argument on $\mathbb{R}^d \times \mathcal{P}_2(\mathbb{R}^d)$, for non-linear coefficients including any bounded continuous drift and diffusion coefficient satisfying some structural assumption in the measure sense that covers a large class of interaction. Under additional assumptions, we then establish the existence and smoothness of the associated density as well as Gaussian type bounds, the derivatives with respect to the measure being understood in the sense introduced by P.-L. Lions. Finally, existence and uniqueness for the related linear Cauchy problem with irregular terminal condition and source term among the considered class of non-linear interaction is addressed.

Schauder Estimates and Strong Uniqueness for Degenerate Kolmogorov Equation

Igor Honore
Université d'Evry, France
Paul-Eric Chaudru de Raynal, Stè phane Menozzi

We provide here some sharp Schauder estimates for degenerate partial differential equations of Kolmogorov type with non-linear first order term. Our method is based on parametrix techniques and provides, even in the non-degenerate framework, an alternative approach to Schauder estimates. As a consequence of our main estimates, we derive through suitable pointwise gradient bounds the well strong posedness of a chain of related degenerate stochastic differential equations.

L^P Estimates for Degenerate Non-Local Kolmogorov Operators

Lorick Huang
INSA Toulouse, France
Enrico Priola, Stephane Menozzi

Let $z = (x, y) \in \mathbb{R}^d \times \mathbb{R}^{N-d}$, with $1 \leq d < N$. We prove a priori estimates of the following type :

$$\|\Delta_x^{\frac{\alpha}{2}} v\|_{L^p(\mathbb{R}^N)} \leq c_p \Big\| L_x v + \sum_{i,j=1}^{N} a_{ij} z_i \partial_{z_j} v \Big\|_{L^p(\mathbb{R}^N)},$$

for $v \in C_0^\infty(\mathbb{R}^N)$, where L_x is a non-local operator comparable with the $\mathbb{R}^d$-fractional Laplacian $\Delta_x^{\frac{\alpha}{2}}$ in terms of symbols, $\alpha \in (0, 2)$.

We require that when L_x is replaced by the classical $\mathbb{R}^d$-Laplacian Δ_x, i.e., in the limit local case $\alpha = 2$, the operator $\Delta_x + \sum_{i,j=1}^N a_{ij} z_i \partial_{z_j}$ satisfy a weak type Hörmander condition with invariance by suitable dilations. Such estimates were only known for $\alpha = 2$. This is one of the first results on L^p estimates for degenerate non-local operators under Hörmander type conditions.
We complete our result on L^p-regularity for $L_x + \sum_{i,j=1}^N a_{ij} z_i \partial_{z_j}$ by proving estimates like

$$\|\Delta_{y_i}^{\frac{\alpha_i}{2}} v\|_{L^p(\mathbb{R}^N)} \le c_p \Big\| L_x v + \sum_{i,j=1}^N a_{ij} z_i \partial_{z_j} v \Big\|_{L^p(\mathbb{R}^N)},$$

involving fractional Laplacians in the degenerate directions y_i (here $\alpha_i \in (0, 1 \wedge \alpha)$ depends on α and on the numbers of commutators needed to obtain the y_i-direction). The last estimates are new even in the local limit case $\alpha = 2$ which is also considered.

Heat Trace Asymptotics for Equiregular Sub-Riemannian Manifolds

Yuzuru Inahama
Kyushu University, Japan
Setsuo Taniguchi

We study a div-grad type sub-Laplacian with respect to a smooth measure and its associated heat semigroup on a compact equiregular sub-Riemannian manifold. We prove a short time asymptotic expansion of the heat trace up to any order. Our main result holds true for any smooth measure on the manifold, but it has a spectral geometric meaning when Popp's measure is considered. Our proof is probabilistic. In particular, we use S. Watanabe's distributional Malliavin calculus. (This is a joint work with Setsuo Taniguchi.)

The Laplacian on Circle Packing Fractals: from Kesten's Renewal Theorem, via Conformal Dynamics, to Weyl's Eigenvalue Asymptotics

Naotaka Kajino
Kobe University, Japan

The purpose of this talk is to present the speaker's recent results on the construction of a "canonical" Laplacian on circle packing fractals invariant under the action of certain Kleinian groups (i.e., discrete groups of Moebius transformations) and on the asymptotic behavior of its eigenvalues. In the simplest case of the Apollonian gasket, the Laplacian was constructed by Teplyaev (2004) as one with respect to which the coordinate functions on the gasket are harmonic, and the author has recently proved its uniqueness and discovered an explicit expression of it in terms of the circle packing structure of the gasket. The expression of the Laplacian actually makes sense on general circle packing fractals and defines (a candidate of) a "canonical" Laplacian on such fractals. When the fractal is the limit (i.e., minimum invariant non-empty compact) set of a certain class of Kleinian groups, some explicit combinatorial structure of the fractal is known and makes it possible to prove Weyl's asymptotic formula for the eigenvalues of this Laplacian, which is of the same form as the circle-counting asymptotic formula by Oh and Shah [Invent. Math., 2012]. Its proof is based on a serious application of Kesten's renewal theorem [Ann. Probab., 1974].

Large Time Behavior of Randomly Perturbed Dynamical Systems

Leonid Koralov
University of Maryland, USA

We will discuss several asymptotic problems for randomly perturbed flows. One class of flows (with regions where a strong flow creates a trapping mechanism) leads to a new class of boundary value problems with non-standard boundary conditions. We will also discuss how large-deviation techniques can be used to study the asymptotic behavior of solutions to quasi-linear parabolic equations with a small parameter at the second order term and the long time behavior of the corresponding diffusion processes.

Continuity and Gaussian Two-Sided Bounds of the Density Functions of the Solutions to Path-Dependent Stochastic Differential Equations Via Perturbation

Seiichiro Kusuoka
Okayama University, Japan

We consider Markovian stochastic differential equations with low-regular coefficients and their perturbations by adding a measurable bounded path-dependent drift term. When we assume the diffusion coefficient matrix is uniformly positive definite, then the solution to the perturbed equation is given by the Girsanov transformation of the original equation. By using the expression we obtain the Gaussian two-sided bounds and the continuity of the density function of the solution to the perturbed equation. The perturbation here is a stochastic analogue to the perturbation in the operator analysis.

Weak Uniqueness and Density Estimates for SDEs with Coefficients Depending on Some Path-Functionals

Libo Li
University of New South Wales, Australia
Noufel Frikha

We develop a general methodology to prove weak uniqueness for stochastic differential equations with coeffcients depending on some path-functionals of the process. As an extension of the technique developed by Bass & Perkins [BP09] in the standard diffusion case, the proposed methodology allows one to deal with processes whose probability laws are singular with respect to the Lebesgue measure. To illustrate our methodology, we prove weak existence and uniqueness in two examples: a diffusion process with coeffcients depending on its running symmetric local time and a diffusion process with coeffcients depending on its running maximum. In each example, we also prove the existence of the associated transition density and establish some Gaussian upper-estimates.

Some Properties of Density Functions on Maxima of One-Dimensional Diffusion Processes

Tomonori Nakatsu
Shibaura Institute of Technology, Japan

In the talk, let X be a solution to one-dimensional stochastic differential equations (SDEs), and consider discrete and continuous time maximum of the solution which are denoted as M_T^n and M_T, respectively. We will show some important properties of $p_{M_T^n}$ and p_{M_T}, where $p_{M_T^n}$ and p_{M_T} denote probability density functions of M_T^n and M_T, respectively. In particular, we shall obtain expressions, upper bounds for $p_{M_T^n}$ and p_{M_T} and that $p_{M_T^n}$ converges to p_{M_T} as $n \to \infty$. The key of the proofs is to show integration by parts formulas for M_T^n and M_T by means of the Malliavin calculus. If time permits, we will consider some other properties of the density functions which have been obtained recently.

The Parametrix Method for Parabolic SPDEs

Andrea Pascucci
University of Bologna, Italy
Antonello Pesce

We consider the Cauchy problem for a linear stochastic partial differential equation. By extending the parametrix method for PDEs whose coefficients are only measurable with respect to the time variable, we prove existence, regularity in Hölder classes and estimates from above and below of the fundamental solution. This result is applied to SPDEs by means of the Itô-Wentzell formula, through a random change of variables which transforms the SPDE into a PDE with random coefficients.

Implicit Euler-Maruyama Scheme for Non-Colliding Particle Systems

Dai Taguchi
Osaka University, Japan
Hoang-Long Ngo

In this talk, we consider a discrete approximation for non-colliding particle systems such as Dyson's Brownian motions with drift and Brownian particle systems with nearest neighbour repulsion. As a numerical analysis of a solution to stochastic differential equation, one often approximates it by using the "explicit" Euler-Maruyama scheme. However, unfortunately, the explicit scheme does not preserve the non-colliding property of non-colliding particle systems. Therefore, we introduce implicit Euler-Maruyama scheme which preserve the non-colliding property, and study its rate of convergence in L^p-norm.

On Solutions of McKean-Vlasov Equations with Irregular Coefficients

Alexander Veretennikov
University of Leeds, England

Stochastic Itô–McKean–Vlasov equation is considered in R^d,

$$dX_t = B[t, X_t, \mu_t]dt + \Sigma[t, X_t, \mu_t]dW_t, \qquad X_0 = x_0,$$

under the convention

$$B[t, x, \mu] = \int b(t, x, y)\mu(dy), \;\; \Sigma[t, x, \mu] = \int \sigma(t, x, y)\mu(dy).$$

Here W is a standard d_1-dimensional Wiener process, b and σ are, respectively, vector and matrix Borel functions of corresponding dimensions d and $d \times d_1$, μ_t is the distribution of the process X at t. The initial data x_0 may be random, but independent of W. New weak and strong existence and weak and strong uniqueness results for the equation are established under relaxed regularity conditions as well as a "propagation of chaos" (convergence of multi-particle approximations). The talk is based on a joint work with Yulia Mishura and on discussions and a work in progress with David Šiska and Lukasz Szpruch.

Small Time Asymptotic for Joint Density of System Driven by Gaussian Process

Tai-Ho Wang
Baruch College, CUNY, USA

In this talk we present a bridge representation for the finite dimensional joint density of stochastic process driven by Gaussian processes. A small time approximation of the joint density is readily obtained by approximate the bridge representation by a single deterministic path, which in the classical case recovers the heat kernel expansion for diffusion processes. Applications of such small time approximations include small time asymptotic for prices and implied volatilities of European or Asian equity options, options on realized variance, as well as an approximate maximum likelihood estimator.

Special Session 75: Mathematics and Materials: Models and Applications

Marco Morandotti, Technical University of Munich, Germany
Marco Barchiesi, University of Naples Federico II, Italy
Thomas Hudson, University of Warwick, England

This special session will provide an overview of recent results in the mathematical modelling of materials and in the description and simulation of their behaviour. The analysis of materials involves understanding processes at a wide variety of length scales, and the development of models that can capture this complexity. Mathematical challenges arise in rigorously verifying that models are well-posed and self-consistent, and in finding rigorous links between them. Meeting some of these challenges has in the past led to the developments in the theories of the Calculus of Variations, homogenisation, gradient flows for dissipative phenomena, and techniques to rigorously link discrete and continuum models. New results which aim to model the evolution of materials both over long time scales and at the smallest length scales continue to generate novel theories which offer the possibility of rich new Mathematics. The scope of this special session is to bring together both leading experts and young researchers in the field to foster discussion and exchange of ideas.

Convergence of Discrete and Continuous Unilateral Flows for Phase-Field Energies

Stefano Almi
TU München, Germany
Sandro Belz, Matteo Negri

In my talk I will present some results regarding the convergence of an alternate minimization scheme for a phase-field model of fracture. This algorithm is characterized by the lack of irreversibility constraints in the minimization of the phase-field variable. From a computational stand point, the advantage of such a choice is in the efficiency of the numerical implementation. Irreversibility is then recovered a posteriori by a simple pointwise truncation. Exploiting a time discretization procedure, I will show, in continuous and discrete settings, the convergence of time-discrete solutions to a unilateral L2-gradient flow with respect to the phase-field variable.

A Simple Discrete Model for Damage Exhibiting Infinitely-Many Phases, and a Continuum Counterpart

Andrea Braides
University of Rome Tor Vergata, Italy

We give a variational interpretation of a model by Novak and Truskinovsky of a discrete systems where microscopic fracture produces macroscopic damage. Depending on the arrangement at a discrete level the overall response corresponds to different damaged materials. We give a formal continuum counterpart depending on a small parameter mimicking the discrete dimension. The homogenized description as this parameter tends to zero departs from the previous one in certain regimes. Work in collaboration with A. Causin, M. Solci and L. Truskinovsky.

Lattice Green's Function in the Anti-Plane Crack Geometry

Maciej Buze
University of Warwick, England
Thomas Hudson, Christoph Ortner

In order to establish qualitatively sharp regularity estimates for solutions of discrete equilibration problems employed to model crystal defects, one needs to first establish existence of a corresponding lattice Green's function and prove its decay properties. In this talk I will first briefly discuss how to do it in a spatially homogeneous setup, explain why it is non-trivial in a non-homogeneous setup of a crack defect and present a way of tackling this problem.

Modeling and Analysis of Nematic Elastomer Membranes

Pierluigi Cesana
Kyushu University, Japan

I will review recent results on modeling the microstructure in nematic liquid crystal elastomers in the thin-film regime.

Field-Driven Domain Wall Motion in Bilayer Piezoelectric Nanostructures with Inertial and Nonlinear Dissipations Effects

Sharad Dwivedi
SRM Insitute of Science & Technology, Chennai, India
Shruti Dubey

In this work, we investigate the field-driven domain wall motion in bilayer piezoelectric nanostructure under the influence of inertial and crystallographic defects. We examine the magnetization dynamics in the framework of the generalized Landau-Lifshitz-Gilbert equation of micromagnetism that includes the effects of nonlinear dissipations, inertia, and piezo-induced strains. The obtained analytical results explicate the threshold and Walker-type breakdown estimates of an applied magnetic field in both the steady-state and precessional regimes. Finally, the interaction of inertial, nonlinear dissipations, and piezo-induced strains and their physical implications are discussed analytically and numerically in both the dynamical regimes.

Limiting Dynamics of a System of N Particles Interacting Via Attractive/Repulsive Potentials

Maria Stella Gelli
University of Pisa, Italy

We study the limit as the number of particles goes to infinity of the dynamics of a system of N particles moving through velocity fields of convolution type and subject to an additional separation constraint (hard-spheres model). This can be formalised by introducing a suitable control problem and assuming that the velocity fields are Lipschitz regular. The limiting dynamics is then described by a particle density $\rho(t, x)$ satisfying a suitable continuity equation of Vlasov type an uniform upper bound.

Gradient Flow Structure of the Maxwell-Zener Model for Viscoelasticity

Masato Kimura
Kanazawa University, Japan
Hirofumi Notsu, Yoshimi Tanaka, Hiroki Yamamoto

Maxwell-Zener type viscoelastic model is studied from mathematical and numerical points of view. It is shown that the model has a gradient flow property with respect to viscoelastic energy. Based on the gradient flow structure, a structure-preserving time-discrete model is proposed and existence of a unique solution is proved. Moreover, a structure-preserving P1/P0 finite element scheme is presented and its stability is also shown by its discrete gradient flow structure. As typical viscoelastic phenomena, two-dimensional numerical examples by the proposed scheme for a creep deformation and a stress relaxation are shown.

Quasiconvex Elastodynamics: Weak-Strong Uniqueness for Measure-Valued Solutions

Konstantinos Koumatos
University of Sussex, England
Stefano Spirito

A weak-strong uniqueness result is presented for dissipative measure-valued solutions to the system of conservation laws arising in elastodynamics. The main novelty of this work is that the underlying stored-energy function is assumed strongly quasiconvex. The proof borrows tools from the calculus of variations to prove a Garding type inequality for quasiconvex functions, and recasts them to adapt the relative entropy method to quasiconvex energies.

Asymptotic Rigidity of Layered Structures and Its Applications in Homogenization Theory

Carolin Kreisbeck
Utrecht University, Netherlands
Fabian Christowiak

Rigidity results in elasticity are powerful statements that allow to derive global properties of a deformation from local ones. The classical Liouville theorem states that every local isometry of a domain corresponds to a rigid body motion. If connectedness of the set fails, clearly, global rigidity can no longer be true. In this talk, I will present a new type of asymptotic rigidity lemma, which shows that if an elastic body contains sufficiently stiff connected components arranged into fine parallel layers, then macroscopic rigidity up to horizontal shearing prevails in the limit of vanishing layer thickness. The optimal scaling between layer thickness and stiffness can be identified using suitable bending constructions. This result constitutes a useful tool for proving homogenization results of variational problems modeling high-contrast bilayered composites. We will finally utilize it to characterize the homogenized Gamma-limits of two models inspired by nonlinear elasticity and finite crystal plasticity.

Variational Modelling of Nematic Elastomer Foundations

Andres A. Leon Baldelli
CNRS FR, IMSIA Lab, France
Pierluigi Cesana

We compute the Γ-limit of energy functionals describing mechanical systems composed of a thin nematic liquid crystal elastomer sustaining a homogeneous and isotropic elastic membrane. We work in the regime of infinitesimal displacements and model the orientation of the liquid crystal according to the order tensor theories of both Frank and De Gennes. We describe the asymptotic regime by analysing a family of functionals parametrised by the thickness of the membranes and the relative ratio of the elastic constants, establishing that, in the limit, the system is represented by a two-dimensional integral functional interpreted as a linear membrane on top of a nematic "active foundation" involving an effective De Gennes optic tensor which allows for low order states. The latter can suppress shear energy by formation of microstructure as well as act as a pre-strain transmitted by the foundation to the overlying film. We then compute a phase diagram of the effective macroscopic behaviour of the homogeneous volume element and finally provide numeric computations solving a boundary value problem for a structure of interest for applications.

A Reshetnyak-Type Lower Semicontinuity Result for Linearised Elasto-Plasticity Coupled with Damage

Gianluca Orlando
Technische Universitaet Muenchen, Germany
Vito Crismale

In this talk I will present a lower semicontinuity result of Reshetnyak type for a class of functionals which appear in models for small-strain elasto-plasticity coupled with damage. The plastic potentials that we consider depend on Sobolev functions α, the damage variables, and on bounded Radon measures p, the plastic strains. The main difficulty in the proof of the lower semicountinuity of the plastic potential lies in the lack of continuity of the damage variable. To prove the result, obtained in collaboration with Vito Crismale, we characterise the limit of measures $\alpha_k \, \mathrm{E}u_k$ with respect to the weak convergence $\alpha_k \rightharpoonup \alpha$ in $W^{1,n}(\Omega)$ and the weak* convergence $u_k \overset{*}{\rightharpoonup} u$ in $BD(\Omega)$, E denoting the symmetrised gradient. A concentration compactness argument shows that the limit has the form $\alpha \, \mathrm{E}u + \eta$, with η supported on an at most countable set. This is the key for the proof of the result.

Heteroclinic Connections and Chaotic Orbits for Nonlocal Equations

Stefania Patrizi
UT Austin, USA
E. Valdinoci, S. Dipierro

We consider a system of nonlocal equations driven by a perturbed periodic potential. We construct multi-bump solutions that connect one integer point to another one in a prescribed way. In particular, heteroclinic, homoclinic and chaotic trajectories are constructed. The description of the stationary positions for the atom dislocation function in a perturbed crystal, as given by the Peierls-Nabarro model, is a particular case of the result presented.

Microscopic Justification of a Model for Epitaxially-Strained Crystalline Films

Paolo Piovano
University of Vienna, Austria
Leonard Kreutz

A discrete-to-continuum analysis for free-boundary problems related to crystalline materials deposited on crystalline substrates is performed by employing the notion of Γ-convergence and rigidity estimates. The discrete models take into account the possibility of a mismatch between the lattice parameters of the film and the substrate at the crystalline equilibrium. In view of a proper scaling regime the resulting continuum limiting model is characterized by a configurational energy that includes both the surface and the bulk elastic energy. As such, it well describes the two competing mechanisms responsible for the profile shape of epitaxially-strained thin films on substrates. On the one hand, the lattice mismatch may generate large stresses in the film forcing film atoms to move in order to release the elastic energy, thus possibly generating profile corrugations. On the other hand, more regular profiles may be preferable to minimize the surface energy.

Stochastic Homogenisation of Free-Discontinuity Problems

Lucia Scardia
University of Bath, England
Filippo Cagnetti, Gianni Dal Maso, Caterina Ida Zeppieri

In this talk I will present a stochastic homogenisation result for free-discontinuity functionals. Assuming stationarity for the random volume and surface integrands, we prove the existence of a homogenised random free-discontinuity functional, which is deterministic in the ergodic case. Moreover, by establishing a connection between the deterministic convergence of the functionals at any fixed realisation and the pointwise Subadditive Ergodic Theorem by Akcoglou and Krengel, we characterise the limit volume and surface integrands in terms of asymptotic cell formulas. Therefore, our qualitative homogenisation result extends to the SBV-setting the classical qualitative results by Papanicolaou and Varadhan, Kozlov, and Dal Maso and Modica, which were formulated in the more regular Sobolev setting.

Well-Posedness of One-Dimensional Nonlinear Viscoelasticity with Limited Strain

Yasemin Sengul
Sabanci University, Turkey
Husnu Ata Erbay, Albert Erkip

We are interested in finding solutions of nonlinear differential equations describing the behaviour of one-dimensional viscoelastic medium with implicit constitutive relations. We focus on a subclass of such models known as the strain-limiting models introduced by Rajagopal. To describe the response of viscoelastic solids we assume a nonlinear relationship among the linearized strain, the strain rate and the Cauchy stress. We first look at traveling wave solutions that correspond to the heteroclinic connections between the two constant states, and establish conditions for the existence of such solutions, and find them explicitly, implicitly or numerically, for various forms of the non-linear constitutive relation. Then we consider corresponding Cauchy and boundary-value problems from both modelling and analysis points of view. This research is partially funded by TUBITAK through the project 116F093.

Analytical and Numerical Approach to a Class of Damage Models

Marita Thomas
Weierstrass Institute Berlin, Germany

This presentation deals with techniques for the spatial and temporal discretization of models for rate-independent damage featuring a gradient regularization. Different methods in dependence of the choice of the gradient term and related convergence results are discussed. The ideas are also demonstrated for a phase-field fracture model at finite strains, which takes into account the anisotropy of damage by applying an anisotropic split and the modified invariants of the right Cauchy-Green strain tensor. Numerical examples in two and three space dimensions are presented.

Overview of Discrete-To-Continuum Limit Passages of Nonlocally Interacting Particles in 1D

Patrick van Meurs
Kanazawa University, Japan

I will present an overview of discrete-to-continuum limit passages of nonlocally interacting particles where the unknowns are the positions of the particles in a one-dimensional domain. Such particle systems can be used as models for dislocations in metals, cellular aggregation, granular media, pedestrian movement, opinion formation and predator–prey models. The interesting common feature of these results is that numerical simulations show striking agreements between the particle systems and the related continuum models. However, any rigorous result beyond Γ-convergence of the particle interaction energy in the many-particle limit calls for novel mathematical ideas.

Special Session 77: Advances in Mathematical Modelling and Numerical Simulation of Superfluids

Ionut Danaila, University of Rouen Normandy, France
Weizhu Bao, National University of Singapore, Singapore

This special session will address some of the most challenging recent problems concerning the mathematical modelling and numerical simulation of superfluid systems (Bose-Einstein condensate and superfluid Helium). The objective of the session is to bring together applied and pure mathematicians, physicists, as well as computational scientists who have contributed to the development of relevant mathematical and computational models in this area. The topics which will be discussed include mathematical models for complex quantum systems (spinor BEC, quantum turbulence), high-order numerical methods for the Gross-Pitaevskii equation, high-performance computing of realistic configurations corresponding to physical quantum systems.

Modeling, Analysis and Simulation for Degenerate Dipolar Quantum Gas

Weizhu Bao
National University of Singapore, Singapore

In this talk, I will present our recent work on mathematical models, asymptotic analysis and numerical simulation for degenerate dipolar quantum gas. As preparatory steps, I begin with the three-dimensional Gross-Pitaevskii equation with a long-range dipolar interaction potential which is used to model the degenerate dipolar quantum gas and reformulate it as a Gross-Pitaevskii-Poisson type system by decoupling the two-body dipolar interaction potential which is highly singular into short-range (or local) and long-range interactions (or repulsive and attractive interactions). Based on this new mathematical formulation, we prove rigorously existence and uniqueness as well as nonexistence of the ground states, and discuss the existence of global weak solution and finite time blowup of the dynamics in different parameter regimes of dipolar quantum gas. In addition, a backward Euler sine pseudospectral method is presented for computing the ground states and a time-splitting sine pseudospectral method is proposed for computing the dynamics of dipolar BECs. Due to the adoption of new mathematical formulation, our new numerical methods avoid evaluating integrals with high singularity and thus they are more efficient and accurate than those numerical methods currently used in the literatures for solving the problem. In addition, new mathematical formulations in two-dimensions and one dimension for dipolar quantum gas are obtained when the external trapping potential is highly confined in one or two directions. Numerical results are presented to confirm our analytical results and demonstrate the efficiency and accuracy of our numerical methods. Some interesting physical phenomena are discussed too.

Energy Preserving Methods for Nonlinear Schrödinger Equations

Christophe Besse
Université de Toulouse, France

The Schrödinger equation is at heart of Bose Einstein Condensates with the celebrated Gross Pitaevskii equation. Such dispersive partial differential equations have many preserved quantities such as the mass, the energy or the momentum. It is therefore crucial to build numerical schemes that preserve these invariants. We will present in this talk various way to take this property into account and the pros and cons of the various methods.

Bose-Einstein Condensates with Higher Order Interactions

Yongyong Cai
Beijing CSRC, Peoples Rep of China
Weizhu Bao, Xinran Ruan

We analyze the ground states of a Bose-Einstein condensate in the presence of higher-order interactions (HOI), modeled by a modified Gross-Pitaevskii equation (MGPE). In fact, due to the appearance of HOI, the ground state structures become very rich and complicated. We establish the existence and nonexistence results under different parameter regimes, and obtain their limiting behaviors and/or structures with different combinations of HOI and contact interactions. Both the whole space case and the bounded domain case are considered, where different structures of ground states are identified. In addition, HOI effects on dynamics are investigated.

Finite-Element Tools for the Simulation of Bose-Einstein Condensates

Ionut Danaila
University of Rouen Normandy, France
F. Hecht, B. Protas, G. Vergez

We present several numerical tools using classical finite elements with mesh adaptivity for solving different models used for the study of Bose-Einstein condensates. The programs are written as a toolbox for FreeFem++ (www.freefem.org), a free finite-element software, allowing to easily implement various numerical algorithms [1]. For solving the stationary (imaginary-time) Gross-Pitaevskii equation, we use two robust and optimised numerical methods: a steepest descent method based on Sobolev gradients and a minimization algorithm based on the state-of-the-art optimization library IPOPT.

A very recent conjugate-gradient method using concepts of Riemannian optimization is also presented [2]. For the Bogoliubov-de Gennes system, representing a linearisation of the Gross-Pitaevskii equation, a Newton method and a fast algorithm based on ARPACK for the calculation of eigenvalues are available. For the real-time Gross-Pitaevskii equation, classical splitting and relaxation methods were implemented and intensively tested. Validations and illustrations are presented for computing difficult configurations with vortices observed in physical experiments: single-line vortex, Abrikosov lattice, giant vortex, dark/anti-dark solitons in one or two-component Bose-Einstein condensates [3].

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On the Modeling and Simulation of Anyons Systems

Romain Duboscq
Institut de Mathematiques de Toulouse, France
Correggi, Lundholm, Rougerie

In this talk, we will present recent results on the modeling and simulation of anyons systems. Our main interest lies in the case of an average-field model where we consider a system with a large number of anyons in the ground state. Thanks to an average-field energy which ressembles the usual Gross-Pitaevskii energy but with a nonlinear magnetic potential, we can apply a gradient method that enables us to numerically compute the ground state. However, due to the nonlinear and nonlocal structure of the energy functional, some numerical difficulties need to be tackled.

Some Theoretical Studies on the Stochastic Gross-Pitaevskii Equation

Reika Fukuizumi
Tohoku University, Japan
Anne de Bouard, Arnaud Debussche

After the realization of Bose-Einstein condensation twenty years ago, one of the interests in the theoretical physics community is the modeling of cold atoms systems. In this talk we focus on the model which describes the influence of thermal effects around the Bose Einstein condensate at positive temperature. The model is a stochastic complex Ginzburg-Landau equation under a harmonic trapping potential, We present some theoretical results on the existence of solution and exponential convergence to the thermal equilibrium.

Modelling Nonlinear Schrödinger Superfluid Turbulence

Michikazu Kobayashi
Kyoto University, Japan

Nonlinear Schrödinger equation (NSE) is the simplest model for superfluid systems such as liquid helium and a dilute gas at ultralow temperatures. Superfluid turbulence can be realized as spatially and dynamically complicated structure of quantized vortices. Quantized vortices are vortices with discrete circulations, and stable topological line defects. Here we show our recent large-scale numerical works for superfluid turbulence with NSE. The first topic is fully-developed turbulence and existences of two scaling regimes with the Kolmogorov's -5/3 power-law spectra separated by the mean inter-vortex spacing. The second topic is the opposite case to the first one; transition from superfluid turbulence to vortex-free dissipationless flow. We can regard this transition as simplest non-equilibrium phase transition having three independent critical exponents.

Computation of Ground States of the Gross-Pitaevskii Functional Via Riemannian Optimization

Bartosz Protas
McMaster University, Canada
Ionut Danaila

This presentation concerns a novel approach to the computation of ground states in Bose-Einstein condensates where we combine concepts from Riemannian Optimization and the theory of Sobolev gradients to derive a new conjugate gradient method for direct minimization of the Gross-Pitaevskii energy functional with rotation. The conservation of the number of particles in the system constraints the minimizers to lie on a Riemannian manifold corresponding to the unit L2 norm. The idea developed in our study is to transform the original constrained optimization problem to an unconstrained problem on this (spherical) Riemannian manifold, so that faster minimization algorithms can be applied. We first obtain Sobolev gradients using an equivalent definition of an H^1 inner product which takes into account rotation. Then, the Riemannian gradient (RG) steepest descent method is derived based on projected gradients and retraction of an intermediate solution back to the constraint manifold. Then, we use the concept of the Riemannian vector transport to propose a new Riemannian conjugate gradient (RCG) method for this problem. It is derived at the continuous level based on the optimize-then-discretize paradigm instead of the usual discretize-then-optimize approach, as this ensures robustness of the method when adaptive mesh refinement is performed in computations. Numerical tests carried out in the finite-element setting based on Lagrangian piecewise quadratic space discretization demonstrate that the proposed RCG method outperforms the simple gradient descent RG method in terms of rate of convergence. The RCG method is extensively tested by computing complicated vortex configurations in rotating Bose-Einstein condensates, a task made challenging by large values of the non-linear interaction constant and the rotation rate. Finally, we will also discuss our on-going work on the design and implementation of the Riemannian Newton method for the minimization of the Gross-Pitaevskii energy functional.

A Robust and Efficient Numerical Method to Compute the Dynamics of the Rotating Two-Component Dipolar Bose-Einstein Condensates

Yong Zhang
University of Vienna, Austria
Qingin Tang, Norbert Mauser

We present a robust and efficient numerical method to compute the dynamics of the rotating two-component dipolar Bose-Einstein condensates (BEC). Using the rotating Lagrangian coordinates transform, we reformulate the original coupled Gross-Pitaevskii equations (CGPE) into new equations where the rotating term vanishes and the potential becomes time-dependent. A time-splitting Fourier pseudospectral method is proposed to numerically solve the new equations where the nonlocal Dipole-Dipole Interactions (DDI) are computed by a newly-developed Gaussian-sum (GauSum) solver which helps achieve spectral accuracy in space within $O(N \log N)$ operations (N is the total number of grid points). The new method is spectrally accurate in space and second order accurate in time - these accuracies are confirmed numerically. Dynamical properties of some physical quantities, including the total mass, energy, center of mass and angular momentum expectation, are presented and confirmed numerically. Interesting dynamical phenomena that are peculiar to the rotating two-component dipolar BECs, such as dynamics of center of mass, quantized vortex lattices dynamics and the collapse dynamics in 3D, are presented.

Special Session 78: Advances in Qualitative Theory of Differential, Difference and Dynamic Equations

Elvan Akin, Missouri University S&T, USA
Billur Kaymakcalan, Cankaya University, Turkey
Agacik Zafer, American University of the Middle East, Kuwait

The subject of differential equations and difference equations has a long and enriched history ranging from their theoretical developments to tremendous applications in a variety of applied fields. Dynamic equations (time scales calculus) unifies the differential equations and difference equations, and equations in between. In this special session, we aim to bring together the leading researchers to highlight the recent achievements in the related fields. The talks will focus on the qualitative and quantitative properties of differential equations, difference equations, fractional equations, impulsive equations, and dynamic equations with a particular emphasis on the following topics:

- Asymptotic behavior
- Boundedness of solutions
- Oscillation theory
- Representations of solutions
- Stability theory
- Boundary value problems
- Inequalities
- Numerical methods

Modeling the Dynamics of HIV-1 Decline in Patients on Protease Inhibitor Monotherapy on Time Scales

Elvan Akin
Missouri S&T, USA
Gulsah Yeni

The theory of time scales, nonempty closed subsets of real numbers, was first introduced by Stefan Hilger in 1988 in order to unify the continuous and discrete analysis. The continuous case of HIV-1 dynamics in vivo is modeled by systems of first order differential equations in 1996. Our main purpose in this study is to analyze patient data on continuous and discrete cases. To achieve our goal, we formulate initial value problems for modeling the dynamics of HIV-1 on time scales and calculate the total concentration of plasma virions on different time scales. Then, we use Mathlab with nonlinear least squares regression to fit these models to the data.

Connecting Caputo-Fabrizio Fractional Derivatives Without Singular Kernel and Proportional Derivatives

Douglas Anderson
Concordia College Moorhead, USA

A direct connection will be made between the new Caputo-Fabrizio definition of fractional derivatives without singular kernel (2015, 2017) and proportional derivatives (2015), which are motivated by proportional-derivative controllers in control theory. The definition and properties of proportional derivatives will be explored, particularly the kind of calculus that they generate.

On Difference Equations of the Type $X_{N+1} = F(\bar{X_n})$

Mohamed Ben Haj Rhouma
Qatar University, Qatar

Difference equations of the form

$$x_{n+1} = f(x_n, x_{n-1}, \ldots x_{n-k})$$

assume that the system has a limited memory and is independent of time.
In this talk we will examine the behavior of difference equations of the form

$$x_{n+1} = f(\bar{x_n})$$

where $\bar{x_n} = \sum_{i=0}^{n} \omega(n,i)x_i$ is an averaged value of all previous values of x_j and $\omega_{n,i} \geq 0$ are appropriate weights.

Fractional Lyapunov-Type Inequalities

Rui Ferreira
University of Lisbon, Portugal

In this talk we will present our recent results, regarding some inequalities of Lyapunov type, in which fractional derivatives appear somehow. Then we will discuss some possible directions for future research.

Harmonic Measure Distribution Functions for a Class of Multiply Connected Symmetrical Slit Domains

Christopher Green
Macquarie University, Australia
Marie A. Snipes, Lesley A. Ward

Consider releasing a Brownian particle from a basepoint z_0 in a planar domain $\Omega \subset \mathbb{C}$. What is the chance, denoted $h_{\Omega,z_0}(r)$, that the particle's first exit from Ω occurs within a fixed distance $r > 0$ of z_0? The function of r suggested by this question, $h_{\Omega,z_0} : [0,\infty) \to [0,1]$, is called the harmonic measure distribution function, or h-function, of Ω with respect to z_0. We can think of the h-function as a signature that encodes the geometry of the boundary of Ω. In the language of PDEs, the h-function can also be formulated in terms of a suitable Dirichlet problem on Ω. For simply connected domains, the theory of h-functions is now quite well developed and several explicit results are known. However, until recently, for multiply connected domains the theory of h-functions has been almost entirely out of reach. We will show how to construct explicit formulae for the h-functions of symmetric multiply connected slit domains $\hat{\Omega}$ whose boundaries consist of an even number of colinear slits. We will employ the special function theory of the Schottky-Klein prime function $\omega(\zeta,\gamma)$ and its associated constructive methods in conformal mapping to build explicit formulae for the h-functions of domains $\hat{\Omega}$ with any finite even number of slits.

Periodic and Almost Periodic Solutions in Gross-Substitute Discrete Dynamical Systems

Yoshihiro Hamaya
Okayama University of Science, Japan

We consider the existence of periodic and almost periodic solutions of a gross-substitute discrete system, which appear as tatonnement processes of mathematical economic models, by using uniformly stable, uniformly asymptotically stable and luxurious Lyapunov functionals.

Asymptotic Behaviour of the Population Models with Piecewise Constant Argument

Fatma Karakoc
Ankara University, Turkey

In this talk, we consider some population models with piecewise constant argument under impulse effect. First, we introduce the relation between the solutions of an impulsive differential equation with piecewise constant argument and difference equations. Then we investigate asymptotic behaviour of the positive equilibrium point of the model by using theory of difference equations. Finally we present impulse effect to the solutions of the population models.

Lyapunov Type Inequality for Linear Dynamic Hamiltonian Systems

Zeynep Kayar
Van Yuzuncu Yil University, Turkey

By using matrix measure approach, we obtain a new Lyapunov type inequality for linear $2n \times 2n$ Hamiltonian systems on arbitrary time scales. The results obtained not only unify the related continuous and discrete ones but also generalize and improve them. As an application, we present new diconjugacy criteria for linear Hamiltonian systems and find new lower bound estimates for eigenvalues of associated eigenvalue problems for such systems.

Bennet and Leindler Type Dynamic Inequalities

Billur Kaymakçalan
Çankaya University, Turkey
Neslihan Nesliye Pelen

This presentation will serve as a survey of the Hardy, Copson, and its converses, which are Bennet, and Leindler type inequalities in the classical continuous, discrete cases as well as their time scales developments in the delta calculus set-up. Nabla time scale versions of these results, recently obtained by the authors, will also be given. Finally, an attempt of a proposition will be made in the more general dynamic inequalities sense of the above mentioned cases, by use of the diamond-alpha type integrals, so that all of the aforementioned results may be obtained as consequences of this most general set-up of time scale calculus.

Best Constant in Hyers–Ulam Stability of First-Order Nonhomogeneous Linear Difference Equations with a Constant Stepsize

Masakazu Onitsuka
Okayama University of Science, Japan

In this talk, we deal with Hyers–Ulam stability of the first-order nonhomogeneous linear difference equation $\Delta_h x(t) - ax(t) = f(t)$ on $h\mathbb{Z}$, where $\Delta_h x(t) = (x(t+h) - x(t))/h$ and $h\mathbb{Z} = \{hk | \, k \in \mathbb{Z}\}$ for the constant stepsize $h > 0$; a is a real number; $f(t)$ is a real-valued function on $h\mathbb{Z}$. The purpose of this talk is to answer the question "how does the stepsize influence the best HUS constant for the linear equation on $h\mathbb{Z}$?"

On the Applications of Impulsive Preadtor-Prey Dynamic Systems

Neslihan Nesliye Pelen
Ondokuz Mayis University, Turkey

Impulsive dynamic systems are very important, since they have many applications in the real life. In this study, one of its examples which is impulsive predator-prey systems are considered and some analytic results about the solutions are given. Mainly, this results are about the periodicity and stability of the solutions.

Non-Monotonic Solutions to Nonlinear Second-Order Differential Equations with Damping

Yuriy Rogovchenko
University of Agder, Norway
Mervan Pašić

We are concerned with the behavior of solutions to the following two classes of nonlinear second-order differential equations with a damping term:

$$(r(t)x')' + p(t)x' + q(t)x + f(t,x) = e(t), \quad t \geq t_0, \quad (1)$$

and

$$(r(t)x')' + p(t)x' + q(t)g(x) = e(t), \quad t \geq t_0, \quad (2)$$

where $e(t)$ is a continuous non-homogeneous term, and nonlinear terms $f(t,x)$ and $g(x)$ satisfy, respectively, conditions

$$f(t,u)u \geq 0 \text{ and } f(t,u) = -f(t,-u) \quad (3)$$

for all $t \geq t_0$ and $u \in \mathbb{R}$, and

$$g'(u) \geq K > 0 \text{ and } g(u) = -g(-u) \text{ for all } u \in \mathbb{R}. \quad (4)$$

Assumptions (3) and (4) hold, for instance, for Emden-Fowler differential equations where $f(t,u) = g(u) = |u|^\nu \mathrm{sgn}(u)$, $\nu > 0$. We start by providing sufficient conditions for the first derivative of a solution $x(t)$ of equation (1) (or equation (2)) to change sign at least once in a given interval (in a given infinite sequence of intervals). These conditions imply global non-monotone behavior of solutions. Recall that a function $h \in C^1(t_0,\infty)$ is called *non-monotone (weakly oscillatory) on* (t_0,∞) if there exists a sequence of points $\{s_n\}_{n\in\mathbb{N}} \in (t_0,\infty)$, $s_n \to \infty$ as $n \to \infty$, such that $h'(t)$ changes sign at $t = s_n$ for all $n \in \mathbb{N}$. We also discuss how oscillation criteria can be turned into non-monotonicity tests.

Oscillation Problems for Second-Order Nonlinear Dynamic Equations of Euler Type on Time Scales

Naoto Yamaoka
Osaka Prefecture University, Japan

In this talk, we consider second-order nonlinear dynamic equations on time scales of the form $x^{\Delta\Delta} + f(x)/(t\sigma(t)) = 0$, where $f(x)$ satisfies $xf(x) > 0$ if $x \neq 0$. By means of Riccati technique and phase plane analysis of a system, (non)oscillation criteria are established. A necessary and sufficient condition for all nontrivial solutions of the Euler-Cauchy dynamic equation $y^{\Delta\Delta} + \lambda/(t\sigma(t))\, y = 0$ to be oscillatory plays a crucial role in proving our results.

Recent Advances on Nonoscillation of Two-Dimensional Time Scale Systems

Özkan Öztürk
Giresun University, Turkey

We provide the long-time behaviors of nonoscillatory solutions of a system of first-order dynamic equations on time scales. Some well-known fixed point theorems and double improper integrals are used to prove the main results. Examples are significant to see the importance of the main results and how the theorems work. Therefore, we also give some examples to validate our theoretical claims.

Special Session 79: Monte Carlo Methods

Kody Law, Oak Ridge National Laboratory, USA
Ajay Jasra, National University of Singapore, Singapore
Kengo Kamatani, University of Osaka, Japan

Uncertainties in the parameters which define differential equations give rise to distributions of solutions, hence distributions of functionals of the solutions. Quantities of interest can be represented as expectations of such functionals. The resulting high-dimensional integrals, involving expensive function evaluations, has lead to a wealth of new Mathematics. Further complexity is introduced if data is available. The Bayesian framework gives rise to a probabilistic interpretation of inverse problems. Monte Carlo methods comprise a robust and versatile class of methods for solving both forward and statistical inference problems. These methods are expected to become only more prevalent as the emerging architecture of modern and future supercomputers becomes progressively more parallel. This special session aims to bring together researchers interested in all aspects of Monte Carlo methods, to share ideas and discuss these interesting problems and the interplay between them.

Ensemble Kalman-Bucy Filtering and Stability

Adrian Bishop
UTS and Data61 (CSIRO), Australia
Pierre Del Moral

The Ensemble Kalman filter is a data assimilation method for filtering high dimensional problems arising in, e.g., fluid mechanics, weather forecasting, and geophysical sciences. In the linear case, this Monte Carlo method can be interpreted as a mean-field McKean-Vlasov type particle interpretation of a particular nonlinear Kalman-Bucy diffusion that is related to the classical linear Kalman-Bucy filter. This talk presents a series of new functional inequalities to quantify the stability of these nonlinear diffusion processes and their mean field approximations. Results focused on the behaviour of the flow of the relevant sample covariance arising in the mean-field approximation are central to this work. This latter idea amounts to the study of particular stochastic matrix Riccati diffusions (that capture the flow of the sample covariance in nonlinear ensemble filtering). This is joint work with Pierre Del Moral at INRIA.

An Efficient Sequential Discrete Optimal Transport Method for Bayesian Inverse Problems

Tan Bui-Thanh
The University of Texas at Austin, USA
Aaron Myers, Alexandre Thiery, Kainan Wang

We present the Sequential Ensemble Transform (SET) method for generating approximate samples from a posterior distribution as a solution to Bayesian inverse problems. The method explores the posterior by solving a sequence of discrete, linear optimal transport problems, resulting in a series of transport maps which map prior samples to posterior samples. This allows us to efficiently characterize statistical properties of quantities of interest, quantify uncertainty, and compute moments. We present theory proving that the sequence of Dirac mixture distributions generated by the SET method converges to the true posterior. Numerically, we show this method can offer superior computational efficiency when compared to resampling-based Sequential Monte Carlo (SMC) methods in the regime of low mutation steps and small ensemble size; the regime where particle degeneracy is likely to occur.

Transport Map-Accelerated Adaptive Importance Sampling

Simon Cotter
University of Manchester, England
Colin Cotter, Yannis Kevrekidis, Paul Russell

In this talk we will consider how transport maps can simplify the problem of sampling from complex posterior distributions using adaptive importance sampling methods. Adaptive importance sampling or population Monte Carlo methods use a mixture distribution to approximate the posterior, in order to produce an efficient importance sampling scheme. However if there are complex structures such as strong correlations or sharp ridges in the posterior, these methods require a large increase in the number of ensemble members, or they may become unstable. In this work, we investigate the use of transport maps to stabilise and speed up sampling using these methods for such problems with no increase in the size of ensemble. The transport map is chosen, following work by Marzouk and Parno, through the sample obtained so far from the posterior, to minimise the KL divergence between the push-forward of the posterior through the map, and a reference Gaussian. This simplifies greatly the problem of sampling from such a distribution. We will demonstrate the approach through some numerical examples.

Spatio-Temporal Multilevel Ensemble Transform Methods for Bayesian Inference

Alastair Gregory
Imperial College London, England
Colin Cotter, Sebastian Reich

Ensemble transform methods provide frameworks for a Bayesian approach to filtering. In the Ensemble Transform Particle Filter (ETPF) for example, a deterministic transform from forecast to analysis ensembles replaces the resampling step in a classical particle filter. This analysis ensemble provides an approximation to the posterior distribution in question. Filtering within high-dimensional systems is of great importance to a wide-range of industries. However, the Bayesian approach to filtering can fail in this setting due to the curse of dimensionality. This work presents two advancements in this area of research, using variants of the ETPF: making the process of propagating ensembles in these filters more efficient, and proposing a localisation scheme for the ETPF when the forecasting system is a spatio-temporal system discretized by Finite Element (FE) methods. The former is achieved via an adaptation of the multilevel Monte Carlo method, a novel variance reduction technique. The latter is achieved by using projection and multi-grid ideas for FE approximations to random spatio-temporal fields. A case-study of the stochastic quasi-geostrophic equations is presented.

Reversible Proposal MCMC with Heavy-Tailed Target Distributions

Kengo Kamatani
Osaka University, Japan

In this talk, we will discuss Markov chain Monte Carlo methods for heavy-tailed target probability distributions, based on a reversible proposal transition kernel. We will study the dimensionality effect using the high-dimensional asymptotic analysis of Roberts, Gelman, and Gilks. We also study ergodic properties for heavy-tailed target distributions.

Lifting the Curse of Dimensionality by Quasi-Monte Carlo Methods

Frances Kuo
UNSW Sydney, Australia

High dimensional problems – meaning problems with hundred or thousands or even infinitely many continuous variables – are now arising in many real world applications. They present major challenges to computational resources, and require serious mathematical efforts in devising new and effective methods. This talk will begin with a contemporary review of Quasi-Monte Carlo (QMC) methods, which offer tailored point constructions for solving high dimensional integration and approximation problems by sampling. By exploiting the smoothness properties of the underlying mathematical functions, QMC methods can achieve higher order convergence rates than standard Monte Carlo sampling, and moreover, QMC error bounds can be independent of the dimension under appropriate theoretical settings. In recent years the modern QMC theory has been successfully applied to a number of application areas including option pricing in financial mathematics, maximum likelihood estimation in statistics, and PDEs with random coefficients in computational physics and uncertainty quantification. Rather than talking about these well-known popular problems, this talk will present some ongoing works where we take QMC methods to new territories such as quantum field problems in high energy physics, neutron transport as a high dimensional PDE eigenvalue problem, high frequency wave scattering in random media, and spatiotemporal estimation in air pollution modelling.

Nonparametric Estimation of Probability Density Functions of Random Persistence Diagrams

Vasileios Maroulas
University of Tennessee, USA

We introduce a nonparametric way to estimate the global probability density function for a random persistence diagram. Precisely, a kernel density function centered at a given persistence diagram and a given bandwidth is constructed. Our approach encapsulates the number of topological features and considers the appearance or disappearance of features near the diagonal in a stable fashion. In particular, the structure of our kernel individually tracks long persistence features, while considering features near the diagonal as a collective unit. The choice to describe short persistence features as a group reduces computation time while simultaneously retaining ac- curacy. Indeed, we prove that the associated kernel density estimate converges to the true distribution as the number of persistence diagrams increases and the bandwidth shrinks accordingly. We also establish the convergence of the mean absolute deviation estimate, defined according to the bottleneck metric. Lastly, examples of kernel density estimation are presented for typical underlying datasets.

Quasi-Monte Carlo Methods in Uncertainty Quantification

Dirk Nuyens
KU Leuven, Belgium

The approximation of very high-dimensional integrals is a common challenge in many uncertainty quantification problems. These integrals pop up as expectations of functionals of the solution of a PDE with random coefficients, or they appear when calculating the Bayesian posterior mean of a quantity of interest in an inverse problem. Sometimes the uncertainties can be modelled by very smooth random fields and then polynomial chaos expansions (PCE) methods can be used to approximate the uncertainty with very few terms (say 10), however, when the needed dimensionality is really high (and we are talking thousands or millions), e.g., for a rough random field with possibly high variance, the PCE method will have problems. In such a context quasi-Monte Carlo (QMC) methods can deliver a solution. Sometimes giving dimension-independent convergence of $1/N$ or even higher algebraic rates.

Parallel Replica Dynamics for Sampling and Sensitivity Analysis Of multi-scale stochastic reaction networks

Petr Plechac
University of Delaware, USA

Stochastic reaction networks that exhibit metastable behavior are common in chemical reaction kinetics, systems biology as well as materials science. Sampling of the stationary distribution is crucial for understanding and characterizing the long term dynamics of stochastic dynamical systems. However, this task is normally hindered by the insufficient sampling of the rare transitions between metastable regions. We present parallel replica dynamics for accelerating simulations of continuous time Markov chains in the presence of metastability. We demonstrate that the proposed method accelerates stationary distribution sampling and yields correct stationary averaging. Furthermore, we show that it can be combined with path-space information bounds on path-dependent functionals and risk sensitive functionals. Such bounds provide error estimates on quantities of interest as well as bounds on parametric sensitivity in the complex reaction networks.

Markov Chain Monte Carlo for Semi-Supervised Learning

Daniel Sanz-Alonso
Brown University, USA
N. Garcia Trillos, Z. Kaplan, T. Samakhoana

Semi-supervised learning concerns the inference of labels on inputs by the use of unlabelled data (inputs) and labelled data (pairs of inputs and labels). In many applications, unlabelled data abounds, but labels are limited. The aim of this talk is to present a new theoretical result on the Bayesian approach to semi-supervised learning that address this regime of interest, and to introduce a Markov Chain Monte Carlo method with rate of convergence independent of the size of the unlabelled data set.

Ensemble Kalman Filter with Localization

Xin Tong
National University of Singapore, Singapore

Ensemble Kalman filter (EnKF) is an important data assimilation method for high dimensional geophysical systems. Efficient implementation of EnKF in practice often involves the localization technique, which updates each component using only information within a local radius. This paper rigorously analyzes the local EnKF (LEnKF) for linear systems, and shows that the filter error can be dominated by the ensemble covariance, as long as 1) the sample size exceeds the logarithmic of state dimension and a constant that depends only on the local radius; 2) the forecast covariance matrix admits a stable localized structure. In particular, this indicates that with small system and observation noises, the filter error will be accurate in long time even if the initialization is not. The analysis also reveals an intrinsic inconsistency caused by the localization technique, and a stable localized structure is necessary to control this inconsistency. While this structure is usually taken for granted for the operation of LEnKF, it can also be rigorously proved for linear systems with sparse local observations and weak local interactions. These theoretical results are also validated by numerical implementation of LEnKF on a simple stochastic turbulence in two dynamical regimes.

Special Session 80: Modern Topics in Nonlinear PDEs and Applications

Alessio Fiscella, IMECC, Universidade Estadual de Campinas, Brazil
Patrizia Pucci, Università degli Studi di Perugia, Italy
Binlin Zhang, Heilongjiang Institute of Technology, Peoples Rep of China

The aim of this session is to present recent progress and future applications in the field of local and nonlocal problems. Besides analysis of classical nonlinear problems, the topics of the session include free boundary problems, calculus of variations and recent trends on local and nonlocal PDEs. In particular, the existence, multiplicity, regularity and qualitative properties of solutions for modern nonlinear PDEs will be discussed.

Nonlocal Scalar Field Equations

Mousomi Bhakta
Indian Institute of Science Education and Research, Pune, India

We study the nonlocal scalar field equations with a vanishing parameter ϵ. For $\epsilon > 0$ small, we prove the existence of a ground state solution and show that any positive solution is a classical solution and radially symmetric and symmetric decreasing. We also obtain the decay rate of solution at infinity. Next, we characterize the asymptotic behavior of ground state solutions and using this we prove the *l*ocal uniqueness of solution.

Multiplicity and Bifurcation of Positive Solutions for Nonhomogeneous Semilinear Fractional Laplacian Problems

Yongqiang Fu
Harbin Institute of Technology, Peoples Rep of China
Bingliang Li

We consider the following nonhomogeneous semilinear fractional Laplacian problem $(-\Delta)^s u + u = \lambda(f(x,u) + h(u))$ in $H^s(\mathbb{R}^n)$. We prove that under suitable conditions on f and h, there exists $\lambda^* \in (0,\infty)$ such that the problem has at least two positive solutions if $\lambda \in (0,\lambda^*)$, a unique positive solution if $\lambda = \lambda^*$, and no solution if $\lambda > \lambda^*$. We also obtain the bifurcation of positive solutions for the problem at (λ^*, u^*) and further analyse the set of positive solutions.

Multiplicity Results and Qualitative Properties for the Lane-Emden Problem

Isabella Ianni
Università degli Studi della Campania, Italy

We consider the semilinear elliptic problem

$$\begin{cases} -\Delta u = |u|^{p-1}u & \text{in } B \\ u = 0 & \text{on } \partial B \end{cases} \qquad (\mathcal{E}_p)$$

where B is the unit ball of $\mathbb{R}^2$ centered at the origin and $p \in (1,+\infty)$. We prove the existence of nonradial sign-changing solutions to $(\mathcal{E}_p)$ having 2 nodal domains and whose nodal line does not touch ∂B. The result is obtained with two different approaches: via nonradial bifurcation from the least energy sign-changing radial solution u_p of $(\mathcal{E}_p)$ at certain values of p and by investigating the qualitative properties, for p large, of the least energy nodal solutions in spaces of functions invariant by the action of the dihedral group generated by the reflection with respect to the x-axis and the rotation about the origin of angle $\frac{2\pi}{k}$ for suitable integers k. We also prove that for certain integers k the least energy nodal solutions in these spaces of symmetric functions are instead radial, showing in particular a breaking of symmetry phenomenon in dependence on the exponent p.

Infinitely Many Sign-Changing Solutions for the Brezis-Nirenberg Problem Involving the Fractional Laplacian

Lin Li
Chongqing Technology and Business University, Peoples Rep of China
Jijiang Sun, Stepan Tersian

In this paper, we consider a Brèzis-Nirenberg problem involving the fractional Laplacian operator $(-\Delta)^s$ on a bounded smooth domain of $\mathbb{R}^N$ $(N > 6s)$ and $2_s^* = \frac{2N}{N-2s}$ is the critical fractional Sobolev exponent. We show that, for each $\lambda > 0$, this problem has infinitely many sign-changing solutions by using a compactness result obtained in [S. Yan, J. Yang and X. Yu 2016 JFA] and a combination of invariant sets method and Ljusternik-Schnirelman type minimax method.

A Maximal Nondegenerate Sign Changing Solution for the Yamabe Problem

Maria Medina
Pontificia Universidad Catolica de Chile, Chile

In this talk we will construct a sequence of nondegenerate (in the sense of Duyckaerts-Kenig-Merle) nodal nonradial solutions to the critical Yamabe problem

$$-\Delta u = \frac{n(n-2)}{4}|u|^{\frac{4}{n-2}}u, \qquad u \in \mathcal{D}^{1,2}(\mathbb{R}^n),$$

which provides the first example in the literature of a solution with maximal rank. This is a joint work with M. Musso and J. Wei.

Boundary Singularities of Solutions to Semilinear Fractional Equations

Tai Nguyen
Masaryk University, Czech Rep
Laurent Veron

In this talk, I will discuss the existence of a solution of $(-\Delta)^s u + f(u) = 0$ in a smooth bounded domain Ω with a prescribed boundary value μ in the class of Radon measures for a large class of continuous functions f satisfying a weak singularity condition expressed under an integral form. I will present the existence of a boundary trace for positive moderate solutions. In the particular case where $f(u) = u^p$ and μ is a Dirac mass, I will show the existence of several critical exponents.

Entire Solutions for Critical P– Fractional Hardy Schrödinger Kirchhoff Equations

Paolo Piersanti
City University of Hong Kong, Hong Kong

This presentation is inspired into a joint work with Professor Patrizia Pucci (University of Perugia) Existence theorems of nonnegative entire solutions of stationary critical p–fractional Hardy Schrödinger Kirchhoff equations are presented in this talk. The equations we treat deal with Hardy terms and critical nonlinearities and the main theorems extend several recent results on the topic.

Hölder Regularity for Bounded Solutions to a Class of Anisotropic Elliptic Equations by Using a Parabolic Approach

Stella Maria Piro (Vernier)
University of Cagliari, Italy
Francesco Ragnedda, Vincenzo Vespri

In this talk we consider bounded local weak solutions to the following class of anisotropic elliptic equations

$$\sum_{i=1}^{N-1} \frac{\partial}{\partial x_i} A_{q,i}(x,u,Du) + \frac{\partial}{\partial x_N} A_p(x,u,Du) = 0 \text{ in } \Omega, \tag{1}$$

with Ω a regular domain in $\mathbb{R}^N$. The functions $A_{q,i}(x,u,Du)$ and $A_p(x,u,Du) : \Omega \times \mathbb{R}^{N+1} \to \mathbb{R}^N$ are assumed to be measurable and satisfying the structure conditions. We prove the Hölder regularity of solutions to 1 when $p > q > 1$ (singular case). A first step to face this problem was made by Liskevich and Skrypnik ([2]) in 2009. Then Duzgum, Marcellini and Vespri ([1]) in 2014 extended the result proved in [2] to the case when in the equation 1 it is assumed $q > p > 1$ (degenerate case). The results have been established in a joint research with Francesco Ragnedda (Cagliari) e Vincenzo Vespri (Firenze).

References

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Sign-Changing Solutions for Critical Equations with Hardy Potential

Giusi Vaira
University of Campania Luigi Vanvitelli, Italy
P. Esposito, N. Ghoussoub, A. Pistoia

We discuss existence issues for a Dirichlet problem in bounded domains with polynomial nonlinearites of critical growth in presence of an Hardy potential. Linear perturbations can produce positive solutions and we aim to construct sign-changing solutions, shaped as a tower of bubbles centered at zero with alternating signs. The construction is optimal as a very fine asymptotic analysis shows for radial towers of bubbles.

Finite Time Blowup for Nonlinear Klein Gordon Equations with Arbitrarily Positive Initial Energy

Runzhang Xu
Harbin Engineering University, Peoples Rep of China
Yanbing Yang

This paper is concerned with the finite time blow up of the solution to the Cauchy problem for the Klein-Gordon equation at arbitrarily positive initial energy level. By introducing a new auxiliary function and an adapted concavity method we establish some sufficient conditions on initial data such that the solution blows up in finite time, which extends the results established in Wang (2008).

Special Session 81: Stochastic Systems, SDEs/SPDEs, and Games with Numerics and Applications

Wanyang Dai, Nanjing University, Peoples Rep of China

We will present the latest achievements of theory, methods, and numerics for general stochastic systems, stochastic differential equations (SDEs), and stochastic partial differential equations (SPDEs). Their interactions with and/or applications in broad areas, e.g., optimal control and game theory, queueing theory, information theory and technology, FinTech, Big Data and intelligent cloud computing, genomics and healthcare, smart material sciences and communication systems will be highlighted.

Solution Properties of a 3D Stochastic Euler Fluid Equation

Dan Crisan
Imperial College London, England
Darryl Holm, Franco Flandoli

We show two important analytical properties of deterministic Euler fluid dynamics in three dimensions possess close counterparts in the stochastic Euler fluid model introduced by Holm [2015]. The first of these analytical properties is the local-in-time existence and uniqueness of deterministic Euler fluid flows. The second property is stochastic counterpart of the classical criterion for blow-up in finite time due to Beale, Kato and Majda. The stochastic model investigated here bodes well for the potential use of this model in, e.g., uncertainty quantification of either observed, or numerically simulated fluid flows.

Simulation and Numerics by Malliavin Calculus for Backward Stochastic Partial Differential Equations and Stochastic Differential Games

Wanyang Dai
Nanjing University, Peoples Rep of China

We develop a generic scheme to simulate the 2-tuple adapted strong solution in a classical sense to a generalized Cauchy (or called terminal-value) problem, i.e., to a unified system of backward stochastic partial differential equations driven by Brownian motions. The scheme is a completely discrete and iterative algorithm in terms of both time and space, and more importantly, its mean-square convergence with supporting numerical examples is established. In doing so, the system is assumed to be high-dimensional and vector-valued, whose drift and diffusion coefficients may involve nonlinear and high-order partial differential operators. Under general local Lipschitz and linear growth conditions, the unique existence of the 2-tuple adapted strong solution to the system is proved by constructing a suitable Banach space to handle the difficulty that the partial differential orders on both sides of these equations are different. Extensions to the case with Levy jumps and applications in stochastic differential games will also be addressed.

A Malliavin-Stein Approach for Multivariate Approximations in Wasserstein Distance

Xiao Fang
The Chinese University of Hong Kong, Hong Kong
Qi-Man Shao, Lihu Xu

Stein's method has been widely used for probability approximations. However, in the multi-dimensional setting, most of the results are for multivariate normal approximation or for test functions with bounded second- or higher-order derivatives. For a class of multivariate limiting distributions, we use Bismut's formula in Malliavin calculus to control the derivatives of the Stein equation solutions by the first derivative of the test function. Combined with Stein's exchangeable pair approach, we obtain a general theorem for multivariate approximations with near optimal error bounds on the Wasserstein distance. We apply the theorem to the unadjusted Langevin algorithm.

Semiclassical Computational Methods for Quantum Dynamics with Uncertain Band-Gap

Shi Jin
Shanghai Jiao Tong University and University of Wisconsin-Madison, Peoples Rep of China

Band-crossing is a quantum dynamical behavior that contributes to important physics and chemistry phenomena such as quantum tunneling, Berry connection, chemical reaction etc. In this talk, we will discuss some recent works in developing semiclassical methods for band-crossing in surface hopping. For such systems we will also introduce an asymptotic-preserving method that is accurate uniformly for all wave numbers, including the problem with random uncertain band gaps.

Portfolio Optimization Under Shortfall Risk Constraint

Qinghua Li
Columbia University (non-academic, PhD Columbia), USA
Oliver Janke (Humboldt)

This paper solves a utility maximization problem under utility-based shortfall risk constraint, by proposing an approach using Lagrange multiplier and con-

vex duality. Under mild conditions on the asymptotic elasticity of the utility function and the loss function, we find an optimal wealth process for the constrained problem and characterize the bi-dual relation between the respective value functions of the constrained problem and its dual. This approach applies to both complete and incomplete markets. Moreover, the extension to more complicated cases is illustrated by solving the problem with a consumption process added. Finally, we give an example of utility and loss functions in the Black-Scholes market where the solutions have explicit forms.

A Markovian Queue with Feedback and Multiple Types of Customers

Hsing Luh
National Chengchi University, Taiwan
Jerim Kim, Bara Kim

Considering a feedback queue of First Come First Served service policy with multi-class customers, we derive functional equations for the stationary probability distribution of the queue size and the total response time for each type of customers. Numerical examples are given to show that moments of the queue sizes and of the total response times can be easily computed for the weighted round-robin queue as well.

Stochastic Control Problems with Delay

Federica Masiero
Milano Bicocca University, Italy
G. Guatteri, C. Orrieri

In this talk we deal with stochastic optimal control problems with delay in the state, and we focus on the Pontryagin Maximum principle. It is well known that when we consider a controlled delay state equation, the adjoint BSDE turns out to be anticipating, see [1]. We introduce a new class of anticipating BSDEs suitable to treat general control problems with delay in the state via the stochastic maximum principle. We show how to handle delay in the control variable. Next we discuss possible approaches to formulate the stochastic maximum principle for controlled delay equations with control-dependent-noise when the space of controls is not convex. Part of the talk is based on [2], and on a joint work in progress with G. Guatteri.

References

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Zero-Sum Stochastic Differential Games of Generalized McKean-Vlasov Type

Huyen Pham
University Paris Diderot, France
Andrea Cosso

We study zero-sum stochastic differential games where the state dynamics of the two players is governed by a generalized McKean-Vlasov (or mean-field) stochastic differential equation in which the distribution of both state and controls of each player appears in the drift and diffusion coefficients, as well as in the running and terminal payoff functions. We prove the dynamic programming principle (DPP) in this general setting, which also includes the control case with only one player, where it is the first time that DPP is proved for open-loop controls. We also show that the upper and lower value functions are viscosity solutions to a corresponding upper and lower Master Bellman-Isaacs equation. Our results extend the seminal work of Fleming and Souganidis to the McKean-Vlasov setting.

Nonlinear Fokker-Planck-Kolmogorov Equations and Stochastic Distribution Dependent SDE

Michael Roeckner
Bielefeld University, Germany

By Itos formula the time marginals of a solution to a distribution dependent SDE solve a nonlinear Fokker-Planck-Kolmogorov equation. This talk is about the converse: we present a general technique how to identify a solution to a nonlinear Fokker-Planck-Kolmogorov equation consisting of probability densities as the time marginals of a solution to a distribution dependent SDE. We apply this to the special case of a porous media equation perturbed by the divergence of a vector field depending nonlinearly on the solution. More precisely, we construct a generalized entropic solution u to this equation and apply the above general technique to find the corresponding distribution dependent SDE which has a weak solution with marginals given by u. We thus gain a probabilistic representation of u.
Reference: arXiv:1801.10510

On Feller and Strong Feller Properties of Regime-Switching Jump Diffusion Processes with Countable Regimes

Chao Zhu
University of Wisconsin-Milwaukee, USA
Fubao Xi

This work focuses on a class of regime-switching jump diffusion processes, in which the switching component has countably infinite many states or regimes and the underlying stochastic differential equations have super linear growth and non-Lipschitz coefficients. Under some mild conditions on the coefficients, the existence and uniqueness of the underlying process are obtained by an interlacing procedure. Then the Feller and strong Feller properties of such processes are derived by the coupling method and an appropriate Radon-Nikodym derivative.

Special Session 82: Recent Advances in Differential Equations with Applications to Biology, Ecology and Epidemiology

Guihong Fan, Columbus State University, USA
Yanyu Xiao, University of Cincinnati, USA

Differential equations plays a substantial role in the modeling process of problems arising from ecology and epidemiology, which includes many types of differential equations such as ordinary differential equations, partial differential equations, functional differential equations, stochastic differential equations, and computational differential equations. The study of differential equations using methods of dynamics analysis, sensitivity analysis, data analysis, or cost-effectiveness analysis can help capture the complex relationships among species, direct or indirect spatial and temporal factors on disease transmissions and dispersals. This special session will focus on a variety of modeling and computational developments using differential equations as a tool and topics cover infectious diseases (in- or within- host modeling) and population dynamics using various types of models with age structure/multiple patches/spatial spreading (reaction-diffusion equation)/seasonality/ density-dependent mortality and growth rates. This special session will unify the strength of scientists and enhance their collaborations in the prediction, prevention, and control of infectious diseases and the protection of endangered species or control of invasive species with a hope to advance the study of differential equations in ecology and epidemiology or other problems with strong biological backgrounds.

Quantification of Seasonal Drivers of Lassa Fever Epidemics in Nigeria

Andrei Akhmetzhanov
Hokkaido University, Japan
Yusuke Asai, Hiroshi Nishiura

Lassa fever (LF) is an acute haemorrhagic viral infection with clear zoonotic origin. Animal reservoir of the virus is the most common rodent species in Africa, natal multimammate mice (*Mastomys natalensis*). Human infection occurs upon exposure to virus-contaminated environment. The disease is widespread in West Africa with about few thousand cases reported each year, including few hundred of deaths. This ranks LF as of a serious public health concern and necessitates further investigation of its epidemiology. In our talk, we present the analysis of surveillance data on LF incidence in Nigeria, one of the most affected countries, for 2012–2017. We explicitly specify main causative factors on seasonality of LF epidemics that are seasonal migration of rodents linked to rainfall pattern, and low awareness on LF pathogenicity among local residents. Using epidemiological modelling (maximum likelihood estimation, dynamic model of infection transmission in rodents), we quantify their relative effects on course of epidemic and assess the overall risk of contacting a virus along the year. Additionally, we apply cross-convergent method to show that rainfall pattern is indeed one of the main factors explaining LF incidence. Our findings will help to design more efficient intervention programs to constrain LF spread in the future.

Models of Dynamic Virus and Immune Response Networks

Cameron Browne
University of Louisiana at Lafayette, USA

The dynamics of virus and immune response within a host can be viewed as a complex and evolving ecological system. For example, during HIV infection, an array of CTL immune response populations recognize specific epitopes (viral proteins) presented on the surface of infected cells to effectively mediate their killing. However HIV can rapidly evolve resistance to CTL attack at different epitopes, inducing a dynamic network of viral and immune response variants. We consider models for the network of virus and immune response populations. Our analysis provides insights on viral immune escape from multiple epitopes. In the "binary allele" setting, we prove that if the viral fitness costs for gaining resistance to each of n epitopes are equal and multiplicative, then the system of 2^n virus strains converges to a "perfectly nested network" with less than $n+1$ persistent virus strains. Overall, our results suggest that immunodominance is the most important factor determining viral escape pathway of HIV against multiple CTL populations.

Modeling Evolution and Spread of Vector-Borne Diseases

Hayriye Gulbudak
University of Louisiana at Lafayette, USA
Vincent Cannataro, Necibe Tuncer, Maia Martcheva

There is recent interest in mathematical models which connect the epidemiological aspects of infectious diseases to the within-host dynamics of the pathogen and immune response. Parasite (and host) fitness depend on both scales of infection, which makes such models a natural setting for studying pathogen-host evolution. Multi-scale modeling of infections allows for assessing how immune-pathogen dynamics affect spread of the disease in the population. Here, we consider a within-host model for immune-pathogen dynamics nested in an age-since-infection structured PDE system for vector-borne epidemics. First, we study pathogen-host coevolution by analytically establishing evolutionary stable strategies for parasite and host, and by utilizing computational methods to simulate the evolution in various settings. We find that vector inoculum size can contribute to virulence of vector-borne diseases in distinct ways. Next, we develop a robust methodology for identifiability and estimation of parameters with multi-scale data, along with sensitivity analysis. The nested multi-scale model is fit to combined within-host and epidemiological data for Rift Valley Fever. An ultimate goal is to accurately model how control measures, such as vaccination, affect both scales of infection.

Modelling the Large-Scale Yellow Fever Outbreak in Luanda, Angola, and the Impact of Vaccination

Daihai He
Hong Kong Polytechnic University, Hong Kong
S. Zhao, L. Stone, D. Gao, D. He

An epidemic model for the transmission of yellow fever virus (YFV) in urban areas is formulated and implemented to study the 2016 yellow fever (YF) outbreak in Luanda, Angola. We explore the complex vector-host dynamics of this system taking into account mosquito abundance, vaccination and asymptomatic infections in the human population, that are generally not included in other modelling studies of YF. The model successfully fits the time series of weekly reported YF cases and deaths during the epidemic. This allows us to study the impact of the vaccination campaign in Luanda and hypothetical "delayed vaccination" scenarios. The transmission of YFV appeared to be oscillatory having a wave-like pattern in the basic reproduction number (R_0). The oscillations are hypothesized to be due to human reaction to the reported deaths, as has been noted for other human infectious diseases, and the second wave also possibly due to El Nino rainfall patterns. We conclude that the lives saved due to the vaccination campaign before August 2016 should have been approximately 370 (i.e., approximately five-fold of the observed 73 deaths), and would have been far larger extrapolating beyond August 2016.

Number and Stability of Relaxation Oscillations for Predator-Prey Systems with Small Death Rates

Ting-Hao Hsu
McMaster University, Canada

Predator-prey models with small predator death rate can be understood as slow-fast systems. Using geometric singular perturbation theory and the theory of Floquet multipliers, I will derive characteristic functions that determine the location and the stability of relaxation oscillations. For systems with prey isocline possessing a single interior local extremum, I will show that the system has a unique nontrivial periodic orbit, which forms a relaxation oscillation. For some systems with prey isocline possessing two interior local extrema, I will show that either the positive equilibrium is globally stable, or the system has exact two periodic orbits. In particular, for the Holling type IV functional response there is a threshold value of the carrying capacity that separates these two outcomes. This result supports the so-called paradox of enrichment.

Stochastic Modelling for Population of Culex Mosquitoes with Temperature

Bing Hu
York University, Canada

The impact of climate change on mosquito distribution and human health have attracted much attention in recent years and it is widely expected to significantly affect the spread, intensity and distribution of mosquito-borne diseases. Linking global and regional climate models with mathematical models of mosquito population dynamics provides a valuable tool towards improving and quantifying our understanding of how future climate change may affect the distribution of vector mosquito population. In this talk I will focus on using stochastic methods to model the dynamics of mosquito population with stochastic climatic factors. I will present some basic properties of the stochastic models and simulations to explore the variations of mosquito population under different weather patterns.

A Hybrid Continuous/Discrete-Time Model for Invasion Dynamics of Zebra Mussels in Rivers

Qihua Huang
Southwest University, Peoples Rep of China
Hao Wang, Mark A. Lewis

While some species spread upstream in river environments, not all invasive species are successful in spreading upriver. Here the dynamics of unidirectional water flow found in rivers can play a role in determining invasion success. We develop a continuous-discrete hybrid benthic-drift population model to describe the dynamics of invasive freshwater mussels in rivers. In the model, a reaction-advection-diffusion equation coupled to an ordinary differential equation describes the larval dispersal in the drift until settling to the benthos, while two difference equations describe the population growth on the benthos. We study the population persistence criteria based on three related measures: fundamental niche, source-sink distribution, and net reproductive rate. We calculate the critical domain size in a bounded domain by analyzing a next generation operator. We analyze the upstream and downstream spreading speeds in an unbounded domain. The model is parameterized by available data in the literature. Combining the results of model parameterization and theoretical analysis, we numerically analyze how the interaction between population growth and dispersal, river flow rate, and water temperature, affects both persistence and the spread of zebra mussels along a river.

Analysis of Two Models on Antimicrobial Usage Strategies in ICUs

Xi Huo
Univeristy of Miami, USA

Previously, we have developed high dimensional mathematical models to compare antimicrobial deescalation and continuation as two drug use strategies in ICUs. Our numerical analysis showed that either of the two strategies is superior than the other one with different parameter settings. As a follow-up mathematical analysis study, we are able to simplify our previous high-dimensional models and quantify the parameter conditions under which one strategy is better than the other. This talk will summarize our math results.

Stability and Controlling of Complex Ecosystems

Maoxing Liu
North University of China, Peoples Rep of China
Yongzheng Sun

In this article, we investigate the stability and control of a biological systems with networks. We show that there is a strength, below which the stability occurs. Specifically, we find a power-law relationship between the convergence time and the density of group. We also investigate the influence of control parameter and an optimal value is found that minimize the convergence time.

Complex Dynamics and Bifurcations of a Mosquito-Borne Disease Model with a Nonlinear Recovery Rate

Chunhua Shan
The University of Toledo, USA
Huaiping Zhu

In this talk I will introduce a three-dimensional model to study the impact of limited health resources on transmission dynamics of mosquito-borne diseases by incorporating a nonlinear recovery rate. The model exhibits multi-steady states and backward bifurcation as common features of vector-borne diseases. We show that the cusp type of Bogdanov-Taken bifurcation of codimension 3 is the organizing center. Multiple limit cycles discovered in this model may help to reveal the recurrence mechanism of mosquito-borne diseases. Relaxation oscillations will also be discussed.

Prediction of Daily $PM_{2.5}$ Concentration in China Using Partial Differential Equation in Spatial-Temporal Dimension

Yufang Wang
Tianjin University of Finance and Economics, Peoples Rep of China
Haiyan Wang, Shuhua Chang

Accurate reporting and forecasting of $PM_{2.5}$ concentration are important for improving public health. We propose a partial differential equation (PDE), specially, a linear diffusive equation, to describe the spatial-temporal characteristics of $PM_{2.5}$ in order to make short-term prediction. We present the temporal and spatial patterns based on a real dataset from China's National Environmental Monitoring and validate the proposed PDE model in terms of predicting the $PM_{2.5}$ concentration of the next day by the former days' history data. Our experiment results show that the PDE model is able to characterize and predict the process of $PM_{2.5}$ transport. To our knowledge, this is the first attempt to use PDE-based model to study the $PM_{2.5}$.

Combining Networks and PDE Models to Improve Influenza Predictions

Haiyan Wang
Arizona State University, USA

The data generated from social media can be used for predictions in a rapid and accurate fashion. In recent years many researchers have explored real-time streaming data from Twitter for a broad range of applications such as predicting flu trends. In this talk, we present our design and implementation of a prototype system to collect flu related twitter data. Further we use partial differential equation models and neural networks to predict epidemic outbreaks based on the twitter data. We correlate the results with official statistics from Center for Disease Control and Prevention (CDC). These results demonstrate that the system can be used to real-timely monitor the spread of influenza

Special Session 83: Recent Advances in the Analysis of Nonlinear Phenomena

Jessica Lin, University of Wisconsin–Madison, USA
Yao Yao, Georgia Tech, USA

This session is dedicated to the analysis of various phenomena modeled by nonlinear partial differential equations (PDE). These models include (but are not limited to) fluid equations, reaction-diffusion equations, Hamilton-Jacobi equations, and fully nonlinear elliptic and parabolic equations. Our goal is to bring together experts and young researchers in various subfields of nonlinear PDE in order to exchange ideas and techniques for analysis.

Diffusion and Mixing in Fluid Dynamics

Michele Coti Zelati
Imperial College London, England

We study diffusion and mixing in different fluid dynamics models, and give a general criterion that links mixing rates to enhanced dissipation time-scales. Applications include passive scalar evolution in both planar and radial settings, linearized two-dimensional Navier-Stokes equations, and even simple examples in kinetic theory.

Well-Posedness of the Magneto-Hydrodynamics in Optimal Sobolev Spaces

Mimi Dai
University of Illinois at Chicago, USA

We will talk about some recent results on the well-posedness problems in Sobolev spaces for the magneto-hydrodynamics with and without Hall effect, i.e., the Hall MHD and classical MHD models. One of the purposes of the work is to search the optimal Sobolev space of well-posedness for the two models. Another purpose is to understand the nonlinear Hall term $\nabla \times ((\nabla \times b) \times b)$ in the Hall MHD, which appears more singular than $u \cdot \nabla u$ in the NSE, but with special geometry.

Boundary Regularity for the Navier-Stokes Equations in the Critical Lebesgue Spaces

Hongjie Dong
Brown University, USA
Kunrui Wang

I will first review some previous results on the conditional regularity of solutions to the incompressible Navier-Stokes equations in the critical Lebesgue spaces, and then discuss a recent work which mainly addressed the boundary regularity issue.

The Influence of Non-Local Interactions on Propagation Speeds

Christopher Henderson
University of Chicago, USA

Reaction-diffusion equations taking into account non-local competition have gathered substantial interest in recent years. A major difficulty of these equations is due to the absence of a maximum principle; however, early results seemed to suggest that, despite this technical challenge, the speed of propagation was unaffected by this non-local term. That is, the front of the non-local equation moved with the same speed as that of the local analogue. We discuss a model arising naturally in ecology for which this is false.

The Langevin Equation with Variable Friction and Smoluchowski-Kramers Approximation

Hitoshi Ishii
Tsuda University, Japan
Panagiotis E. Souganidis, Hung V. Tran

I will discuss some recent results on two asymptotic problems for the Langevin equation with variable friction coefficient. The first is the small mass asymptotic behavior, known as the Smoluchowski-Kramers approximation, of the Langevin equation with strictly positive variable friction. The second result is about the limiting behavior of the solution when the friction vanishes in regions of the domain. The results generalize previous works due to Freidlin-Hu and Freidlin-Hu-Wentzell. My talk will be based on joint work with P. E. Souganidis (University of Chicago) and H. V. Tran (University of Wisconsin-Madison).

A Shape Theorem for a Discrete Cane Toads Equation with Random Motility

Tau Shean Lim
Duke University, USA
James Nolen

We study the propagation of a discrete cane toads equation with random, unbounded diffusion coefficients. Under appropriate assumptions on the growth of the diffusion coefficients, we prove the shape theorem for both quenched and averaged solutions. The method is based on the analysis of large deviation for stochastic processes in random environment.

Asymptotic Coupling and Uniqueness of Invariant Measures for Hydrodynamic and Related Equations

Vincent Martinez
Tulane University, USA
Nathan Glatt-Holtz, Geordie Richards

In their 1967 seminal paper, Foias and Prodi captured precisely a notion of finitely many degrees of freedom in the context of the two-dimensional (2D) incompressible Navier-Stokes equations (NSE). In particular, they proved that if a sufficiently large spectral projection of the difference of two solutions converge to 0 asymptotically in time, then the corresponding complementary projection of their difference must also converge to 0 in the infinite-time limit. In other words, the high modes are "eventually enslaved" by the low modes. One could thus define the number of degrees of freedom to be the smallest number of modes needed to guarantee this convergence for a given flow, insofar as it represents as a solution to the NSE. This property has since led to several developments in the long-time behavior of solutions to the NSE, such as, for instance, to turbulence, data assimilation, and the existence of determining forms. In this talk, we will discuss this asymptotic enslavement property with regard to the issue of uniqueness of invariant measures for stochastically forced equations, specifically those in the context of hydrodynamic and related equations.

Space-Time Discrete Numerical Schemes for a Feedback-Control Data Assimilation Algorithm

Cecilia Mondaini
Tulane University, USA
H. Ibdah, E. S. Titi

We consider a feedback-control (nudging) approach for data assimilation that works for a general class of dissipative dynamical systems and observables. As a model example, we consider the 2D incompressible Navier-Stokes equations (NSE). Our purpose is to present an estimate of the error between a numerical approximation of the solution to the nudging equation and a reference solution of the 2D NSE, representing the truth. We consider a spatial discretization given by the Postprocessing Galerkin method and two types of implicit Euler schemes for the time discretization: fully implicit and semi-implicit. Our results show that the time-discrete schemes are unconditionally stable and the error estimates are uniform in time. This is based on joint works with H. Ibdah and E.S. Titi.

Feeble Fish in Time-Dependent Waters and Homogenization of the G-Equation

Alexei Novikov
Penn State University, USA

We study the following control problem. Fish with bounded aquatic locomotion speed swims in fast waters. Can the fish, under reasonable assumptions, get to a desired destination? It can, even if the flow is time-dependent. Moreover, given a prescribed sufficiently large time t, it can be there at exactly the time t. We give an application of this result to homogenization of the G-equation.

Large-Time Behavior of the Anisotropic Stefan Problem in Nonuniform Media

Norbert Pozar
Kanazawa University, Japan

In this talk, I will discuss the large-time behavior of the solutions of the one-phase Stefan problem with oscillating diffusion coefficients and latent heat of phase transition. After appropriate rescaling, the asymptotic limit of the solutions can be viewed as a homogenization limit. We show that the rescaled solutions converge to a solution of the Hele-Shaw problem with a point source and an anisotropic homogeneous diffusion. The singularity that appears at the origin in the limit is handled using careful barrier arguments with fundamental solutions of the elliptic equation. This is joint work with Giang Thi Thu Vu.

Coupling Distance Between Levy Measures and Uniqueness of Viscosity Solutions of Non-Local HJB Equations

Andrzej Swiech
Georgia Institute of Technology, USA
Nestor Guillen, Chenchen Mou

We will discuss a new approach to the proof of comparison principle for viscosity solutions of nonlocal Hamilton-Jacobi-Bellman equations. Comparison principle for such equations is still a largely open problem when the Levy measures appearing in the equations depend on the spacial variable. Our approach is based on the use of an optimal transport based distance between Levy measures and it allows to prove comparison results for a significantly larger class of equations. This is a joint work with Nestor Guillen and Chenchen Mou.

(-1)-Homogeneous Solutions of Stationary Incompressible Navier-Stokes Equations with Singular Rays

Xukai Yan
Georgia Institute of Technology, USA
Li Li, Yanyan Li

In 1944, L.D. Landau discovered explicit (-1)-homogeneous solutions of 3-d stationary incompressible Navier-Stokes equations (NSE) with precisely one singularity at the origin, which are axisymmetric with no swirl. These solutions are now called Landau solutions. In 1998 G. Tian and Z. Xin proved that all solutions which are (-1) homogeneous, axisymmetric with one singularity are Landau solutions. In 2006 V. Sverak proved that with just the (-1)-homogeneous assumption Landau solutions are the only solutions with one singularity. He also proved that there are no such solutions in dimension greater than 3. Our work focuses on the (-1)-homogeneous solutions of 3-d incompressible stationary NSE with finitely many singularities on the unit sphere. In this talk we will first classify all (-1)-homogeneous axisymmetric no-swirl solutions of 3-d stationary incompressible NSE with one singularity at the south pole on the unit sphere as a 2-d solution surface. We will then present our results on the existence of a one parameter family of (-1)-homogeneous axisymmetric solutions with non-zero swirl and smooth on the unit sphere away from the south pole, emanating from the two dimensional surface of axisymmetric no-swirl solutions. We will also present asymptotic behavior of general (-1)-homogeneous axisymmetric solutions in a cone containing the south pole with a singularity at the south pole on the unit sphere. We also constructed families of solutions smooth on the unit sphere away from the north and south poles.

Special Session 84: Analysis of Mathematical Modeling Arising from Population Biology

Yu Jin, University of Nebraska-Lincoln, USA
Sze-Bi Hsu, National Tsing Hua University, Taiwan
Feng-Bin Wang, Chang Gung University, Taiwan

In the past century, Mathematical models have been increasingly developed to describe and study population dynamics and behaviors in spatially and temporally homogeneous and heterogeneous environments. On one hand, investigations of mathematical models provide theoretical foundation for long and short term population dynamics as well as management strategies. On the other hand, realistic biological problems lead to new, interesting, and challenging mathematical modeling and theories. In this special session, we will bring together researchers to present and discuss recent advances of mathematical theories and analyses, especially in differential equations and dynamical systems, and their applications in population biology.

Quality of Life in Diabetics Could Be Improved by Integrated Dynamical Systems for an Artificial Pancreas

Jiaxu Li
University of Louisville, USA

Multi-millions of people suffer diabetes mellitus, including type 1 diabetes and type 2 diabetes, that is caused by the lack of, or relatively insufficient, insulin produced from pancreatic beta-cells. Exogenous insulin or its analogues must be daily administered to utilize and lower the chronic high glucose level, ideally, though an artificial pancreas, a medical device consisting of an insulin pump, a glucose monitoring system, and a set of closed loop control (CLC) algorithms. An effective CLC algorithm is still lacking in handling the delayed effects of insulin in delivery mechanisms, GMS and the hepatic glucose production (HGP). The timing discrepancies and dose inaccuracies often cause undesired glucose fluctuations including both hyperglycemia and dangerous hypoglycemia. We aim to formulate an integrated system, consisting of several sub dynamical system models, through which we develop and validate effective CLC algorithms for artificial pancreas with clinical data.

Multiple Invasion Speeds in a Two-Species Integro-Difference Competition Model

Bingtuan Li
University of Louisville, USA

We discuss an integro-difference competition model for the case that two species consecutively invade a habitat. We show that if a species spreads into a traveling wave of its rival, or if two species expand their spatial ranges in both directions, in a direction where open space is available, the species with larger invasion speed can always establish a wave moving into open space with its own speed. We demonstrate that when one species is stronger in competition, under appropriate conditions, the speeds at which the boundaries between two species move can be analytically determined. We find that in general there are multiple invasion speeds in the model. It is possible for a species to develop two separate waves propagating with different invasion speeds. It is also possible for each species to establish a single wave spreading with distinct speeds in both directions.

Modelling Diapause in Mosquito Growth

Yijun Lou
Hong Kong Polytechnic University, Hong Kong
Kaihui Liu, Daihai He, Daozhou Gao, Shigui Ruan

Diapause (or named as dormancy traditionally) as a process of physiological rest is widespread in insects and other invertebrate organisms, which serves as a key survival mechanism in response to adverse environmental conditions. In this paper, a novel modelling framework is proposed to investigate the effects of diapause on population growth, where diapause period is taken as an independent dynamic process, during which the population dynamics is completely different from that in the normal developmental and post-diapause periods. To explicitly describe population growth with different diapausing stages, either immature or adult ones, two different delay differential equation models are constructed with an emphasis on mosquitoes. These two models can be further unified into one with different death rates during the diapause period. In addition to the theoretical analysis, numerical simulations are performed to investigate the seasonality of population abundances of two temperate mosquito species and the sensitivity analysis of the diapause-related parameters, which validate theoretical models and further identify the key role on population persistence that diapause plays.

Coexistence and Bistability of a Competition Model in Open Advective Environments

Hua Nie
Shaanxi Normal University, Peoples Rep of China
Yuan Lou, Yane Wang

The community composition in open advection environments, where individuals are exposed to unidirectional flow, is formed by the complex interplays of hydrological and biological factors. We investigate the coexistence mechanism of species by a reaction-

diffusion-advection competition model proposed by Lutscher et al.. It turns out that the locations of two critical curves, which separate the stable region of the semi-trivial solutions from the unstable one, determines whether coexistence or bistability happens. Furthermore, the analytical and numerical results suggest a tradeoff driven coexistence mechanism. More precisely, there is a tradeoff between the dispersal strategy and growth competence which allows the transition of competition outcomes, including competition exclusion, coexistence and bistability. This shifting may have an effect on the community composition in aquatic habitat.

Effects of Macroalgal Toxicity and Overfishing on the Resilience of Coral Reef

Samares Pal
University of Kalyani, India
Joydeb Bhattacharyya

Competition between macroalgae and corals for occupying the available space in sea bed is an important ecological process underlying coral-reef dynamics. While herbivorous reef-fish play a beneficial role in decreasing the growth of macroalgae, macroalgal toxicity and overfishing of herbivores leads to the proliferation of macroalgae in coral reef ecosystem, which eventually changes the community structure towards macroalgae-dominated reef ecosystem. We analyze a mathematical model of interactions between coral, macroalgae and herbivores to investigate coral-macroalgal phase shifts by assuming that the growth of herbivorous Parrotfish is limited by coral cover. It is observed that in presence of macroalgal toxicity and overfishing of Parrotfish the system exhibits hysteresis through saddle-node bifurcation and transcritical bifurcation. We examine the effects of macroalgal toxicity and herbivore-harvesting in the resilience of coral-macroalgae coexistence steady state. Further, we study the non-autonomous version of the model by incorporating synchronous or asynchronous seasonal variations in different parameters. By using of Mawhin's continuous theorem of coincidence degree theory, a sufficient condition is obtained for the existence of a positive periodic solution. Computer simulations have been carried out to illustrate different analytical results.

Modeling Pharmacodynamics on HIV Latent Infection

Libin Rong
University of Florida and Xinyang Normal Univ., USA
Naveen Vaidya

Highly active antiretroviral therapy has successfully controlled HIV replication in many patients. The treatment effectiveness may depend on the pharmacodynamics of antiretroviral drugs. We integrate several drug-related parameters into an HIV infection model to investigate the effects of drug pharmacodynamics on the HIV latent reservoir and viral load dynamics. We show that pharmacodynamic characteristics of drugs and the dosing schedule can significantly affect the outcome of either early or late treatment. This is a joint work with Naveen Vaidya.

Persistence and Extinction of Population in Reaction-Diffusion-Advection Model with Allee Effect Growth

Junping Shi
College of William and Mary, USA
Yan Wang, Jinfeng Wang

A reaction-diffusion-advection equation with strong or weak Allee effect growth rate is proposed to model a single species stream population in a unidirectional flow. Here random undirected movement of individuals in the environment is described by passive diffusion, and an advective term is used to describe the directed movement in a river caused by the flow. Under biologically reasonable boundary conditions, the existence of multiple positive steady states are shown when both the diffusion coefficient and the advection rate are small, which lead to different asymptotic behavior for different initial conditions. On the other hand, under different conditions, the extinction of population occurs.

Global Bifurcation Analysis of a Single Species Population Model

Hongying Shu
Shaanxi Normal University, Peoples Rep of China

We first consider a class of stage-structured differential equations. By using the time delay as a bifurcation parameter, we analytically prove that these local Hopf bifurcation values are neatly paired, and each pair is jointed by a bounded global Hopf branch. We use the well-known Mackey Glass equation with a stage structure as an illustrative example to demonstrate that bounded global Hopf branches can induce interesting and rich dynamics. We then study the dynamics of a delayed diffusive hematopoiesis model with Dirichlet boundary conditions. we show that the only positive steady state is a constant solution. By using the delay as a bifurcation parameter, we show that the model has infinite number of Hopf bifurcation values and the global Hopf branches bifurcated from these values are unbounded, which indicates the global existence of periodic solutions.

Spatial Propagation for a Parabolic System with Multiple Species Competing for Single Resource

Zhiguo Wang
Shaanxi Normal University, Peoples Rep of China
Hua Nie, Jianhua Wu

In this talk, we focus on a model of multi-species competing for a single growth-limiting resource. We aim to use the dynamics of such a problem to describe the invasion and spread of species which are introduced localized in space. The existence, uniqueness and uniform boundedness of the Cauchy problem are investigated. Our result shows that the asymptotic speed of spread for multi-species is characterized by the minimum wave speed of the positive traveling wave solutions associated with this system.

Impact of Bacterial Hyperinfectivity on Cholera Epidemics in Spatially Heterogeneous Environments

Xueying Wang
Washington State University, USA
Feng-Bin Wang

The transmission of cholera, a water- and food-borne intestinal infection, involves complex interactions among human hosts, pathogens, and the environment. This talk will address the epidemic dynamics of cholera in non-homogenous environments, with a focus on the spatial variation and bacterial hyperinfectivity, using partial differential equation models. In this work, we develop a new modeling framework to study the effect of bacterial hyperinfectivity on cholera epidemics in a spatially non-homogeneous environment. First, the global threshold dynamics is established based on the derived basic reproduction number. Secondly, the global attractivity of the unique endemic equilibrium is obtained in a special case. Thirdly, the dependence of the basic reproduction number on model parameters is investigated by using theoretical and numerical means. Our results indicate that the prevention and intervention strategies need to take into account the non-homogeneity of the environments in order to effectively control cholera while optimize the use of available resources.

Spreading with Two Speeds and Mass Segregation in a Diffusive Competition System with Free Boundaries

Chang-Hong Wu
National University of Tainan, Taiwan
Yihong Du

In this talk, we focus on the spreading behavior of two invasive species modeled by a Lotka-Volterra diffusive competition system with two free boundaries in a spherically symmetric setting. We show that, for the weak-strong competition case, under suitable assumptions, both species in the system can successfully spread into new environment, but their spreading speeds are different, and their population masses tend to segregate. This is a joint work with Yihong Du (University of New England).

Special Session 86: Recent Advances in Mathematical Modeling in Health and Disease

Yi Jiang, Georgia State University, USA
James A. Glazier, Indianan University Bloomington, USA
Yangjin Kim, Konkuk University, Korea

Mathematical models in biology and physiology integrate known biological constructs to understand a particular process or phenomenon, to fill in the gaps of the unknown, to create new hypotheses, and to redesign the models based on experimental outcomes and measures. The aim of this special session is to present the state of the art in modeling of development and diseases, in particular, cancer. The session has an emphasis in the integration of models from multiple scales into a framework that can make predictions at scales relevant to physiology and clinical applications.

Modeling the Role of Gradients in the Somatic Evolution of Solid Tumors

James Glazier
Indiana University, USA

Tumor cells and structure both evolve due to heritable variation of cell behaviors and selection over periods of weeks to years (somatic evolution). Microenvironmental factors exert selection pressures on tumor-cell behaviors, which influence both the rate and direction of evolution of specific behaviors, especially the development of tumor-cell aggression and resistance to chemotherapies. We present a multicell, virtual-tissue model of tumor somatic evolution, which includes essential cell behaviors, microenvironmental components and their interactions. The self organized gradients created by the growing tumor lead to regions with different cell survival rates, driving the evolution of less cohesive cells with lower levels of cadherins and higher levels of integrins. Such reduced cohesivity is a key hallmark in the progression of many types of solid tumors.

Somites With and Without a Clock

James Glazier
Indiana University, USA

The formation of body segments (somites) in vertebrate embryos is accompanied by molecular oscillations (the segmentation clock). Interaction of this oscillator with a posterior-moving molecular gradient along the body axis is generally believed to control somite number, size, and axial identity according to the clock-and-wavefront model. We present simulations that replicate many observed features of segmentation based on these hypotheses. However, experiment using the differentiation inducer Noggin, show that the clock-and-wavefront mechanism is not necessary for somite formation and we present an alternative mechanical instability mechanism which also explains many of the observed features of somitogenesis. The moral is not that one mechanism is right and the other wrong. Rather, Occam's Razor does not always apply in biology–in many cases fundamentally distinct mechanisms each of which would be independently sufficient, act simultaneously during development to build specific structures.

Leading the Pack: Collective Invasion in Cancer

Yi Jiang
Georgia State University, USA
Bin Zhang, Adam Marcus

A major reason for cancer treatment failure and disease progression is a heterogeneous composition of tumor cells at the genetic, epigenetic, and phenotypic levels. While tremendous efforts have tried to characterize the makeups of single cells, much less is known about interactions between heterogeneous cancer cells and between cancer cells and the microenvironment in the context of cancer invasion. Clinical studies show that invasion predominantly occurs via collective invasion packs, which invade more aggressively and result in worse outcomes. Using non-small cell lung cancer spheroids in collagen, we show that the invasion packs consist of leaders and followers. In vitro and in silico experiments show that leaders and followers engage in mutualistic social interactions during collective invasion. Many fundamental questions remain: What is the division of labor within the heterogeneous invasion pack? How does the leaders emerge? How do the invasion packs interact with the stroma? Can the social interaction network be exploited to devise novel treatment strategies? I will present the recent experimental and modeling efforts that address these questions. Analyzing this social interaction network can potentially reveal the "weak-links", which when perturbed can disrupt collective invasion.

The Shapes of Cell Migration

Yi Jiang
Georgia State University, USA
Xiuxiu He, Kuangcai Chen, Ning Fang

Cell shape is determined by the interaction of many elements such as the cytoskeleton, cell membrane and adhesion to the substrate. Cell shape changes during migration. Can we discriminate cell migration patterns from cell shape? We addressed this question by analyzing a large number of cell migration images over time, in the absence of symmetry breaking perturbation. Our findings suggest that 1. Effective cell migration is characterized with long cellular persistence time, low speed variation, spatial-temporally coordinated protrusion and contraction; 2. The cell

shape variation space is low dimensional; and 3. Migration behavior can be determined by a single image projected in the low dimensional cell shape variation space. Our findings provide a quantitative underpinning for the general practice of using cell morphology to differentiate cell phenotype and states.

Cytoskeletal Dynamics and Mechanosensing in Immune Cells

Arpita Upadhyaya
University of Maryland, USA

Lymphocyte activation is an essential step in the adaptive immune response, and involves the binding of specialized receptors with antigen on the surface of antigen presenting cells. This leads to changes in cell morphology and assembly of receptors and signaling proteins into microclusters, which are essential for immune cell activation. During activation, immune cells interact with structures possessing a diverse range of physical properties. We have examined how T cells and B cells respond to physical cues such as stiffness, topography and ligand mobility. I will discuss the roles of the cytoskeleton in the exertion of mechanical stresses that support signaling activation and microcluster assembly. We have used traction force microscopy to measure the forces exerted by T cells during activation on elastic substrates. Forces exerted are largely due to actin dynamics and T cells are mechanosensitive to substrate stiffness. Further, these forces are regulated by microtubule dynamics through Rho activity and myosin filament assembly. Our studies highlight the importance of cytoskeletal forces in T cell receptor activation. More recently, we have examined the role of actin dynamics on forces exerted by cytotoxic T lymphocytes. These studies may have implications for understanding the effectiveness of lymphocytes in killing cancer cells.

Landscape Quantification of Cancer

Jin Wang
Stony Brook University, USA
Chong Yu, Wenbo Li, Chunhe Li,

We provide a physical and quantitative approach to cancer. We consider cancer as a state of the underlying gene networks. The stability of the cancer state can be characterized by the depth of the basin of the attraction, or more precisely the barrier or the time to escape from the cancer basin. We explore the underlying landscape for the underlying gene networks to reveal the presence of normal, premalignant and cancer states. We uncover the relationship and switching among these states as well as the underlying biological implications. The interplay between the cancer and development will also be explored. Finally the relationship between the cancer and immune responses will be investigated. Our study shows certain key genes and gene regulations in determining the cancer progresses. This facilitates the drug discovery from the network medicine perspectives.

Special Session 88: Geometric Analysis

Paul Laurain, Université Paris VII, France
Jorge Lira, Universidade Federal do Ceará, Brazil
Luciano Mari, Scuola Normale Superiore, Italy

Geometric Analysis consists in the use of analytic tools such as PDE theory or geometric measure theory to solve natural geometric questions. It has been a growing field over the past decades, with some spectacular achievements such as the proofs of the Poincaré Conjecture using geometric flows, of the classical conjectures by Willmore and Lawson in the realm of Differential Geometry of surfaces, and of the Positive Mass Theorem in General relativity. The goal of this special session is to bring together researchers of the field from all around the world to work together and explore new perspectives and foster scientific interchanges.

The Mass of Asymptotically Hyperbolic Manifolds with Non-Compact Boundary

Sergio Almaraz
Universidade Federal Fluminense (UFF), Brazil
Levi Lopes de Lima

We define a mass-type geometric invariant for Riemannian manifolds asymptotic to the hyperbolic half-space and discuss a positive mass theorem.

Ends of Immersed Minimal and Willmore Surfaces in Asymptotically Flat Spaces

Yann Bernard
Monash University, Australia
Tristan Riviere

We study ends of an oriented, immersed, noncompact, complete Willmore surfaces, which are critical points of the integral of the square of the mean curvature, in asymptotically flat spaces of any dimension; assuming the surface has L^2-bounded second fundamental form and satisfies a weak power growth on the area. We give the precise asymptotic behavior of an end of such a surface. This asymptotic information is very much dependent on the way the ambient metric decays to the Euclidean one. Our results apply in particular to minimal surfaces in any codimension.

Geometric Analysis on Metric Measure Spaces with Uniform Ricci Bounds from Below and Applications

Shohei Honda
Tohoku University, Japan

In this talk we will discuss some spectral properties on metric measure spaces with uniform Ricci bounds from below. Applications include Weyl's law on such spaces, and continuities of geometric PDEs (including p-Laplace equations for all p) with respect to measured Gromov-Hausdorff convergence. The later one implies estimates for solutions of such PDEs which are new even for smooth manifolds.

On the 1/H Flow Via P-Laplace Approximation Under Ricci Lower Bounds

Luciano Mari
Scuola Normale Superiore, Italy
Alberto Setti, Marco Rigoli

In this talk, we consider the existence problem for weak solutions of the Inverse Mean Curvature Flow on a complete manifold with a Ricci lower bound. Solutions either issue from a point or from the boundary of a relatively compact open set. To prove their existence in the sense of Huisken-Ilmanen, we follow the strategy pioneered by J. Moser that uses approximation by p-Laplacian kernels. In particular, we prove new and sharp gradient estimates for the kernel of the p-Laplacian on M via the study of the fake distance associated to it. We address the compactness of the flowing hypersurfaces, as well as monotonicity formulas in the spirit of Geroch's one.

Prescribing the Tangent Plane of a Minimal Place

Laurent Mazet
Université de Paris-Est Creteil, France

In this talk we will explain how minimal plane with prescribed tangent space at some point can be constructed in asymptoticaly Euclidean 3-manifold. This is a joint work with H. Rosenberg.

The Willmore Functional and Ambient Scalar Curvature

Jan Metzger
Univerity of Potsdam, Germany
Tobias Lamm, Felix Schulze

In this talk I will consider the Willmore functional and related variational problems for surfaces embedded into targets with non-flat Riemannian metrics. The main focus is the interaction of intrinsic and extrinsic curvature quantities. More specifically, we study the position of surfaces critical for the Willmore functional subject to an area constraint with respect to the ambient scalar curvature.

Stratified Spaces and Synthetic Curvature Bounds

Ilaria Mondello
Université de Paris-Est Creteil, France
J. Bertrand, C. Ketterer, T. Richard

Singular metric spaces appear naturally in geometry, for example as Gromov-Hausdorff limits of smooth manifolds, quotients, or singularities in geometric flows. One way to study such singular spaces is to introduce weak notions of curvature and dimension, as successfully done in the work of K. T. Sturm and J. Lott and C. Villani by means of entropy and optimal transport. The aim of this talk is to present a new class of geometric examples of metric spaces satisying the curvature-dimension condition. Such class consists in manifolds with cone-like singularities, more precisely stratified spaces, satisfying an appropriate lower bound on the Ricci curvature.

Examples of Compact Einstein Four-Manifolds with Negative Curvature

Bruno Premoselli
Université Libre de Bruxelles ULB, Belgium
J. Fine

We construct Einstein metrics of negative sectional curvature on ramified covers of compact hyperbolic four-manifolds with symmetries, initially considered by Gromov and Thurston. These metrics are obtained through a deformation procedure. The approximate Einstein metric is an interpolation between a black-hole Einstein metric near the branch locus and the pulled-back hyperbolic metric. The deformation relies on an involved bootstrap procedure. Our construction yields the first example of compact Einstein manifolds with negative sectional curvature which are not locally homogeneous.

On the Moving Plane Method

Berardino Sciunzi
UNICAZ, Italy

We shall discuss the applicability of the moving plane method to singular solutions to semilinear and quasilinear elliptic PDEs.

Necessary Conditions for Submanifolds to Be Connected in a Riemannian Manifold

Keomkyo Seo
Sookmyung Women's University, Korea

It is well-known that any simple closed curve in $\mathbb{R}^3$ bounds at least one minimal disk, which was independently proved by Douglas and Radò. However, for any given two disjoint simple closed curves, we cannot guarantee existence of a compact connected minimal surface spanning such boundary curves in general. From this point of view, it is interesting to give a quantitative description for necessary conditions on the boundary of compact connected minimal surfaces. In this talk, we give various necessary conditions and nonexistence results for compact connected minimal submanifolds, Bryant surfaces, and surfaces with small L^2 norm of the mean curvature vector in a Riemannian manifold.

A Quantitative Version of a Theorem of Alexandrov

Luigi Vezzoni
Università di Torino, Italy
Giulio Ciraolo

Let M be the Euclidean space or the hyperbolic space. A celebrated theorem of Alexandrov asserts that spheres are the only closed constant mean curvature hypersurfaces embedded in M. The talk mainly focuses on the following quantitive version of the Alexandrov theorem:

Theorem [Ciraolo-V.]. *Let S be an n-dimensional, C^2-regular, connected, closed hypersurface embedded in M. There exist constants ϵ, $C > 0$ such that if*

$$\operatorname{osc}(H) \leq \epsilon,$$

then there are two concentric balls B_{r_i} and B_{r_e} such that

$$S \subset \overline{B}_{r_e} \setminus B_{r_i},$$

and

$$r_e - r_i \leq C \operatorname{osc}(H).$$

The constants ϵ and C depend only on n and upper bounds on the C^2-regularity and the area of S.

The proof of the theorem makes use of a quantitive study of the method of the moving planes and the result implies a new pinching theorem for hypersurfaces. Furthermore, the theorem is optimal in a sense that it will be specified in the talk. The last part of the talk will be about an on-going study on the generalization of the result.

On the Mean Curvature Flow with Free Boundary Supported on a Double Cone

Valentina-Mira Wheeler
University of Wollongong, Australia
Glen Wheeler

In this talk we discuss some recent progress on the analysis of the mean curvature flow with free boundary supported on a double cone. This includes results on convergence and finite-time singularities. Singularities can be classified by their type, as in the case of the compact flow. Here, we have a new kind of singularity, that we call type 0. We give examples of support hypersurfaces that yield the development of singularities of a specified type for certain initial data. This includes examples of the elusive Type 2 singularities.

Special Session 89: Advances in Analysis of Mathematical Problems Arising from Materials and Biological Science

Toyohiko Aiki, Japan Women's University, Japan
Sander Hille, Leiden University, Netherlands
Adrian Muntean, Karlstad University, Sweden

This special session focuses on technologically important and mathematically interesting problems arising from material and biological sciences. Besides well-posedness and asymptotic studies of nonlinear PDEs and related particle systems, the session also includes presentations on averaging processes in porous media, non-periodic homogenization, statistical mechanics of particle systems, and multiscale dynamics. Interactions between different branches of mathematics are particularly welcome (e.g. measure theory and PDEs, probability and analysis of evolution equations.

Weak Formulation of a Free Boundary Problem Describing Adsorption Phenomena Appearing Concrete Carbonation Process

Toyohiko Aiki
Japan Women's University, Japan

Recently, we have investigated a multi-scale system consisting of a nonlinear diffusion equation and a free boundary problem. The system was proposed as a mathematical model to concrete carbonation process in a three-dimensional domain.
We note that the free boundary problem describes adsorption phenomena appearing carbonation process.
In this talk we focus on a mathematical treatment for the free boundary problem.
Here, let u and s be the relative humidity and the degree of saturation, respectively.
Then, these variables satisfy

$$\begin{aligned}
&\rho_g u_t - \kappa u_{xx} = 0 \text{ in } Q_s(T),\\
&u(t,1) = h(t) \text{ for } 0 < t < T,\\
&u(0,x) = u_0(x) \text{ for } s_0 < x < 1, s(0) = s_0,\\
&s'(t) = a(u(t,s(t) - \varphi(s(t))) \text{ for } 0 < t < T,\\
&\kappa u_x(t,s(t)) = (\rho_a - \rho_g u(t,s(t)))\alpha(s(t),u(t,s(t))) \text{ for } 0 < t < T,
\end{aligned}$$

where $Q_s(T) := \{(t,x)|s(t) < x < 1, 0 < t < T\}$, s_0 and u_0 are initial values of s and u, h is a given boundary data, ρ_g (resp. ρ_a) is the density of water in liquid (resp.air), κ is a diffusion coefficient, a is a positive constant, $\varphi : \mathbb{R}^2 \to \mathbb{R}$ is a non-negative continuous function.
In our previous works we have established uniqueness and existence of a solution, globally in time, under the assumption $0 \le h \le 1 - \delta$, where δ is a small positive constant.
However, when we consider the multi-scale system, it is not easy to prove existence of a solution because of the assumption for h. Then, in order to prove the existence without this assumption we provide a weak formulation for the free boundary problem. Our aims of this talk are to formulate a weak solution of the free boundary problem and give a result on existence of an approximate solution.

A Model for Thermal Diffusion by Plasmonic Heating

Martijn Anthonissen
Eindhoven University of Technology, Netherlands
Toyohiko Aiki, Miu Takahashi, Ryoko Shimad, Hitomi Sakai

We present a mathematical model for thermal diffusion induced by plasmonic heating using silver nanoparticles. We consider a two-dimension domain with a liquid region and areas where silver is deposited. The silver areas are illuminated with laser light, which causes local hot spots and temperature gradients. The liquid contains a species that is assumed to be temperature-sensitive, meaning that a flow of matter will occur due to the temperature gradient. This phenomenon is called the *Soret effect.* Because we are interested in modeling and simulating the Soret effect in this paper, we do not use a full Maxwell equations based description of the plasmonic heating. Instead we present a simplified heat source that has the same characteristics. The heat equation for temperature and advection-diffusion equation for the species concentration are discretized using a finite volume method. The resulting nonlinear system is solved using Newton's method.

Perspectives in Nonlinear Diffusion Equations As Asymptotic Limits of Cahn–Hilliard Systems

Takeshi Fukao
Kyoto University of Education, Japan
Taishi Motoda

In this talk, recent advances in nonlinear diffusion equations as asymptotic limits of Cahn–Hilliard systems is treated. The weak formulation of the Stefan problem, the porous media equation, the fast diffusion equation, the weak formulation of Hele-Shaw equation and many other nonlinear diffusion equations of the form

$$\frac{\partial u}{\partial t} - \Delta\beta(u) = g$$

are target problems.

In Fukao (2016) and Colli–Fukao (2016), an idea of asymptotic limits of Cahn–Hilliard system is introduced for the characterization of the solution. More precisely, as the level of approximation the term of nonlinear diffusion $\beta(u)$ is treated as a monotone term in the Cahn–Hilliard system.

$$\frac{\partial u}{\partial t} - \Delta\mu = 0, \quad \mu = -\varepsilon\Delta u + \beta(u) + \pi_\varepsilon(u) - f.$$

This approach has an advantage to improve the growth condition of $\hat{\beta}$, the primitive of nonlinear term β. The problem is considered under the Robin type boundary condition in this talk. This study is based on the joint work with Taishi Motoda, Kyoto University of Education.

Stefan/Navier-Stokes Problems – Quasi-Variational Inequality Approach

Nobuyuki Kenmochi
Chiba University, Japan
M. Gokieli, M.Niezgodka

This work is motivated by modelling the formation of ice fields. In the begining of winter one can see a lot of pieces of ice floating on the sea in the north. They reach the beach and accumulate there, and form a large ice-field. We are interested in the dynamics of such a natural phenomenon.

On a Free Boundary Problem Describing Swelling Process in Porous Materials

Kota Kumazaki
Nagasaki University, Japan
Adrian Muntean

In this talk, we propose a mathematical model for water swelling process in concrete materials. Concrete material has infinite microscopic holes, and water swelling occurs in each hole by the influence of moisture in the whole material. As the first investigation of this process, we focus on water swelling process in one hole. Our model is a free boundary problem consisting of a diffusion equation for water in a one microscopic hole and an ordinary differential equation describing the growth rate of the front of water region. In this talk, we discuss the existence and uniqueness of a solution for this problem.

A Cahn-Hilliard Type System Coupled with a Heat Equation on Unbounded Domains

Shunsuke Kurima
Tokyo University of Science, Japan

In this talk we deal with vanishing viscosity for a Cahn-Hilliard type system coupled with a heat equation on unbounded domains with smooth bounded boundary. Colli-Gilardi-Rocca-Sprekels (2017) studied it in the case of bounded domains by using the Aubin-Lions lemma. However, this lemma does not work directly in the case of unbounded domains. In this talk we will try to discuss vanishing viscosity for the above system in unbounded domains.

A Two-Scale Pressure Model: Well-Posedness and Inverse Stability Estimates

Martin Lind
Karlstad University, Sweden
Adrian Muntean, Omar Richardson

We consider a coupled micro-macro parabolic-elliptic system modeling the interplay between two pressures in a gas-liquid mixture close to equilibrium that is filling a porous media with distributed microstructures. We establish well-posedness as well as other properties of the system. We also obtain local stability of the inverse micro-macro Robin transfer coefficient.

Simultaneous Homogenization and Dimension Reduction of Nonlinear Transport Through Thin Heterogeneous Membranes

Adrian Muntean
Karlstad University, Sweden
Emilio Cirillo, Ida de Bonis

We study the diffusion of particles through a thin heterogenous membrane under a one–directional nonlinear drift. Using mean-field equations derived from a Monte Carlo lattice dynamics for the problem at hand, we study the possibility to upscale the system and to compute the effective transport coefficients accounting for the presence of the membrane. For a special scaling regime, we perform a simultaneous homogenization asymptotics and dimension reduction, allowing us to replace completely the heterogenous membrane by an homogeneous obstacle line provided with effective transmission conditions. The heterogeneities we account for in this context are assumed to be arranged periodically, but the same working techniques can cover also the locally periodic case.

Existence of Solutions for Brewing Model for Japanese Sake of Quasi-Variational Type

Yusuke Murase
Meijo University, Japan

The following system is a mathematical model for the first stage of brewing Japanese Sake. It's configured by differential equations (with initial conditions, boundary conditions), and a constraint condition. This model is studied by A. Ito and Y. M.

$$\begin{cases} \theta_t - d_0\Delta\theta + g_0(\theta, u_1, u_2) = f_0 & \text{a.e. in } Q \\ (u_1)_t - d_1\Delta u_1 + g_1(\theta, u_1, u_2) = f_1 & \text{a.e. in } Q \\ (u_2)_t - d_2\Delta u_2 + g_2(\theta, u_1, u_2) = f_2 & \text{a.e. in } Q \\ (u_1, u_2) \in K(\theta) & \text{a.e. in } Q \\ (v)_t - d_3\Delta v = -c_3 v u_1 + f_3 & \text{a.e. in } Q \\ (w)_t - d_4\Delta w = c_6 v u_1 - (c_7 u_1 + c_8 u_2)w + f_4 & \text{a.e. in } Q \\ \frac{\partial\theta}{\partial\mathbf{n}} + c_9\theta = h & \text{a.e. on } \Sigma \\ \frac{\partial u_1}{\partial\mathbf{n}} = \frac{\partial u_2}{\partial\mathbf{n}} = \frac{\partial v}{\partial\mathbf{n}} = \frac{\partial w}{\partial\mathbf{n}} = 0 & \text{a.e. on } \Sigma \\ \theta(0) = \theta_0, \quad u_1(0) = u_{1,0}, \quad u_2(0) = u_{2,0} & \\ v(0) = v_0, \quad w(0) = w_0 & \text{a.e. on } \Omega \end{cases}$$

The constraint condition is one of the characteristic point of our model. We can see that equations for u_1, u_2 and the constraint condition configure "Quasi-variational inequality". In my talk, we discuss existence of weak solutions for our model, and some numerical simulations are presented.

A Linear and Structure-Preserving Scheme for a Non-Local Conservative Allen–Cahn Equation

Makoto Okumura
Osaka University, Japan

We propose a linear difference scheme for a non-local conservative Allen–Cahn equation based on a combination of the discrete variational derivative method (DVDM) and a linearization technique. DVDM is a numerical method for designing schemes for PDEs. DVDM schemes inherit conservative or dissipative properties from the original PDEs in a discrete sense. By this approach, we obtain a nonlinear scheme in general, in which the current state is decided by a nonlinear relation concerning the previous state. Then, we need some iterative solver such as the Newton method to solve the system. This means that the computational cost is expensive. Therefore, we also use a linearization technique. The basic idea of our linearization technique is the decompositions of nonlinear terms by introducing extra time steps of numerical schemes. We expect that the proposed linear scheme is faster than the nonlinear one. In this talk, we show the existence and uniqueness of a solution for the proposed scheme and stability. We also show numerical experiments.

Semidiscrete Finite Element Approximation and A-Priori Feedback Strategy Estimates for a Two-Scale Pressure Problem

Omar Richardson
Karlstad University, Sweden
Martin Lind, Adrian Muntean

We consider a coupled micro-macro parabolic-elliptic system of partial differential equations (PDEs) modelling the interplay between two pressures in a gas-liquid mixture close to equilibrium that is filling a porous medium with distributed microstructures. We prove well-posedness for the space-discrete time-continuous problem using a semidiscrete Galerkin scheme. We obtain a priori error estimates and convergence rates for the discretized problem. Additionally, we design an a-priori feedback strategy that refines the mesh in a pre-computational stage to reduce the overall numerical error.

Gradient Systems for Anisotropic Energies Associated with Image Processings

Ken Shirakawa
Faculty of Education, Chiba University, Japan
Hiroshi Watanabe

In this talk, we consider gradient systems of nonconvex functionals, defied by integrals on a bounded spatial domain $\Omega \subset \mathbb{R}^2$. Each functional is based on a governing energy for anisotropic image processing, proposed by [Berkels et -al, pp. 293–301, Vision Modeling and Visualization 2006 (2006)], and the principal part of the corresponding gradient system is descried in a nonstandard form of *partial differential inclusions*, which contains a composition $\partial\gamma \circ R$ of: a (possibly) set-valued subdifferential $\partial\gamma$ of an anisotropic metric $\gamma \in W^{1,\infty}(\mathbb{R}^2)$; and a rotation matric $R \in C^\infty(\mathbb{R}; \mathbb{R}^{2\times 2})$. Under appropriate assumptions, some mathematical observations for the gradient system will be provided on the basis of the time-discretization approach.

Spin-Diffusions and Diffusive Molecular Dynamics

Gideon Simpson
Drexel University, USA
Brittan Farmer, Mitchell Luskin, Petr Plechac

Metastable configurations in condensed matter typically fluctuate about local energy minima at the femtosecond time scale before transitioning between local minima after nanoseconds or microseconds. This vast scale separation limits the applicability of classical molecular dynamics (MD) methods and has spurned the development of a host of approximate algorithms. One recently proposed method is diffusive MD which aims at integrating a system of ordinary differential equations describing the likelihood of occupancy by one of two species, in the case of a binary alloy, while quasistatically evolving the locations of the atoms. While diffusive MD has shown itself to be efficient and provide agreement with observations, it is fundamentally a model, with unclear connections to classical MD. In this work, we formulate a spin-diffusion stochastic process and show how it can be connected to diffusive MD. The spin-diffusion model couples a classical overdamped Langevin equation to a kinetic Monte Carlo model for exchange amongst the species of a binary alloy. Under suitable assumptions and approximations, spin-diffusion can be shown to lead to diffusive MD type models. The key assumptions and approximations include a well-defined time scale separation, a choice of spin-exchange rates, a low temperature approximation, and a mean field type approximation. We

derive several models from different assumptions and show their relationship to diffusive MD. Differences and similarities amongst the models are explored in a simple test problem.

Control Problem for a Concrete Carbonation Process with Hysteresis

Sergey Timoshin
SRM University AP, India
Toyohiko Aiki

A nonlinear partial differential control system arising in the moisture transport in concrete carbonation modelling is considered. The system consists of a diffusion equation for moisture and an ordinary differential equation accounting for the hysteresis effect appearing in the process. The control function is subject to a state-dependent and nonconvex constraint. We prove the existence of solutions to the system and establish some of its properties such as relaxation and bang-bang principle: the density in an appropriate topology of solutions and extreme solutions of the original system among those of the system with the convexified control constraint.

Solvability of Problems for Charged Particles in Plasmas with Angle Error in Magnetic Field

Yutaka Tsuzuki
Hiroshima Shudo University, Japan

We deal with initial–boundary problems for Vlasov–Poisson systems in a half-space with external magnetic force horizontal to a wall. In 2013, Skubachevskii gives local-in-time solvability to the system. Moreover, in 2017, solutions with larger time were obtained by effectively using the magnetic force whose direction is horizontal to the wall. This talk provides an existence result for the system where the magnetic force has angle error in the vertical direction.

Application of Homogenization Structures to Sulfate Corrosion of Concrete

Arthur Vromans
Eindhoven University of Technology & Karlstads Universitet, Netherlands
Adrian Muntean

Homogenization is a mathematical theory that takes a micro scale system and upscales it. Applications of homogenization are multiple, e.g. in understanding biological tissue behavior, oil and gas extraction and geothermal energy systems. In 1989 Gabriel Nguetseng introduced in [1] the concept of two-scale convergence in the homogenization context. This has significantly simplified proofs and has, therefore, facilitated the applicability of the theory. In 2003 and 2004 he generalized this concept in [2, 3] even further with the notion of a homogenization structure. He showed that two-scale convergence is intimately linked with the mathematical description of the structure of the microscopic domain (such as porous rock). Currently his theory has only been applied to deterministic and static stochastic systems. In this talk we provide further developments concerning the homogenization of a PDE system connected to sulfate corrosion of concrete. This represents joint work with Adrian Muntean (Karlstad, Sweden).

References

[1] G. Nguetseng, 1989, *A General Convergence Result for a Functional Related to the Theory of Homogenization*, SIAM J. Math. Anal. 20 (3), 608-623

[2] G. Nguetseng, 2003, *Homogenization Structures and Applications I*, Z. Anal. Anwend. 22 (1), 73-107

[3] G. Nguetseng, 2004, *Homogenization Structures and Applications I*, Z. Anal. Anwend. 23 (3), 483-508

Solvability of Some Systems for Parabolic-Hyperbolic Conservation Laws

Hiroshi Watanabe
Oita University, Japan

Systems for parabolic-hyperbolic conservation laws are interesting research object in the sense of mathematics and applications. In a mathematical point of view, the systems have both properties of hyperbolic equations and those of parabolic equations. Therefore, it has discontinuous solutions in general. From this, the unified well-posedness theory is not given. In application point of view, the systems can be applied to many mathematical models (fluid dynamics, traffic flow, aggregation phenomena, crowd dynamics and so on). In this talk, we formulate initial value problems for the systems and discuss solvability of the problem.

Double Quasi-Variational Evolution Equations Governed by Time-Dependent Subdifferentials and Applications

Noriaki Yamazaki
Kanagawa University, Japan
Nobuyuki Kenmochi, Ken Shirakawa

We consider the following double quasi-variational evolution equations governed by time-dependent subdifferentials in a uniformly convex Banach space V^*:

$$\text{(QP)}\quad \partial_* \psi^t(u; u'(t)) + \partial_* \varphi^t(u; u(t)) + g(t, u(t))$$

such that $f(t)$ in V^* for a.a. $t \in (0, T)$. where V is a (real) uniformly convex Banach space with the uniformly convex dual space V^*, $0 < T < \infty$, $u' = du/dt$ in V, $g(t, \cdot)$ is a single-valued Lipschitz operator in V^*, and f is a given V^*-valued function. The time-dependent function $\psi^t(v; z)$ is proper, lower semi-continuous (l.s.c.), and convex in $z \in V$. Also, $\varphi^t(v; z)$ is a time-dependent, non-negative, continuous convex function in $z \in V$. Note that $(t, v) \in [0, T] \times C([0, T]; H)$ is a parameter that determines the convex functions $\psi^t(v; \cdot)$ and $\varphi^t(v; \cdot)$ on V. The dependence of function v upon $\psi^t(v; \cdot)$ and $\varphi^t(v; \cdot)$ is, in general, allowed to be non-local. In addition, the subdifferentials $\partial_* \psi^t(v; z)$ of $\psi^t(v; z)$ with respect to $z \in V$ is a multivalued operator in V^*, and $\partial_* \varphi^t(v; z)$ of $\varphi^t(v; z)$ with respect to $z \in V$ is a single-valued linear operator in V^*. In this talk, we establish the existence theory of abstract double quasi-variational evolution equations (QP). We also give some applications to nonlinear PDEs with gradient constraint for time-derivatives. This is a joint work with Nobuyuki Kenmochi (ICM, University of Warsaw, Warsaw, Poland) and Ken Shirakawa (Chiba University, Chiba, Japan).

Long Time Behavior of a Finite Volume Scheme for Concrete Carbonation Model

Antoine Zurek
Lille University, France

In this talk, we consider a free-boundary model in one space dimension which appears in the modeling of concrete carbonation. The unknowns u and v represent the mass concentration of CO_2 respectively in aqueous and gaseous phase and s represents the penetration depth which measures the size of the carbonated zone. The model consists in a system of two weakly coupled diffusion-reaction equations in a varying domain $(0, s(t))$ where s solves an ordinary differential equation. It has been shown that the interface of the domain follows a $\sqrt{t}$-law of propagation (see Aiki and Muntean, Commun. Pure Appl. Anal. 2010 and Interfaces Free Bound. 2013). Our goal, is to define a numerical scheme which ensures that the approximate penetration depth behaves like $\sqrt{t}$. To this end, we propose a scheme obtained by an implicit Euler discretization in time and a finite volume discretization in space. We first prove the existence of a solution to the scheme. Then, we establish some estimates satisfied by the approximate solutions. Finally, we show that the approximate penetration depth follows a $\sqrt{t}$-law of propagation.

Special Session 91: Recent Advances in Mathematical Biology, Ecology, Epidemiology, and Oncology

Sophia Jang, Texas Tech University, USA
Hsiu-Chuan Wei, Feng-Cia University, Taiwan
Ting-Hui Yang, Tankang University, Taiwan
Jui-Ling Yu, Providence University, Taiwan
Alain Miranville, University of Poitiers, France

Applications of mathematics have become increasingly important to the understanding and solutions of biological problems. This special session focuses on topics of current interest and recent advances in the theory and applications in mathematical biology. Deterministic models, stochastic models, and recent mathematical methods developed to study these models will be presented including problems in the spread and control of emerging and re-emerging diseases, in the control of invasive species, in resource related competition, and in oncology. This session will provide a forum for discussion and exchange of innovative ideas among new investigators and active researchers on recent advances in the areas of mathematical biology.

Design of Optimal Strategies for Tumor Management with Reversible Mechanism of Resistance

Andrei Akhmetzhanov
Hokkaido University, Japan
Chen-Hsiang Yeang (Institute of Statistical Sciences, Academia Sinica, Taipei, Taiwan)

First, we formulate a mathematical model for tumor growth based on clinical observations of melanoma progression, and then we study how the tumor can be managed by a single drug agent, an inhibitor of a particular mutated pathway. In the model, a state of each cell is described by activities of two distinctive genetic programs. The main program affected by a drug conforms a higher proliferative ability to a cell, whereas the alternative program leads to slower division rate, but protects a cell from the action of a drug. Activation of either program and transition between them are of stochastic nature. Our main result is in finding an optimal control strategy that minimizes a tumor size at the end of a given treatment period. The obtained optimal pattern contains a singularity, where a drug is applied at intermediate dosage. In our work we describe the methodology on how to construct such optimal strategy by using the method of singular characteristics.

Cahn-Hilliard-Hele-Shaw Systems As Simple Mixture Models for Tumor Growth

Maurizio Grasselli
Politecnico di Milano, Italy

S.M. Wise et al. proposed a continuous mixture model for tumor growth [J. Theor. Biology 253 (2008), 524-543]. This model consists of four advective Cahn-Hilliard equations with reaction terms whose velocities are given by Darcy's type laws. Such equations govern the water phase, the viable tumor cell phase, the dead tumor cell phase and the host tissue phase. The simplest version reduces to a single equation for the volume fraction of the tumor cells with an advective velocity field depending on the pressure gradient and on the Korteweg force (i.e. the divergence of a suitable adhesion flux). This model is also known as Cahn-Hilliard-Hele-Shaw system. We present an overview of some recent theoretical results on such a system.

An Invariant Loop in Four-Dimensional Nonlinear Semelparous Leslie Matrix Models

Ryusuke Kon
University of Miyazaki, Japan

A semelparous Leslie matrix model is an age-structured population model described by a system of difference equations. In a certain class of semelparous Leslie matrix models, either a positive equilibrium is stable and an invariant set on the boundary of the nonnegative cone is unstable or vice versa generically if the number of age-classes is two or three. However, it was shown, recently, that if the number of age-classes is four, this dynamic dichotomy is failed [1]. In this talk, we review this result and examine an attractive invariant loop that emerges when a positive equilibrium is unstable and the boundary of the nonnegative cone does not have an attractor.
[1] R. Kon (2017), Non-synchronous oscillations in four-dimensional nonlinear semelparous Leslie matrix models, Journal of Difference Equations and Applications, 10 pp.1747–1759.

Mathematical Modeling of Macroalgal Allelopathy in the Emergence of Coral Diseases

Samares Pal
University of Kalyani, India
Joydeb Bhattacharyya

Competition between macroalgae and corals for occupying the available space in sea bed is an important ecological process underlying coral-reef dynamics. Several benthic macroalgae species produce allelopathic chemical compounds that hinder the settlement and survival of coral larvae. Toxic macroalgae species damage coral tissues when in contact by transferring hydrophobic allelochemicals present on macroalgal surfaces. This leads to the reduction in fecundity of corals and even coral mortality. Also,

the release of allelochemicals by toxic-macroalgae influences the microbes associated with corals by transmitting pathogens. We investigate coral-macroalgal phase shift in presence of macroalgal allelopathy and microbial infection on corals under the assumption that the transmission of infection occurs through both contagious and non-contagious pathways. We found that the system is capable of exhibiting the existence of two stable configurations of the community under the same environmental conditions by allowing saddle-node bifurcations that involves in creation and destruction of fixed points and associated hysteresis effect. It is observed that in presence of low coral recruitment rate on algal turf and reduction in herbivory, the system exhibits hysteresis through a saddle-node bifurcation and a transcritical bifurcation.

What about Lactate Kinetic in a (Gliomatous) Brain?

Angelique Perrillat-Mercerot
Poitiers University, France
C. Guillevin, R. Guillevin, A. Miranville

Lactate used to be known as a a molecular waste in the brain. But today it is fully accepted that lactate is an important source of energy for neurons. Like other cancers, gliomas lead to alterations of cells energy management. In particular, lactate creation, consumption, import and export of a glioma cell seem to play a key role in the cancer development. But because energy management in healthy and tumoral cells can be difficult to observe and explain experimentally, we use mathematical modeling to help to describe and understand cells energy changes. We present here a fast-slow system describing the locals mechanisms of interest. This model was first proposed by A. Aubert, R. Costalat, P.J. Magistretti & L. Pellerin. In particular, we will give bounds on the solutions, compare global ($\varepsilon > 0$) and limit ($\varepsilon = 0$) dynamics and give simulations for several parameters of interest. We will also compare simulations with MRS data and discuss our results.

Joint Impact of Cell-Free and Cell-To-Cell Transmissions in Viral Dynamics

Xiang-Sheng Wang
University of Louisiana at Lafayette, USA

We study a viral model coupling cell-free and cell-to-cell transmissions. The joint effect of direct and indirect infection mechanisms will be investigated via analysis of viral dynamics. Especially, we will calculate the basic reproduction number and compare it with the basic reproduction numbers for uncoupled models with only one type of infection mechanism. Local and global stability analysis of the infection-free equilibrium as well as positive equilibrium will also be provided.

Mathematical Modelling of Breast Tumor Growth: MCF-7 Cell Line

Hsiu-Chuan Wei
Feng Chia University, Taiwan

Breast cancer is the second most commonly diagnosed cancer in women worldwide. MCF-7 cell line is an extensively studied human breast cancer cell line. This cell line expresses estrogen receptors, and the growth of MCF-7 cells is hormone dependent. In this study, I will present a mathematical model governing MCF-7 cell growth and interaction among tumor cells, estradiol, and immune cells. Experimental data will be used for the development of functional forms and estimation of parameter values. Breast tumor formation and growth will then be studied.

Special Session 92: Dynamics of Fluids and Nonlinear Waves

Zhiwu Lin, Georgia Institute of Technology, USA
Roman Shvydkoy, University of Illinois at Chicago, USA
Chongchun Zeng, Georgia Institute of Technology, USA

This special session focuses on the dynamic properties of fluids and nonlinear waves arsing in models such as Euler and Navier-Stokes equations, water wave and the model equations (KDV, Bousinesq etc.), nonlinear Schrödinger equations for Bose-Einstein condensates. The main aspects include the global well-posedness of solutions, stability/instability and nearby local qualitative structures (e.g. invariant manifolds) of coherent states, long time behaviors of general solutions etc. The theory and the methods applied on these problems address some of the most advanced development in the field.

Energy Equality for the Navier-Stokes Equations in Weak-In-Time Onsager Spaces

Alexey Cheskidov
University of Illinois at Chicago, USA
Xiaoyutao Luo

Onsager's conjecture for 3D Navier-Stokes equations concerns the validity of energy equality of weak solutions with regards to their smoothness. We establish energy equality for weak solutions in a large class of function spaces. These conditions are weak-in-time with optimal space regularity and therefore weaker than all previous classical results. Heuristics using intermittency argument suggests the possible sharpness of our results.

Enhanced Existence Time in Fractional KdV Equations

Mats Ehrnstrom
NTNU Norwegian University of Science and Technology, Norway
Yuexun Wang, NTNU Norwegian University of Science Technology

The fractional KdV equation $u_t + uu_x - |D|^\alpha u_x = 0$ contains the KdV ($\alpha = 2$), the Benjamin–Ono ($\alpha = 1$), the Burgers (an exceptional case, ($\alpha = 0$)), the Burgers–Hilbert ($\alpha = -1$) and the reduced Ostrovskii equation ($\alpha = -2$), and has been proposed as a family of dispersive model equations suitable to the study of the balance between dispersive and nonlinear effects. Classical solutions exist globally in time for not too small values of $\alpha > 0$, whereas wave-breaking comes into play as the dispersion gets weaker (α negative). Hunter and Ifrim showed that, for $alpha = -1$, the time of existence of classical solutions in the Burgers-Hilbert equation can be extended from $1/\varepsilon$ to $1/\varepsilon^2$ when the initial data is of size ε. Using a normal-form transformation inspired by theirs, but working almost completely in Fourier space, we extend this result to all $\alpha \in (-1, 0) \cup (0, 1)$.

Dynamics of Expanding Gases

Juhi Jang
University of Southern California, USA

I will present global existence results for compressible fluids governed by Euler and Euler-Poisson systems in the vacuum free boundary framework. The solutions are constructed near particular expanding solutions by taking advantage of stabilizing effect of those background expanding solutions. The talk is based on a joint work with Mahir Hadzic.

Turning Point Principle for the Stability of Stellar Models

Zhiwu Lin
Georgia Institute of Technology, USA

I will discuss some recent results on the linear stability criterion of spherically symmetric equilibria of several stellar models, including Euler-Poisson, Einstein-Euler and Einstein-Vlasov models. For Euler-Poisson and Einstein-Euler models, a turning point principle for the sharp stability criterion will be given. For Vlasov-Einstein model, the stability part of the turning point principle is obtained and the linear instability in the strong relativisitic limit will also be discussed. For all these models, a combination of first order and 2nd order Hamiltonian formulations is used to derive the stability criterion and study the linearized equation for initial data in the energy space. This is joint work with Chongchun Zeng (on Euler-Poisson) and with Hadzic and Rein (on Einstein-Euler and Einstein-Vlasov).

Special Session 93: Recent Trends in Nonlinear PDEs

Isabella Ianni, Università degli Studi della Campania, Italy
Angela Pistoia, La Sapienza Università di Roma, Italy
Giusi Vaira, Università degli Studi della Campania, Italy

The aim of the session is to bring together some recent emergent ideas in the study of asymptotic patterns of solutions to elliptic partial differential equations possessing a variational structure. It will mainly focus on the formation of singularities, spatial segregation and blow-up phenomena of solutions of some singular limit problems.

Blow Up Phenomena for Liouville Systems

Luca Battaglia
Roma Tre University, Italy

We consider a general systems of two coupled Liouville-type PDEs on a bounded planar domain. Using a perturbation argument we will construct a family of blowing-up solutions. In particular, we will consider blow-up profiles known as tower of bubbles or cluster of bubbles, which look like a superposition of rescaled entire solutions of a Liouville equation.

Morse Index and Uniqueness of Positive Solutions of the Lane-Emden Problem

Francesca de Marchis
University of Rome Sapienza, Italy

We present a Morse index computation for positive solution of the classical Lane-Emden problem in planar domains, obtained via an accurate asymptotic analysis of the solutions as the exponent tends to infinity. When the domain in convex this result allows to prove uniqueness of positive solutions (for large values of the exponent), giving a first general answer to a longstanding open problem raised by Gidas-Ni-Nirenberg [CMP, 1979]. The results are obtained in collaboration with M. Grossi, I. Ianni and F. Pacella.

On Fractional Kirchhoff Problems Involving Singular and Critical Terms

Alessio Fiscella
Universidade Estadual de Campinas, Brazil

In this talk, we discuss about Kirchhoff type problems driven by nonlocal fractional operators, involving singular terms and critical Sobolev nonlinearities. In particular, we focus our attention on singular terms of type u^γ, with a negative exponent γ. Our variational problems present some difficulties due to the bi-nonlocal nature of the elliptic part, the nondifferentiability of the related functional and the lack of compactness at critical level. For this, in order to state existence and multiplicity results, we introduce different proof techniques based on approximation and minimization arguments. Finally, we present some interesting open questions, concerning the study of our problems.

Blowing-Up Solutions to Perturbed Fractional Yamabe Equations

Seunghyeok Kim
Hanyang University, Korea
Woocheol Choi, Wenjing Chen, Shengbing Deng, Angela Pistoia

We address the stability issue on the fractional Yamabe equation by considering blowing-up solutions to its perturbations. Remarkably, even though the fractional conformal Laplacian is defined in a nonlocal manner, its conformal covariance property produces highly localized solutions to the perturbed fractional Yamabe equations. We provide several conditions that guarantee the existence of one-bubble, clustered-bubbles and bubble-tower type solutions.

Principal Eigenvalue of Mixed Problem for the Fractional Laplacian

Tommaso Leonori
Università di Roma Sapienza, Italy
M. Medina, A. Primo, I. Peral, F. Soria

We analyze the behavior of the eigenvalues of the following nonlocal mixed problem

$$\begin{cases} (-\Delta)^s u &= \lambda_1(D)\, u & \text{in } \Omega, \\ u &= 0 & \text{in } D, \\ \mathcal{N}_s u &= 0 & \text{in } N. \end{cases}$$

Our goal is to construct different sequences of problems by modifying the configuration of the sets D and N, and to provide sufficient and necessary conditions on the size and the location of these sets in order to obtain sequences of eigenvalues that in the limit recover the eigenvalues of the Dirichlet or Neumann problem. We will see that the nonlocality plays a crucial role here, since the sets D and N can have infinite measure, a phenomenon that does not appear in the local case.

Monotonicity Properties of Solutions to Quasilinear Elliptic Equations in Half Spaces

Luigi Montoro
UNICAL, Italy

I will present some new results regarding the monotonicity properties of solutions to quasilinear elliptic equations in half spaces.

Asymptotically Radial Solutions to an Elliptic Problem on Expanding Annular Domains in Riemannian Manifolds with Radial Symmetry

Filippo Morabito
KAIST, Korea

Many authors studied in the past the following boundary value problem:

$$\begin{cases} \Delta u + u^p = 0 & \text{in} \quad A, \\ u > 0 & \text{in} \quad A, \\ u = 0 & \text{on} \quad \partial A, \end{cases} \tag{1}$$

where $A \subset \mathbb{R}^n$, $n \geq 2$, is an annulus, that is $A = \{x \in \mathbb{R}^n : R_1 < r(x) < R_2\}$, with $r(x)$ equal to the distance to the origin. The radial solution always exists for any $p > 1$, it is unique and radially non-degenerate.

If A is replaced by an expanding domain in $\mathbb{R}^n$ which is diffeomorphic to an annulus, then it is known (see [2],[3]) that there exists an increasing number of solutions as the domain expands.

Furthermore in [3] it is shown such solutions are not close to the radial one, indeed they exhibit a finite number of bumps.

In [1] Bartsch, Clapp, Grossi, Pacella show instead the existence of a positive solution to the problem (1) on an expanding annular domain Ω_R diffeomorphic to $A_R = \{x \in \mathbb{R}^n : R < r(x) < R+1\}$, which is close to the radial solution to the corresponding problem on the annulus A_R.

In our work [4] we show the result of [1] holds true for an unbounded Riemannian manifold M of dimension $n \geq 2$ with metric $g := dr^2 + S^2(r) g_{S^{n-1}}$, where $g_{S^{n-1}}$ denotes the standard metric of the $(n-1)$-dimensional unit sphere S^{n-1}; $r \in [0, +\infty)$ is the geodesic distance measured from a point O.

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On the Impulse Response for Solutions of Two-Dimensional Liouville Type Equations

Hiroshi Ohtsuka
Kanazawa University, Japan
Tadatsugu Hatori, Yuichi Yatsuyanagi

Motivated by the experimental facts observed in confined non-neutral plasma, we are interested in the impulse response for solutions of two-dimensional Liouville type equations, that is, the asymptotic behavior for solutions of Liouville type equations with one singular source as the singularity vanishes. In this talk, we observe some basic facts on the exact solution for singular Liouville type equation in $\mathbf{R}^2$ established by Prajapat and Tarantello (2001) and try to generalize some of them.

Unbounded Non-Differentiable Functionals and Quasi-Linear Schroedinger Equations in Bounded Domains

Benedetta Pellacci
Università della Campania “Luigi Vanvitelli”, Italy
Lucio Boccardo, Marco Squassina

We will present some existence results concerning Mountain Pass solutions of quasi-linear Schroedinger equations in bounded domains via variational methods. In order to do this we will study non-differentiable, and possibly unbounded functionals.

Part of the presented results have been obtained in collaboration with Marco Squassina and Lucio Boccardo

Quasilinear Elliptic (P, Q) Systems: New Results and Open Questions

Patrizia Pucci
University of Perugia, Italy

The importance of studying problems involving (p, q) operators, or operators with non-standard growth conditions, begins with the pioneering papers of Marcellini and Zhikov. Since then the subject has been attracting increasing attention on existence, regularity and qualitative properties of solutions of different problems. The talk is based on very recent results contained in a series of papers. In particular, existence of nontrivial entire solutions for critical Hardy quasilinear systems, driven by general (p, q) elliptic operators of Marcellini types, are briefly presented. The results also raise, and leave open, a number of other intriguing questions.

Elliptic Equations in $\mathbb{R}^2$ with Exponential Growth and Vanishing Weights

Federica Sani
Milano University, Italy

We consider a class of elliptic equations in the whole space $\mathbb{R}^2$ with weights vanishing at infinity. The decay of the weights that we prescribe enables to study the problem in suitable (limiting) weighted Sobolev space where the maximal growth allowed for the nonlinear term is governed by a weighted version of the Trudinger-Moser inequality. We are concerned with the existence and concentration of solutions in the case of nonlinearities with exponential growth. The results are obtained in collaboration with Joao Marcos do O, Elisandra Gloss and Jianjun Zhang.

Rigidity of Nonlocal Phase Transitions

Yannick Sire
Johns Hopkins University, USA
Mostafa Fazly

Motivated by a conjecture of De Giorgi, we present several recent results and progress on proving 1D symmetry for nonlocal Allen-Cahn type equations in low dimensions. The methods that we use are of variational type and of geometric nature. We emphasize on the case of nonlinear operators.

Hardy's Inequality in a Limiting Case on General Bounded Domains

Futoshi Takahashi
Osaka City University, Japan
Jaeyoung Byeon

We study Hardy's inequality in a limiting case: $\int_\Omega |\nabla u|^N dx \geq C_N(\Omega) \int_\Omega \frac{|u(x)|^N}{|x|^N \left(\log \frac{R}{|x|}\right)^N} dx$ for functions $u \in W_0^{1,N}(\Omega)$, where Ω is a bounded domain in $\mathbb{R}^N$ with $R = \sup_{x\in\Omega} |x|$. We study the attainability of the best constant $C_N(\Omega)$ in several cases. We provide sufficient conditions that assure $C_N(\Omega) > C_N(B_R)$ and $C_N(\Omega)$ is attained, here B_R is the N-dimensional ball with center the origin and radius R. Also we provide an example of $\Omega \subset \mathbb{R}^2$ such that $C_2(\Omega) > C_2(B_R) = 1/4$ and $C_2(\Omega)$ is not attained.

Special Session 94: Fluid-Structure Interactions in Medicine and Biology: Modeling, Analysis, and Experiments

Sookkyung Lim, University of Cincinnati, USA
Boyce Griffith, University of North Carolina at Chapel Hill, USA

Fluid-structure interaction is ubiquitous in nature and occurs at a wide range of scales, for example swimming microorganisms and the cardiovascular system. Modeling and simulating fluid-structure interaction remains a major mathematical and computational challenge, however, and contributions from many different fields are needed. This special session will focus on recent developments in biological fluid-structure interaction. This research is highly interdisciplinary and brings together areas including engineering, medicine, biology, and mathematics.

Effects of Dynein Activation and Viscosity on the Emergent Waveform of an Elastic, Internally-Actuated, Model Flagellum

Robert Dillon
Washington State University, USA
Robert Dillon, Charlotte Omoto, Lisa Fauci

We describe a fluid-mechanical model of an individual sperm which incorporates discrete representations of the dynein arms, the passive elastic structure of the axoneme including the microtubules and nexin links. This model couples the internal force generation of the molecular motors through the passive elastic structure with the external fluid mechanics governed by the Navier-Stokes equations. The representation of the dynein motors gives us the flexibility to incorporate various dynein activation theories. Here we use a simple activation mechanism based on local curvature with a time delay. The flagellar beat is not preset and is an emergent property of the interacting components of the coupled fluid-axoneme system. Numerical results demonstrate strong qualitative agreement with sperm experimental data and show a highly nonlinear response to increasing viscosity.

Dynamic Instabilities in the Hook-Flagellum System and Bacterial Flicks

Henry Fu
University of Utah, USA
Mehdi Jabbarzadeh

Dynamical bending, buckling, and polymorphic transformations of the flagellum are known to affect bacterial motility, but run-reverse-flick motility of monotrichous bacteria also involves the even more flexible hook connecting the flagellum to its rotary motor. Although flick initiation has been hypothesized to involve either static Euler buckling or dynamic bending of the hook, the precise mechanism of flick initiation remains unknown. We find that flicks initiate via a dynamic instability requiring flexibility in both the hook and flagellum. We obtain accurate estimates of forces and torques on the hook that suggest that flicks occur for stresses below the (static) Euler buckling criterion, then provide a mechanistic model for flick initiation that requires combined bending of the hook and flagellum. We calculate the triggering torque:stiffness ratio and find that our predicted onset of dynamic instability corresponds well with experimental observations. We discuss current and future efforts to model hook and flagellum dynamics during flicks.

In Silico Simulation of Glioma Invasion Including the Role of Myosin II

Wanho Lee
National Institute for Mathematical Sciences, Korea
Sookkyung Lim, Yangjin Kim

Gliomas are malignant tumors that are commonly observed in primary brain cancer. Glioma cells migrate through a dense network of normal cells in microenvironment and spread long distances within brain. In this poster we present a two-dimensional hybrid model in which a glioma cell is surrounded by normal cells and its migration is controlled by cell-mechanical components in the harsh microenvironment via the regulation of myosin II in response to chemoattractants. Our simulations reveal that the myosin II plays a key role in deformation of the cell nucleus as the glioma cell passes through the narrow intercellular space smaller than its nuclear diameter. In addition, our results suggest that in the presence of myosin II the strong signal of chemoattractants may retract invasive glioma cells back to the resection site so that they can be removed completely. This study sheds lights on the understanding of glioma infiltration through the narrow intercellular spaces and a potential approach for the development of anti-cancer invasion strategies.

Single-Flagellated Bacterial Swimming: Run, Reverse, and Flick

Sookkyung Lim
University of Cincinnati, USA
Yongsam Kim, Yunyoung Park

Single-flagellated bacteria propel themselves by rotating a flagellar motor, translating rotation to the filament through a compliant hook and subsequently driving the rotation of the flagellum. The flagellar motor alternates the direction of rotation between counterclockwise and clockwise, and this leads to the forward and backward directed swimming. Such bacteria can change the course of swimming as the hook experiences its buckling caused by the change of bending rigidity. In this paper, we present a comprehensive model of a monotrichous bacterium as a free swimmer in a viscous fluid. We describe a cell body as a rigid body using the penalty method and a flagellum as an elastic rod using the Kirchhoff rod theory. The hydrodynamic interaction of the bacterium is described by the regularized Stokes formulation. Our model of a single-flagellated microorganism is able to mimic the swimming pattern that is well matched with the experimental observation. Furthermore, we find the critical thresholds of the rotational frequency of the motor and the bending modulus of the hook for the buckling instability, and investigate the dependence of the buckling angle and the reorientation of the swimming cell after buckling on the physical and geometrical parameters of the model.

Modeling Hydrodynamic Effects on Choanoflagellate Feeding

Hoa Nguyen
Trinity University in San Antonio, USA
C. Oakes, L. Fauci, M.A.R. Koehl

Choanoflagellates are unicellular organisms whose intriguing morphology includes a set of collar/microvilli emanating from the cell body, surrounding the beating flagellum. As the closest living relative to animals, they are important for both ecological and evolutionary studies. We consider two unicellular types: slow swimmers and thecate cells (attached to a wall by a stalk). Assuming they have similar morphologies, we use the method of regularized Stokeslets to (i) simulate cell-fluid interactions of the slow swimmers and thecate cells with the surrounding environment and (ii) show hydrodynamic effects on the amount of fluid flow across a capture zone around the collar (net flux). The results shed light on how each morphological feature of the cell aids in bacteria captures during feeding. We have found that the existence of the collar not only attracts more fluid particles but also impedes the fluid flow close to the microvilli. Among the two choanoflagellate types, slow swimmers gain the most net flux which shows an advantage of being motile. Due to the wall effect, thecate cells have less net flux but the interactions of cell-fluid-wall-stalk create small eddies around the stalk which can be used to explain bacterial gathering in that area.

An Immersed Boundary Method for Simulating Vesicle Dynamics

Yunchang Seol
National Chiao-Tung University, Taiwan
Wei-Fan Hu, Yongsam Kim, Ming-Chih Lai

A vesicle is a liquid droplet with a radius of about $10\mu m$ enclosed by a phospholipid membrane suspended in an incompressible viscous fluid. The understanding of vesicle behaviors in fluid flows might lead to a better knowledge of red blood cells (RBCs). The dynamics of vesicle in fluids can be determined by the membrane inextensibility, bending, and hydrodynamical forces. There are two different approaches to enforce the local inextensibility constraint in literature. The first one needs to discretize the whole equations first and then to solve the discretized equations simultaneously for the tension and fluid variables. There usually exists a trade-off between the time-step stability and efficiency in those algorithms simply because iterative procedures are needed. Another approach is called a penalty idea. Instead of keeping the vesicle membrane locally inextensible, the penalty idea makes the vesicle surface patch nearly inextensible by introducing a modified elastic tension energy. This approach replaces the unknown tension by a spring-like tension depending on the surface configuration so that we can avoid solving the whole system to obtain the variable tension. In this talk, we present the extension of our previous immersed boundary method for simulating inextensible vesicles to general three dimensions.

Modeling and Simulation of Blood Flow Past the Distal Anastomosis of the Arteriovenous Graft for Hemodialysis

Luoding Zhu
Indiana University - Purdue University Indianapolis, USA
Zengding Bai

Arteriovenous grafting (AVG) is a common device applied in hemodialysis for kidney failure patients. It is often failed because of the intimal hyperplasia formed around the AVG anastomosis. In order to help understand the mechanism of the formation of intimal hyperplasia, we investigate the flow patterns in the AVG anastomosis and forces distribution on the vein near the anastomosis by modeling and simulation. The anastomosis structure (graft and vein) is modeled by elastic fibers. The surrounding tissue is modeled by elastic springs. The blood is modeled by viscous incompressible flow and the flow is numerically simulated by the lattice-Boltzmann method. The fluid-structure-interaction is treated by the immersed-boundary method. We perform a series of simulations using different flow Reynolds numbers, different AVG configuration including attached angles and vein-graft diameter ratio. Both rigid and deformable cases are considered. Flow fields are visualized and compared. Wall shear stress, wall normal stress, etc. on the vein/AVG are computed and analyzed.

Special Session 95: Kinetic and Related Equations: Collisions, Mean Field, and Organized Motion

Hung-Wen Kuo, National Cheng Kung University, Taiwan
Kazuo Aoki, National Center for Theoretical Sciences, Taiwan
Seok-Bae Yun, Sungkyunkwan University, Korea
Young-Pil Choi, Inha University, Korea

Kinetic equations describe the evolution of dynamics of a particle system at mesoscopic level. Traditionally, the study of kinetic equations has been around the study of Boltzmann type equations, Vlasov type equations and related models. Recently, it is seeing new applications in the study of emergence of organized motion in a system where only a simple local rules are imposed between particles. In this session, we aim to bring together experts and young researchers in these traditional and new fields so that they can report their recent progress, exchange ideas and enjoy fruitful discussions.

Boltzmann-Type Optimal Control Problems in Self-Organizing Systems

Giacomo Albi
University of Verona, Italy

We present a Boltzmann-type framework to deal with the optimal control of large particle systems. We will propose two main approaches, the first one based on the construction of a control hierarchy at level of the binary interactions, the second one which tackle directly the optimal control of the Boltzmann equation. In particular, we construct a control hierarchy for a finite time horizon. We will show that, under a suitable scaling limit, this class of problems converges towards corresponding mean-field optimal control problems. A set of stochastic particle approximations is derived, coupling Monte-Carlo methods with model predictive control, and dynamic programming. This novel approach permits to tackle the curse of dimensionality, and to perform efficient numerical simulations. Several examples from consensus dynamics to swarming models are presented. This is a joint work with Y-P. Choi, M. Fornasier, D. Kalise, and with L. Pareschi.

A Kinetic Approach of the Non Conservative Bitemperature Model

Stephane Brull
Institut des mathematiques de Bordeaux, France
Corentin Priget, Bruno Dubroca

This talk is devoted to the numerical approximation of the bitemperature Euler system that has been derived from an underlying kinetic model by performing an hydrodynamic limit. This fluid model describes a plasma in an out of equilibrium situation. This system enters into the framework of the non conservative hyperbolic systems, whose theory is up to now not well understood. In the situation of shocks the numerical solutions of such systems show plateaux depending on the viscosity of the schemes. In the present work, we propose a discretisation of the kinetic model which is conservative in order to select the non regular solutions of the system.

Swarming Models with Local Alignment Effects: Phase Transition & Hydrodynamics

Jose Antonio Carrillo
Imperial College London, England

I will make a review of swarming models with repulsive-attractive effects focusing on two new aspects: phase transitions for the local Cucker-Smale type model and self-organized hydrodynamics of the Vicsek model with fixed speed from asymptotic speed Cucker-Smale models by penalization. In short, we will show that the asymptotic speed Cucker-Smale model behaves in terms of hydrodynamics and phase transitions as the Vicsek model with fixed speed for large friction parameter.

Regularity for Diffuse Reflection Boundary Problem to the Stationary Linearized Boltzmann Equation in a Convex Domain

I-Kun Chen
National Taiwan University, Taiwan
Chun-Hsiung Hsia, Daisuke Kawagoe

We consider the diffuse reflection boundary problem for linearized Boltzmann equation for hard sphere potential, cutoff hard potential, or Maxwellian molecular gases in a C^2 convex bounded domain with positive Gaussian curvature. We obtain a pointwise estimate for the derivative of the solution provided the boundary temperature is bounded differentiable and the solution is bounded.

On a Derivation of the Polyatomic Vlasov Equation with Vibratory and Rotational Motions

Sun-Ho Choi
Kyung Hee University, Korea
Seok-Bae Yun

In this talk, we discuss a mathematical theory for polyatomic gas. Polyatomic gas is common in our environment. For examples, hydrogen, nitrogen, oxygen. However, in the mathematical theory of gas, one assumes that each particle of gas is monoatomic for simplicity. Here, we derive a polyatomic Vlasov equation with the self-consistence Poisson force field and prove the global existence of the solution to the polyatomic Vlasov-Poisson equation with vibratory and rotational motions.

Uncertainty Quantification for Kinetic Equations: a Monte Carlo Approach

Giacomo Dimarco
University of Ferrara, Italy
Lorenzo Pareschi, Mattia Zanella

In this talk we will discuss some recent results concerning the construction of efficient numerical methods for uncertainty quantification (UQ) in kinetic equations. In spite of the vast amount of existing research, both theoretically and numerically, the study of kinetic equations has mostly remained deterministic and ignored uncertainty. In reality, there are many sources of uncertainties that can arise in these equations as for instance incomplete knowledge of the interaction mechanism between particles/agents, imprecise measurements of the initial and boundary data, other sources of uncertainty like forcing and geometry, etc. Understanding the impact of these uncertainties is critical to the simulations of the complex kinetic systems to validate the kinetic models, recently we developed novel approaches to UQ of kinetic equations based on micro-macro Monte Carlo techniques which using control variate estimators based on the local equilibrium are capable to accelerate the slow statistical convergence of Monte Carlo methods.

Local Sensitivity Analysis for Flocking and Synchronization Models

Seung-Yeal Ha
Seoul National University, Korea
Shi Jin, Jinwook Jung

In this talk, we introduce two random flocking and synchronization models, namely random Cucker-Smale model and random Kuramoto model, and then present local sensitivity analysis for those particle and kinetic models based on the uniform stability analysis and propagation of Sobolev regularity in random parameter space. This is a joint work with Shi Jin (UW-Madison) and Jinwook Jung (SNU).

Cucker-Smale Model with Normalized Communication Weights and Time Delay

Jan Haskovec
KAUST, Saudi Arabia
Young-Pil Choi

We study a Cucker-Smale-type system with time delay in which agents interact with each other through normalized communication weights. We construct a Lyapunov functional for the system and provide sufficient conditions for asymptotic flocking, i.e., convergence to a common velocity vector. Moreover, we carry out a rigorous limit passage to the mean-field limit of the particle system as the number of particles tends to infinity. For the resulting Vlasov-type equation we prove the existence, stability and large-time behavior of measure-valued solutions. This is, to our best knowledge, the first such result for a Vlasov-type equation with time delay. We also present numerical simulations of the discrete system with few particles that provide further insights into the flocking and oscillatory behaviors of the particle velocities depending on the size of the time delay.

Dynamics of Singularities in Dissipative Kinetic Equations

Hyung Ju Hwang
POSTECH, Korea
Yan Guo, Juhi Jang, Jinoh Kim, Juan Velazquez

We describe the structure of solutions of dissipative kinetic equations in domains with boundaries near the singular set. Representative equations are the kinetic Fokker-Planck and Landau equations. We discuss in particular the well-posedness and the regularity of solutions of these equations with various boundary conditions.

Asymptotic Stability of the Relativistic Boltzmann Equation for Soft Potentials Without Angular Cut-Off

Jin Woo Jang
IBS - Center for Geometry and Physics, Korea
Robert M. Strain

We establish here global-in-time well-posedness and stability results for solutions nearby the relativistic Maxwellian to the special relativistic Boltzmann equation without angular cutoff. We work in the case of a spatially periodic box. We assume the generic soft-potential conditions on the collision kernel in that were derived by Dudyǹski and Ekiel-Jezewska (Commun. Math. Phys. **115**(4):607–629, 1985). In this physical situation, the angular function in the collision kernel is not locally integrable, and the collision operator behaves like a non-isotropic fractional diffusion operator.

On the Cauchy Problem for the Boltzmann Equation

Jin-Cheng Jiang
National Tsing Hua University, Taiwan
Lingbing He

In this talk, we will present some recent progress on the Cauchy problem for the Boltzmann equation without cutoff. We prove the local well-posedness for the Cauchy problem with general initial data in weighted Sobolev spaces with polynomial weights. We do not assume that the solution initially is a small perturbation of some background solution, for instance, the equilibrium.

Cucker-Smale Model with Bonding Force and Singular Interaction Kernel

Jeongho Kim
Seoul National University, Korea
Jan Peszek

We prove the lack of asymptotic collisions between particles following Cucker-Smale flocking model with a bonding force (CSB model) and its simplification. Moreover, we prove that in the case of the CSB model with sufficiently singular communication weight (e.g., $\psi(s) = s^{-\alpha}$ for $\alpha \geq 1$), finite-in-time collisions are impossible. Consequently, we establish existence of the global-in-time minimal distance between the particles. Finally, we present asymptotic distribution of particles which is confined within a ball of given radius.

Self-Similarity Breaking of Cosmological Solutions with Collisionless Matter

Ho Lee
Kyung Hee University, Korea
Ernesto Nungesser

We consider the Einstein-Vlasov system with Bianchi VII_0 symmetry. Under the assumption of small data we show that self-similarity breaking occurs for reflection symmetric solutions. This generalizes the previous work concerning the non-tilted fluid case to the Vlasov case, and we obtain detailed information about the late-time behaviour of metric and matter terms.

A BGK Model for Polyatomic Gas Flows at High Temperature

Luc Mieussens
University of Bordeaux, France
C. Baranger, G. Marois, J. Mathe, J. Mathiaud

High temperature gases, for instance in hypersonic reentry flows, show complex phenomena like excitation of rotational and vibrational energy modes, and even chemical reactions. For flows in the continuous regime, simulation codes use analytic or tabulated constitutive laws for pressure and temperature. In this talk, we propose a BGK model which is consistent with any arbitrary constitutive laws, and which is designed to make high temperature gas flow simulations in the rarefied regime. A Chapman-Enskog analysis gives the corresponding transport coefficients. Our approach is illustrated by a numerical comparison with a compressible Navier-Stokes solver with rotational and vibrational non equilibrium. The BGK approach gives a deterministic solver with a computational cost which is close to that of a simple monoatomic gas.

Asymptotic Stability of a Rarefaction Wave for a Symmetric System of Hyperbolic-Parabolic Coupled Equations

Shinya Nishibata
Tokyo Institute of Technology, Japan
Tetsuya Mitsuhori, Tohru Nakamura

In this talk we discuss a large time behavior of a solution to a coupled system of viscous and inviscid conservation laws. The system of equations appears in compressible fluid dynamics. We, mainly, talk about an asymptotic stability of a rarefaction wave under assuming the existence of an entropy function. This assumption enables us to rewrite the original system in a normal form consisted of symmetric hyperbolic and parabolic systems. In asymptotic analysis, we derive an a priori estimate by an energy method. In order to derive the basic estimate, we make use of an energy form defined by substituting a smooth approximation of the rarefaction wave in the entropy function. The symmetric system is utilized in deriving the estimates of the higher order derivatives of the solution. In this computation, we suppose the stability condition, which ensures dissipation.

Emergence of Aggregation in the Swarm Sphere Model with Adaptive Coupling Law

Se Eun Noh
Myongji University, Korea
Eung-Yeal Ha, Dohyun Kim, Jaeseung Lee

In this talk, we study aggregation properties of the swarm sphere model equipped with an adaptive coupling law on a sphere. The temporal evolution of coupling strength is determined by the balance between a feedback law due to relative positions and damping. For the analytical treatment, we employ two types of adaptive feedback laws, namely anti-Hebbian and Hebbian laws. For the anti-Hebbian law, we provide a sufficient condition leading to the complete aggregation in which all particles aggregate together and behave like one big cluster asymptotically. Our conditions are given in terms of initial positions and coupling strengths. For the Hebbian law, we provide subsets of basin of attractions to the complete synchronization and bi-polar synchronization in which the particles aggregate toward the north pole and south pole separately. We also present a uniform lp-stability of the swarm sphere model with adaptive couplings with respect to initial data when the complete aggregation occurs exponentially fast.

Cross-Coupling Effect in a Slow Rarefied Flow Past a Heated Sphere

Satoshi Taguchi
Kyoto University, Japan
Takuya Kakehashi

When a sphere is moving slowly in a gas, it experiences a resistive force (drag). Considering the case where the moving sphere is uniformly heated (or cooled), we investigate the effect of the temperature difference on the drag on the basis of the Boltzmann equation, which provides a mesoscopic description of the motion of the surrounding medium. More precisely, we consider the time-independent behavior of a slow flow of a rarefied gas past a heated (or cooled) sphere in the following situations: (i) the normalized temperature difference is of the same order as the (small) Mach number, (ii) the Knudsen number (the mean free path) is finite, and (iii) gas molecules are diffusely reflected on the sphere. A matched asymptotic analysis for small Mach numbers is employed to derive a formula for the drag up to the second order of the Mach number, whose second-order term expresses the coupling effect of the linearized uniform flow and heat transfer problems. As the result, we show that the drag is increased by the heating of the sphere. The effect is attributed to the ballistic motion of molecules, which enhances the effect of temperature dependency of the viscosity and the pressure effect.

Pointwise Estimate of the Linearized Boltzmann Equation

Haitao Wang
Shanghai Jiao Tong University, Peoples Rep of China
Yu-Chu Lin, Kung-Chien Wu

We study the pointwise behavior of the linearized Boltzmann equation for hard potentials and soft potentials. Our results extend the classical result of Liu-Yu for hard sphere to hard and soft potentials by imposing suitable exponential velocity weight on the initial condition.

Contact Discontinuity for Compressible Navier-Stokes Equation

Shih-Hsien Yu
National University of Singapore, Singapore

In this talk we present a synthysis procedure to construct the Green's function of the compressible Navier-Stokes equation linearized around a hyperbolic contact discontiuity with exponentially pointwise space-time structure. With the precise structure of the Green's function for the problem around the hyperbolic contact discontiuity, the space-time pointwise structure of the compressible Navier-Stokes equation follows.

Special Session 96: Quantization in Stochastic, Fuzzy System and Nonstandard Analysis

Kiyoyuki Tchizawa, Professor, Dept of Maths, Tokyo City University, Japan
Shuya Kanagawa, Professor, Dept of Maths, Tokyo City University, Japan
Hiroaki Uesu, Kanazawa Institute of Technology, Japan

In case blowing up the space or time, we sometimes find new aspects in the differential equations and even in the stochastic process. Blowing up the delta function, obviously it relates to the normal distribution in nonstandard analysis. Discretizing the distribution in a sense of quantization, there exists an essential problem: which conditions we need to justify for the continuous one. Recently, we can confirm it in physical or social phenomena not only in mathematics. In this session, we take up such problems including electric circuits and Stochastic, Fuzzy Systems.

Study of a Binocular Rivalry Model Represented by a Field in R4 with Two Fast Components

Cheriet Djamel Eddine
USTHB Algeria, Algeria

We investigate here a binocular rivalry model. It is about two neuronal populations' response to two stimuli with linearly dependent forces. This work generalizes work done previously for two neuronal populations responding to the same constant stimulus. Our work is devoted in the first to the singular points research of the representative system, secondly to the possible search for duck solutions. The use of nonstandard notions is useful to determine the nature of the singular solutions.

The Global Optimization Algorithm for the Sum of the Certain Nonlinear Nonconvex Functions

Mio Horai
Isegakuen High School, Japan
Takashi Gyoshin Nitta

This research presents an approximation algorithm for a certain nonlinear optimization problem whose objective functions are the sum of the composite functions, second differential functions and polynomial fractional functions. In order to solve the problems, we divide the domain into 4 parts, where the first and second differential functions are positive or negative. The nonlinear functions are converted into the linearized function in the each divided domain. We compute the linear optimization problem by Simplex method, and obtain the optimal value by using Branch and Bound algorithm. We illustrate some numerical experiments to demonstrate the feasibility of our approach.

Estimation of Volatility of Share Prices of Stock Index Using a Jump Diffusion Model

Shuya Kanagawa
Tokyo City University, Japan

We investigate the daily share prices of stock index to estimate its random volatility using a jump diffusion model. The volatility is estimated by the historical volatility from the observation of daily share prices. We focus on the relation between the historical volatility and the number of jumps of the share prices and estimate the optimal number of observed days for the historical volatility.

Road Traffic Simulation System Using Bird's Eye View Images by Image Recognition

Mitsuhiro Namekawa
Kaetsu University, Japan
Taisuke Hori, Shuya Kanagawa

We predict traffic congestion to use the system simulation. To perform the system simulation, the movement of the car is described as a model. As a general method to making the model of automobiles, we use the characteristics of automobiles and our experience. However, when we use this method, for example, it cannot cope with the difference in characteristics between narrow urban roads and wide suburban roads. In this research, we adopt a modeling method to take a bird's-eye view of an intersection connecting roads and analyze it by machine learning. In this method, a model of a car is constructed statistically. So, we use this method to build a model close to reality. We will discuss this method and system in our presentation.

On the Application of Probability Theory to Radiation Biology and Insect Ecology

Yasumasa Saisho
Hiroshima University, Japan

In mathematical biology, researches to explain various natural phenomena using analytical or stochastic results are actively conducted and many excellent re-

sults are obtained. In this presentation, I would like to introduce examples of application of probability theory to radiation biology and ecology and to point out mathematical problems appearing there. Radicals are generated along the track of radiation and diffuse through the aqueous solution to approach the DNA and generate damages. Since the spacing of the radicals on the track is considered to approximate the exponential distribution, this problem can be thought of as the relation between a counting process and DNA damages. On the other hand, it is known that the ecology of emergence of insects is closely related to mating, the phenomenon in which the peak of emergence of males precedes that of females (protandry) is said to be male's tactic. However, as for cicadas, little is known about the relationship between emergence ecology and mating. Therefore, investigating the relationship between the temporal distribution of emergence of cicada and the copulation rate is important in considering the the evolution of copulation system.

Sociometry Analysis Applying Fuzzy Graph Theory and Fuzzy Core Index Method

Kimiaki Shinkai
Tokyo Kasei Gakuin University, Japan
Ei Tsuda, Hajime Yamashita

The Sociometry developed by Moreno is one of the measurement and evaluation methods of social structure which we could analyze by applying the fuzzy graph theory. Applying fuzzy graph theory, we could measure the preferring degree and amicable degree among the members of a group. Indicator that can measure the importance of members in a group can be analyzed more effectively because we can order the members based on the indicator. In this paper, we would introduce Fuzzy Core Index Method that can sequence members totally in fuzzy graph. Moreover, we show the effectiveness of this proposed method through the sociometry analysis.

Random Processes on the Cantor Set

Hiroshi Takahashi
Tokyo Gakugei University, Japan

On the Cantor set, random processes can be defined as limit of suitably scaled random walks on disconnected self-similar set. The scaled random walks lead to a super-diffusion, that is, the diffusion exponent is larger than one. In this talk, we consider several random process processes on the Cantor set, which are arisen from independent randomness from the random walks, and study their self-similarities.

Bubble Chart Analysis of Mathematics Class Applying Type-2 Fuzzy Contingency Table

Hiroaki Uesu
Kanazawa Institute of Technology, Japan

Increasing motivation of students in lectures is strongly linked to academic achievement, and teachers are required to have educational skills to motivate students. Indeed, modern pedagogy points out that motivation of students themselves has a great influence on learning effect. Also, improvement of student motivation is very important. In this paper, we would propose a type-2 contingency table, conduct a questionnaire to grasp students' motivation and investigate each factor by using bubble chart analysis. Furthermore, we try to extract improvement items and their priorities in the lesson.

Special Session 97: Analysis and Dynamics on Boundaries of Manifolds and Related Topics

Hiroshige Shiga, Tokyo Institute of Technology, Japan
Hiroaki Aikawa, Hokkaido University, Japan

This special session is concerned with mathematical analysis on the structure of boundaries of manifolds and their dynamical properties. Related topics are potential theory, complex analysis and complex dynamics.

Parabolic and Elliptic Boundary Behavior on a Non-Smooth Domain

Hiroaki Aikawa
Hokkaido University, Japan

We give an estimate of the principal frequency of a non-smooth domain in terms of capacity. It is related to the torsion function, survival probability, boundary Harnack principle and intrinsic ultracontractivity. As an application we derive the global integrability of positive superharmonic functions and supertemperatures.

The Type Problem and Geometric Structures on Hyperbolic Surfaces

Ara Basmajian
City University of New York, USA
Hrant Hakobyan, Dragomir Saric

We describe new results involving the relationship between Fenchel-Nielsen coordinates and a version of the classical type problem (whether or not the surface carries a Green's function). In particular, we study so called tight flute surfaces- possibly incomplete hyperbolic surfaces constructed by linearly gluing infinitely many tight pairs of pants along their cuffs- and the relationship between their type and geometric structure. This is joint work with Hrant Hakobyan and Dragomir Saric.

Variational Formulas for Hydrodynamic Differentials and Its Application

Sachiko Hamano
Osaka City University, Japan

We establish the variational formulas of hydrodynamic differentials (i.e., L_s-canonical semi-exact differentials; $-1 < s \leq 1$) for deforming open Riemann surfaces $R(t)$ of genus one with complex parameter t, and give an application to the rigidity theorem for pseudoconvex domains.

A Carleson-Type Estimate for P-Superharmonic Functions

Takanobu Hara
Hokkaido University, Japan

Let Ω be a domain in $\mathbb{R}^n$ $(n \geq 2)$ and u be a nonnegative p-superharmonic function in Ω. In 1994, Kilpelainen and Maly proved that there exists a constant $C > 0$ such that

$$u(x) \leq C \left(\inf_{B(x,R)} u + \mathbf{W}_p^\mu(x, 2R) \right)$$

whenever $B(x, 2R) \subset \Omega$, where μ is the Riesz measure of u and $\mathbf{W}_p^\mu(x, 2R)$ is the Wolff potential of μ. In this talk, we extend this inequality to near the boundary of Ω. More precisely, we give a pointwise estimate for p-superharmonic functions which vanish on the boundary and a global integrability estimate of p-superharmonic functions. Combining the two estimates, we give an analog of the Carleson estimate.

Removable Isolated Boundary Singularities of Positive Solutions of Semilinear Elliptic Equations in a Lipschitz Domain

Kentaro Hirata
Hiroshima University, Japan

In a Lipschitz domain, we discuss the behavior of positive weak solutions of a superlinear elliptic equation $-\Delta u = a(x)u^p$ satisfying zero Dirichlet boundary condition except for one point, say 0. In particular, we present sufficient conditions for solutions to be extendable continuously at 0 in the case where p is close to 1. Moreover, two sided estimates for such extensions are given.

A Generalized Conservation Property of Brownian Motion with Killing Inside

Jun Masamune
Hokkaido University, Japan
Marcel Schmidt

One of the most important characterizations of the conservation property of the heat of a Riemannian manifold is a Liouville type property of the solution of the heat equation. This is called the Khasminskii test. A heat equation with killing inside is never conservative; however, it may enjoy the Liouville type property. Namely, the characterization fails. In this talk, we will propose a generalized conservation property and show that it is characterized by the Khasminskii test.

A Moduli Space of a Riemann Surface of Infinite Topological Type

Katsuhiko Matsuzaki
Waseda University, Japan

We investigate the dynamics of the Teichmüller modular group on the Teichmüller space of a Riemann surface of infinite topological type. We introduce the set of points where the action of the Teichmüller modular group is stable, and we prove that this region of stability is generic in the Teichmüller space.
By taking the quotient and completion with respect to the Teichmüller distance, we obtain a moduli space of the quasiconformally equivalent complex structures admitted on a topologically infinite Riemann surface.

Extremal Length Geometry on Teichmuller Space and Its Application

Hideki Miyachi
Kanazawa University, Japan

In this talk, I will give the recent progress on the extremal length geometry on Teichmuller space. Indeed, I will talk about an exchange between the topological and the complex analytical aspects in Teichmuller theory with the extremal length geometry as a broker.

Hyperbolic Surfaces with the Largest Maximal Injectivity Radius

Gou Nakamura
Aichi Institute of Technology, Japan

On the moduli space of closed Riemann surfaces of genus $g \geq 2$, we define a function which assigns to each surface its maximal injectivity radius. In this talk we consider surfaces attaining the maximum of the function for $g = 2$ and their coordinates in a certain Teichmüller space based on hyperbolic polygons.

Reproducing Kernels with Respect to Function Spaces of Polyharmonic and Polyparabolic Functions

Masaharu Nishio
Osaka City University, Japan
Katsunori Shimomura

We consider Hilbert spaces of polyharmonic functions on half spaces and the iterates of parabolic operators of fractional order. In this talk, after discussing the relation of harmonic functions with the Poisson operator, we introduce the corresponding Hilbert spaces. And we shall discuss the reproducing kernels.

Caloric Morphisms for Rotation Invariant Metrics on Semi-Euclidean Spaces

Katsunori Shimomura
Ibaraki University, Japan

Caloric morphism is a transformation which preserve the solutions of the heat equation. In this talk, we shall discuss the explicit forms of caloric morphism for rotation invariant (general Lorentz invariant in indefinite metric case) metrics on semi-Euclidean spaces of dimension greater than 2.

Polynomial Solution to Dirichlet Problem for the Heat Equation

Noriaki Suzuki
Meijo University, Japan
Gou Nakamura

We study the boundary value problem of the heat equation in the space of all real polynomials of two variables x and t. Our interest is to determine a polynomial $d(x,t)$ such that for every polynomial $f(x,t)$, there exists a heat polynomial $u(x,t)$ which is equal to $f(x,t)$ on the curve $d(x,t) = 0$ in the xt-plane, where heat polynomial means a polynomial which satisfies the heat equation. We give all such d with degree at most two, and prove that there exist no such d of degree greater than 3. The Hermite polynomial $H_n(x)$ plays an essential role in our argument. In particular, the irreducibility of $H_{2n-1}(x)/x$ and $H_{2n}(x)$ in the field of rational numbers is very important.

Estimate for the Weighted Polyharmonic Bergman Kernel and Its Application

Kiyoki Tanaka
Daido University, Japan

In this talk we discuss the weighted polyharmonic Bergman space on the unit ball. We give estimates for the weighted polyharmonic Bergman kernel. As its application, we give a characterization of positive bounded Toeplitz operators on the polyharmonic Bergman space and solve Gleason's problem for the polyharmonic Bergman space.

Special Session 99: Problems and Challenges in Financial Engineering and Risk Managment

Hong-Ming Yin, Washington State University, USA
Jin Liang, Tongji University, Peoples Rep of China
Kenneth K. Palmer, National Taiwan University, Taiwan

This special session will focus on current problems and challenges in financial engineering and risk management. The topics will include (a) Option pricing models for various securities, (b) Portfolio management and optimization, (c) Risk analysis and control on commodities and securities, (d) Numerical analysis and simulation, and (e) Program trading.

Option Price Error Formula in Flexible Binomial Trees

Guillaume Leduc
American University of Sharjah, United Arab Emirates

We study the convergence of the price of European options when the underlying asset is approximated by a general class of binomial trees. We show that under mild conditions, when the payoff is a C^1 function, the convergence is smooth and occurs at a rate of $1/n$. More importantly, we find an expression for the coefficient of $1/n$ in terms of the derivatives of the price in the Black-Scholes model. Using a known formula for call options, we extend our formula to payoffs which are merely continuous.

Robust Portfolio Selection Using Dynamic Copulas

Ping Li
Beihang University, Peoples Rep of China
Yingwei Han, Ping Lia, Jie Li

In this paper, we construct a dynamic portfolio selection model based on dynamic copulas. We first determine the existence and the number of changes in the dependence structure using change point analysis method. Then the whole sample data is split into 10 periods according to the change points. For each period, we solve robust portfolio optimiztion problem and compute the weights. In the empirical part we choose four representative assets from Chinese market to construct a portfolio. The results show that the method performs the best in out-of-sample tests when considering the dynamic dependence between assets and the uncertainty in the estimated model

Free Boundary and Traveling Wave in Credit Rating Migration

Jin Liang
Tongji University, Peoples Rep of China
Bei Hu, Yuan Wu

An asymptotic traveling wave solution of a free boundary model for pricing a corporate bond with credit rating migration risk is considered. The credit rating migration risk is measured by pricing a corporate bond. Under the structure framework, the moudel is turned to a free boundary problem, whose solution is convergent to a traveling wave, which has an explicit form. The existence, uniqueness and regularity of the solution are obtained. Furthermore, numerical examples are presented.

Optimal Dividend Strategy with Endogenous Bankruptcy Boundary Under Chapter 11 of the US Bankruptcy Code

Jianwei Lin
Putian University, Peoples Rep of China
Min Dai, Steven Kou, H.M. Soner

This paper concerns the optimal dividend strategy problem with endogenous bankruptcy boundary under the presence of Chapter 11 of the US bankruptcy code. For any given declaring bankruptcy boundary under Chapter 11, we model the value function as the solution to a singular stochastic control problem and its corresponding closed-form solution is obtained by partial differential equation. Meanwhile, we prove that the optimal dividend strategy for any given bankruptcy boundary is a barrier strategy constructed by a unique reflecting barrier. Furthermore, the optimal bankruptcy boundary is numerically determined by maximizing the value function. Numerical results show that Chapter 11 has a role impact on the optimal dividend strategy problem. The company can get the bigger wealth value by filing for Chapter 11. Furthermore, in the presence of Chapter 11, the company usually pays dividends to shareholders earlier so as to maximize the wealth value of the company, but the corresponding life cycle is shorter.

Stability Analysis of Local Volatility Model

Yuh-Dauh Lyuu
National Taiwan University, Taiwan
Lok U. Hou

The local-volatility (LV) model is an option model that attempts to fit the implied-volatility surface while retaining the preference-freedom feature of the Black-Scholes model. But the calibration of the LV model has been plagued by numerical problems. This talk shows that the existence of repelling fixed points may be one major reason.

Path Independence of Exotic Options and Convergence of Binomial Approximations

Kenneth Palmer
National Taiwan University, Taiwan
Guillaume Leduc

We present ways in which barrier and lookback options can be regarded, in some sense, as *path-independent options*. Exploiting this, we derive closed form formulae for the coefficients of $1/\sqrt{n}$ and $1/n$ in the expansion of the error of our *path-independent pricing* when the underlying is approximated by the Cox, Ross, and Rubinstein model. This yields a convergence of order $n^{-3/2}$ to the price of barrier and lookback options in the Black-Scholes model. Our results are supported and illustrated by numerical examples.

Mean-Risk Portfolio Selection with Expectiles

David Saunders
University of Waterloo, Canada
Hongcan Lin, Chengguo Weng

We consider the mean-risk portfolio selection problem using the expectile as the risk measure in a continuous-time diffusion model. Taking advantage of the close relationship between expectiles and the Omega performance measure, we propose an alternative optimization problem with the Omega measure as an objective and show the equivalence between both problems. After showing the solution for the mean-expectile problem is not attainable, we modify the problem with an upper bound constraint imposed on the terminal wealth and solve the problem via Lagrangian duality. The global expectile minimizing portfolio and mean-expectile efficient frontier will also be discussed.

Optimal Consumption/Investment and Retirement with Necessities and Luxuries

Yong Hyun Shin
Sookmyung Women's University, Korea
Hyeng Keun Koo, Kum-Hwan Roh

We study an optimal consumption of necessity and luxury, investment, and voluntary retirement choice model. The utility function in the objective function is given by the weighted sum of a quadratic utility function and a HARA utility function. We use the martingale method to derive a closed-form solution for optimal consumption of necessity and luxury, and investment. We also give some numerical implications.

The Valuation of Vulnerable European Options with Risky Collateral

Guanying Wang
Tianjin University, Peoples Rep of China
Xingchun Wang

In this paper, we investigate vulnerable European options with risky collateral. We describe the underlying asset and the risky collateral using geometric Brownian motions, and assume the option issuer's default intensity follows an Ornstein-Uhlenbeck process. An integral-form pricing formula for call options is derived. Numerical results show that collateral can effectively cover credit losses, especially for in-the-money options with high default risk.

Optimal Search for Parameters in Monte Carlo Simulation for Derivative Pricing

Chuan-Ju Wang
Academia Sinica, Taiwan
Ming-Yang Kao

This paper provides a novel and general framework for the problem of searching parameter space in Monte Carlo simulations. We propose a deterministic online algorithm and a randomized online algorithm to search for suitable parameter values for derivative pricing which are needed to achieve desired precisions. We also give the competitive ratios of the two algorithms and prove the optimality of the algorithms. Experimental results on the performance of the algorithms are presented and analyzed as well.

Pairs Trading

Qing Zhang
University of Georgia, USA
Jingzhi Tie

This paper is concerned with an optimal strategy for simultaneously trading of a pair of stocks. The idea of pairs-trading is to monitor their price movements and compare their relative strength over time. A pairs trade is triggered by their prices divergence and consist of a pair of positions to short the strong stock and to long the weak one. Such a strategy bets on the reversal of their price strengths. From the viewpoint of technical tractability, typical pairs trading models usually assume a difference of the stock prices satisfies a mean reversion equation. In this paper, we consider the optimal pairs-trading problem by allowing the stock prices to follow general geometric Brownian motions. The objective is to trade the pairs over time to maximize an overall return with a fixed commission cost for each transaction. The optimal policy is characterized by threshold curves obtained by solving the associated HJB equations. Numerical examples are included to demonstrate the dependence of our trading rules on various parameters and to illustrate how to implement the results in practice.

Special Session 100: Models and Numerical Methods in Kinetic Theory

Giacomo Dimarco, University of Ferrara, Italy
Andrea Tosin, Politecnico di Torino, Italy
Mattia Zanella, Politecnico di Torino, Italy

This special session is concerned with innovative models and numerical methods having roots in kinetic theory. One of the key aspect of this theory is its flexibility in the description of phenomena at different scales, leading to mean-field equations and hydrodynamic equations as well. Nowadays countless applications, ranging from plasma physics to socio-economic and biological models, utilize this theoretical framework, posing new and exciting questions from both the analytical and the numerical point view.

Boltzmann Games in Heterogeneous Consensus Dynamics

Giacomo Albi
University of Verona, Italy
Lorenzo Pareschi, Mattia Zanella

We consider a constrained hierarchical opinion dynamics in the case of leaders' competition and with complete information among leaders. Each leaders' group tries to drive the followers' opinion towards a desired state accordingly to a specific strategy. By using the Boltzmann-type control approach we analyze the best-reply strategy for each leaders' population. Derivation of the corresponding Fokker-Planck model permits to investigate the asymptotic behaviour of the solution. Heterogeneous followers populations are then considered where the effect of knowledge impacts the leaders' credibility and modifies the outcome of the leaders' competition.

Shock Wave Structure for a Polyatomic Gas with a Large Bulk Viscosity

Kazuo Aoki
National Cheng Kung University, Taiwan
Shingo Kosuge

The structure of a standing plane shock wave in a polyatomic gas is investigated on the basis of kinetic theory, with special interest in gases with large bulk viscosities, such as the CO2 gas. The polyatomic version of the ellipsoidal statistical (ES) model is employed, and the shock structure is obtained numerically for different upstream Mach numbers. The double-layer structure consisting of a thin upstream layer with a steep change and a much thicker downstream layer with a mild change is obtained. An analytical solution, consisting of a jump condition and a slowly varying solution, is also obtained by an asymptotic analysis. The analytical solution shows good agreement with the numerical solution. An extension of the ES model to a gas with temperature dependent specific heats (a non-polytropic or thermally perfect gas) is also mentioned briefly.

Leader Formation with Mean-Field Birth and Death Models

Mattia Bongini
Université Paris Dauphine, France
Giacomo Albi, Francesco Rossi, Francesco Solombrino

We provide a mean-field description for a leader-follower dynamics with mass transfer among the two populations. This model allows the transition from followers to leaders and vice versa with transition rates depending nonlinearly on the measures of the two populations. Under certain assumptions on the interaction kernels and the transition rates, we reformulate the problem as a continuity equation over the state space and a system of ODEs for the change of label follower-leader. We then introduce a stochastic process approximating the PDE, together with a jump process over the space of labels that models the transition between the two populations. Using a propagation of chaos argument, we show that the particle system generated by these two processes converges in probability to a solution of the PDE-ODE system. Our approach can be easily generalized to multiple (even countable) populations.

Stability of Stationary Inverse Transport Equation in Diffusion Scaling

Ke Chen
University of Wisconsin-Madison, USA
Qin Li, Li Wang

We consider the inverse problem of reconstructing the optical parameters for stationary radiative transfer equation (RTE) from velocity-averaged measurements. The RTE describes the dynamics of the distribution for photon particles. It often contains multiple scales characterized by the magnitude of a dimensionless parameter – the Knudsen number (Kn). In the diffusive scaling ($\mathrm{Kn} \ll 1$), the stationary RTE is well approximated by an elliptic equation in the forward setting. However, the inverse problem for the elliptic equation is acknowledged to be severely ill-posed as compared to the well-posedness of inverse transport equation, which raises the question of how uniqueness being lost as $\mathrm{Kn} \to 0$. We tackle this problem by examining the stability of inverse problem with varying Kn. We show that, the discrepancy in two measurements is amplified in the reconstructed parameters at the order of Kn^{-p} ($p = 1$ or 2), and as a result lead to ill-posedness in the zero limit of Kn. Our results apply to both continuous and discrete settings. Some numerical tests are performed in the end to validate these theoretical findings.

Mean-Field Limit for Collective Behavior Models with Sharp Sensitivity Regions

Young-Pil Choi
Inha University, Korea

Emergent aggregation and flocking phenomena appearing in many biological systems are simple instances of collective behavior. Recently, they have been an active research in applied mathematics, biology, engineering, and physics. In this talk, we discuss the mean-field limit for a large class of swarming individual based models with local sharp sensitivity regions. We provide a quantitative error estimate between the solutions to the differential inclusion system corresponding to the particle descriptions and weak solutions to the expected limiting kinetic equation by employing tools from optimal transport theory.

Time Diminishing Asymptotic Preserving Scheme for Kinetic Equations in the Diffusion Limit

Nicolas Crouseilles
Inria, France
Crestetto, Dimarco, Lemou

In this work, we introduce a new class of numerical schemes for collisional kinetic equations in the diffusion limit. The idea consists in reformulating the problem using a micro-macro decomposition and successively in solving the microscopic part by using asymptotically stable Monte Carlo methods. In addition, the particle method which solves the microscopic part is designed in such a way that the global scheme becomes computationally less expensive as the solution approaches the equilibrium state as opposite to standard methods for kinetic equations which computational cost increases with the number of interactions. At the same time, the statistical error due to the particle part of the solution decreases as the system approach the equilibrium state. This causes the method to degenerate to the sole solution of the macroscopic diffusion equation in the asymptotic regime. In a last part, we will show the behaviors of this new approach in comparisons to standard methods for solving the kinetic equation (Monte Carlo or grid based methods) by testing it on different problems in 2d or 3d.

Kinetic Description of Collision Avoidance in Pedestrian Crowds by Sidestepping

Adriano Festa
INSA Rouen, France
Andrea Tosin, Marie Therese Wolfram

We study a kinetic model for pedestrians, who are assumed to adapt their motion towards a desired direction while avoiding collisions with others by stepping aside. These minimal microscopic interaction rules lead to complex emergent macroscopic phenomena, such as velocity alignment in unidirectional flows and lane or stripe formation in bidirectional flows. We start by discussing collision avoidance mechanisms at the microscopic scale, then we study the corresponding Boltzmann-type kinetic description and its hydrodynamic mean-field approximation in the grazing collision limit. In the spatially homogeneous case we prove directional alignment under specific conditions on the sidestepping rules for both the collisional and the mean-field model. In the spatially inhomogeneous case we illustrate, by means of various numerical experiments, the rich dynamics that the proposed model is able to reproduce.

Ray Effect Mitigation by Rotating Angular Grids

Martin Frank
KIT, Germany
T. Camminady, K. Kuepper, J. Kusch

Discrete-ordinate radiation transport simulations suffer from so-called ray effects. These are artefacts in the solution which come from the discretization into finitely many directions of flight. We present and analyze a method that successively rotates the angular grid and interpolates the solution onto it. The method is simple to implement, and we show promising preliminary numerical results.

Velocity Distribution Function of Spontaneously Evaporating Atoms

Livio Gibelli
University of Warwick, England
D.A. Lockerby, J.E. Sprittles

The standard kinetic theory studies on evaporation and condensation processes focus on the vapor dynamics in the Knudsen layer and take into account the molecular exchanges with the liquid phase through phenomenological boundary conditions. These prescribe the velocity distribution function of atoms spontaneously leaving the liquid bulk and the ones back-scattered into the vapor after impinging on the liquid-vapor interface. A lot of effort has been expended in assessing the physical appropriateness of kinetic boundary conditions by using different approaches, including molecular dynamics and mean-field kinetic approximations of simple liquids. The present work complements previous studies on single-component liquids evaporating into near-vacuum conditions. In this process the back scattered vapor component is virtually absent and, therefore, one can evaluate the distribution function of evaporated atoms without any ambiguity. Preliminary results, based on the numerical solution of the Enskog-Vlasov equation, show that, no matter how low the temperature, the distribution function of evaporated atoms is approximated by an anisotropic Maxwellian with different characteristic temperatures for the velocity components normal and parallel to the liquid-vapor interface.

Multiscale Convergence Properties for Spectral Approximations of a Kinetic Model

Cory Hauck
Oak Ridge National Laboratory, USA
Zheng (Leslie) Chen

In this work, we prove rigorous convergence properties for a semi-discrete, moment-based approximation of a model kinetic equation in one dimension. This approximation is equivalent to a standard spectral method in the velocity variable of the kinetic distribution and, as such, is accompanied by standard algebraic estimates of the form N^{-q}, where N is the number of modes and $q > 0$ depends on the regularity of the solution. However, in the multiscale setting, the error estimate can be expressed in terms of the scaling parameter ϵ, which measures the ratio of the mean-free-path to the characteristic domain length. We show that, for isotropic initial conditions, the error in the spectral approximation is $\mathcal{O}(\epsilon^{N+1})$. More surprisingly, the coefficients of the expansion satisfy super convergence properties. In particular, the error of the ℓ^{th} coefficient of the expansion scales like $\mathcal{O}(\epsilon^{2N})$ when $\ell = 0$ and $\mathcal{O}(\epsilon^{2N+2-\ell})$ for all $1 \leq \ell \leq N$. This result is significant, because the low-order coefficients correspond to physically relevant quantities of the underlying system. Numerical tests will also be presented to support the theoretical results.

Control Concepts for Kinetic Equations

Michael Herty
RWTH Aachen, Germany

We present control concepts for kinetic equations. This includes model predicitive control as well as consistent optimal control methods using a novel method for differentiating kinetic differential equations. Extension to meanfield games as well as sparse control will be discussed. Further, numerical results on opinion formation models are presented.

Asymptotic-Preserving and Positivity-Preserving Implicit-Explicit Schemes for the Stiff BGK Equation

Jingwei Hu
Purdue University, USA
Ruiwen Shu, Xiangxiong Zhang

We develop a family of second-order implicit-explicit (IMEX) schemes for the stiff BGK kinetic equation. The method is asymptotic-preserving (can capture the Euler limit without numerically resolving the small Knudsen number) as well as positivity-preserving — a feature that is not possessed by any of the existing second or high order IMEX schemes. The method is based on the usual IMEX Runge-Kutta framework plus a key correction step utilizing the special structure of the BGK operator. Formal analysis is presented to demonstrate the property of the method and is supported by various numerical results. Moreover, we show that the method satisfies an entropy-decay property when coupled with suitable spatial discretizations. Additionally, we discuss the generalization of the method to some hyperbolic relaxation system and provide a strategy to extend the method to third order.

Uncertainty Quantification in Kinetic Theory

Shi Jin
Shanghai Jiao Tong University and University of Wisconsin-Madison, Peoples Rep of China

We first extend the paradigm of asymptotic-preserving schemes to the random kinetic equations, and show how it can be constructed in the setting of the stochastic Galerkin approximations. We then extend the hypocoercivity theory, developed for deterministic kinetic equations, to the random case, and establish in the random space regularity, long-time sensitivity analysis, and uniform (in Knudsen number) spectral convergence of the stochastic Galerkin methods, for general linear and nonlinear random kinetic equations in various asymptotic-including the diffusion, incompressible Navier-Stokes, high-field, and acoustic regimes.

Controlling Collective Dynamics: the Boltzmann-Bellman Approach

Dante Kalise
Imperial College London, England
Giacomo Albi

We study the synthesis of optimal control policies for large-scale multi-agent systems. The optimal control design induces a stabilizing action via an optimal feedback mapping. We study a wide class of optimal feedback controller which can be obtained from dynamic programming techniques. In order to circumvent the dimensionality issues associated to the control of large-scale agent-based models, we follow a

Boltzmann approach. We generate (sub)optimal controls signals for the kinetic limit of the multi-agent dynamics, by sampling of the optimal solution of the control problem associated to two-agent dynamics. Numerical experiments assess the performance of the proposed design.

A Positive Asymptotic Preserving Scheme for Linear Kinetic Transport Equations

M. Paul Laiu
Oak Ridge National Laboratory, USA
Martin Frank, Cory Hauck

We present a positive and asymptotic preserving numerical scheme for solving linear transport equations. The proposed scheme is developed based on a standard spectral angular discretization and classical micro-macro decomposition. The three main ingredients are a semi-implicit temporal discretization, a dedicated finite difference spatial discretization, and positivity limiters in the angular discretization. We show that the proposed scheme is asymptotic preserving in the sense that when the mean-free-path of the particles goes to zero, the scheme achieves a standard numerical scheme for the limiting diffusion equation. We also prove that the proposed scheme preserves positivity of the spatial particle concentration in the solution, which fixes a common defect of spectral angular discretizations. The proposed scheme is tested on two benchmark problems, one with uniform material and one with two types of material embedded as a checkerboard. We also perform a space-time accuracy test and compare the numerical results with theoretical estimates.

A Micro-Macro Method for a Kinetic Graphene Model

Mohammed Lemou
CNRS, University of Rennes, France
N. Crouseilles, S. Jin, F. Mehats

We propose a numerical method that combines micro-macro decomposition techniques with nonlinear geometric optics based methods to solve a highly oscillatory kinetic graphene model. The method solves the highly oscillatory model in the original coordinates, yet can capture numerically the oscillatory space-time quantum solution pointwisely even without numerically resolving the frequency. We prove that the underlying micro-macro equations have smooth (up to certain order of derivatives) solutions with respect to the frequency, and then prove the uniform accuracy of the numerical discretization for a scalar model equation exhibiting the same oscillatory behavior. We finally present some numerical experiments to illustrate the theory.

Kinetic Theory of Gas Mixtures

Silvia Lorenzani
Politecnico di Milano, Italy

The kinetic description of a mixture of gases with different particle masses (and possibly with different internal energies) is not a trivial generalization of the classical Boltzmann theory for a single gas, since the collision operators have to take into account exchanges of momentum and energy among the different species (and also mass exchanges, in the case of reacting mixtures). Moreover, the analysis of the gas-mixture equations is more difficult than the comparable single component gas theory because of the many different scales which now enter in the approach to equilibrium. There is the approach of the distribution function to a Maxwellian distribution (referred to as Maxwellization) and, in addition, there is the equilibration of the species (i.e., the vanishing of differences in velocity and temperature among the species). In kinetic theory, the evolution of a mixture of N elastically scattering gases is usually described by a set of N integro-differential equations of Boltzmann type for the species distribution functions. Since it is difficult, in general, to manage the collision integral operator as such, simplified kinetic models have been developed in the literature and widely used in practice. In the present work, we will analyze different linearized kinetic model equations for binary gas mixtures describing time-dependent problems and we will focus in particular on those allowing a semi-analytical representation of the solution.

Numerical Boundary Conditions for the Simulation of Rarefied Flows Along Solid Boundaries

Luc Mieussens
University of Bordeaux, France
C. Baranger, N. Herouard, J. Mathiaud

We present a numerical comparison between two standard finite volume schemes and a discontinuous Galerkin method applied to the BGK equation of rarefied gas dynamics. We pay a particular attention to the numerical boundary conditions in order to preserve the rate of convergence of the method. Most of our analysis relies on a 1D problem (Couette flow), but we also present some results for a 2D aerodynamical flow.

Fast Kinetic Scheme : Efficient MPI Parallelization Strategy for 3D Boltzmann Equation

Jacek Narski
Toulouse University, France

In the kinetic theory of gases, the state of the system is described by the distribution function defined in seven independent dimensions: the physical space, the velocity space and time. Moreover, the interac-

tion between particles requires multiple integrals over velocity space to be evaluated at every space point and for every time step of the numerical method. This makes the kinetic theory very challenging from numerical view point. In this talk, we present an efficient parallelization strategy to solve full Boltzmann equation in 6D by means of a deterministic semi-Lagrangian scheme and a fast spectral method for computing the collision integral in $O(N \log N)$. The spatial degrees of freedom are distributed over computational nodes, keeping on every mode a complete velocity space so the integrals involved in the Boltzmann collision operator can be solved with no additional communication between nodes. Moreover, the collision operator is GPU/OpenMP parallelized on each node. As a result, the strong scaling is close to ideal in the tested range (up to 1024 computational nodes). Numerical examples include fine grid $3D \times 3D$ numerical simulations of the Boltzmann equation.

Instabilities in Kinetic Traffic Models

Gabriella Puppo
University of Insubria, Italy

I will describe the onset of unstable modes, such as stop and go waves, from the point of view of kinetic models for traffic flow. Stop and go waves can be viewed as non equilibrium phenomena which arise in the congested phase of traffic. These modes propagate backwards with respect to the microscopic velocities of individual vehicles, and they are signalled by the presence of a non positive diffusion coefficient in the Chapman-Enskog expansion around equilibrium states. I will also discuss the numerical issues linked to the capture of these waves, and the construction of asymptotic preserving schemes to capture the macroscopic equation describing traffic at equilibrium. Some numerical tests will illustrate the onset and development of unstable waves.

A Kinetic Approach to the Solution of Large Multi-Scale Reaction Diffusion Systems with Applications to Solar Cell Design

Christian Ringhofer
Arizona State University, USA
D. Brinkman

The evolution of charged defects in polycrystalline solar cells leads to a large system of reaction diffusion equations exhibiting largely different time and spatial scales. The basic structure of the system is quite general, and equivalent to a kinetic system given on a general network. The same structure arises in many applications such as biological systems with many species. We discuss asymptotically preserving computational methods based on an operator splitting approach.

Microscopic Approximations of Macroscopic Models for Traffic Flows

Massimiliano Rosini
Ferrara University, Italy
M. di Francesco, S. Fagioli, G.Russo

We present a many-particle approximation of Conservation Laws (CLs). The unique entropy solution to the IVP for a scalar CL was rigorously approximated in [1, 2] by a discrete density constructed from the follow-the-leader model. The IBVP for a scalar CL has been considered in [4]. The results in [1] have been extended in [3] to a 2×2 system of CLs. Finally, we present the extensions obtained in [5] for the one dimensional Hughes model for pedestrian flows.

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Kinetic Model for the Phase Transition of the Van Der Waals Fluid

Shigeru Takata
Kyoto University, Japan
Takashi Noguchi

A simple kinetic model for the phase transition of the van der Waals fluid is presented. In the model, intermolecular collisions for a dense gas are not treated faithfully. Instead, the expected interactions as the non-ideal gas effect are confined in a self-consistent force term. Collision term plays just a role of thermal bath. Accordingly, it conserves neither momentum nor energy, even globally. It is demonstrated that (i) by a natural separation of the mean-field self-consistent potential, the potential for the non-ideal gas effect is determined fro the equation of state for the van der Waals fluid, with the aide of the balance equation of momentum, (ii) a monotonically decreasing function in time is found from the H theorem and turns out to have a close relation to the Helmholtz free energy Collision term plays just a role of thermal bath. Accordingly, it conserves neither momentum nor energy in thermodynamics, and (iii) the Cahn-Hilliard type equation is obtained in the continuum limit of the proposed kinetic model. Numerical simulations based on the Cahn-Hilliard type equation are also performed.

Transport of Congestion in Two-Phase Compressible/Incompressible Flow

Ewelina Zatorska
University College London, England
Didier Bresch, Charlotte Perrin, Pierre Degond, Piotr Minakowski, Laurent Navoret

Can the fluid equations be used to model pedestrian motion or traffic? In this talk, I will present the compressible-incompressible two phase system describing the flow in the free and in the congested regimes. I will show how to approximate such system by the compressible Navier-Stokes equations with singular pressure for the fixed barrier densities, together with some recent developments for the barrier densities varying in the space and time. At the end, I will present the numerical results showing that our macroscopic system captures some features characteristic for microscopic models of collective behaviour.

Special Session 101: Structure of Solutions for Nonlinear Elliptic Equations

Satoshi Tanaka, Okayama University of Scinece, Japan
Yuki Naito, Ehime University, Japan

In this session, we will discuss properties of solutions to nonlinear elliptic equations and the related problems. We will study qualitative properties, such as existence, uniqueness, stability, bifurcation and symmetry of solutions and related methods.

Radially Symmetric Singular Solutions of Semilinear Elliptic Supercritical Equations

Soohyun Bae
Hanbat National University, Korea

We consider radially symmetric singular solutions for semilinear elliptic supercritical equations and prove the existence under extra conditions including the supercriticality. The asymptotic behavior at the singularity is described by self-similarity. After explaining known results, we show the uniqueness by analyzing the corresponding integral equation. We also discuss a generalization of the result and related questions.

On Some Properties of Eigenvalues and Eigenfunctions of the P-Laplacian

Vladimir Bobkov
University of West Bohemia, Czech Rep

Consider the nonlinear eigenvalue problem for the p-Laplacian

$$-\Delta_p u = \lambda |u|^{p-2} u \quad \text{in} \quad \Omega, \qquad u = 0 \quad \text{on} \quad \partial\Omega,$$

and denote by $(\lambda(p), \varphi_p) \in \mathbb{R} \times W_0^{1,p}(\Omega)$ its eigenpairs. In the present talk, we give a survey of various properties of $\lambda(p)$ and φ_p obtained recently in coauthorship with B. Audoux, P. Drabek, S. Kolonitskii, E. Parini, M. Tanaka.
In particular, we show that φ_p and φ_q are linearly independent provided $p \neq q$; the nodal set of any second eigenfunction φ_p intersects with the boundary $\partial\Omega$ provided Ω is Steiner symmetric; if Ω is a disk, then there can occur eigenfunctions φ_p whose nodal structure is impossible in the linear case $p = 2$. Also, we show that $\lambda(p)$ can be non-monotone with respect to p, and if Ω is a disk, then radial and nonradial eigenfunctions can be associated with the same eigenvalue, which is again impossible for $p = 2$.

Remarks on Separation Property of Positive Radial Solutions to Matukuma Type Equations

Shoichi Hasegawa
Tokyo Institute of Technology, Japan

We devote this talk to discussing separation property of positive radial solutions to Matukuma type equations. For the Matukuma equation

$$-\Delta u = \frac{1}{1+|x|^2} u^p \quad \text{in} \quad \mathbb{R}^N, \tag{M}$$

it is already known that if the exponent p is greater than or equal to the Joseph-Lundgren exponent p_{JL}, then the family of positive radial solutions has separation property, i.e., any two positive radial solutions do not intersect each other. In this talk, we prove that p_{JL} is critical with respect to separation property of positive radial solutions to (M), i.e., if $p \in (p_S, p_{JL})$, then there exist two positive radial solutions intersecting each other, where p_S denotes the Sobolev exponent. Moreover, we extend the result to general Matukuma type equations.

Minimization Problem on the Hardy-Sobolev Inequality in Interior Singularity Case

Masato Hashizume
Ehime University, Japan

We consider a minimization problem on the Hardy-Sobolev inequality. Concerning this problem, position of singularity on bounded domain plays a crucial role. In this talk, we consider interior singularity case. In this case, we can see that existence and nonexistence of a minimizer depend on scale of domain. In addition to existence and nonexistence results, we prove uniqueness result and symmetry property of minimizer when domain is an unit ball.

Global Bifurcation of a Problem Arising in Porous-Medium Combustion

Kuo-Chih Hung
National Chin-Yi University of Technology, Taiwan

We study the bifurcation curve and exact multiplicity of positive solutions of the two-point boundary value problem arising in porous-medium combustion.

Infinitely Many Solutions for the (P, Q)-Laplace Equation

Ryuji Kajikiya
Saga University, Japan

This lecture is a joint work with Dr. Komiya. We study the (p, q)-Laplace equation under the Dirichlet boundary condition. We give a sufficient condition of the nonlinear term for the existence of a sequence of solutions converging to zero or diverging to infinity. Moreover, we give a priori estimates of the C^1-norms of solutions.

Positive Solutions of a Semilinear Elliptic Equation with Singular Dirichlet Boundary Data

Tatsuki Kawakami
Ryukoku University, Japan
Marek Fila, Kazuhiro Ishige

The purpose of this talk is to construct positive solutions of the semilinear elliptic equation $-\Delta u = u^p$ in $\mathbb{R}^N_+$ with a singular Dirichlet boundary condition. We show that for $p > (N+1)/(N-1)$ there exists a positive singular solution which behaves like $|x|^{-2/(p-1)}$ as $|x| \to 0$ and like the Poisson kernel as $|x| \to \infty$.

Multiplicity of Positive Solutions for Elliptic Equations Arising in a Theory of Thermal Explosion

Eunkyung Ko
Keimyung University, Korea

In this talk we consider a mathematical model of thermal explosion which is described by the boundary value problem

$$\begin{cases} -div(|\nabla u|^{N-2}\nabla u) &= \lambda e^{u^\alpha}, \quad x \in \Omega, \\ |\nabla u|^{N-2}\frac{\partial u}{\partial \nu} + g(u) &= 0, \quad\quad x \in \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N, N \geq 2$, $\partial\Omega$ is the smooth boundary of Ω with outward normal ν, $\alpha \in (0, \frac{N}{N-1}]$ and λ is a positive parameter. Using variational methods we show that there exists $0 < \Lambda < \infty$ such that the problem has at least two positive solutions if $0 < \lambda < \Lambda$, no solution if $\lambda > \Lambda$ and at least one positive solution when $\lambda = \Lambda$.

Singularly Weighted Generalized Laplacian Systems and Applications

Yong-Hoon Lee
Pusan National University, Korea
X. Xu

We study the homogeneous Dirichlet boundary value problem of generalized Laplacian systems with a singular weight which may not be integrable. Some explicit intervals which correspond to the existence and nonexistence of positive solutions for the system with the finite asymptotic behaviors of the nonlinearities at 0 and infinity are obtained. We show the relationship between the eigenvalue region and the number of positive solutions under various assumptions on the nonlinearities. Main tool for the proof is the fixed point theorem on a cone.

Exact Eigenvalues and Eigenfunctions for a One-Dimensional Gel'Fand Problem

Yasuhito Miyamoto
The University of Tokyo, Japan
Tohru Wakasa

It is known that every positive solution of a one-dimensional Gel'fand problem can be written explicitly. In this talk we give exact expressions of all the eigenvalues and eigenfunctions of the linearized eigenvalue problem at each solution. We study asymptotic behaviors of eigenvalues and eigenfunctions as the L^∞-norm of the solution goes to the infinity. We also study the problem $u'' + \lambda e^{-u} = 0$ and the associated linearized problem.

Blow-Up Analysis for Nodal Radial Solutions in Trudinger-Moser Critical Equations in $\mathbb{R}^2$

Daisuke Naimen
Muroran Institute of Technology, Japan

We consider low energy nodal radial solutions of Trudinger-Moser critical equations in $\mathbb{R}^2$. We study the asymptotic behavior of them as the growth rate of the nonlinearity goes to a threshold between the existence and nonexistence of nodal radial solutions. The solution exhibits a multiple concentration behavior together with a convergence to the least energy solution of a critical problem. We also observe that each concentration part, with an appropriate scaling, converges to a solution of the classical Liouville problem in $\mathbb{R}^2$. This talk is based on a joint work with Massimo Grossi at Sapienza University of Rome.

Positive Singular Solutions for Semilinear Elliptic Equations with Supercritical Nonlinearity

Yuki Naito
Ehime University, Japan
Yasuhito Miyamoto

We study positive singular solutions to the Dirichlet problems for the semilinear elliptic equation in the unit ball.
We first show the uniqueness of the singular solution to the problem, and then study the convergence of positive regular solutions to the singular solution.

Minimization Problem Related to the Critical Hardy Inequality with Non-Decreasing Potential Function

Megumi Sano
Tokyo Institute of Technology, Japan

Let $a, q, \beta > 1$, and $N \geq 2$. We consider the minimization problem associated with the optimal constant of the generalized critical Hardy inequalities as follows:

$$G_a := \inf_{0 \neq u \in W_0^{1,N}(B_1)} \frac{\int_{B_1} |\nabla u|^N dx}{\left(\int_{B_1} |u|^q f_a(x) dx \right)^{\frac{N}{q}}},$$

where $f_a(x) := \frac{1}{|x|^N (\log \frac{a}{|x|})^\beta}$. In this talk, we show some results concerning the attainability of G_a when $\beta = \frac{N-1}{N} q + 1$ and $q > N$. Note that the potential function f_a is not radially decreasing for a close to 1. This is an open problem mentioned by T. Horiuchi in 2016.

Oscillatory Structures of Bifurcation Curves for Nonlinear Eigenvalue Problems

Tetsutaro Shibata
Hiroshima University, Japan

We consider the bifurcation problem $-u''(t) = \lambda (u(t) + g(u(t)))$, $u(t) > 0$, $t \in I := (-1, 1)$, $u(-1) = u(1) = 0$, where $g(u)$ is a nonlinear term and $\lambda > 0$ is a bifurcation parameter.
It is known that under some suitable conditions on $g(u)$, λ is parameterized by the maximum norm $\alpha = \|u_\lambda\|_\infty$ of the solution u_λ associated with λ and is written as $\lambda = \lambda(g, \alpha)$.
We show that if $g(u)$ satisfies some oscillatory conditions, then the bifurcation diagram of $\lambda(g, \alpha)$ intersects the line $\lambda = \pi^2/4$ infinitely many times by establishing the precise asymptotic formulas for $\lambda(g, \alpha)$ as $\alpha \to \infty$.
We also establish the precise asymptotic formulas for $\lambda(g, \alpha)$ as $\alpha \to 0$.

Total P-Powered Curvature of Closed Curves

Naoki Shioji
Yokohama National University, Japan
Kohtaro Watanabe

We consider a problem concerning a generalization of elastic curves in a two-dimensional sphere or hyperbolic plane. We consider a functional on the manifolds which includes p-powered curvature with $p > 1$. We give its first variation formula, and we realize a solution of the first variation formula as a stationary curve of the functional.

On $P(X)$-Laplace Equations Involving Schrödinger Term and Critical Growth in the Whole Space $\mathbb{R}^N$

Inbo Sim
University of Ulsan, Korea
Ky Ho, Yun-Ho Kim

We obtain concentration-compactness principles for weighted variable exponent spaces that generalize known results. Employing these concentration-compactness principles and suitable imbeddings we obtain some existence results for $p(x)$-Laplace equations in the whole space $\mathbb{R}^N$ involving Schrödinger term and critical growth.

Integral Formulas of Generalized Trigonometric Functions with Two Parameters

Shingo Takeuchi
Shibaura Institute of Technology, Japan
Hiroyuki Kobayashi

Generalized trigonometric functions (GTFs) are simple generalization of the classical trigonometric functions. GTFs are deeply related to the p-Laplacian, which is known as a typical nonlinear differential operator, and there are a lot of works on GTFs concerning the p-Laplacian. However, few formulas for GTFs with two parameters are known. In this talk, we will give integral formulas for the functions, e.g. Wallis-type formulas, and apply the formulas to the lemniscate function and the lemniscate constant.

Symmetry-Breaking Bifurcation for the Moore-Nehari Differential Equation

Satoshi Tanaka
Okayama University of Science, Japan

This is a joint work with Ryuji Kajikiya and Inbo Sim. The bifurcation problem of positive solutions for the Moore-Nehari differential equation $u'' + h(x, \lambda) u^p = 0$ in $(-1, 1)$ with $u(-1) = u(1) = 0$ is considered, where $p > 1$, $h(x, \lambda) = 0$ for $|x| < \lambda$ and $h(x, \lambda) = 1$ for $\lambda \leq |x| \leq 1$ and $\lambda \in (0, 1)$ is a bifurcation parameter. The problem has a unique even positive solution $U(x, \lambda)$ for each $\lambda \in (0, 1)$. It is shown that there exists a unique $\lambda_* \in (0, 1)$ such that a non-even positive solution bifurcates at λ_* from the curve $(\lambda, U(x, \lambda))$, where λ_* is explicitly represented as a function of p.

Solutions for (P,Q)-Laplace Equations with Two Parameters

Mieko Tanaka
Tokyo University of Science, Japan
Vladimir Bobkov

I present the existence and non-existence of solutions for (p,q)-Laplace equations with two parameters under the Dirichlet boundary condition. The results are relevant to the eigenvalues of p-Laplacian and q-Laplacian.

Classification and Evolution of Bifurcation Curves for the One-Dimensional Perturbed Gelfand Equation with Mixed Boundary Conditions

Shin-Hwa Wang
National Tsing Hua University, Taiwan
Yu-Hao Liang

We study the classification and evolution of bifurcation curves of positive solutions for a one-dimensional perturbed Gelfand equation with mixed boundary conditions. Our results substantially improve and generalize those given by Hung, Wang and Yu (J. Math. Anal. Appl. 392 (2012) 40-54) and answer with rigorous proofs some of the computational results provided by Goddard II, Shivaji and Lee (Bound. Value Probl. 2010, Art. ID 357542, 23 pages).

Special Session 102: Asymptotics for Nonlinear Diffusion Equations and Related Topics

Tatsuki Kawakami, Ryukoku University, Japan
Yohei Fujishima, Shizuoka University, Japan

The aim of this special session is to bring together experts in the research area of nonlinear diffusion equations, in order to exchange recent results, ideas and techniques. In particular we are interested in the asymptotic behavior of solutions for nonlinear PDE with respect to anomalous diffusion.

Global in Time Existence of Solutions for the Heat Equation with a Superlinear Source Term

Yohei Fujishima
Shizuoka University, Japan
Norisuke Ioku

This talk is devoted to the study of the global in time existence of solutions for a nonlinear heat equation with general nonlinearity. We introduce a generalization of a self-similar transformation, and exhibit a scale invariant integral under this transformation. As a result, we show the global in time existence of solutions with small initial data.

Existence of Solutions for a Fractional Semilinear Parabolic Equation with Singular Initial Data

Kotaro Hisa
Tohoku University, Japan
Kazuhiro Ishige

In this talk we obtain necessary conditions and sufficient conditions on the initial data for the solvability of the Cauchy problem

$$\partial_t u + (-\Delta)^{\frac{\theta}{2}} u = u^p, x \in \mathbf{R}^N, t > 0,$$

$u(0) = \mu \geq 0$ in $\mathbf{R}^N$, where $N \geq 1$, 01 and μ is a Radon measure or a measurable function in $\mathbf{R}^N$. Here $(-\Delta)^{\theta/2}$ denotes the fractional power of the Laplace operator $-\Delta$ in $\mathbf{R}^N$. This is defined by

$$\mathcal{F}[(-\Delta)^{\frac{\theta}{2}} u](\xi) = |\xi|^{\theta} \mathcal{F}[u](\xi), \quad \xi \in \mathbf{R}^N,$$

where $\mathcal{F}[v]$ is the Fourier transform of v. Our conditions identify the strongest singularity of initial data for the solvability of the problem. And as an application we obtain optimal estimates of the life span of the solution with small initial data.

On the Compactness of the Sobolev Embedding Involving Variable Exponent and Related Topics

Michinori Ishiwata
Osaka University, Japan

In this talk, we are concerned with the compactness of the embedding $H_0^1(\Omega) \hookrightarrow L^{p(\cdot)}(\Omega)$, where Ω is a bounded domain in $\mathbb{R}^N$ with $N \geq 3$ and $p(\cdot)$ is a function called a variable exponent satisfying $p(0) = 2^* = \frac{2N}{N-2}$, $p(\cdot) \leq 2^*$ in $\Omega \setminus \{0\}$. It is well known that the embedding above is continuous but fails to be compact if $p(\cdot) \equiv 2^*$ due to the action of the dilation. In this talk, we give an "almost necessary and sufficient" condition on the decay rate of $2^* - p(x)$ as $|x| \to 0$ which assures the compactness of (S). Our approach is based on the profile-decomposition of bounded sequences in H_0^1 together with the scaling argument. Other related topics are also discussed.

Mathematical and Numerical Studies on Blow-Up Rate of Solutions to Some Quasilinear Parabolic Equation

Tetsuya Ishiwata
Shibaura Institute of Technology, Japan
Koichi Anada, Takeo Ushijima

We consider the blow-up problem of positive solutions to a quasilinear parabolic partial differential equation with zero Dirichlet boundary condition. This problem contains the case which is related to the curvature blow-up to the classical curvature flow. This case is a critical case, that is, it is well-known that the blow-up type is type II for this critical case and super critical case. And recently so-called loglog-type blow-up rate is proved for the critical case under very special conditions. So, there is a question: Is this loglog-type blow-up generic? In this talk, we review the key observation for the critical case and show present mathematical results for super critical case. (We haven't got the estimates of the blow-up rate for this case.) Also we introduce a numerical method for estimating a blow-up rate by using scale invariance of the equation and show numerical observations for the critical and the super critical cases.

Analyticity and Large Time Behavior of Solutions for the Burgers Equation with the Critical Dissipation

Tsukasa Iwabuchi
Tohoku University, Japan

We study the Cauchy problem of the Burgers equation, where the derivative order of the linear part and the nonlinear part are balanced. We will discuss the space-time analyticity of solutions and the large time behavior that smooth solutions behave like the Poisson kernel as time tends to infinity.

Heat Equation with a Dynamical Boundary Condition

Tatsuki Kawakami
Ryukoku University, Japan
Marek Fila, Kazuhiro Ishige

We consider the heat equation with a dynamical boundary condition in N-dimensional half space. For the Laplace equation case it is well known that the solution exists global-in-time for any measurable initial data. In this talk, for suitable initial data we construct global-in-time solution of our problem, and show that this solution converge to a solution for the case of the Laplace equation when the coefficient of the time derivative for the heat equation tends to zero. This talk is based on the joint work with M. Fila (Comenius Univ.) and K. Ishige (Tohoku Univ.).

A Phase Transition Approach to Boundary Value Problems for Bending Energy

Tatsuya Miura
The University of Tokyo, Japan

Some typical diffusion equations are directly connected with phase transition models. In this talk we present that a phase transition viewpoint is also useful for understanding the mechanism of the shape formation of solutions to boundary value problems for bending energy in a certain case. In particular, this approach enables us to reach a first uniqueness result of non-closed Euler's elastica of least energy under the clamped boundary condition.

Sobolev Inequalities on Cartan-Hadamard Manifolds and Applications to Nonlinear Diffusions

Matteo Muratori
Politecnico di Milano, Italy
G. Grillo, A. Roncoroni, J.L. Vazquez

The classical Sobolev inequality not only holds in Euclidean space, but also on Cartan-Hadamard manifolds (with the same optimal constant), that is complete and simply connected Riemannian manifolds having nonpositive sectional curvatures at every point. On the other hand, the Poincaré (or spectral gap) inequality fails on Euclidean space but holds on hyperbolic space or more in general on any Cartan-Hadamard manifold with sectional curvatures bounded from above by a negative constant: this is a result due to McKean (1970). However, almost nothing seems to be known in between, namely when curvatures are negative but allowed to vanish at infinity. Here we establish some results in this direction, along with related consequences regarding smoothing effects for certain nonlinear diffusions of porous medium type. We are able to prove suitable Sobolev-type inequalities in the radial setting when curvatures are allowed to vanish with a power-type rate at infinity. Surprisingly enough, such inequalities tend to fail for nonradial functions, as counterexamples show.

Threshold Behavior of Solutions for Semilinear Heat Equations with Slowly Decaying Initial Data

Yuki Naito
Ehime University, Japan

We study the Cauchy problem for the semilinear heat equation with polynomial decaying initial data. We will discuss the behavior of global solutions by means of the potential well with forward self-similar transformation.

Global Existence in Critical Space for a Nonlinear Diffusion Equation with a Nonlinear Source

Ryuichi Sato
Tohoku University, Japan

In this talk, we shall consider global existence of a nonlinear diffusion equation of porous medium type with a nonlinear source.
We obtain the existence of a solution in scaling critical space.

Optimal Theory of Existence and Uniqueness for the Fractional Heat Equation

Yannick Sire
Johns Hopkins University, USA
Matteo Bonforte, Juan-Luis Vazquez

We consider the fractional heat equation in the whole Euclidean space. We develop an optimal theory in terms of growth of the initial data leading to a solution and its uniqueness. This is a fractional analogue of a celebrated result by Widder.

Existence of Solutions with Moving Singularities for Equations of Porous Medium Type

Jin Takahashi
Tokyo Institute of Technology, Japan

We consider singular solutions for equations of porous medium type $u_t = \Delta u^m$ $(m > 0)$. Under some restriction on the range of m and some smallness assumption on the velocity of the singular point, we prove that there exists a solution with a moving singularity for the equations. This talk is based on a joint work with Professor Marek Fila (Comenius University) and Professor Eiji Yanagida (Tokyo Institute of Technology).

Weighted Energy Estimates for Wave Equation with Space-Dependent Damping Term for Slowly Decaying Initial Data

Yuta Wakasugi
Ehime University, Japan
Motohiro Sobajima

This talk is based on a joint work with Professor Motohiro Sobajima (Tokyo University of Science). We study weighted energy estimates for solutions to wave equation

$$\partial_t^2 u - \Delta u + a(x)\partial_t u = 0$$

with space-dependent damping term $a(x) = |x|^{-\alpha}$ ($\alpha \in [0,1)$) in an exterior domain $\Omega \subset \mathbb{R}^N$ having a smooth boundary. The main result asserts that the weighted energy estimates with weight function like polynomials are given and these decay rate are almost sharp, even when the initial data do not have compact support in Ω. The crucial idea is to use a special solution of the corresponding parabolic equation

$$\partial_t u = |x|^{\alpha} \Delta u$$

including Kummer's confluent hypergeometric functions.

Maximum Principles and Comparison Principles for Time-Fractional Partial Differential Equations

Masahiro Yamamoto
The University of Tokyo, Japan
Yuri Luchko

We consider the time fractional diffusion equation in a bounded domain.
Since Luchko proved an extremum principle for Caputo derivatives in 2009, many results on the maximum principle and comparison principles have been proved. I survey recent results and show applications of them to an inverse problem and nonlinear fractional diffusion equations.

Special Session 103: Recent Advances in Numerical Methods for Parital Differential Equations

Long Chen, University of California at Irvine, USA
Jun Hu, Peking University, Peoples Rep of China
Xuehai Huang, Wenzhou University, Peoples Rep of China

Partial differential equations play a key role in modeling science and engineering problems. Although many numerical methods have been developed for various partial differential equations, new challenges appear in designing efficient numerical methods for partial differential equations, including novel nonstandard discrete methods, fast solvers, adaptive algorithms, nonlinear problems, interface problems, and fractional order equations. This session aims to bring together researchers to exchange ideas of the recent progress in the numerical methods for parital differential equations and explore new directions in the future research.

An HDG Method for Stationary Magnatohydrodynamics

Ke Shi
Old Dominion University, USA
Weifeng Qiu, City University of Hong Kong

In this talk we introduce an hybridizable DG method for the stationary Magnetohydrodynamics (MHD) equations with two types of boundary (or constraint) conditions. The method can be hybridized so that the global computational cost is significantly reduced comparing with traditional DG methods. In addition, the method provides exactly divergence-free velocity field. In contrast, the magnetic field is weakly div-free. We provide a priori error estimates for the method on the nonlinear MHD equations. In the smooth case, we have optimal convergence rate for the velocity, magnetic field and pressure in the energy norm, the Lagrange multiplier only has suboptimal convergence order. With the minimal regularity assumption on the exact solution, the approximation is optimal for all unknowns. To the best of our knowledge, this is the first a priori error estimates of HDG methods for nonlinear MHD equations.

Finite Elements on Prisms and Cones with Polygonal Bases

Yanqiu Wang
Nanjing Normal University, Peoples Rep of China
Wenbin Chen

We discuss the construction and efficient implementation of some lowest-order conforming finite elements on prisms and cones with polygonal bases.
Note that 3D convex polytopes can be divided into cones by connecting its vertices with the center. Combined with our finite element on cones, this provides a practical discretization on convex polytopal meshes. This is a collaborative work with Prof. Wenbin Chen.

Weak Virtual Element Methods for Biot's Consolidation Model

Feng Wang
Nanjing Normal University, Peoples Rep of China

In this talk, we shall present a weak virtual element methods for the poroelasticity problem on polytopal meshes. The pressure is approximated by discontinuous polynomials, while the velocity is discretized by H(div) virtual elements with some tangential polynomials on element boundaries. We will show the error estimates, which are verified by some numerical results.

On the Advances in Order Reduced Methods of Fourth Order Problems

Shuo Zhang
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Peoples Rep of China

More and more fourth order problems have come into the vision of applied and computational mathematics. Order reduced methods have been being designed for various problems. In this talk, the speaker will present some recent advance in the design and implementation of the order reduced finite element schemes on both source problems and eigenvalue problems. The talk is mainly about works by the speaker and his collaborators, while some review of relevant works will be included. Also, both general frameworks and specific examples will be included with observation and thought on the development.

Fast Solvers for Edge Element Discretizations of Time-Harmonic Maxwell Equations

Liuqiang Zhong
South China Normal University, Peoples Rep of China

We will introduce fast solvers for edge element discretizations of time-harmonic Maxwell equations. We first present the classical two grid method, iterative two gird method and improved two-gird method. Secondly, we design and analysis the corresponding additive Schwarz preconditions, the key is to construct a special "coarse mesh" space, which adds the kernel of the curl-operator in a fine space to a coarse mesh space, to solve the original problem, and then uses the fine mesh space to solve the H(curl)-elliptic problem. Numerical experiments show the efficiency of the proposed approach.

Special Session 104: Recent Advances and Applications of Differential Equations

Lingju Kong, University of Tennessee at Chattanooga, USA
Min Wang, Kennesaw State University, USA

The aim of this session is to provide an opportunity for the mathematicians to introduce their new results and exchange ideas on the theory and applications on differential equations including ordinary differential equations, partial differential equations, fractional differential equations, and stochastic differential equations.

Sufficient Conditions of Existence of Integral Solution for Non-instantaneous Impulsive Fractional Evolution Equation

Swaroop Bora
IIT Guwahati, India
Jayanta Borah

In this work, we establish sufficient conditions for existence and uniqueness of integral solution of a non-densely defined closed linear operator satisfying the Hille-Yosida conditions of a non-instantaneous impulsive evolution equation on a Banach space involving Caputo fractional derivative. Our analysis is based on fractional calculus, Banach contraction method and Darbo-Sadvoskii's fixed point theorem. The results are obtained by means of characteristic functions based on probability density. Finally, the main results are illustrated through examples.

Continuation Theorems for the Periodic Φ-Laplacian Equation and Applications

Guglielmo Feltrin
University of Turin, Italy

Using Mawhin's coincidence degree theory, we obtain some new continuation theorems which are designed to have as a natural application the study of the periodic problem for cyclic feedback type systems. Applications to vector ordinary differential equations with a ϕ-Laplacian operator will be discussed. In particular, a continuation theorem for the periodic problem associated with $(\phi(u'))' + \lambda f(t,u,u') = 0$, under the only assumption that ϕ is a homeomorphism, is the key ingredient to prove multiplicity results for positive periodic solutions to an indefinite Minkowski-curvature equation. This talk is based on a joint works with A. Boscaggin (University of Turin) and F. Zanolin (University of Udine).

The Effect of a Nonstandard Cone on Existence Theorems in Nonlocal Boundary Value Problems

Christopher Goodrich
Creighton Preparatory School, USA

In this talk I will consider a perturbed Hammerstein integral equation of the form

$$y(t) = \gamma_1(t)H_1\big(\varphi_1(y)\big)+\gamma_2(t)H_2\big(\varphi_2(y)\big)+\lambda\int_0^1 G(t,s)f\big(s,y(s)\big)\,ds.$$

Since φ_1 and φ_2 are linear functionals, solutions of this type of integral equation can be related to solutions of nonlocal boundary value problems. I will show that by introducing a nonstandard order cone one can equip the linear functionals with coercivity conditions that are useful for the improvement of existence results for both the integral equation and associated boundary value problems for ODEs and elliptic PDEs on annuli.

Infinitely Many Small Energy Solutions for Equations Driven by Nonlocal integro-differential operators in $\mathbb{R}^N$

Yun-Ho Kim
Sangmyung University, Korea

We are concerned with elliptic equations in $\mathbb{R}^N$, driven by a non-local integro-differential operator, which involves the fractional p-Laplacian. The main aim of this paper is to prove the existence of small solutions for our problem with negative energy in the sense that the sequence of solutions converges to 0 in the L^∞-norm by employing the regularity type result on the L^∞-boundedness of solutions and the modified functional method.

Degenerate Elliptic Systems with Variable Exponents

Lingju Kong
University of Tennessee at Chattanooga, USA

We establish several criteria for the existence of non-negative solutions of a degenerate elliptic system with $p_i(x)$-Laplacian operators defined on a bounded domain. Our analysis mainly utilizes variational arguments based on some recent theory on the weighted variable exponent Lebesgue-Sobolev spaces and critical point theories in Calculus of Variations.

Existence and Asymptotic Behaviour of Ground State Solution for Quasilinear Schrödinger-Poisson Systems in $\mathbb{R}^3$

Lin Li
Chongqing Technology and Business University, Peoples Rep of China
Ling Ding, Yi-Jie Meng, Chang-Ling Zhuang

In this paper, we are concerned with the existence and asymptotic behavior of ground state in the whole space $\mathbb{R}^3$ for quasilinear Schrödinger-Poisson systems

$$\begin{cases} -\Delta u + u + K(x)\phi(x)u = a(x)f(u), & x \in \mathbb{R}^3, \\ -\text{div}[(1+\varepsilon^4|\nabla\phi|^2)\nabla\phi] = K(x)u^2, & x \in \mathbb{R}^3, \end{cases}$$

where $f(t)$ is asymptotically linear with respect to t at infinity. Under appropriate assumptions on K, a and f, we establish existence of a ground state solution $(u_\varepsilon, \phi_{\varepsilon,K}(u_\varepsilon))$ of the above system. Furthermore, for all ε sufficiently small, we show that $(u_\varepsilon, \phi_{\varepsilon,K}(u_\varepsilon))$ converges to $(u_0, \phi_{0,K}(u_0))$ which is the solution of the corresponding system for $\varepsilon = 0$.

Positive Solutions for a Class of Boundary Value Problems with Singularity and Critical Exponents

Yanbin Sang
North University of China, Peoples Rep of China
Xiaorong Luo

In this talk, we consider a class of elliptic boundary value problems with singularity and critical exponents, the existence of multiple positive solutions for above problem is established by means of variational methods.

Existence and Multiplicity of Periodic Solutions to Local Coercive Equations with a Φ-Laplacian Type Operator

Elisa Sovrano
University of Udine, Italy
Guglielmo Feltrin, Fabio Zanolin

We discuss the existence of zero, one or two T-periodic solutions for the parameter dependent ϕ-Laplacian equation of the form $(\phi(u'))' + f(u)u' + g(t,u) = s$, where s is a real number, f and g are continuous functions and g is T-periodic in the variable t. Nowadays, a phenomenon modulated by the parameter s of this kind is called "Ambrosetti-Prodi type alternative", with reference to the pioneering work of [Ambrosetti and Prodi, Ann. Mat. Pura Appl., 1972]. Inspired by the results carried out in [E.S. and F. Zanolin, Adv. Nonlinear Stud., 2018], we investigate the periodic problem with ϕ-Laplacian operator for a wide family of nonlinearities under local coercivity conditions on g. As a result, we give a generalization, along with a standardization, of various classical and recent results on parameter-dependent nonlinear equations. This is a joint work with G. Feltrin (University of Turin, Italy) and F. Zanolin (University of Udine, Italy).

Existence of Solutions of a Fractional Compartment Model with Periodic Boundary Condition

Min Wang
Kennesaw State University, USA

In this talk, we consider a type of fractional order single compartment models with periodic boundary conditions. An explicit solution for the associated linear model is first derived. Then a series of conclusions on the existence and uniqueness of solutions for the nonlinear model are proved.

An H-Adaptive RKDG Method and Its Applications

Hongqiang Zhu
Nanjing University of Posts and Telecommunications, Peoples Rep of China

In this talk we present an h-adaptive Runge-Kutta discontinuous Galerkin method which is based on mesh refinement and coarsening. First, the framework of this method is presented, together with the implementation details. After that, we show its applications to different problems, including hyperbolic conservation laws, detonation wave simulations, Vlasov-Possion system and 2D incompressible Euler equations in the vorticity-stream function formulation. Numerical results of classical test problems are given to illustrate the effectiveness and the capability of this method.

Special Session 105: Nonlinear Functional Analysis and its Applications to Nonlinear Elliptic Equations/Fractional Laplacian Equations/Integral Equations

Mei Yu, Northwestern Polytechnical University, Peoples Rep of China
Qianqiao Guo, Northwestern Polytechnical University, Peoples Rep of China
Wenxiong Chen, Yeshiva University, USA

We will exchange new developments and ideas of nonlinear functional analysis and pay attention to its applications to nonlinear elliptic equations/fractional Laplacian equations/integral equations.

Sign-Changing Solutions of Fractional P-Laplacian Problems

Xiaojun Chang
Northeast Normal University, Peoples Rep of China

In this talk, we report some recent results on the sign-changing solutions of fractional p-Laplacian problems. By applying the method of invariant sets of descending flow and minimax theory, we obtain the existence and multiplicity of sign-changing solutions of the fractional p-Laplacian problems. In addition, by using the Nehari manifold method, we prove that the problem admits one least energy sign-changing solution, and its energy exceeds twice the least energy solutions. This work is joint with Prof. Zhi-Qiang Wang and Prof. Zhaohu Nie.

Negative Power Nonlinear Integral Equations on Bounded Domains

Qianqiao Guo
Northwestern Polytechnical University, Peoples Rep of China
Jingbo Dou, Meijun Zhu

We introduce and study some negative power nonlinear integral equations on bounded domains that are related to the sharp reversed Hardy-Littlewood-Sobolev inequality obtained recently by Dou and Zhu. The existence results are obtained. Blowup behavior of the minimizing energy solutions to the subcritical problem is also studied. This is a joint work with Prof. Jingbo Dou and Prof. Meijun Zhu.

Various Shadowing Properties for Partial Hyperbolicity

Manseob Lee
Mokwon University, Korea
Jiweon Ahn

Let $f : M \to M$ be a diffeomorphism on C^∞ manifold $M^d (d \geq 3)$. If f is transitive with a partially hyperbolic splitting on M, then f does not have the shadowing property. Moreover, if f admits a partially hyperbolic splitting on M then f does not have the limit shadowing and the ergodic shadowing properties.

The Properties of Positive Solutions to Nonlinear Fractional Schodinger System with Three Wave Interaction

Zhongxue Lv
Jiangsu Normal University, Peoples Rep of China

In this talk, we consider the following system of nonlinear fractional Schrödinger equations with three wave interaction:

$$\begin{cases} (-\Delta)^{\alpha/2}u_1(x) + w_1u_1(x) - u_1^p(x) = \gamma u_2(x)u_3(x), x \in \mathbb{R}^n, \\ (-\Delta)^{\alpha/2}u_2(x) + w_2u_2(x) - u_2^p(x) = \gamma u_1(x)u_3(x), x \in \mathbb{R}^n, \\ (-\Delta)^{\alpha/2}u_3(x) + w_3u_3(x) - u_3^p(x) = \gamma u_1(x)u_2(x), x \in \mathbb{R}^n. \end{cases}$$

By establishing the direct method of moving planes, we obtain radially symmetric and monotone decreasing of positive solutions for this system.

Nonradial Solutions of Nonlinear Scalar Field Equations

Jaroslaw Mederski
Institute of Mathematics, Polish Academy of Sciences, Poland

We look for nonradial solutions of the following nonlinear scalar field equation

$$\{ \ -\Delta u = g(u), \qquad u \in H^1(\mathbb{R}^N),\ N \geq 3,$$

with a nonlinearity g satisfying the general assumptions due to Berestycki and Lions. In particular, we find at least one nonradial solution for any $N \geq 4$ minimizing the energy functional on the Pohozaev constraint. If in addition $N \neq 5$, then there are infinitely many nonradial solutions. Moreover, we build a critical point theory on a topological manifold, which enables us to solve the above equation as well as to treat new elliptic problems.

Weak Measure Expansive Homoclinic Classes of C^1 Robust Vector Fields

Jumi Oh
Sungkyunkwan University, Korea

Let X be a C^1 vector field of a closed smooth Riemannian manifold M with $\dim M \geq 3$ and γ be a hyperbolic closed orbit of X. In this talk, we introduce the notion of C^1 stably weak measure expansiveness of closed invariant set $\Lambda \subset M$. And we show that the homoclinic class $H_X(\gamma)$ of X associated to γ is C^1 stably weak measure expansive if and only if it is hyperbolic.

Expansive Homoclinic Classes of C^1 Vector Fields

Junmi Park
Chungnam National Unversity, Korea
Manseob Lee

Let M be a closed n-dimensional smooth Riemmanian manifold, and let X be a C^1-vector field of M. Let γ be a hyperbolic closed orbit of X_t. In this talk, we show that (i) the chain recurrent set $\mathcal{R}(X_t)$ is C^1-stably expansive for flows if and only if X_t satisfies both Axiom A and the no-cycle condition. (ii) the homoclinic class $H_X(\gamma)$ is C^1-stably expansive for flows if and only if $H_X(\gamma)$ is hyperbolic.

On Nonexistence and Existence of Positive Solutions to Semilinear Elliptic and Parabolic Problems on Manifolds

Yuhua Sun
Nankai University, Peoples Rep of China
Alexander Grigor'yan, Fanheng Xu

We determine the critical exponent for certain semilinear elliptic problem on Riemannian manifolds assuming the volume regularity and Green function estimates. By using a sharp volume condition, we also reinvestigate nonexistence and existence of global positive solutions to semilinear parabolic equation on Riemannian manifolds. This talk is based on joint works with Prof. Grigor'ian, and Fanheng Xu.

On the Existence of Groundstate for Nonlinear Choquard Equation with Lower Critical Exponent

Jiankang Xia
Northwestern Polytechnical University, Peoples Rep of China
Jean Van Schaftingen

In this talk, I will introduce some recent progresses on the nonlinear Choquard equation with the lower critical exponent

$$-\Delta u+Vu=\big(I_\alpha*|u|^{\frac{\alpha}{N}+1}\big)|u|^{\frac{\alpha}{N}-1}u+f(x,u) \quad \text{in } \mathbb{R}^N$$

where $N\geq 1$, $\alpha\in(0,N)$ is the order of the Riesz potential I_α. The exponent $\frac{\alpha}{N}+1$ is critical with respect to the Hardy–Littlewood–Sobolev inequality. Our study contains two existence results on the ground state solutions for Choquard equations with the lower critical exponent, one is the case that potential V is a confining potential, the other is when the local perturbation f satisfies some suitable assumptions. Our approaches are based on variational methods. These are joint works with Prof. Jean Van Schaftingen.

Existence and Nondegeneracy of Ground States in Critical Free Boundary Problems

Yang Yang
Jiangnan University, Peoples Rep of China
Kanishka Perera, Zhitao Zhang

Existence and regularity of minimizers in elliptic free boundary problems have been extensively studied in the literature. The corresponding study of higher critical points was recently initiated in Jerison and Perera [D. Jerison and K. Perera. Existence and regularity of higher critical points in elliptic free boundary problems.J. Geom. Anal. 28 (2018), no. 2, 1258-1294], in which, the existence and nondegeneracy of a mountain pass point in a superlinear and subcritical free boundary problem related to plasma confinement was proved. In this paper we study ground states of a critical free boundary problem related to the Brézis-Nirenberg problem. We extend the results by combining the method introduced there with the concentration compactness principle to overcome the difficulties arising from lack of compactness.

A Liouville Theorem for a Class of Fractional Systems in $\mathbb{R}^N_+$

Mei Yu
Northwestern Polytechnical University, Peoples Rep of China

In this talk, we will introduce a class of fractional elliptic systems. Applying the direct method of moving planes for the fractional Laplacian, without any decay assumption on the solutions at infinity, we prove the Liouville theorem of nonnegative solutions under some natural conditions on f and g.

Existence and Multiplicity of Sign-Changing Solutions of Nonlocal Problems with Integrodifferential Operators

Fukun Zhao
Yunnan Normal University, Peoples Rep of China
Guanze Gu, Yuanyang Yu, Wei Zhang

This talk is based on some recent joint works with Dr. Guangze Gu, Mr. Yuanyang Yu and Dr. Wei Zhang. We will report some new results on the existence and multiplicty of sign-changing solutions of nonlocal problems with integrodifferential operators including fractional Laplace operators.

Extremal Functions of Moser-Trudinger Inequality Involving Finsler-Laplacian

Chunqin Zhou
Shanghai Jiao Tong University, Peoples Rep of China
Changliang Zhou

In this talk, we will introduce the Moser-Trudinger inequality when it involves a Finsler-Laplacian operator that is associated with functionals containing $F^2(\nabla u)$. Here F is convex and homogeneous of degree 1, and its polar F^o represents a Finsler metric on $\mathbb{R}^n$. We will show an existence result on the extremal functions for this sharp geometric inequality.

Some Results on Semilinear Elliptic Equations with Hardy-Leray Potentials

Feng Zhou
CPDE, East China Normal University, Peoples Rep of China

In this talk, we will present some results on the isolated singular solutions for some nonlinear elliptic equations with Hardy- Leray potentials. We present some suitable distributional identities of the solution for the equation and we obtain the qualitative properties for the minimal singular solutions. As applications we prove also some nonexistence results on some principle eigenvalue problems. This is based on joint works with H.Y. Chen.

Special Session 106: Variational Methods and Nonlinear Partial Differential Equations

Jun Wang, Faculty of Sciences, Jiangsu University, Peoples Rep of China
Zhitao Zhang, AMSS of Chinese Academy of Sciences, Peoples Rep of China
Maochun Zhu, Faculty of Sciences, Jiangsu University, Peoples Rep of China

The aim of this section is to bring together mathematicians who work on variational methods and nonlinear partial differential equations to report recent advances and stimulate collaborative research activities.

A Pohozaev-Type Inequality and Its Applications

Tianqing An
Hohai University, Peoples Rep of China
Binyu Kou

We derive a Pohozaev-type inequality for p-Laplacian equations, and then prove some non-existence results for the positive solution of the p-Laplacian equations with supercritical growth in a class of domains that are more general than star-shaped ones.

Cusps and a Converse to the Ambrosetti-Prodi Theorem

Marta Calanchi
Milan University, Italy
C. Tomei and A. Zaccur

By the Ambrosetti-Prodi theorem, the map $F(u) = -\Delta u - f(u)$ between appropriate functional spaces is a global fold. Among the hypotheses, the convexity of the function f is required. We show that convexity is indeed necessary. If f is not convex, there is a point with at least four preimages under F. Even more, F generically admits cusps among its critical points.

Multiple Periodic Solutions for the Nonlinear Wave Equation in a Ball

Jianyi Chen
Qingdao Agricultural University, Peoples Rep of China
Zhitao Zhang

This talk is devoted to the existence of multiple time-periodic solutions for the nonlinear wave equation

$$u_{tt} - \triangle u = g(t, x, u)$$

in an n-dimensional ball with radius R. An interesting feature is that the solvability of the problem depends on n, R and the time-period T. Based on the spectral properties of the radially symmetric wave operator, we use the reduction arguments and variational methods to construct at least three radially symmetric solutions with time-period T, when T is a rational multiple of R and $g(t, x, u)$ satisfying some growth conditions. This is joint work with Zhitao Zhang.

On Positive Solutions of a Schrödinger System

Xiyou Cheng
Lanzhou Universtiy, Peoples Rep of China

In this talk, we are concerned with existence and uniqueness of positive solutions of the coupled nonlinear Schrödinger system arising in nonlinear optics and in Hartree-Fock theory for a double condensate. Under some conditions, we obtain some interesting solution structure theorems in the parameter plane.

Existence and Concentration of Positive Solutions for Coupled Schrödinger Equations

Rong Cheng
Nanjing University of Information Science and Technology, Peoples Rep of China

In this paper, we study the existence and concentration of positive solution of a class of coupled Schrödinger equations. We admit that the potentials may not be non-negative and suppose that the intersection of the sets has positive Lebesgue measure. By studying the modified functional of the associated functional carefully, we establish the existence of positive least energy solutions for the coupled Schrödinger system. Moreover, we prove the concentration phenomenon of the positive solution when the parameter goes to infinity.

Resonant (P, Q)-Equations with Robin Condition

Michael Filippakis
University of Piraeus, Greece
N.S. Papageorgiou

We consider a nonlinear nonhomogeneous Robin problem driven by the sum of a p-Laplacian and of q-Laplacian(a (p, q)-equation). The reaction term is a Caratheodory function which is resonant at $\pm\infty$ with respect to any nonprincipal variational eigenvalue of the Robin p-Laplacian. Using variational methods together with Morse theory (critical groups), we show the existence of at least three nontrivial smooth solutions.

The publication of this paper has been partly supported by the University of Piraeus Research Center.

Time Periodic Solutions of Nonlinear Wave Equation with X-Dependent Coefficients

Shuguan Ji
Jilin University, Peoples Rep of China

In this talk, we consider the time periodic solutions of nonlinear wave equation with x-dependent coefficients. Such a model arises from the forced vibrations of an inhomogeneous string and the propagation of seismic waves in nonisotropic media. We shall talk about some recent results on the existence and multiplicity of time periodic solutions for the nonlinear wave equation with x-dependent coefficients.

General KAM Theorems and Their Applications to Invariant Tori with Prescribed Frequencies

Xu Junxiang
Southeast University, Peoples Rep of China
Xuezhu Lu

In this paper we develop some new KAM technique to prove two general KAM theorems for nearly integrable Hamiltonian systems without assuming any non-degeneracy condition. Many of KAM-type results (including the classical KAM theorem) are special cases of our theorems under some non-degeneracy condition and some smoothness condition. Moreover, we can obtain some interesting results about KAM tori with prescribed frequencies.

A Heat Equation with Exponential Nonlinearity and with Singular Data in R^2

Bernhard Ruf
University of Milano, Italy
N. Ioku, E. Terraneo

We consider a semilinear heat equation with exponential nonlinearities and singular data in R^2.

In R^N, $N \geq 3$, critical growth related to singular initial data is polynomial and has been studied by several authors. Existence and non-existence results for singular initial data in suitable L^p-spaces were obtained by F. Weissler and H. Brezis-T. Cazenave; furthermore, non-uniqueness results for certain singular initial data were given by W.M. Ni, P. Sacks and E. Terraneo. In $N = 2$ critical growth is given by nonlinearities of exponential type (cf. N. Trudinger-J. Moser). We prove that similar phenomena, namely existence, non-existence and non-uniqueness, occur for suitable exponential nonlinearities and singular initial data in certain Orlicz spaces.

Applications of Morse Theory to Some Nonlinear Elliptic Equations with Resonance at Zero

Rushun Tian
Capital Normal University, Peoples Rep of China
Mingzheng Sun, Leiga Zhao

In this paper we study the existence and multiplicity of solutions for some nonlinear elliptic boundary value problems with resonance at zero by applying Morse theory. We do not impose additional global condition on the nonlinearities, except for a subcritical growth condition.

Standing Waves Solutions for the Coupled Hartree-Fock Type Nonlocal Elliptic System

Jun Wang
Jiangsu University, Peoples Rep of China

Standing waves solutions for a coupled Hartree-Fock type nonlocal elliptic system are considered. This nonlocal type problem was considered in the basic quantum chemistry model of small number of electrons interacting with static nucleii which can be approximated by Hartree or Hartree-Fock minimization problems. First, we prove the existence of normalized solutions for different ranges of the positive (attractive case) coupling parameter for the stationary system. Then we extend the results to systems with an arbitrary number of components. Finally, the orbital stability of the corresponding solitary waves for the related nonlocal elliptic system is also considered.

Persistence of Hyperbolic-Type Degenerate Lower Dimensional Invariant Tori with Prescribed Frequency in Reversible System

Xiaomei Yang
Southeast University, Peoples Rep of China
Junxiang Xu

In this paper we consider the reversible system with small perturbations. Using the KAM technique we prove the persistence of hyperbolic-type degenerate lower dimensional invariant tori with prescribed frequency in reversible system for sufficiently small perturbations.

Long-Time Behavior of Stochastic Reaction-Diffusion Equations on Time-Varying Domains

Lu Yang
Lanzhou Universtiy, Peoples Rep of China

Random attractors and their higher order regularity properties are studied for stochastic reaction-diffusion equations on time-varying domains. Some new a priori estimates for the difference of solutions near the initial time and the continuous dependence in initial data are proved.

Some New Results on Henon- Lane-Emden Conjecture and Schrödinger Systems

Zhitao Zhang
Chinese Academy of Sciences, Peoples Rep of China

By Sobolev embeddings on S^{N-1} and the scaling invariant of the solutions, we show Lane-Emden conjecture holds in a new region; and we prove Henon-Lane-Emden conjecture is true for space dimension $N = 3$. We also show new results on some Schrödinger systems, such as symmetry and asymptotic behavior of ground state solutions; bifurcation and multiple existence of positive solutions; uniqueness and structure of positive solutions; symmetry breaking via Morse index.

Existence of Solutions for Schrödinger-Poisson Systems with Saturable Nonlinearity

Fubao Zhang
Southeast University, Peoples Rep of China
Haihua Jiang

In this talk, we are mainly concerned with the Schrödinger-Poisson system with saturable nonlinearity, and prove the existence of bound state and ground state, and nonexistence of nontrivial solution. Furthermore, we consider the existence and nonexistence of standing waves with prescribed L^2-norm for a class of Schrödinger-Poisson-Slater equations.

Existence of Solutions to Nonlinear Schrödinger Equations Involving N Laplacian Operator

Maochun Zhu
Jiangsu University, Peoples Rep of China
Jun Wang, Xiaoyong Qian

In this talk, we give the existence of solutions for the following class of nonlinear Schrödinger equations $-\Delta_N u + V(x) u = K(x) f(u)$ in $\mathbb{R}^N$ where V and K are bounded and decaying potentials and the nonlinearity $f(s)$ has exponential critical growth. The approaches used here are based on a version of the Trudinger–Moser inequality and a minimax theorem. This is a joint work with Jun Wang and Xiaoyong Qian.

Special Session 107: Optimal Control and Differential Games: Recent Developments in Theory and Applications

Khai T. Nguyen, North Carolina State University, USA
Hien T. Tran, North Carolina State University, USA

This session will focus on recent developments on optimal control problems and non-cooperative differential games. One of the focuses of the session will be on optimal feedback controls, including the Hamilton-Jacobi Bellman equations for the value functions and Nash equilibrium solutions. The second focus will be on applications in finance, engineering and biological sciences.

A Strategy of Optimal Efficacy of T11 Target Structure in the Treatment of Brain Tumor

Sandip Banerjee
Indian Institute of Technology Roorkee, India
Subhas Khajanchi

We propose a mathematical model portraying glioma and immune system interactions, considering the role of immunotherapeutic drug T11 target structure. The mathematical model comprises of a system of coupled nonlinear ordinary differential equations involving glioma cells, macrophages, activated cytotoxic T-lymphocytes (CTLs), immunosuppressive cytokine TGF-β (Transforming growth factor-β), immunostimulatory cytokine interferon-γ (IFN-γ) and the concentrations of immunotherapeutic agent T11 target structure. For the better understanding of the circumstances under which the gliomas can be eradicated, we use optimal control theory. We design the objective functional by considering the biomedical goal, which minimizes the glioma burden and maximize the macrophages and activated CTLs. The existence and the characterization for the optimal control is established. The uniqueness of the quadratic optimal control problem is also analyzed. We demonstrate numerically that the optimal treatment strategies using T11 target structure reduce the glioma burden and increase the cell count of activated CTLs and macrophages.

Optimal Control of Immunosuppressants in Renal Transplant Recipients Susceptible to BKV Infection

H. Thomas Banks
N.C. State University, USA
Neha Murad, H.T. Tran

Finding the optimal balance between over-suppression and under-suppression of the immune response is difficult to achieve in renal transplant patients, all of whom require lifelong immunosuppression. We use a mathematical model of the immune response to kidney transplant and BKV to enhance personalized treatment by application of control theory, and determine an optimal individualized treatment strategy. We design a feedback optimal control using the Receding Horizon Control (RHC) methodology. To address the non-linearity and lack of observations for all model states associated with the dynamical system we use non-linear Kalman Filtering state estimation techniques.

Control of a Cardiovascular-Respiratory System Model, Sensitivity Analysis and Parameter Identification

Aurelio de los Reyes V
Institute of Mathematics, University of the Philippines, Philippines
Pio Gabrielle B. Calderon, Franz Kappel, Lean V. Palma

A cardiovascular-respiratory system (CVRS) model is developed to describe its response to different ergometric workloads. An optimal control for time-varying respectively constant workloads is obtained using the Euler-Lagrange formulation of the optimal control problem. Essential CVRS controls considered in the study are variations in the heart rate and the alveolar ventilation in which arterial pressure of CO2 is regulated close to 40 mmHg. Included in the cost functional are penalization terms to match the metabolic need for O2 and metabolic production of CO2 with O2- and CO2- transport by blood. Sensitivity analysis on the parameters of CVRS model under a constant ergonometric workload is performed to identify which are most/least influential to the arterial systemic pressure (Pas), for which experimental data are available. In addition to Pas, alveolar ventilation measurements are used to identify model parameters.

A Hybrid Control Approach to the Route Planning Problem for Sailing Boats

Adriano Festa
INSA Rouen, France
Roberto Ferretti

We present an optimal control approach to the problem of route planning for sailing boats. The problem presents many difficulties connected to the non convexity of the dynamics and the complexity of the variables involved. The approach that we propose is based on hybrid control. We show the effectiveness of the latter and we show how some aspects of the phenomenon, not possible to consider with other frame-

works, are included in the study. We extend some of those ideas to a multiplayer scenario where an optimal strategy is derived in relation to the choices of the other players using a mean field games model.

Model Development and Treatment Control of Acute Inflammatory Response to Endotoxin Challenge

Dennis Frank-Ito
Duke University, USA
Hien T. Tran

In this work, we developed a reduced ordinary differential equation model of acute inflammatory response to endotoxin challenge. To modulate inflammation, an open-loop optimal control based treatment strategy was formulated to regulate endotoxin-induced inflammatory response. As open-loop control methods do not have the ability to incorporate disturbances in the system as time progresses, we implemented a feedback scheme known as Nonlinear Model Predictive Control (NMPC) in conjunction with Unscented Kalman Filter (UKF). In conclusion, we demonstrated our methodology with an example where noise was added to our simulated results to create an experimental data with noise. Further, UKF was used to filter out the noisy data and then estimate the unobserved states at every recalculation step in the NMPC scheme.

Feedback Control of an HBV Model Based on Model Predictive Control and Kalman Filter

Hee-Dae Kwon
Inha University, Korea

In this talk, we study a feedback control problem to drive efficient drug treatment strategies for hepatitis B virus (HBV) infection. We introduce and analyze a mathematical model that describes the HBV infection during antiviral therapy. The reproduction number R_0 is determined. The local/global stability of virus-free steady state is investigated. We formulate a control problem which minimizes the viral load as well as treatment costs. In order to reflect the status of patients not only at the initial time but also at the follow-up visits, we consider the model predictive control based on ensemble Kalman filter and differential evolution. The ensemble Kalman filter is employed to estimate full information of the state from incomplete observation data. We derive piecewise constant drug schedule applying techniques of differential evolution algorithm. Numerical simulations are performed using various weights in the objective functional to suggest optimal treatment strategies in different situations.

On Uniqueness Sets of Additive Eigenvalue Problems and Applications

Hiroyoshi Mitake
Hiroshima University, Japan
Hung V. Tran

We provide a simple way to find uniqueness sets for additive eigenvalue problems of Hamilton–Jacobi equations by using the nonlinear adjoint method. This way is versatile and we can deal with second-order Hamilton-Jacobi-Bellman equations. This talk is based on a joint work with Hung V. Tran (Univ. Wisconsin-Madison).

Lack of BV Bounds for Impulsive Control Systems

Monica Motta
Dep. of Mathematics, University of Padua, Italy
Caterina Sartori

For a nonlinear impulsive control system we introduce a notion of generalized solution x associated to a control u whose total variation is bounded on $[0, t]$ for every $t < T$ but possibly unbounded on $[0, T]$, and prove existence, consistency with classical solutions and well-posedness of this solution. It provides the natural setting for controllability questions and for some non-coercive optimal control problems, where chattering phenomena at the final time are expected. More in general, it is well suited to describe the evolution of control systems subject to a train of impulses where no a-priori bounds on the number and the amplitude of the impulses are imposed.

Variational Analysis of Two Convex Optimization Problems in Duality Applied to Some Mean Field Game Systems

Daniela Tonon
Paris Dauphine University, France

Mean field games (MFG) systems have been introduced to describe Nash equilibria in differential games with infinitely many players. In some simple cases, the model collapses into a system consisting on a backward Hamilton-Jacobi equation coupled with a forward Fokker-Plank equation. The starting point of the current study is that in some cases the MFG system can be understood as the optimality system of two convex optimization problems in duality. This leads to a variational analysis strategy to study the well-posedness of the PDE system. Following this methodology, we will discuss first the existence and uniqueness of weak solutions of some possibly degenerated Mean Field Games and then the existence of solutions of a modified problem prescribing the final distribution of the agents.

Rate of Convergence in Periodic Homogenization of Hamilton-Jacobi Equations: the Convex Setting

Hung Tran
UW Madison, USA

We obtain optimal rate of convergence in various situations in periodic homogenization of Hamilton-Jacobi equations in convex setting. We use the optimal control formula, and find a natural connection between how fast the average of backward characteristic converging to its rotation vector and rate of convergence in homogenization. Joint work with H. Mitake, Y. Yu.

Game Theory Application for Contract Optimization

Hien Tran
North Carolina State University, USA
Tien Nguyen, Daniel Chertock, Brittany Dyer, Claire Goldhammer, Scott Mahan

In this talk, we present a novel modeling and simulation approach for the acquisition of complex systems. In particular, we will present an innovative system engineering framework to model the department of defense acquisition process and offer a number of optimization modules including simulation models using game theory and war-gaming concepts. As a result, we suggest a set of acquisition strategies that can (i) optimally select the most affordable and least risky program and technical baseline solution, (ii) identify the most favorable contract type to acquire the selected program and technically baseline solution, and (iii) optimize the contract parameters and incentives for the selected contract type. Simulation results will be presented for the cost-plus incentive fee contract type.

Special Session 108: Water Waves and Other Dispersive Phenomena

J. Douglas Wright, Drexel University, USA
David Ambrose, Drexel University, USA

Many problems of interest in science exhibit dispersive, or wave-like phenomena. Examples include waves in the ocean or in optical fibers, among others. Questions of interest include existence of solutions for initial value problems, existence and stability of coherent structures, and dynamics over long time scales. In this session, we discuss recent advances in these areas.

Semiclassical Soliton Ensembles in Dispersive and Non-Dispersive Equations

Robert Buckingham
University of Cincinnati, USA
Robert Jenkins, Peter Miller

Lax and Levermore showed that solutions of the Korteweg-de Vries equation in the small-dispersion limit exhibit rapid oscillations within slowly modulated envelopes. Similar behavior has been shown for other integrable soliton equations, such as focusing NLS and sine-Gordon, using so-called semiclassical soliton ensembles, which are pure soliton intial data intended to approximate more general initial data in the zero-dispersion limit. We will present recent analytical and numerical results on semiclassical solitons ensembles for the three-wave resonant interaction equations. Despite the fact that these equations are non-dispersive, many of the qualitative behaviors are the same as for dispersive equations. We will also show how our results for the three-wave resonant interaction equations can be used to better understand solutions of the focusing NLS equation with compactly supported initial data.

Stability and Instability of Solitary Wave Solutions for Systems of Nonlinear Dispersive Equations

Hongqiu Chen
University of Memphis, USA
Xiaojun Wang

Considered here is a system

$$\partial_t u + \partial_x u - \partial_{xxt} u + \partial_x \partial_u H(u,v) = 0,$$

$$\partial_t u + \partial_x u - \partial_{xxt} u + \partial_x \partial_v H(u,v) = 0$$

of nonlinear dispersive equations, where $u = u(x,t)$, $v = v(x,t)$ are real-valued functions, and H is a homogeneous polynomial function of degree $p \geq 3$. We present existence of explicit solitary wave solutions. A simple algebraic condition for stability of the explicit solitary wave solution is derived. Criteria for instability of explicit solitary wave solutions are obtained as well.

The Zero Surface Tension Limit of Three-Dimensional Interfacial Darcy Flow

Shunlian Liu
Hunan University of Technology, Peoples Rep of China
David M. Ambrose

We study the zero surface tension limit of three-dimensional interfacial Darcy flow. To begin, we prove well-posedness of three-dimensional interfacial Darcy flow for any fixed, positive coefficient of surface tension. The primary tool for this well-posedness proof is an energy estimate. The time of existence for these solutions will, in general, go to zero with the surface tension parameter. However, in the case that a stability condition is satisfied by the initial data, we prove an additional energy estimate, establishing that the time of existence can be made to be uniform in the surface tension parameter. Then, an additional estimate allows the limit to be taken as surface tension vanishes, demonstrating that three-dimensional interfacial Darcy flow without surface tension is the limit of three-dimensional interfacial Darcy flow with surface tension as surface tension vanishes. This provides a new proof of existence of solutions for the problem without surface tension.

Using Exponential Asymptotics to Compute Nanopteron Behaviour and Free-Surface Waves

Christopher Lustri
Macquarie University, Australia

Many physical systems contain waves that are exponentially small in some asymptotic limit within the system. These waves are invisible to classical asymptotic power series methods, and require the application of sophisticated mathematical techniques known as exponential asymptotics. I will explain what exponential asymptotic techniques are, and how they can be used to extract and isolate these apparently inaccessible features of the physical systems. I will then outline how these techniques can be applied in order to:

1. Find nonlocal solitary waves (or nanoptera) in diatomic particle chains with nearest-neighbour interactions, using the Toda lattice as a particular example, and
2. Compute wave patterns caused by ships and submarines in the small Froude number limit (ie. when gravity dominates inertia).

An Evans Function for 2D Steady Flows of the Euler Equations on the Torus

Robert Marangell
University of Sydney, Australia
H. Dullin

This talk will consider the stability of time independent solutions to the incompressible, inviscid Euler equations on the torus whose stream functions have the form $\psi = U(\xi) = U(p_1x + p_2y)$ for fixed integers p_1 and p_2. By an appropriate change of coordinates and separation of variables, the linearised spectral problem is reduced to the study of a Hill's equation with a complex potential. By using Hill determinants, an Evans function of the original linearised Euler equation can be constructed. For certain, well-known shear flows, the form of the Hill determinant makes such an Evans function numerically straightforward to compute.

Instabilities of Two-Stratified Fluids Under Linear Shear

Katie Oliveras
Seattle University, USA

In this talk, we discuss the stability of periodic traveling wave solutions describing the interface between two fluids of varying density and vorticity trapped between to rigid lids. Using a generalization of a non-local formulation of the water wave problem due to Ablowitz, *et al.*, and Ashton

Instability of Peaked Waves in the Reduced Ostrovsky Equation

Dmitry Pelinovsky
McMaster University, Canada
Anna Geyer

Stability of the peaked periodic wave in the reduced Ostrovsky equation has remained open for long time. We have obtained sharp bounds on the exponential growth of the L^2 norm of co-periodic perturbations to the peaked periodic wave, from which it follows that the peaked periodic wave is orbitally unstable. We also prove that the peaked periodic wave with the parabolic profile is a unique weakly singular wave in the space of mean-zero periodic L^2 functions.

Existence Theory for Magma Equations in Dimension Two and Higher

Gideon Simpson
Drexel University, USA
David M. Ambrose, J. Douglas Wright, Dennis G. Yang

We examine a degenerate, dispersive, nonlinear wave equation related to the evolution of partially molten rock in dimensions two and higher. This simplified model, for a scalar field capturing the melt fraction by volume, has been studied by direct numerical simulation where it has been observed to develop stable solitary waves. In this work, we prove local in time well-posedness results for the time dependent equation, on both the whole space and the torus, for dimensions two and higher. We also prove the existence of the solitary wave solutions in dimensions two and higher.

Global Well-Posedness and Higher Sobolev Norm Bounds for Non-Focusing Schrödinger Equations on Mixed Domains

Nathan Totz
University of Miami, USA

We consider the long time well-posedness of the Cauchy problem with large Sobolev data for a class of nonlinear Schrödinger equations (NLS) on mixed flat/periodic domains of spatial dimension at least 3, and with power nonlinearities of arbitrary odd degree. Specifically, the method applies to those NLS equations having either elliptic signature with a defocusing nonlinearity, or else having an indefinite signature. We argue by contradiction that, if any scaling-subcritical Sobolev norm of a solution increases faster than a certain threshold of exponential growth, we can directly construct a perturbation of the solution that grows slower than this exponential growth rate, violating classical stability results. This establishes unconditional global well-posedness with exponential bounds on all subcritical homogeneous Sobolev norms. The perturbed NLS solution at the core of the argument is constructed as a modulational limit of a specific artificial evolution equation.

Traveling Waves in Diatomic Fermi-Pasta-Ulam-Tsingou Lattices

J. Douglas Wright
Drexel University, USA

Consider an infinite chain of masses, each connected to its nearest neighbors by a (nonlinear) spring. This is an FPUT lattice. In the instance where the masses are identical, there is a well-developed theory on the existence, dynamics and stability of solitary waves and the system has come to be one of the paradigmatic examples of a dispersive nonlinear equation. In this talk, I will discuss recent rigorous results of mine (together with T. Faver, A. Hoffman, R. Perline, A. Vainchstein and Y. Starosvetsky) on the existence of traveling waves in the setting where the masses alternate in size. In particular I will address in the limit where the mass ratio tends to zero. The problem is inherently singular and as such the existence theory becomes rather complicated. In particular, we find that the traveling waves are not true solitary waves but rather "nanopterons", which is to say, waves which asymptotic at spatial infinity to very small amplitude periodic waves. Moreover, we can only find solutions when the mass ratio lies in a certain open set. The difficulties in the problem all revolve around understanding Jost solutions of a non-local Schrödinger operator in its semi-classical limit.

Performance Analysis and Optimization of a Water Tank with Oscillating Walls for Wave Energy Harvesting

Tian-Shiang Yang
National Cheng Kung University, Taiwan
Po-Hsun Chen

In this talk we shall discuss the performance of a novel conceptual design for water-wave energy convertors. The model system in question consists of a water tank with two hinged side walls that oscillate when the water surface within the tank is subjected to a temporally periodic, spatially distributed pressure variation. Then, through transmissions, the two oscillating walls are connected to electric generators. A linearized two-dimensional potential flow problem is formulated for such a model system, and the steady periodic system response is solved for analytically. A comprehensive parameter study then is carried out, so as to clarify how the system's performance is affected by its design and operation parameters. Briefly, it is found that, in addition to a series of resonant forcing frequencies that produce large electric power output, there also exist certain anti-resonant forcing frequencies that generate zero power. Moreover, with optimally tuned parameters, the maximized electric power output of the model system studied here can be comparable with, and even higher than, that of preexisting systems of similar nominal size.

Special Session 109: Multiscale Methods for Highly Oscillatory Partial Differential Equations

Qinglin Tang, National University of Singapore, Singapore
Yongyong Cai, Beijing Computational Science Research Center, Peoples Rep of China
Weizhu Bao, National University of Singapore, Singapore

Partial differential equations with highly oscillatory solutions arise in vast areas of science and engineering, such as quantum mechanics, plasma physics and acoustics, with important applications in semiconductors and low-temperature physics. These equations bring great challenges to both mathematical analysis and numerical simulations. This session aims to bring together prominent international experts who have been active in this field to share their ideas and experiences. Emphasis is placed on reviewing, developing and promoting interdisciplinary researches on the multi-scale methods and beyond for higly oscillatory systems, including Schrödinger-type equation, Dirac equation, Klein-Gordon equation, Zakharov system, etc.

Multiscale Methods and Analysis for the Dirac Equation in the Nonrelativistic Limit Regime

Weizhu Bao
National University of Singapore, Singapore

In this talk, I will review our recent works on numerical methods and analysis for solving the Dirac equation in the nonrelativistic limit regime, involving a small dimensionless parameter which is inversely proportional to the speed of light. In this regime, the solution is highly oscillating in time and the energy becomes unbounded and indefinite, which bring significant difficulty in analysis and heavy burden in numerical computation. We begin with four frequently used finite difference time domain (FDTD) methods and the time splitting Fourier pseudospectral (TSFP) method and obtain their rigorous error estimates in the nonrelativistic limit regime by paying particularly attention to how error bounds depend explicitly on mesh size and time step as well as the small parameter. Then we consider a numerical method by using spectral method for spatial derivatives combined with an exponential wave integrator (EWI) in the Gautschi-type for temporal derivatives to discretize the Dirac equation. Rigorous error estimates show that the EWI spectral method has much better temporal resolution than the FDTD methods for the Dirac equation in the nonrelativistic limit regime. Based on a multiscale expansion of the solution, we present a multiscale time integrator Fourier pseudospectral (MTI-FP) method for the Dirac equation and establish its error bound which uniformly accurate in term of the small dimensionless parameter. Numerical results demonstrate that our error estimates are sharp and optimal. Finally, these methods and results are then extended to the nonlinear Dirac equation in the nonrelativistic limit regime. This is a joint work with Yongyong Cai, Xiaowei Jia, Qinglin Tang and Jia Yin.

Transparent Boundary Conditions for Dispersive PDEs

Christophe Besse
Université de Toulouse, France

Dispersive partial differential equation are usually set on an unbounded domain. When building a numerical scheme, it is therefore mandatory to extract a bounded computational domain and to set boundary conditions. The derivation of adapted boundary conditions is therefore of matter importance to obtain a solution that may be the restriction of the solution existing on the initial unbounded domain. We will review in this talk the various ways to obtain what is known as transparent boundary conditions and we will explain to approximate them to obtain convergent numerical schemes.

Numerical Methods for the Zakharov System

Yongyong Cai
Beijing CSRC, Peoples Rep of China
Cheng Wang

We present the commonly used numerical methods for the Zakharov system and give a construction of higher order in time numerical methods with provable stability and error estimates.

A Generic Technique for Constructing Uniformly Accurate Methods for Oscillating Evolution Equations

Philippe Chartier
INRIA, France
Mohammed Lemou, Florian Méhats

In this talk, I will present a new methodology for the numerical solution of oscillating equations considered in varying regimes (from smooth to highly-oscillatory). The strategy we propose strongly relies on averaging theory and allows to transform any oscillatory problem into a smooth one, irrespectively of the stiffness of the original equations. This is joint work with Mohammed Lemou and Florian Méhats.

On the Nonlinear Schrödinger Equation with White Noise Dispersion

Romain Duboscq
Institut de Mathematiques de Toulouse, France
Renaud Marty, Anthony Reveillac

In this talk, I will present some results on the Cauchy problem and the simulation of a randomly modulated nonlinear Schrödinger equation. This equation models the propagation of light pulses in fiber optics with a random dispersion management. In an asymptotic regime, this random modulation gives rise to the white noise dispersion which has a strong stabilizing effect and improves the well-posedness of the equation. In particular, new Strichartz estimates will be discussed. Moreover, in order to account for the small scale fluctuations of the noise, one has to rely on asymptotic preserving schemes to obtain accurate simulations. We will discuss a Lie splitting scheme which well suited for this task.

Solid-State Dewetting: Modeling and Numerics

Wei Jiang
Wuhan University, Peoples Rep of China
Quan Zhao, Weizhu Bao

In this talk, I will talk about our work about modeling solid-state dewetting problems. Taking the 2D case for example, I will explain the main idea behind these approaches; some extensions to the 3D case will be also presented. If time permits, I will talk about the recent work about using Onsager's principle to derive reduced models with applications to solid-state dewetting.

Non-Relativistic Limit of Klein-Gordon Equations and Stability Analysis in Geometric Optics

Yong Lu
Nanjing University, Peoples Rep of China
Zhifei Zhang

The Klein-Gordon equation is a relativistic form of the Schrödinger equation. In the non-relativistic limit (as the speed of light goes to infinity) of Klein-Gordon equations, one derives, at least formally, Schrödinger equations. We find a strong connection between the stability analysis in geometric optics and non-relativistic limit of Klein-Gordon equations. By employing the techniques in geometric optics, we obtain the optimal convergence rates. Moreover, for quadratic nonlinearities, we show the long time approximation of Klein-Gordon equations by Schrödinger equations in the non-relativistic limit regime. Even in the framework of geometric optics, we find that the strong transparency conditions are not satisfied. We introduce a compatible condition and a singular localization method which allows us to prove the stability of WKB solutions over long time intervals. This compatible condition is weaker than the strong transparency condition. The singular localization method allows us to do delicate analysis near resonances.

Computing Ground States of Spin 2 Bose–Einstein Condensates by the Normalized Gradient Flow

Qinglin Tang
SiChuan University, Peoples Rep of China
Weizhu Bao, Yongjun Yuan

In this paper, an efficient and accurate numerical method is proposed to compute the ground state of spin-2 Bose-Einstein condensates (BECs) by using the normalized gradient flow (NGF) or imaginary time method (ITM). The key idea is twofold. One is to find the five projection or normalization conditions that are used in the projection step of NGF/ITM, while the other one is to find a good initial data for the NGF/ITM. Based on the relations between chemical potentials and the two physical constrains given by the conservation of the total mass and magnetization, these five projection or normalization conditions can be completely and uniquely determined in the context of the back-Euler finite difference (BEFD) scheme that discretized the NGF/ITM, which allows one to successfully extend the most powerful and popular NGF/ITM to compute the ground state of spin-2 BECs. Additionally, the structures and properties of the ground states in a spatial uniform system are analysed so as to construct efficient initial data for NGF/ITM. Extensive numerical results on ground states of spin-2 BECs with ferromagnetic/nematic/cyclic interaction and harmonic/optical lattice potential in one/two dimensions are reported to show the efficiency of our method and to demonstrate some interesting physical phenomena.

Quasineutral Limit of Drift-Diffusion Models for Semiconductors and the Related Models

Shu Wang
Beijing University of Technology, Peoples Rep of China

In this talk, I will review quasineutral limit of drift-diffusion models for semiconductors and the related models. Some new results are given.

Convergence of Multi-Revolution Composition Time-Splitting Methods for Highly Oscillatory Differential Equations of Schrödinger Type

Yong Zhang
University of Vienna, Austria
Philippe Chartier, Florian Mehats, Mechthild Thalhammer

The convergence behaviour of multi-revolution composition methods combined with time-splitting methods is analysed for highly oscillatory linear differential equations of Schrödinger type. Numerical experiments illustrate and complement the theoretical investigations.

The Gaussian Wave Packets Transform for the Semi-Classical Schrödinger Equation with Vector Potentials

Zhennan Zhou
Peking University, Peoples Rep of China
Giovanni Russo

In this work, we reformulate the semi-classical Schrödinger equation in the presence of electromagnetic field by the Gaussian wave packets transform. With this approach, the highly oscillatory Schrödinger equation is equivalently transformed into another Schrödinger type wave equation, the w equation, which is essentially not oscillatory and thus requires much less computational effort. We propose two numerical methods to solve the w equation, where the Hamiltonian is either divided into the kinetic, the potential and the convection part, or into the kinetic and the potential-convection part. The convection, or the potential-convection part is treated by a semi-Lagrangian method, while the kinetic part is solved by the Fourier spectral method. The numerical methods are proved to be unconditionally stable, spectrally accurate in space and second order accurate in time, and in principle they can be extended to higher order schemes in time. Various one dimensional and multidimensional numerical tests are provided to justify the properties of the proposed methods.

Special Session 111: Nonlinear Evolution Equations

Maria Pia Gualdani, George Washington University, USA
Natasa Pavlovic, University of Texas at Austin, USA

This session will focus on common issues associated with analytical methods for dispersive and dissipative nonlinear partial differential equations, including wave and dispersive equations which have been proposed as models for many basic wave phenomena from Bose-Einstein condensation to formation of freak waves in an ocean, and kinetic equations that describe dynamics of a dilute gas or plasma, and are at the core of applied analysis, probability and statistical physics. The aim of the session is to allow exchange of ideas which could help in building an interdisciplinary perspective, including addressing derivation of these equations from microscopic complex interacting systems.

Ill-Posedness of Truncated Series Models of Water Wave

David Ambrose
Drexel University, USA

The full (irrotational Euler) equations for the motion of water waves can be difficult to implement computationally. A popular alternative is to study truncated series models, in which the Dirichlet-to-Neumann operator is expanded as a series, and an approximate system is formed by truncating this series after finitely many terms. While the full equations of motion are famously known to have a well-posed initial value problem, in joint work with Jerry Bona, David Nicholls, and Mike Siegel, we have shown that the truncated series models in fact have ill-posed initial value problems. More specifically, we identify a nonlinear backward parabolic term left in the equations after truncation which causes catastrophic growth. The work presented includes a mix of analysis and numerical results.

A Rigorous Derivation of a Boltzman-Type Cubic Equation

Ioakeim Ampatzoglou
The University of Texas at Austin, Greece
Natasa Pavlovic

In this talk we will present a rigorous derivation of a Boltzman-type cubic equation describing the motion of a classical system of particles. The equation serves as a kinetic model for a dense gas in non-equilibrium, and is for the first time derived from laws of three particle interactions, preserving momentum and energy.

Coupled Systems for Internal Wave Propagation

Jerry Bona
University of Illinois at Chicago, USA
Angel Duran, Dimitrios Mitsotakis

The discussion will feature coupled systems of evolution equations that serve as models for long-crested internal wave motion. Especial interest will be focused upon their solitary-wave solutions.

Antithesis of the Stokes Paradox on the Hyperbolic Plane

Chi Hin Chan
National Chiao Tung University, Taiwan
Magdalena Czubak

We show there exists a nontrivial H 1 solution to the steady Stokes equation on the 2D exterior domain in the hyperbolic plane. Hence we show there is no Stokes paradox in the hyperbolic setting. We also show the existence of a nontrivial solution to the steady Navier-Stokes equation in the same setting, whereas the analogous problem is open in the Euclidean case.

Higher Order Nonlinear Dispersive Equation on a Quarter Plane

Hongqiu Chen
University of Memphis, USA

The focus of the talk is the higher order nonlinear dispersive equation

$$u_t + u_x - \gamma_1 \beta u_{xxt} + \gamma_2 u_{xxx} + \delta_1 \beta^2 u_{xxxxt} + \delta_2 u_{xxxxx} + \frac{3}{4}\alpha (u^2)_x + \alpha\beta\Big(\gamma (u^2)_{xx} - \frac{7}{48} u_x^2\Big)_x - \frac{1}{8}\alpha^2 (u^3)_x = 0$$

which models unidirectional propagation of small amplitude long waves in dispersive media.
The dependent variable $u = u(x,t)$ is a real-valued function of $x \in \mathbb{R}, t \geq 0$. It represents the deviation of the free surface relative to its undisturbed state at the space point x and at time t. The subscripts connote partial derivatives while $\gamma_1, \delta_1, \alpha, \beta > 0$ and $\gamma_2, \delta_2, \gamma \in \mathbb{R}$ are modeling constants.
The specific interest of this talk is in the initial-boundary value problem where both spatial and time variables lie in $\mathbb{R}^+$, namely, quarter plane problem. With proper requirements on initial and boundary condition, we show local and global well posedness.

Ground States of Spin-1 Bose-Einstein Condensates

I-Liang Chern
National Taiwan University, Taiwan
Liren Lin, Tien-Tsan Shieh

The ultra-cold dilute boson gases have apparent macroscopic quantum ground state, called Bose-Einstein condensates (BECs). In this talk, I will report analytical results on such ground state patterns and their phase transitions of spin-1 BECs confined in a harmonic or box potential under the influence of a homogeneous magnetic field, based on a mean field model–a generalized Gross-Pitaevskii equation. First, we show necessary and sufficient condition for the existence of ground states in arbitrary dimensions. Second, we have developed a Γ-convergence theory in the semi-classical regime for antiferromagnetic systems on the whole parameter plane. In the first part, we define an effective interaction parameter and show that the ground state can exist if and only if this effective interaction is repulsive in 3D, weakly attractive or repulsive in 2D, and no condition in 1D. For the second part, the ground states and bifurcation curves are given explicitly in the Thomas-Fermi regime. Further, the limiting ground state patterns are determined by the constant mean curvature interfaces with contact angle determined by Young's relation, a generalization of classical wetting theory to the quantum cases.

On Local Well-Posedness for Boltzmann's Equation and the Boltzmann Hierarchy

Ryan Denlinger
University of Texas at Austin, USA
Natasa Pavlovic, Thomas Chen

A key problem for kinetic theorists is to rigorously derive Boltzmann's equation starting from a system of N particles (e.g., hard spheres), obeying Newton's laws, in a natural scaling limit where N tends to infinity. Lanford showed that a hard sphere gas in the so-called Boltzmann-Grad limit, subject to a molecular chaos assumption, will evolve according to Boltzmann's equation for a short time. One step in Lanford's proof involves comparing solutions of the Liouville equation against solutions of an infinite system of equations known as the Boltzmann hierarchy; this hierarchy is directly analogous to the Gross-Pitaevskii hierarchy, which appears in several well-known approaches to deriving certain dispersive equations such as cubic NLS. We will write down some toy Boltzmann hierarchies, not directly related to any known N-particle system, and discuss techniques for solving these hierarchies (existence and uniqueness) on a short time interval. The results employ Hilbert-Schmidt type norms (unusual in the literature for Boltzmann hierarchies), and the proof makes use of the (inverse) Wigner transform, space-time estimates à la Klainerman-Machedon, and combinatorial arguments (in boardgame form) originally due to Erdős-Schlein-Yau.

The Schrödinger Map Problem with Small Besov Norm

Benjamin Dodson
Johns Hopkins, USA

In this talk we discuss a recent result for the high dimensional Schrödinger map problem with small Besov norm. The argument uses bilinear Morawetz estimates.

The Isotropic Landau Equation

Maria Gualdani
George Washington University, USA
Nestor Guillen

In this talk we provide an overview of the homogeneous Landau equation and present recent results concerning regularization effect for weak solutions. We show that for moderately soft potentials the estimates only depends on the mass, energy and entropy. For general potentials, a similar regularization takes place provided an additional condition is satisfied uniformly over time. The main feature of the proofs is the analysis of the Landau operator as a Schroedinger operator whose coefficients are Ap-weights. This is a joint work with Nestor Guillen.

Growth of Sobolev Norms for the Nonlinear Schrödinger Equation Near Some Quasiperiodic Tori

Zaher Hani
Georgia Tech, USA
Marcel Guardia, Emanuele Haus, Alberto Maspero, Michela Procesi

We consider the 2D cubic nonlinear Schrödinger equation. This equation admits a family of quasiperiodic solutions coming from the complete integrability of the 1D model. We prove the existence of solutions to the 2D NLS equation that start very close to those quasiperiodic solutions (in appropriate Sobolev norms) and then become very far away in the same topology.

Discrete Schrödinger Equation

Younghun Hong
Chung-Ang University, Korea
Changhun Yang

We consider the linear and nonlinear Schrödinger equations on the lattice $h\mathbb{Z}^d$, where h is the (small) distance between two adjacent lattice points. For fixed $h > 0$, it is known that weaker Strichartz estimates are available due to the lattice resonance. In this talk, we prove uniform (in h) Strichartz estimates with some loss of derivative, which is indeed necessary. As an application, we obtain local well-posedness of the corresponding nonlinear problem with a uniform bound.

The Kinetic Fokker-Planck Equation in Bounded Domains

Juhi Jang
University of Southern California, USA

I will discuss the kinetic Fokker-Planck equation with inelastic boundary conditions in one-space dimension with focus on the structure of solutions near the singular set. The inelastic boundary conditions are characterized by a restitution coefficient r describing the amount of energy lost in the collisions of the particles with the boundaries of the domain. I will present the non-uniqueness result for r less than the critical value and the uniqueness for r greater than the critical value. The talk is based on a joint work with Hyung Ju Hwang and Juan J.L. Velázquez.

Invariant Manifolds for Stationary Boltzmann Equation and Applications

Tai-Ping Liu
Academia Sinica, Taiwan
Shih-Hsien Yu

We consider the stationary Boltzmann equation in one spatial dimension and use the Green's function approach to construct invariant manifolds. The enter manifold corresponds to fluid-like waves. Of particular interest is the coupling of Knudsen-type boundary layer with fluid-like waves. As a corollary of our analysis, it is shown that Boltzmann shock profiles are monotone. There is a striking bifurcation phenomenon on the change of wave patterns with respect to small perturbation of the flux.

Probabilistic Well-Posedness and Scattering Results for Nonlinear Wave and Schrödinger Equations on Euclidean Space

Dana Mendelson
University of Chicago, USA
B. Dodson and J. Luhrmann

We will discuss recent progress on the probabilistic local well-posedness of the nonlinear Schrödinger equation. The main ingredient in our proof is the introduction of a functional framework for the study of the associated forced cubic nonlinear Schrödinger equation, which is inspired by certain function spaces used in the study of the Schrödinger maps problem, and is based on Strichartz spaces as well as variants of local smoothing, inhomogeneous local smoothing, and maximal function spaces. We will also discuss certain probabilistic scattering results for nonlinear Schrödinger and wave equations.

Scattering for Dispersive PDE with Potentials

Jason Murphy
Missouri S&T, USA

In this talk, we will discuss recent results on scattering for dispersive PDE with external potentials.

Global Wellposedness and Scattering for the Davey-Stewartson System at Critical Regularity

Matthew Rosenzweig
University of Texas at Austin, USA

In this talk, I will discuss a two-dimensional nonlinear dispersive PDE arising in the study of water waves called the Davey-Stewartson system (DS)

$$i\partial_t u+(\sigma\partial_{x_1}^2+\partial_{x_2}^2)u=\mu|u|^2u+\beta\frac{\partial_{x_1}^2}{(\partial_{x_1}^2+\alpha\partial_{x_2}^2)}(|u|^2)u,$$

with $(t,x)\in\mathbb{R}\times\mathbb{R}^2$, $\sigma,\mu\in\{\pm1\},\alpha,\beta\in\mathbb{R}$, which for $\sigma=+1$ is formally similar to the L^2-critical cubic nonlinear Schrödinger equation (NLS) but differs by an additional nonlocal term. Specifically, I will discuss recent work on the global wellposedness and scattering for a particular case of DS with initial data in the critical L^2 space, which is inspired by Benjamin Dodson's breakthrough work on the cubic NLS. Finally, I will discuss the question of the rigorous justification of DS as a multiple scales approximation for wave packet solutions to the water waves equation.

Existence of Blow-Up Solutions in KdV-Type Equations

Svetlana Roudenko
George Washington University, USA
Luiz Farah, Justin Holmer, Kai Yang

While the KdV equation and its generalizations with higher power nonlinearities (gKdV) have been long studied, a question about existence of blow-up solutions in the critical and supercritical cases has posed lots of challenges and far from being answered. One of the main obstacles is that unlike other dispersive models such as the nonlinear Schrödinger or wave equations, the gKdV equation does not have a suitable virial quantity, which is the key for showing existence of the finite time blow-up in other models. Only at the dawn of this century the groundbreaking works of Martel and Merle lead to showing the existence of finite-time blow-up solutions in the critical gKdV equation. We consider a higher dimensional extension of the gKdV equation, called Zakharov-Kuznetsov equation (the gKdV is limited as a spatially one-dimensional model), and ask if blow-up solutions exist in the corresponding critical ZK equation. We positively answer this question for the two-dimensional critical (cubic) Zakharov-Kuznetsov equation. The new main ingredients include the Liouville-type theorem, pointwise decay estimates, virial-type quantities and understanding the corresponding spectral properties. This is a joint work with Luiz Farah, Justin Holmer and Kai Yang.

Smoothing Results for the Landau Equation

Stanley Snelson
Florida Institute of Technology, USA
Christopher Henderson, Andrei Tarfulea

The Landau equation is an integro-differential kinetic model that describes the evolution of a particle density in phase space, in a regime where grazing collisions predominate. This talk will focus on recent results that provide sufficient conditions for weak solutions to be C^∞ in all variables. First, by iteratively applying local regularity estimates, we show that solutions are smooth as long as the mass, energy, and entropy are bounded above, and the mass is bounded away from zero. Next, using a probabilistic argument, we show that the mass is always bounded below by a positive quantity, so this lower bound can be removed as a hypothesis. We will also briefly discuss the implications of these results for the existence theory of the Landau equation.

On the Relativistic Landau Equation

Maja Taskovic
University of Pennsylvania, USA
Robert M. Strain

Classical Landau equation, introduced in 1936 by Landau, models a dilute plasma in which charged particles interact via binary Coulomb collisions. When particle velocities are not small compared to the speed of light, relativistic effects play a significant role. A model that captures these effects was proposed by Bubker and Beliaev in 1956, and it is known as the relativistic Landau equation. In this talk we will describe recent results on the relativistic Landau equation. This is joint work with Robert M. Strain.

Global Flows with Invariant Measures for a Family of Almost Inviscid SQG Equations

Nathan Totz
University of Miami, USA
A. Nahmod, N. Pavlovic, G. Staffilani

With Andrea Nahmod, Natasa Pavlovic, and Gigliola Staffilani, we consider some results in constructing extremely low regularity flows for the inviscid surface quasi-geostrophic equation (SQG) in the periodic setting. Our first result is the construction of solutions globally in time almost surely for a family of modified SQG equations by constructing an invariant Gibbs measure and extracting a solution by Galerkin approximation; our method here requires some positive amount of smoothing and so does not include the inviscid case. To understand whether such smoothing is necessary or an artifact of our method, we will therefore also present results toward constructing global solutions to a variation of SQG with stochastic forcing using the theory of controlled solutions developed by M. Gubinelli and M. Jara.

Explicit Structure of the Fokker-Planck Equation with Flat Confinement

Kung-Chien Wu
National Cheng Kung University, Taiwan
Yu-Chu Linb, Haitao Wang

We study the pointwise (in the space and time variables) behavior of the Fokker-Planck Equation with flat confinement. The solution has very clear description in the xt-plane, including large time behavior and asymptotic behavior. Moreover, the structure of the solution highly depends on the potential function.

Special Session 114: Electrodiffusion and Ion Channel Problems: Modeling, Analysis, and Numerics

Tai-Chia Lin, National Taiwan University, Taiwan
Chun Liu, Penn State University, USA
Weishi Liu, University of Kansas, USA

Electrodiffusion – diffusion and migration of charged particles – plays a critical role in understanding of natures and in inventions of modern electronic devices. Ionic flow through ion channels is an important particular process of electrodiffusion that depends on many physical parameters such as channel structures (channel shape and spatial distribution of permanent charges), boundary concentrations, electric potential differences, diffusion coefficients, dielectric properties, ionic sizes, etc. It is not surprising that ionic flow exhibits extremely rich dynamics and its study is of great challenge. The special session will provide an opportunity for researchers with various backgrounds to share ideas, methods, approaches, findings on modeling, analysis, numerics, and their applications to electrodiffusion phenomena in general and ion channel properties in particular. Through this activity, it is hoped to strengthen existing collaborations and generate new collaborations on this exciting research topics.

A GPU Poisson-Fermi Solver for Ion Channel Simulations

Jen-Hao Chen
National Tsing Hua University, Taiwan
Ren-Chuen Chen, Jinn-Liang Liu

The graphic processing unit (GPU) with enormous arithmetic capability and streaming memory bandwidth is now a powerful engine for scientific as well as industrial computing. We propose two parallel GPU algorithms, one for linear solver and the other for nonlinear solver, for solving the Poisson-Fermi equation approximated by the standard finite difference method in 3D to study biological ion channels with crystallized structures from the Protein Data Bank, for example. The results show that the parallel algorithms on GPU over the sequential algorithms on CPU (central processing unit) can achieve 22.8× and 16.9× speedups for the linear solver time and total runtime, respectively.

Do Bi-Stable Steric PNP Models Describe Single Channel Gating?

Nir Gavish
Technion, Israel
Bob Eisenberg, Chun Liu

Experiments measuring currents through single protein channels show unstable currents, a phenomena called the gating of a single channel. Channels switch between an "open" state with a well defined current level and "closed" states with nearly zero current. The existing mean-field theory of ion channels focuses almost solely on the open state, while the physical modeling of the dynamical features of ion channels is still in its infancy, and does not describe the transitions between open and closed states. One hypothesis is that gating corresponds to noise-induced fast transitions between multiple steady states of the underlying system. In this work, we aim to test this hypothesis. Particularly, our study focuses on the (high order) steric Poisson-Nernst-Planck-Cahn-Hilliard model since it has been successful in predicting permeability and selectivity of ionic channels in their open state, and since it gives rise to multiple steady states. We show that this system gives rise to a gating-like behavior that does not have the defining features of gating in biological systems. Furthermore, we show that noise prohibits switching in the system of study. The above phenomena strongly suggests that one has to go beyond over-damped (gradient flow) dynamics to explain the spontaneous gating of single channels.

Poisson-Nernst-Planck Models with Singular Permanent Charges

Chia-Yu Hsieh
National Center for Theoretical Sciences, Taiwan
Yong Yu

The Poisson-Nernst-Planck (PNP) system is widely used to model charge transport in many physical and biological problems, for example, semiconductors and ion channels. In this talk, we will introduce a PNP system with singular permanent charges. When the permanent charge distribution is an ensemble of point charges in the domain, we transform the original system into a parabolic-elliptic system with weights. We will show the well-posedness of the system in some weighted Sobolev spaces.

Flux Ratios and Channel Structures Via Poisson-Nernst-Planck Systems

Shuguan Ji
Jilin University, Peoples Rep of China
Bob Eisenberg, Weishi Liu

We talk about the joint work with Bob Eisenberg (Rush Medical Center) and Weishi Liu (University of Kansas). In this work, we investigate Ussing's unidirectional fluxes and flux ratios of charged tracers motivated particularly by the insightful proposal of Hodgkin and Keynes on a relation between flux ratios and channel structure. Our study is based on analysis of quasi-one-dimensional Poisson-Nernst-Planck type models for ionic flows through membrane channels. We focus on two treatments of tracer flux measurements that serve as estimators of important properties of ion channels. The first estimator determines the flux of the main ion species from measurements of the flux of its tracer. The second treatment of tracer fluxes concerns ratios of fluxes and experimental setups that try to determine some properties of channel structure. This work is a first step showing how measurements of fluxes and flux ratios can give important insights into channel structure and function.

On the Predator-Prey Systems with Nonrandom Motion

Hai-Yang Jin
South China University of Technology, Peoples Rep of China

In this talk, we establish the global boundedness and stability of the predator-prey system with nonrandom motion in a two-dimensional bounded domain with Neumman boundary conditions. We first establish the existence of classical solution with uniform-in time bound. Then by constructing Lyapunov functionals, we establish the global stability of the prey-only steady states and coexistence steady states under certain conditions on parameters. This is a joint work with Zhi-An Wang (Polyu).

Biological Ion Channels: Theory and Simulation

Jinn-Liang Liu
Tsing Hua University, Taiwan

Ion channels are porous membrane proteins that control the flow of ions across cell membranes and play vital roles in various physiological processes in human body. Ion channel dysfunction can cause many diseases such as cardiac, neurological, renal, endocrine, and bone disorders. Single channel recordings are performed in thousands of laboratories worldwide on a daily basis using the patch clamp technique invented by the Nobel laureates Sakmann and Neher in the 1970s. However, there are relatively very few simulation tools for calculating single ion currents under physiological or experimental conditions, which can assist experimentalists in the study of normal or mutated ion channels based on the crystal structures provided by the Protein Data Bank before experimentation. The Poisson-Nernst-Planck-Fermi theory we proposed in recent years is a Continuum-Molecular theory that can be used to simulate ion currents in biological ion channels under physiological or experimental conditions. PNPF treats ions and water as non-uniform hard spheres, and accounts for interstitial voids between spheres, as well as important physical properties such as water polarization and ion correlation.

Structural Analysis of Steady States in Chemical Reaction Networks

Je-Chiang Tsai
National Tsing Hua University, Taiwan
Atsushi Mochizuki, Takashi Okada

In living sells, large numbers of reactions are connected by sharing substrates or product chemicals, forming a large and complex network. Due to the large number of reactions, it would be difficult to perform stability analysis of steady states in networks if it was not impossible. In this talk, we will present a theory, based on the structure of networks and not on the detailed information of reaction kinetics of networks, to analyze steady states of large network systems. Our approach works well for a class of systems satisfying a specific topological relation. This is a joint work with Atsushi Mochizuki (Kyoto University) and Takashi Okada (RIKEN).

Qualitative Properties of Ionic Flows Via Poisson-Nernst-Planck Models: Selectivity of Cations

Mingji Zhang
New Mexico Institute of Mining and Technology, USA

We study a quasi-one-dimensional Poisson-Nernst-Planck system for ionic flows through a membrane channel. We consider three ion species, two positively charged with the same valence and one negatively charged, and assume zero permanent charge. Bikerman's local hard-sphere potential is included to account for finite ion size effects. Under the framework of a geometric singular perturbation theory, together with specific structures of this concrete model, the existence of solutions to the boundary value problem for small ion sizes is established. Furthermore, treating the ion sizes as small parameters, we derive an approximation of individual fluxes, from which one can further study the qualitative properties of ionic flows and extract concrete information directly related to biological measurements. Of particular interest is the selectivity between two cations due to finite ion sizes for open ion channels with given protein structures. Furthermore, we are able to characterize the distinct effects of the nonlinear interplays between physical parameters, such as ion sizes, diffusion coefficients, boundary concentrations and boundary potentials.

Special Session 116: Recent Advances on Numerical Methods and Applications of Phase-Field Methods

Chuanju Xu, Xiamen University, Peoples Rep of China
Mejdi Azaiez, Bordeaux INP, France
Jie Shen, Purdue University, USA

Interfacial dynamics in complex fluids presents tremendous challenges to science. From a fluid mechanical viewpoint, the essential physics is the coupling between interfacial movement and the flow of the bulk fluids. Phase field (diffuse-interface) methods start from a multi-scale point of view and treat the interface as a microscopic transition zone of small but finite width. Then a set of governing equations can be derived that are thermodynamically consistent and mathematically well-posed. This principle is very powerful and flexible. It has been applied successfully to describe complicated interfaces in various complex fluids. Well designed numerical methods with the diffuse-interface approach can be highly robust and accurate, as long as the interface is well resolved. Phase field methods are now widely used in many branches of science and engineering, such as the material science, biomedical science, biology, chemical engineering. This mini-symposium will bring together numerical analysts and computational scientists working on phase field methods to present their recent advances in algorithm designs and applications of phase field methods. The main purposes of this mini-symposium are to review the current status, identify problems and future directions, and to promote phase field methods to a wider scientific and engineering community.

A Generalized Scalar Auxiliary Variable Approach for L^2 Gradient Flows

Mejdi Azaiez
Institut Polytechnique of Bordeaux, France
Dianming Hou, Chuanju Xu

In this paper, we propose and analyze a new generalized scalar auxiliary variable (GSAV) approach to deal with nonlinear terms in L^2 gradient flows. This new approach can be viewed as an extension of scalar auxiliary variable (introduced by J. Shen et al) without assumption that $\int_\Omega F(\phi)d\mathbf{x}$ is bounded from below. We construct an efficient and robust first and second order unconditionally energy stable GSAV schemes for gradient flows. Optimal error estimates are established. The explain how GSAV approach is not restricted to the special forms of the nonlinear terms and only requires to solve decoupled linear system with constant coefficients that can solved using any fast solver for Poisson equation. Finally several numerical experiments were carried out to verify the theoretical claims and illustrate the efficiency of our method.

Numerical Schemes for the Viscous Cahn-Hilliard-Navier-Stokes Equations with Dynamic Boundary Conditions

Laurence Cherfils
University of La Rochelle, France
M. Petcu

In this talk, we will present numerical schemes for the viscous Cahn-Hilliard-Navier-Stokes equations with two kinds of dynamic boundary conditions. Then, we will focus on the moving contact line model introduced and studied in [1]. In this case, we will discuss the unconditional solvability and the energy stability for the time discretization scheme, as well as for the fully discrete scheme. We will also give numerical simulations illustrating our results.

References

[1] C.G. Gal, M. Grasselli and A. Miranville, "Cahn-Hilliard-Navier-Stokes system with moving contact lines", Calculus of variations and Partial differential equations, Vol. 55, Issue 3, 2016

A Derivation of a Thermodynamically Consistent Phase Field Model and Its Mass Conservative and Energy Stable Finite Difference Method

Ping Lin
University of Dundee, Scotland

In this talk we briefly show the derivation of a thermodynamically consistent phase field model. Then we will show how to design a mass conservative, energy stable, finite difference method on a staggered grid for the system of the two-phase flow equations with variable density and viscosity. We also present an efficient, practical nonlinear multigrid method - comprised of a standard FAS method for the Cahn-Hilliard equation, and a method based on the Vanka-type smoothing strategy for the Navier-Stokes equation - for solving these equations. We test the scheme in the context of Capillary Waves, rising droplets and Rayleigh-Taylor instability. Quantitative comparisons are made with existing analytical solutions or previous numerical results that validate the accuracy of our numerical schemes. This is a joint work with Z.L. Guo et al.

Thermal Effects in General Diffusions

Chun Liu
Illinois Institute of Technology, USA
Pei Liu, Simo Wu

In this talk, I will present a general framework in deriving nonlinear coupled PDE systems in modeling the thermal effects for various diffusion and transport processes. The system will be consistent with the general thermodynamics laws and the energetic variational framework for the isothermal situations.

The Cahn-Hilliard Equation in Image Inpainting

Alain Miranville
University of Poitiers, France

Our aim in this talk is to discuss variants of the Cahn-Hilliard equation in view of applications to image inpainting. We will present theoretical results as well as numerical simulations

Modeling and Simulation of Drops, Bubbles and Moving Contact Lines Using the Dynamic Van Der Waals Theory

Tiezheng Qian
Hong Kong University of Science and Technology, Hong Kong

The dynamic van der Waals theory has been presented as a diffuse-interface model to describe one-component fluids with liquid-vapor coexistence and transition. In this theory, mass transport, momentum transport, and heat transport processes are coupled in the presence of latent-heat release/absorption. In this talk I will present our recent works on the modeling and simulation of drops, bubbles and moving contact lines using the dynamic van der Waals theory.

Accurate and Efficient Numerical Methods for Coupled Surface-Groundwater Flow

Xiaoming Wang
Fudan University and Florida State University, Peoples Rep of China

We present several accurate and efficient numerical methods for coupled Cahn-Hilliard-Navier-Stokes-Darcy system which is a phase-field model for two-phase flows in karstic geometry or coupled surface-groundwater flow. These algorithms are unconditionally stable and decoupled. Ample numerical results will be presented as well.

A New Phase-Field Model Using Fractional Laplacians: Algorithm and Simulations

Chuanju Xu
Xiamen University, Peoples Rep of China

We discuss a fractional mass-conserving Allen-Cahn phase-field model that describes the mixture of two incompressible fluids. The new fractional model allows controlling the sharpness of the interface, which is typically diffusive in integer-order phase-field models. The spatial discretization is based on a Galerkin spectral method whereas the temporal discretization is based on a stabilized ADI scheme. Specially we focus on how to efficiently compute the eigen-spectrum and fractional Laplacians in prototype complex-geometry domains. A number of numerical examples are provided to demonstrate the accuracy of the method and the ability to control the interface thickness between two phases.

Special Session 117: Propagation Phenomena and Nonlinear Free Boundary Problems

Yihong Du, University of New England, Australia
Bendong Lou, Shanghai Normal University, Peoples Rep of China
Maolin Zhou, University of New England, Australia

This special session is devoted to recent advances in the research on the propagation phenomena described by reaction-diffusion equations, including spreading in heterogeneous media, and spreading involving free boundaries. Specific topics covered include traveling waves, long-time dynamical behavior, spreading speed, and biological/ecological applications. We aim to bring together leading researchers working on these topics to report current research status and discuss future research directions, from both the theoretical and applied points of view.

A Logistic Model with Starvation-Driven Dispersal Under a Free Boundary

Inkyung Ahn
Korea University, Korea
Wonhyung Choi

In many cases, the movement of species within a region depends on the availability of food and other resources necessary for its survival. Starvation-driven diffusion (SDD) is a dispersal strategy that increases the motility of biological organisms in unfavorable environments i.e., a species moves more frequently in search of food if resources are insufficient (Cho and Kim, 2013). In this study, the proposed model represents the dispersion of an invasive species undergoing SDD, where the free boundary represents the expanding front. We observed that the spreading-vanishing dichotomy, which holds in the random dispersal model (Zhou and Xiao, 2013), also holds in the model undergoing SDD. We also provided the range of estimates for the spreading speed of the free boundary during the spreading process. Finally, our results were compared with the results of the random dispersal model to investigate the advantages of this strategic dispersal with respect to survival in new environments i.e., the conditions that allowed the species undergoing SDD to spread and the random diffusers vanish were monitored.

Asymptotic Behavior of Solutions of Free Boundary Problems for Fisher-KPP Equation

Jingjing Cai
Shanghai University of Electric Power, Peoples Rep of China
Hong Gu

We study a free boundary problem for Fisher-KPP equation: $u_t = u_{xx} + f(u)$ $(g(t) < x < h(t))$ with free boundary conditions $h'(t) = -u_x(t, h(t)) - \alpha$ and $g'(t) = -u_x(t, g(t)) + \beta$ for $0 < \beta < \alpha$. This problem can model the spreading of a biological or chemical species. We investigate the affects of α and β on the asymptotic behavior of bounded solutions.

The Stefan Problem for the Fisher-KPP Equation with Unbounded Initial Range

Weiwei Ding
Meiji University, Japan
Yihong Du, Zongming Guo

We consider the nonlinear Stefan problem

$$\begin{cases} u_t - d\Delta u = au - bu^2 & \text{for } x \in \Omega(t),\ t > 0, \\ u = 0 \text{ and } u_t = \mu|\nabla_x u|^2 & \text{for } x \in \partial\Omega(t),\ t > 0, \\ u(0, x) = u_0(x) & \text{for } x \in \Omega_0, \end{cases}$$

where $\Omega(0) = \Omega_0$ is an unbounded smooth domain in $\mathbb{R}^N$, $u_0 > 0$ in Ω_0 and u_0 vanishes on $\partial\Omega_0$. When Ω_0 is bounded, the long-time behavior of this problem has been rather well-understood. Here we reveal some interesting different behavior for certain unbounded Ω_0. We also give a unified approach for a weak solution theory to this kind of free boundary problems with bounded or unbounded Ω_0.

Propagation Models in Evolutionary Epidemiology

Quentin Griette
Meiji University, Japan
Matthieu Alfaro, Gaél Raoul, Sylvain Gandon

I will talk about a system of two coupled reaction-diffusion equations modeling the spread of evolving diseases. In this scenario, a pathogen propagates within a population of susceptible hosts while a fast mutation process allows its phenotype to change in the same time scale as the invasion process. I will consider a special case where only two phenotypes exists, leading to a system of two coupled KPP-type equations. I will first talk about the case of a homogeneous space, where the reaction coefficients do not depend on the space variable, and present a construction of traveling waves that allow us to characterize the propagation. Then, I will investigate the case of a periodically heterogeneous space, and show how we constructed pulsating fronts in this situation. In both cases, there is competition between the two pathogens, which we treated as a non-local term; in particular, we are not in a situation where a comparison principle is available, which is a challenging mathematical problem.

Ecological Invasion in Competition-Diffusion Systems When the Exotic Species Is Either Very Strong Or Very Weak

Danielle Hilhorst
CNRS/Univ. Paris-Sud, France
Lorenzo Contento, Masayasu Mimura

We consider a m-component competition-diffusion system, and prove the following results: If the carrying capacity of the population of density u_m tends to infinity, then all populations except for the mth-population disappear while the mth population not only remains but becomes infinitely large; if the carrying capacity of the population of density u_m is large enough, then all populations except for the mth-population disappear while the mth population not only remains but become infinitely large as time tends to infinity. If the carrying capacity of the population of density u_m tends to zero, then all populations probably remain except for the mth-population which vanishes. If the carrying capacity of the population of density u_m is small enough, and if all the other populations diffuse at an equal rate, then all populations probably remain except for the mth-population which disappears in large time.

Propagation Curves of a Curvature Flow in a Cylinder

Bendong Lou
Shanghai Normal University, Peoples Rep of China
Lixia Yuan

I will talk about the propagation curves of a (mean) curvature flow in a cylinder. In case there is no stationary curves in the cylinder, we show the existence of entire solutions propagating from $-\infty$ to ∞, connecting one (periodic) traveling wave with another; in case there are stationary solutions, we show the existence of entire solutions growing out of or tending to one stationary solution.

Spreading and Vanishing in a Free Boundary Problem for Nonlinear Diffusion Equations with a Given Forced Moving Boundary

Hiroshi Matsuzawa
National Institute of Technology, Numazu College, Japan
Yuki Kaneko

In this talk I will present the recent study on a free boundary problem of the nonlinear diffusion equations $u_t = u_{xx} + f(u)$, $t \in (0, \infty)$, $x \in (ct, h(t))$. The nonlinearity f is C^1 function satisfying $f(0) = 0$, $c > 0$ is a given constant, that is $c = ct$ is a given forced moving boundary and $h(t)$ is a free boundary which is determined by a Stefan-like condition. At left boundary $x = ct$, zero Dirichlet boundary condition is imposed. When f is logistic nonlinearity, [Matsuzawa, to appear, arXiv:1708.01995] dealt with this problem. In this talk I will present the extension of the earlier study to much more general nonlinear functions f. I will also give the The approach here is quite different from that used in my previous work.

Entire Solutions of Reaction-Diffusion Equations in Multiple Semi-Infinite Intervals with a Junction

Yoshihisa Morita
Ryukoku University, Japan
Shuichi Jimbo

In this talk we consider a bistable reaction-diffusion equation in a domain of multiple semi-infinite intervals joined at a junction. We show that there exist entire solutions exhibiting front dynamics as time goes backward.

Asymptotic Stability of Traveling Waves for Bistable Lattice Dynamical Systems of Cooperation Type

Ken-Ichi Nakamura
Kanazawa University, Japan
Jong-Shenq Guo, Toshiko Ogiwara, Chin-Chin Wu

We consider traveling front solutions connecting two stable equilibria for general cooperative lattice dynamical systems. We will study the monotonicity and (asymptotic) stability of the traveling fronts satisfying some non-decoupling condition.

Logarithmic Corrections in Fisher-KPP Problems for the Porous Medium Equation

Fernando Quiros
Universidad Autonoma de Madrid, Spain
Yihong Du, Maolin Zhou

We consider the large time behaviour of solutions to the Porous Medium Equation with a Fisher-KPP type reaction term

$$u_t = \Delta u^m + u - u^2 \quad \text{in } \mathbb{R}^N \times \mathbb{R}_+, \quad u(\cdot, 0) = u_0 \text{ in } \mathbb{R}^N,$$

$m > 1$, for nonnegative, nontrivial, radially symmetric, bounded and compactly supported initial data u_0. It is well known that in spatial dimension one there is a minimal speed $c_* > 0$ for which the equation admits a traveling wave profile Φ_{c_*} with a finite front. We prove that there exists a second constant $c^* > 0$ independent of the dimension N and the initial function u_0, such that

$$\lim_{t\to\infty} \left\{ \sup_{x\in\mathbb{R}^N} |u(x,t) - \Phi_{c_*}(|x| - c_* t + (N-1)c^* \log t - r_0)| \right\} = 0$$

for some $r_0 \in \mathbb{R}$ (depending on u_0). Moreover, the radius, $h(t)$, of the support of the solution at time t satisfies

$$\lim_{t\to\infty} \left[h(t) - c_* t + (N-1)c^* \log t\right] = r_0.$$

Thus, in contrast with the semilinear case $m = 1$, we have a logarithmic correction only for $N > 1$. If the initial function is not radially symmetric, then there exist $r_1, r_2 \in \mathbb{R}$ such that the boundary of the spatial support of the solution $u(\cdot, t)$ is contained in the spherical shell $\{x \in \mathbb{R}^N : r_1 \le |x| - c_* t + (N-1)c^* \log t \le r_2\}$ for all $t \ge 1$. Moreover, as $t \to \infty$, $u(x,t)$ converges to 1 uniformly in $\{|x| \le c_* t - (N-1)c \log t\}$ for any $c > c^*$.

Fisher-KPP Equation with Free Boundaries and Time-Periodic Advections

Ningkui Sun
Shandong Normal University, Peoples Rep of China
Bendong, Lou, Maolin, Zhou

We consider the Fisher-KPP equation with free boundary conditions and time-periodic advections. We are interested in the long time behavior of the solutions. When the advection is small, we give a vanishing-spreading dichotomy result; when the advection is medium-sized, we show a vanishing-transition-virtual spreading trichotomy result; when advection is large, we prove that vanishing happens for all solutions. Moreover, we give the estimate for the spreading speed of the spreading/virtual spreading solutions. This work is joint with Bendong Lou and Maolin Zhou.

Multidimensional Traveling Fronts in Reaction-Diffusion Equations

Masaharu Taniguchi
Okayama University, Japan

Traveling fronts to reaction-diffusion equations in the N-dimensional Euclidean space have been studied recently by many mathematicians. Here N is an integer that is larger or equals 2. If the nonlinear term is unbalanced, there exists an N-dimensional traveling front associated with an given (N-1)-dimensional convex compact set as in [T2015, SIAM J. Math. Anal., T2016, JDE]. In this talk I will review multi-dimensional traveling fronts in reaction-diffusion equations.

Spatial Propagation for a Predator-Prey Model Via a Moving Boundary Formalism

Zhiguo Wang
Shaanxi Normal University, Peoples Rep of China
Hua Nie, Yi Cao

In this talk, we will report our recent research results on a diffusive predator-prey model with a free boundary in a radially symmetric setting, which arises in a biological invasion problem. The spreading-vanishing dichotomy is obtained for this model, and sharp criteria for spreading and vanishing are given. The numerical simulations reveal that the average speed of the free boundary varies linearly over large rang, which inspires us to consider semi-wave solutions of the same system and establish the existence and uniqueness of such solution. Our result shows that the asymptotic spreading speed of invasion species in the free boundary problem is no more than the asymptotic spreading speed of the Cauchy problem associated with this system.

Asymptotic Estimates of Solutions for a Certain Class of One-Dimensional Free Boundary Problems

Yoshio Yamada
Waseda University, Japan

This talk is concerned with a free boundary problem which consists of a reaction-diffusion equation in a one-dimensional interval and a free boundary condition of Stefan type at one of boundary points of the interval. We put Neumann or Dirichlet condition at the other boundary point. When the reaction term is given by a certain class of bistable function, such a free boundary problem exhibits different types of spreading behaviors with free boundaries going to infinity as time tends to infinity. Asymptotic spreading speeds of free boundaries are different or same depending on nonlinear function. We will study the structure of spreading solutions and derive precise asymptotic estimates on their profiles.

Traveling Waves for Nonlinear Diffusion Problems with Free Boundaries in a One-Dimensional Heterogeneous Medium

Guanghui Zhang
Huazhong University of Science & Technology, Peoples Rep of China
Maolin Zhou

We consider solutions of a reaction-diffusion equation of monostable or combustion type with free boundaries. The reaction rate is assumed to be random, stationary and ergodic. We establish existence of generalized random traveling waves.

Special Session 120: New Developments in the Variational Analysis of Elastic and Complex Media

Jonathan Bevan, University of Surrey, England
Caterina Zeppieri, University of Münster, Germany
David Bourne, Durham University, England

This special session will highlight some of the recent advances in the variational modelling and analysis of elastic and complex materials. Our understanding of materials ranging from liquid crystals to elastic composites has benefited from careful study using tools from the Calculus of Variations, including PDE analysis and *Gamma*-convergence techniques, as well as Homogenisation theory. The topics of this session will be drawn from these areas and will showcase the often very fruitful interaction between analysis and materials modelling.

Modelling Steel Grains Using Optimal Transport Theory

David Bourne
Durham University, England

The steel industry is entering a new era of virtual engineering, where expensive physical experiments are replaced by crystal plasticity simulations and computational homogenization to design and test new alloys. An important part of these simulations is the generation of realistic representative volume elements (RVEs) to model the grain structure of metals. In this talk I will describe a new approach to this problem using semi-discrete optimal transport theory.

Variational Integrals with Linear Growth in Small Strain Elasticity

Miroslav Bulicek
Charles University, Czech Rep

We investigate the properties of certain elliptic systems leading, a priori, to solutions that belong to the space of Radon measures. We show that if the problem is equipped with a so-called Uhlenbeck structure, then the solution can in fact be understood as a standard weak solution, with one proviso: analogously as in the case of minimal surface equations, the attainment of the boundary value is penalized by a measure supported on (a subset of) the boundary, which, for the problems under consideration here, is the part of the boundary where a Neumann boundary condition is imposed. Finally, we will connect such elliptic systems with certain problems in elasticity theory- the limiting strain models.

Chirality Transitions in Two-Dimensional Frustrated Spin Systems

Marco Cicalese
TU Munich, Germany
Marwin Forster, Gianluca Orlando

In the last years the analysis of the properties of edge-sharing cuprates has motivated scientists to introduce and study models of frustrated spin systems in which the frustration comes from the competition between ferromagnetic and antiferromagnetic behaviour. In some cases such a frustration suggests the introduction of a chiral order parameter in terms of which phase transitions can be analyzed. In this framework we consider the atomistic-to-continuum limit of a two-dimensional Villain-type spin system leading to the emergence of chirality phase transitions and discuss the dependence of some of their geometric properties on the anisotropy of the lattice.

Gradient-Polyconvex Materials

Martin Kruzik
Czech Academy of Sciences, Czech Rep

Modern approaches to elasticity are based on the assumption that the first Piola-Kirchhoff stress tensor possesses a potential called stored energy density, W, which depends on the deformation gradient. Such materials are then called hyperelastic. If we additionally assume that external forces applied on a body are conservative, equilibrium equations of elasticity are formally Euler-Lagrange equations for minimizers of the elastic-energy functional. Existence of minimizers can be ensured if W is polyconvex, for instance. Polyconvexity also allows for physically realistic behavior of W, i.e., orientation-preservation of deformations and that $W(F) \to +\infty$ if $\det F \to 0$. Nevertheless, many materials cannot obey polyconvex stored energy density. A prominent example are e.g. shape-memory alloys. A possible solution, often found in literature, is to assume that the stored energy density depends also on the second deformation gradient and is convex in it. We show the existence of minimizers under weaker assumptions, namely, we make the energy density depend on gradients of nonlinear minors of the deformation gradients. Moreover, we outline some interesting properties of minimizers and a few applications to modeling of shape memory materials and plasticity. This talk is based on a joint work with B. Benešovà and A. Schlömerkemper (both from Würzburg).

The Effect of Forest Dislocations on the Evolution of a Phase-Field Model for Plastic Slip

Matthias Kurzke
University of Nottingham, England
Patrick W. Dondl, Stefan Wojtowitsch

We consider dynamics of a phase-field model for crystal dislocations in the large body/small Burgers vector limit. In the one-dimensional Peierls-Nabarro setting without a forest dislocation background, the limit of the gradient flows of the energies is the gradient flow of the Γ-limit, similar to related problems in ferromagnetic materials. Forest dislocations introduce an extra strange term into the Γ-limit. Although this term may speed up the evolution of the Γ-limit, we show that it does not represent an additional driving force: instead, the presence of forest dislocations introduces a wiggliness into the system that actually slows down the observed evolution.

Derivation of a Bending-Torsion Theory for Rods with Microstructural Prestrain

Stefan Neukamm
TU Dresden, Germany
Robert Bauer, Mathias Schaeffner

We consider a nonlinear elastic composite material with ε-periodic microstructure that occupies a thin cylindrical domain (in $\mathbb{R}^3$) with small thickness h. We are interested in the situation when the different phases of the composite are prestrained (i.e. the reference configuration is not a stress-free state for the individual phases). As a consequence the rod will show a non-flat equilibrium shape that depends in a non-linear and non-local way on the heterogeneity of the material and the distribution of the prestrain. By combining homogenization and dimension reduction, we derive a one-dimensional nonlinear bending-torsion theory for rods that invokes a spontaneous curvature/torsion tensor that captures the macroscopic effect of the microstructural prestrain. The spontaneous curvature/torsion tensor characterizes the equilibrium shape in the asymptotic limit $(h, \varepsilon) \downarrow 0$. It can be computed by solving linear elliptic systems.

Michell Trusses in Two Dimensions As a Gamma-Limit of Optimal Design Problems in Linear Elasticity

Heiner Olbermann
Leipzig University, Germany

We reconsider the minimization of the compliance of a two dimensional elastic body with traction boundary conditions for a given weight. It is well known how to rewrite this optimal design problem as a nonlinear variational problem. The limit of vanishing weight is taken by sending a suitable Lagrange multiplier to infinity in the variational formulation. We show that the limit, in the sense of Γ-convergence, is a certain Michell truss problem. This proves a conjecture by Kohn and Allaire.

Lower Semicontinuity of a Class of Integral Functionals on the Space of Functions of Bounded Deformation

Gianluca Orlando
Technische Universitaet Muenchen, Germany
Gianni Dal Maso, Rodica Toader

In this talk I will present a result concerning the lower semicontinuity of some free discontinuity functionals with linear growth defined on the space of functions with bounded deformation BD. The functionals in analysis feature a volume term that is convex and depends only on the Euclidean norm of the symmetrized gradient. I will introduce a suitable class of surface terms, which make the functional lower semicontinuous. The proof of the result is based on an unusual slicing argument.

Global and Local Minimizers in Liquid Crystals

Jinhae Park
Chungnam National University, South Korea, Korea

In this talk, we briefly introduce Landau-de Gennes theory for liquid crystals and consider interface problems between two different liquid crystal phases. We study profiles of global or local minimizers of the energy functional. If time permits, we plan to discuss various unsolved mathematical questions. Some part of this talk is joint work with Z. Zhang, P. Zhang and W. Wang.

A Compactness and Structure Result for a Discrete Multi-Well Problem with $SO(N)$ Symmetry in Arbitrary Dimension

Angkana Rueland
Max-Planck Institute for Mathematics in the Sciences, Germany
G. Kitavtsev, G. Lauteri, S. Luckhaus

In this talk I present a new, self-contained compactness argument for a discrete multi-well problem with $SO(n)$ symmetry of the type which typically arises in applications to phase transformations in physics. As the main ingredients I discuss a spin-type argument and a reduction of the compatible multi-well problem to an incompatible single-well problem. Relying on the formulation as an incompatible single-well problem, I then deduce compactness and an accompanying structure result. This result for instance applies to martensitic phase transformations in the surface energy regime, but also remains valid for a much more general class of phase transformation problems, in which the ground states can for instance be periodic. The talk is based on joint work with G. Kitavtsev, G. Lauteri and S. Luckhaus.

Equilibrium Measures for Nonlocal Energies: the Effect of Anisotropy

Lucia Scardia
University of Bath, England
J.A. Carrillo, J. Mateu, M.G. Mora, L. Rondi, J. Verdera

Nonlocal energies are continuum models for large systems of particles with long-range interactions. Under the assumption that the interaction potential is radially symmetric, several authors have investigated qualitative properties of energy minimisers.
Motivated by the example of dislocation interactions in materials science, we pushed the methods developed for nonlocal energies beyond the case of radially symmetric potentials, and discovered interesting connections with random matrices and fluid dynamics.

Effective Theories and Energy Minimizing Configurations for Heterogeneous Multilayers

Bernd Schmidt
Universitaet Augsburg, Germany
Miguel de Benito Delgado

We will report on recent advances in deriving effective theories for thin sheets consisting of multiple layers with (slightly) mismatching equilibria. While the regime of finite bending energy is well understood by now, the talk will focus on energy scaling regimes beyond Kirchhoff's theory leading to linearized Kirchhoff, von Kármán and linearized von Kármán functionals with a spontaneous curvature term. We will also investigate optimal energy configurations and find that the von Kármán scaling is critical for their generic shape.

The Vortex Filament Conjecture for Euler Flows

Christian Seis
University of Munster, Germany
Robert L. Jerrard

We study the evolution of vortex filaments in ideal fluids. A conjecture, dating back to da Rios in 1906, states that if the the vorticity is intially concentrated around a closed curve, it remains concentrated for some time and the evolution of the curve is geometrically described by the binormal curvature flow. In a joint work with Bob Jerrard we focus on the second part of this conjecture and derive the binormal curvature flow under a weak vorticity concentration condition. Our proof relies on estimates for the underlying Hamiltonian structures.

An Approach to Nonlinear Viscoelasticity Via Metric Gradient Flows

Yasemin Sengul
Sabanci University, Turkey
Alexander Mielke, Christoph Ortner

We formulate quasistatic nonlinear viscoelasticity of rate type as a gradient system. Our focus is on nonlinear dissipation functionals and distances that are related to metrics on weak diffeomorphisms and that ensure time-dependent frame indifference of the viscoelastic stress. In the multi-dimensional case we discuss which dissipation distances allow for the solution of the time-incremental problem. Because of the missing compactness the limit of vanishing time steps can be obtained only by proving some kind of strong convergence. We show that this is possible in the one-dimensional case by using a suitably generalized convexity in the sense of geodesic convexity of gradient flows. For a general class of distances we derive discrete evolutionary variational inequalities and are able to pass to the time-continuous limit in a specific case.

Special Session 121: Stability of Solitary Waves in Nonlinear PDEs

Dmitry Pelinovsky, McMaster University, Canada
Yusuke Shimabukuro, Institute of Mathematics, Taiwan

Nonlinear waves are of great importance in various physical applications ranging from optics to fluids, to mention a few. Nonlinear waves can be described by non-integrable and integrable PDEs. One of the most important solution of the nonlinear PDEs is a solitary wave, stability of which is a fundamental problem of analysis and applications. Many analytical methods to study stability of solitary waves have been developed and improved significantly in the past decades. In our session, we focus on stability questions for solitary waves in both non-integrable and integrable dispersive PDEs. The list of nonlinear PDEs includes the generalized derivative nonlinear Schrödinger equation, the Zakharov-Kuznetsov equation, the water wave equations, among the others.

Stability Issues for Schrödinger Models : from Quasilinear Versions to Systems

Mathieu Colin
Bordeaux INP and INRIA CARDAMOM, France
L. Di Menza, L. Jeanjean, J.C. Saut, M. Squassina

Schrödinger models can be used to modelize different physical phenomena in many situations: self-channeling of a high-power ultra short laser in matter, dissipative quantum mechanics, plasma physics and fluid mechanics, propagation in quadratic media, ... However, little is known about Cauchy problem and the question of global well-posedness is still an open problem in many cases. In this direction, solitary waves and their stability properties is of great interest. In this talk, we propose to investigate two different types of models: a quasilinear Schrödinger equation and a system of semilinear equations. Our aim is to prove the stability or instability of ground states, to exhibit the specificity of each model and to discuss some open problems. In particular, concerning solitonic structure arising in quadratic media, we will present, in the context of normal or anomalous dispersion regimes, the case of elliptic and nonelliptic systems.

Orbital Stability of Domain Walls in Coupled Gross-Pitaevskii Systems

Andres Contreras
NMSU, USA
Dmitry Pelinovsky, Michael Plum

Domain walls appear as sharp interfaces separating species in several Hamiltonian systems arising in Physics. In this talk I will describe how these structures that correspond to heteroclinic solutions to a related variational problem are orbitally stable when placed in a suitable functional space adapted to the the domain wall. This is joint work with D. Pelinovsky and M. Plum.

Strong Instability of Standing Waves for Nonlinear Schrödinger Equations with an Attractive Inverse Power Potential

Noriyoshi Fukaya
Tokyo University of Science, Japan
Masahito Ohta, Yusuke Shimabukuro

We consider strong instability of the standing wave $e^{i\omega t}\phi_\omega(x)$ for N-dimensional nonlinear Schrödinger equations with L^2-supercritical nonlinearity and an attractive inverse power potential, where ω is the frequency of the standing wave, and ϕ_ω is a ground state of the corresponding stationary equation. Recently, Ohta proved that if $\partial_\lambda^2 E(\phi_\omega^\lambda)|_{\lambda=1} \leq 0$, then the standing wave for NLS with a harmonic potential is strongly unstable, where E is the energy, and $\lambda \mapsto v^\lambda(x) := \lambda^{N/2} v(\lambda x)$ is the scaling, which does not change L^2-norm. In this talk, we prove strong instability under the same assumption as the above-mentioned in inverse power potential case. Our proof can be applicable to NLS with other potentials such as an attractive Dirac delta potential.

Scattering in the Schrödinger Equation with a Point Nonlinearity

Reika Fukuizumi
Tohoku University, Japan
Riccardo Adami, Justin Holmer

In this talk we consider the nonlinear Schrödinger equation with a point nonlinearity in 1d, where the point nonlinearity is described as a jump condition at a point in space in the linear Schrödinger equation. H^1 local well-posedness theory to this equation is available in [1], the authors in [2] established L^2 supercritical global existence, and blow-up dichotomy. We address in this talk a L^2 supercritical scattering result applying the Kenig-Merle method [3], but it is required to use an appropriate function space according to the smoothing properties of the corresponding Duhamel form, which is in fact independent of the space variable.

References

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[3] C. Kenig and F. Merle, *Global well-posedness, scattering, and blow-up for the energy-critical focusing nonlinear Schrödinger equation in the radial case,* Invent. Math. **166** (2006) no.3 645-675

Stability of Topological Solitons of 2D Landau-Lifshitz Equations

Stephen Gustafson
University of British Columbia, Canada

Landau-Lifshitz equations are the basic dynamical equations in a micromagnetic description of a ferromagnet. They are naturally viewed as geometric evolution PDE of dispersive ("Schrödinger map") or mixed dispersive-diffusive type, which scale critically with respect to the physical energy in two dimensions. We describe some results on existence and stability of important topological soliton solutions known as "chiral magnetic skyrmions". This includes joint work with Li Wang.

Global Existence of Solutions for the Derivative Nonlinear Schrödinger Equation

Masayuki Hayashi
Waseda University, Japan

We give a sufficient condition for global existence of the solutions to the derivative nonlinear Schrödinger equation (DNLS) by a variational argument. Our approach can be also applied to the generalized derivative nonlinear Schrödinger equation.

Orbital Stability of Solitary Waves of Derivative Nonlinear Schrödinger Equations

Soonsik Kwon
Korea Advanced Institute of Science and Technology, Korea
Yifei Wu

We show the orbital stability of solitons arising in the cubic derivative nonlinear Schrödinger equations. We consider the endpoint case the the gauge transform has zero mass. As opposed to other cases, this case enjoys L^2 scaling invariance. So we expect the orbital stability in the sense up to scaling symmetry, in addition to spatial and phase translations. For the proof, we are based on the variational argument and extend a similar argument that was used for the proof of global existence for solutions with mass less than 4π. Moreover, we also show a self-similar type blow up criteria with critical mass 4π. This is a joint work with Yifei Wu.

On Scattering for NLS-ODE Model Having Metastable Solution

Masaya Maeda
Chiba University, Japan
Scipio Cuccagna

In this talk, we consider a Hamiltonian system combining a nonlinear Schrödinger equation (NLS) coupled to an ordinary differential equation (ODE). This system is a simplified model of the NLS around soliton solutions. Following Nakanishi, we show scattering of L^2 small H^1 radial solutions. The proof is based on Nakanishi's framework and Fermi Golden Rule estimates on L^4 in time norms.

Remarks on Strong Instability of Standing Waves for Nonlinear Schrödinger Equations

Masahito Ohta
Tokyo University of Science, Japan

We study the instability of standing wave solutions $e^{i\omega t}\phi_\omega(x)$ for some nonlinear Schrödinger equations (NLS) with or without potentials, where ω is a real parameter, and ϕ_ω is a ground state of the corresponding stationary problem. We first review some results on strong instability by blowup of standing wave solutions for NLS with double power nonlinearity (M. Ohta and T. Yamaguchi, SUT J. Math. 51 (2015), 49–58), for NLS with an attractive delta function potential in one space dimension (M. Ohta and T. Yamaguchi, RIMS Kokyuroku Bessatsu B56 (2016), 79–92), and for NLS with a harmonic potential (M. Ohta, Funkcial. Ekvac. 61 (2018), 135–143). Then, we introduce our recent developments in this direction.

Asymptotic Stability of Solitons in the Massive Thirring Model

Aaron Saalmann
University of Cologne, Germany

Very recently, inverse scattering for the massive Thirring model was developed in a rigorous setting. Thanks to that innovation, new proof possibilities for long-time behavior open up. In this talk, we see how the steepest descent method for oscillatory Riemann-Hilbert problems presented by P. Deift and X. Zhou in 1993 can be applied in order to prove dispersion for pure radiation solutions. This method improves earlier results on this subject and is readily extended to prove the soliton conjecture for the massive Thirring model and in particular, we can show stability of (multi-) solitons.

Ground States of Second Order PDEs with Mixed Power Non-Linearities

Atanas Stefanov
University of Kansas, USA

We consider the ground states for Schroedinger equation with mixed power non-linearities in any spatial dimension. The question for the stability of these waves, in spatial dimension one, is well-understood and depends on a sign of a (hard) integral. In higher spatial dimensions however, there are few results, even for two different powers, as the equation suffers a loss of homogeneity. We provide a variational construction of some of these waves (note that we even allow focusing and defocusing powers). As a consequence, we show their spectral stability and in the case of two powers, their orbital stability as well.

Stability of Periodic Waves of 1D Cubic Schroedinger Equations

Tai-Peng Tsai
University of British Columbia, Canada
Stephen Gustafson, Stefan Le Coz

We consider the stability of periodic waves of the 1D cubic Schroedinger equations. The profiles of these periodic waves are rescaled Jacobi elliptic functions sn, cn and dn with parameter 0.

Instability of the Solitary Wave Solutions for Some Dispersive Equations

Yi Wu
Tianjin University, Peoples Rep of China

In this talk, we discuss the instability of the solitary wave solutions for some dispersive equations, which include the nonlinear Klein-Gordon equations, generalized Boussinesq equations and the derivative nonlinear Schrödinger equations. In particular, we discuss the instability of the solitary wave solutions in the degenerated case.

Stability for Line Solitary Waves of Zakharov–Kuznetsov Equation

Yohei Yamazaki
Hiroshima University, Japan

We consider the stability for line solitary waves of the two dimensional Zakharov–Kuznetsov equation on cylindrical spaces which is one of a high dimensional generalization of Korteweg–de Vries equation. The orbital and asymptotic stability of the one soliton of Korteweg–de Vries equation on the energy space was proved by Benjamin, Pego–Weinstein and Martel–Merle. We regard the one soliton of Korteweg–de Vries equation as a line solitary wave of Zakharov–Kuznetsov equation on cylindrical spaces. In this talk, we talk about the orbital and asymptotic stability and the transverse instability of the line solitary waves of Zakharov–Kuznetsov equation.

Bifurcation of Relative Equilibria in Infinite-Dimensional Hamiltonian Systems

Shotaro Yamazoe
Kyoto University, Japan
Kazuyuki Yagasaki

Bifurcations of relative equilibria in perturbed infinite- dimensional Hamiltonian systems are studied. We assume that the unperturbed systems have symmetries and some of them are broken by the perturbations. Using the Lyapunov-Schmidt method, we detect saddle-node and pitchfork bifurcations along with the linear stability of bifurcated relative equilibria. Our theory is illustrated for solitary waves of the nonlinear Schrödinger equations and the theoretical results are demonstrated with the numerical ones.

Special Session 122: Partial Differential Equations Encircling Geometric Structures : Riemannian geometry (Ricci and Scalar Curvature), CR Geometry and Complex Geometry

Man Chun Leung, National University of Singapore, Singapore
Pak Tung Ho, Sogang University, Korea
Xingwang Xu, National University of Singapore/Nanjing University, Singapore

Partial differential equations associated with Riemannian geometry (Ricci and scalar curvature), CR manifolds, complex geometry, mean curvature (hypersurfaces), and applications to physics.

S^1-Equivariant Index Theorems and Morse Inequalities on Complex Manifolds with Boundary

Rung-Tzung Huang
National Central University, Taiwan
Chin-Yu Hsiao, Xiaoshan Li, Guokuan Shao

Let M be a complex manifold of dimension n with smooth connected boundary X. Assume that $\overline{M}$ admits a holomorphic S^1-action preserving the boundary X and the S^1-action is transversal and CR on X. We show that the $\bar{\partial}$-Neumann Laplacian on M is transversally elliptic and as a consequence, the m-th Fourier component of the q-th Dolbeault cohomology group $H^q_m(\overline{M})$ is finite dimensional, for every $m \in \mathbb{Z}$ and every $q = 0, 1, \cdots, n$. This enables us to define $\sum_{j=0}^{n}(-1)^j \dim H^j_m(\overline{M})$ the m-th Fourier component of the Euler characteristic on M and to study large m-behavior of $H^q_m(\overline{M})$. In this talk, we will present an index formula for $\sum_{j=0}^{n}(-1)^j \dim H^j_m(\overline{M})$ and Morse inequalities for $H^q_m(\overline{M})$. This is based on a joint work with Chin-Yu Hsiao, Xiaoshan Li and Guokuan Shao.

Self-Dual Einstein ACH Metric and CR GJMS Operators in Dimension Three

Taiji Marugame
Academia Sinica, Taiwan

Let M be a three dimensional strictly pseudoconvex CR manifold. By refining Matsumoto's construction, we construct a one parameter family of ACH metrics g^{λ}_{IJ} ($\lambda \in \mathbb{R}$) on $M \times [0, \infty)$, which solve the Einstein equation to infinite order. When $\lambda = 0$, the metric g^0_{IJ} is also self-dual to infinite order. As an application, we give another proof of the fact that a three dimensional CR manifold admits CR invariant powers of the sublaplacian of all orders, which has been shown by Gover–Graham.

On Local Stabilities of P-Kahler Structures

Sheng Rao
Wuhan University, Peoples Rep of China
Xueyuan Wan, Quanting Zhao

By use of a natural extension map and a power series method, we obtain a local stability theorem for p-Kahler structures with the (p,p+1)-th mild ddbar-lemma under small differentiable deformations.

Evolution of Scalar Curvature Flow on S^N to a Prescribed Sign-Changing Function

Hong Zhang
University of Science and Technology of China, Peoples Rep of China

We employ the well-known scalar curvature flow to study the problem of prescribing scalar curvature on n-sphere. Assume that the prescribed function f, which is allowed to change sign, satisfies certain kind of Morse index counting condition or symmetry condition. We then prove that the scalar curvature flow exists for all time and converges, for a suitable time sequence, to a conformal metric having f as its scalar curvature. As direct consequences, various existence theorems can be derived for the prescribed scalar curvature problem.

Conformal Scalar Curvature Equation on S^N : Functions with Twin Pseudo-Peaks

Feng Zhou
Nankai University, Peoples Rep of China
Man Chun Leung

In this talk we present existence result for the conformal scalar curvature equation on S^n with $n \geq 3$. Our result is based on the Lyapunov-Schmidt reduction method without perturbation and the construction of the prescribed function (after being projected to R^n) with twin pseudo-peaks in the sense that the two close critical points have the same positive value, equal flatness, and exhibit maximal behavior in certain directions. The process relies on a balance between the two main contributions to the reduced functional, one from the critical points and the other from the interaction of the two bubbles. This is a joint work with Professor Man Chun Leung.

Special Session 123: Asymptotic Theory in Probability and Statistical Physics

Qi-Man Shao, The Chinese University of Hong Kong, Hong Kong
Wei-Kuo Chen, University of Minnesota, USA

Some Recent Progress in Mean Field Spin Glasses

Antonio Auffinger
Northwestern University, USA
Wei-Kuo Chen, Aukosh Jagannath, Qiang Zeng

In this talk, I will survey the progress in mean field spin glasses in the past couple of years. I will focus on properties of the functional order parameter for the SK model, the TAP equations and their connections to complexity, dynamics and optimization theory. The talk contains results obtained in joint works with Wei-Kuo Chen, Aukosh Jagannath and Qiang Zeng.

Critical Two-Point Function for Long-Range Self-Avoiding Walks with Power-Law Couplings: the Marginal Case for $d \geq 4$

Lung-Chi Chen
National Cheng-Chi University, Taiwan
Shu-Chiuan Chang

Consider the long-range self-avoiding walks on $\mathbb{Z}^d$, whose one step distribution $D(x)$ decays as $|x|^{-d-\alpha}$ for some $\alpha > 0$. In our previous work (2015), we have shown that, for $\alpha \neq 2$, the critical two-point function $G_{p_c}(x)$ decays as $|x|^{\alpha\wedge 2-d}$ above the upper-critical dimension $d_c := 2(\alpha \wedge 2)$. In this talk, we show that $G_{p_c}(x)$ for $\alpha = 2$ decays as$|x|^{2-d}/\log|x|$ whenever $d \geq d_c$ (including equality). This solves the conjecture in (2015), extended all the way down to $d = d_c$, and confirms a part of predictions in physics (2014). The proof relies on the lace expansion and new convolution bounds on power functions with log corrections.

Phase Transition in the Spiked Random Tensors

Wei-Kuo Chen
University of Minnesota, USA

The problem of detecting a deformation in a symmetric Gaussian random tensor is concerned about whether there exists a statistical hypothesis test that can reliably distinguish a low-rank random spike from the noise. Recently Lesieur et al. (2017) proved that there exists a critical threshold so that when the signal-to-noise ratio exceeds this critical value, one can distinguish the spiked and unspiked tensors and weakly recover the spike via the minimal mean-square-error method. In this talk, we will show that in the case of the rank-one spike with Rademacher prior, this critical value strictly separates the distinguishability and indistinguishability of the two tensors under the total variation distance. Our approach is based on a subtle analysis of the high temperature behavior of the pure p-spin model, arising initially from the field of spin glasses. In particular, the signal-to-noise criticality is identified as the critical temperature, distinguishing the high and low temperature behavior, of the pure p-spin model.

Limiting Mean-Field Diffusions in Spatial Death-Birth Models

Yu-Ting Chen
University of Tennessee, USA

The Moran process is the canonical death-birth probability model in population genetics and impacts the development of Markov processes for decades. The model allows for celebrated diffusion limits as the Wright-Fisher diffusions or Fleming-Viot processes with rich mathematical structures, and so do its spatial generalizations, usually known as the voter models, in the context of integer lattices, which lead to super-Brownian motions. In this talk, I will discuss mean-field diffusions in certain asymmetric generalizations of the voter models on general large finite sets. These models are considered in a pathbreaking work of Ohtsuki et al. (2006) in evolutionary game theory. There, a physics method was successfully applied to obtain the diffusion limits as one-dimensional Wright-Fisher diffusions with fully explicit coefficients. I will report on recent mathematical results for the corresponding prediction by Ohtsuki et al. This talk is in part based on joint work with J. Theodore Cox (Syracuse).

Self-Normalized Cramer Type Moderate Deviations for Martingales

Xiequan Fan
Tianjin University, Peoples Rep of China
Ion Grama, Quansheng Liu, Qi-Man Shao

Let $(X_i, \mathcal{F}_i)_{i\geq 1}$ be a sequence of martingale differences. Set $S_n = \sum_{i=1}^n X_i$ and $[S]_n = \sum_{i=1}^n X_i^2$. We prove a Cramer type moderate deviation expansion for $\mathbf{P}(S_n/\sqrt{[S]_n} \geq x)$ as $n \to +\infty$. Our results partly extend the earlier work of Jing, Shao and Wang (2003, Ann. Probab.) for independent random variables.

Scaling Limits for Wiener Sausages in Random Environments

Chien-Hao Huang
National Taiwan University, Taiwan

A d-dimensional random walk gathers random energy on lattice. The walk gains energy only on their visit to the site. In the continuum and weak disorder regime, the partition function of our model as a random variable converges weakly to a Wiener Chaos expansion for dimension less than 4.

Moments of the (2+1)-Dimensional Directed Polymer in the Critical Window

Rongfeng Sun
National University of Singapore, Singapore
F. Caravenna, N. Zygouras

Recently, we have shown that the partition function of the directed polymer model on Z^{2+1} admits a phase transition in a suitable continuum and weak disorder limit. In particular, the partition function converges in law to a log-normal distribution below the critical point, and converges to 0 at and above the critical point. Here we focus on a suitable window around the critical point, and we prove that the space-averaged point-to-plane partition function has a uniformly bounded third moment. As a consequence, when interpreted as a random measure on R^2, the rescaled point-to-plane partition functions have non-trivial limit points, and each limit point has the same explicit covariance structure.

A Malliavin-Stein Approach for Multivariate Approximations in Wasserstein Distance

Lihu Xu
University of Macau, Peoples Rep of China
Xiao Fang, Qi-Man Shao

Stein's method has been widely used for probability approximations. However, in the multi-dimensional setting, most of the results are for multivariate normal approximation or for test functions with bounded second or higher order derivatives. For a class of multivariate limiting distributions, we use Bismut's formula in Malliavin calculus to control the derivatives of Stein equation solutions by the first derivative of the test function. Combining with Stein's exchangeable pair approach, we obtain a general theorem for multivariate approximations with near optimal error bounds on the Wasserstein distance. This is a joint work with Xiao Fang and Qi-Man Shao.

Special Session 125: Theoretical and Numerical Advances in Classical and Geophysical Fluid Dynamics

Qingshan Chen, Clemson University, USA
Ming-Cheng Shiue, National Chiao Tung University, Taiwan

Complex fluid phenomena in engineering or geosciences typically involve a wide range of temporal and spatial scales, and these scales interact with each other in a highly nonlinear fashion. A proper understanding of these phenomena is only possible through a combination of theoretical, numerical, and/or experimental techniques. First, there is the fundamental issue of well-posedness of the mathematical models for these phenomena. The theoretical analysis of these models often leans on the asymptotic technique to isolate the predominant scale from the rest of the system. For the nonlinear systems governing the fluid flows, analytic solutions are almost certainly off the table, and thus the numerical approximation and the analysis of the numerical schemes are of vital importance for both theoretical and practical reasons. This session brings together experts from areas such as theoretical PDEs, fluid dynamics, geosciences, scientific computing, etc, to exchange ideas and share experiences in tackling these hard problems.

On the Well-Posedness of Inviscid Quasi-Geostrophic Equations of Large-Scale Geophysical Flows

Qingshan Chen
Clemson University, USA

When the length scale of the flow is on the same order as the Rossby deformation radius, which is often the case in the interior of the flow, the classical rigid-lid assumption is no longer valid, the impact of the free surface/interface deformations on the the vorticity field is no longer negligible, and therefore it has to be accounted for in the model. In this talk, we present some new results concerning the well-posedness of the barotropic quasi-geostrophic equation and the multi-layer QG equations, where the top surface, and the layer interfaces for the multi-layer QG, are left free to evolve. It will be shown that, when the free surface/interfaces are included as components of the potential vorticity, the models remain globally well-posed, under certain generic assumptions on the initial state and the boundary of the domain.

Center Manifold Computations and Application to Fluid Flows in Cylindrical Domains

Yu-Min Chung
University of North Carolina at Greensboro, USA
Emily Schaal

A center manifold, if existed, is useful tool to study long time behavior of a dynamical system. It contains interesting dynamics, such as periodic orbits, and limit cycles. A main application of the center manifold theory is the dimension reduction, also known as center manifold reduction. However, finding the representation of the center manifold is a challenging problem numerically. In this work, we develop algorithms based on a general framework to compute the center manifold. As an application, we apply our methods to a quasilinear elliptic equation in cylindrical domains, which models a steady stratified fluid flow.

Boundary Layers for the Navier-Stokes Equations Linearized Around a Stationary Euler Flow

Gung-Min Gie
University of Louisville, USA

We study the viscous boundary layer that forms at small viscosity near a rigid wall for the solution to the Navier-Stokes equations (NSE) linearized around a smooth and stationary Euler flow in a 3D smooth bounded domain under no-slip boundary conditions. The linearized NSE are supplemented with smooth initial data and smooth external forcing, assumed ill-prepared, that is, not compatible with the no-slip boundary condition. We construct an approximate solution to the linearized NSE obtained via an asymptotic expansion in the viscosity parameter, such that the difference between the linearized Navier-Stokes solution and the proposed expansion vanishes as the viscosity tends to zero in the energy norm uniformly in time. The zero-viscosity limit for the linearized NSE then naturally follows from the validity of this asymptotic expansion. This is a joint work with James Kelliher and Anna Mazzucato.

Stochastic Models for Turbulent Convection

Nathan Glatt-Holtz
Tulane University, USA

Buoyancy driven convection plays a fundamental role in diverse physical settings: from cloud formation to large scale oceanic and atmospheric circulation processes to the internal dynamics of planets and stars. Typically, such fluid systems are driven by heat fluxes acting both through boundaries (i.e. heating from below) and in the bulk (i.e. internal volumetric heating sources) both of which can have an essentially stochastic nature. In this talk I will discuss some recent mathematical developments concerning ergodicity, singular parameter limits and the onset of instability in the stochastic Boussinesq and Magnetohydrodynamics equations.

High-Order Semi-Lagrangian Method for Incompressible Navier-Stokes Equations

Daniel Guo
University of North Carolina Wilmington, USA

In this talk, we focused on the incompressible Navier-Stokes equations with boundary and initial values. The semi-Lagrangian idea with Runge-Kutta method was incorporated with fractional method to obtain the high-order explicit methods for the Navier-Stokes equations. It was one step forward method. The numerical analysis was done based on the first order method. The numerical experiment showed that this high-order method worked effectively.

The Role of Rotation in the Primitive Equations: the Barotropic Mode

Makram Hamouda
Imam Abdulrahman Bin Faisal University, Saudi Arabia
Daozhi Han, Chang-Yeol Jung, Roger Temam

It has been shown that the rotation plays an important role in the study of the Navier-Stokes Equations (NSE); see e.g. the works of Y. Giga et al. Derived from the full NSE, the Primitive Equations seem not to counter this affirmation although the differences arising in the two sets of equations. In this talk, we investigate the influence of the rotating term in the behavior of a fluid near the boundary. Such a phenomenon is called the boundary layer of which the study is of great importance both numerically and theoretically. That's what we intend to do for one mode in the Primitive Equations, namely the Barotropic mode.

Decoupled, Energy-Law Preserving Numerical Schemes for Cahn-Hilliard-Darcy Equations

Daozhi Han
Missouri University of Science and Technology, USA
Xiaoming Wang

In this talk we discuss the construction of decoupled, energy law preserving numerical schemes for solving the Cahn-Hilliard-Darcy equations modeling two-phase flows in porous medium. Both first-order and second-order (in time) numerical methods will be presented. Numerical results including Saffman-Taylor instability will be demonstrated.

Conservative Explicit Local Time-Stepping Schemes for the Shallow Water Equations

Lili Ju
University of South Carolina, USA
Thi-Thao-Phuong Hoang, Wei Leng, Zhu Wang, Konstantin Pieper

In this talk we present explicit time-stepping schemes up to third-order accuracy for the shallow water equations, in which different time step sizes are used in different regions of the domain and are only restricted by respective local CFL conditions. The system is discretized in space by a C-grid staggering method, namely the TRiSK scheme (Ringler et al. 2010) adopted by MPAS-Ocean, a global ocean model with the capability of resolving multiple resolutions within a single simulation. The proposed local time-stepping schemes preserve all important properties of the spatial discretization, such as exact conservation of the mass and potential vorticity and conservation of the total energy within time-truncation errors. Numerical tests are presented to illustrate the performance of the proposed algorithms.

New Time Differencing Methods for Stiff Problems and Applications

Chang-Yeol Jung
UNIST, Korea
Thien Binh Nguyen

A new semi-analytical time differencing is applied to spectral methods for partial differential equations which involve higher spatial derivatives. The basic idea is approximating analytically the stiffness (fast part) by the so-called correctors and numerically the non- stiffness (slow part) by the integrating factor (IF) and exponential time differencing (ETD) methods. It turns out that rapid decay and rapid oscillatory modes in the spectral methods are well approximated by our corrector methods, which in turn provides better accuracy in the numerical schemes presented in the text. We investigate some nonlinear problems with a quadratic nonlinear term, which makes all Fourier modes interact with each other. We construct the correctors recursively to accurately capture the stiffness in the mode interactions. Polynomial or other types of nonlinear interactions can be tackled in a similar fashion.

Time-Stepping Numerical Schemes for Three-Dimensional Viscous Primitive Equations

Ming-Cheng Shiue
National Chiao Tung University, Taiwan
Chun-Hsuing Hsia

In this talk, three-dimensional viscous primitive equations which govern equations of the motion of flows in geophysical fluid dynamics are considered. The global in time strong solution for the model has been established by Cao and Titi and Kobelkov independently. In this talk, several time-stepping numerical schemes have been studied. We are interested in the study and discussion of the long-time stability of these numerical schemes, which is important for the study of the climate. In this talk, we will introduce the primitive equations, present the semi-discretized schemes and analyze and discuss the long time stability of the numerical scheme.

Long-Time Stability of a Regularized Family of Models for Homogeneous Incompressible Two-Phase Flows

Florentina Tone
University of West Florida, USA
T. Tachim Medjo, C. Tone

In this talk we present results on the stability of the fully implicit Euler scheme for a regularized family of models for an incompressible two-phase flow model. More precisely, we consider the time discretisation scheme and with the aid of the discrete Gronwall lemma and of the discrete uniform Gronwall lemma we prove that the numerical scheme is stable.

Navier-Stokes Equations on the Beta Plane: Degrees of Freedom

Djoko Wirosoetisno
Durham University, UK, England
Naoko Miyajima

We revisit the Navier-Stokes equations on a (differentially rotating) β plane and present bounds on the number of determining modes and nodes of the flow.

Special Session 126: Ergodic Theory and Dynamical Systems

Scott Kaschner, Butler University, USA
Tamara Kucherenko, City University of New York - City College, USA
Hiroki Sumi, Kyoto University, Japan

The theory of dynamical systems describes the changes that occur in real physical as well as man-made systems over time. Examples of such systems include e.g. planetary motions, weather, crystal growth, stock prices, and traffic jams. These phenomena can be unified conceptually in the mathematical notion of a dynamical system. To study dynamical systems at a sophisticated level, ergodic theory has emerged as an important tool. Historically, ergodic theory dealt primarily with averaging problems that appeared to be of largely technical nature and addressing general qualitative questions. By now, however, this area has become a powerful combination of methods that can be leveraged for the analysis of deeper statistical properties of dynamical systems. The aim of this session is to bring together scientists including the young researchers to discuss and exchange ideas in the areas of Ergodic Theory and Dynamical Systems.

Large Deviation Principle for Unimodal Maps with Flat Critical Point

Yong Moo Chung
Hiroshima University, Japan

We consider the stochastic property of the dynamical system given by a unimodal map with flat critical point. Under suitable assumptions we establish the large deviation principle of level 2 and give a characterization of the zeros of the corresponding rate function. This talk is based on joint work with Hiroki Takahasi.

The Amazing Universal Fatou Component

Mark Comerford
University of Rhode Island, USA
Christopher Staniszewski

The possibilities for limit functions on a Fatou component for the iteration of a single polynomial or rational function are well understood and quite restricted. In non-autonomous iteration, where one considers compositions of arbitrary polynomials with suitably bounded degrees and coefficients, one should observe a far greater range of behaviour. We show this is indeed the case and we exhibit a bounded sequence of quadratic polynomials which has a bounded Fatou component on which one obtains as limit functions every member of the classical Schlicht family of normalized univalent functions on the unit disc. The main idea behind this is to make use of dynamics on Siegel discs where high iterates of a single polynomial with a Siegel disc U approximate the identity closely on compact subsets of U. Do almost nothing and you can do almost anything!

Asymptotic Properties and Perturbations of Markov Operators on Based Norm Spaces

Nazife Erkursun Ozcan
Hacettepe University, Turkey
Farrukh Mukhamedov, United Arab Emirates University

It is well-known that the transition probabilities $P(x, A)$ of Markov processes naturally define a linear operator by $Tf(x) = \int f(y)P(x, dy)$, which is called *Markov operator* and acts on L^1-spaces. The study of the entire process can be reduced to the study of the limit behavior of the corresponding Markov operator. When we look at quantum analogous of Markov processes, it naturally appear in various directions of quantum physics. In these studies it is important to elaborate with associated quantum dynamical systems which eventually converge to a set of stationary states. From the mathematical point of view, ergodic properties of quantum Markov operators were investigated by many authors. Since the study of several properties of physical and probabilistic processes in abstract framework is convenient and important, some applications of this scheme in quantum information have been discussed We emphasize that the classical and quantum cases confine to this scheme. We point out that in this abstract scheme one considers a based normed spaces and mappings of these spaces. In this talk, in this setting certain ergodic properties of Markov operators are considered and investigated. Also the question about the sensitivity of stationary states and perturbations of the Markov chain are explored well.

On-Off Intermittency and Chaotic Walks

Ale Jan Homburg
University of Amsterdam, Netherlands
Vahatra Rabodonandrianandraina

We consider a class of skew product maps of interval diffeomorphisms over the doubling map. The interval maps fix the end points of the interval. It is assumed that the system has zero fiber Lyapunov exponent at one endpoint and zero or positive fiber Lyapunov exponent at the other endpoint. We discuss the appearance of on-off intermittency. A main ingredient is the equivalent description in terms of chaotic walks: random walks driven by the doubling map.

One-Dimensional Random Maps with Sigma-Finite Invariant Measures

Tomoki Inoue
Ehime University, Japan

We consider a family of transformations and study a random dynamical system such that one transformation is randomly selected from the family and then applied on each iteration. For this kind of random dynamical systems, we study the first return random map and make clear the relation between an invariant measure of the first return random map and it of the original random map. We apply our result to the random iterations of one-dimensional maps with indifferent fixed points and study invariant measures for these random maps. In this talk, we give a result on the existence of absolutely continuous sigma-finite invariant measures. After that, we give a condition such that the sigma-finite invariant measure of a random map is a finite measure. We also give a condition such that the sigma-finite invariant measure of it is an infinite measure.

Values of Hausdorff Measure and Packing Measure of the Limit Sets of Infinite Conformal IFSs Related to Complex Continued Fractions

Kanji Inui
Kyoto University, Japan
Hiroki Sumi, Hikaru Okada

Many famous fractal sets (for example, Cantor set, Sierpinski gasket and so on) are defined as the limit sets of contractive iterated function systems (for short IFSs) with finitely many mappings. But, recently D. Mauldin and M. Urbanski studied limit sets of conformal IFSs (for short CIFSs) with infinitely many mappings. And, they showed that there exists a CIFS such that the Hausdorff measure of the limit set corresponding to the Hausdorff dimension is zero and the packing measure of the limit set corresponding to the same dimension is positive. Note that the limit sets of CIFSs with finitely many mappings do not have the above properties. In this talk, we introduce an analytic family of CIFSs with infinitely many mappings related to complex continued fractions such that the limit set of each system in the family has the above strange properties and such that the Hausdorff dimension of the limit set is a real analytic and subharmonic function of the parameter. This study is a joint work with Hiroki Sumi (Kyoto University) and Hikaru Okada (Osaka University).

Monte Carlo and Quasi Monte Carlo Approach to Ulam's Method for Position Dependent Random Maps

Md Shafiqul Islam
University of Prince Edward Island, PE, Canada

Let

$$T = \{\tau_1(x), \tau_2(x), \ldots, \tau_K(x); p_1(x), p_2(x), \ldots, p_K(x)\}$$

be a position dependent random map on $I = [0, 1]$ which posses a unique absolutely continuous invariant measure (acim) μ^* with density function f^*. In this paper, first, we present a general numerical algorithm for the approximation of the density function f^*. Then, we show that Ulam's method can be derived as a special case of the general method. Finally, we describe a Monte-Carlo and a Quasi Monte Carlo implementations of Ulam's method for the approximation of f^*. In an Ulam's method, the entries of the Ulam's marix are calculated using inverse images of intervals under the transformations $\tau_k, k = 1, 2, \ldots, K$ of the randm map T. The main advantage of Monte-Carlo and Quasi Monte Carlo approach to Ulam's method is that we do not need to find the inverse images of subsets under the transformations of the random map T. We consider different types of random maps and compare the performances of the Monte Carlo and the Quasi Monte Carlo approached to Ulam's method with examples. Our numerical schemes are generalizations of numerical schemes of single deterministic maps to numerical schemes for position dependent random maps.

Spectral Gap Property for Random Dynamics on the Real Line and Multifractal Analysis of Generalised Takagi Functions

Johannes Jaerisch
Shimane University, Japan
Hiroki Sumi

We derive the spectral gap property for the random iteration of finitely many expanding diffeomorphisms on the real-line without a common fixed point. As an applilcation, we introduce generalised Takagi functions on the real-line and investigate their Hoelder regularity by using the multifractal analysis in ergodic theory.

Regularity of Superstable Manifolds of Invariant Circles

Scott Kaschner
Butler University, USA

In this talk, I will discuss the dynamics of dominant, meromorphic self-maps of complex manifolds of dimension $n > 1$. Specifically, I will focus on the situation in which there is an invariant embedded copy of CP^1 that is transversally superattracting and also contains an invariant real circle. I will describe the regularity the of superstable manifolds of this circle and how they relate to properties of the map restricted to a neighborhood of the embedded CP^1. Also, there is a physical interpretation to one of the maps described; I will explain how this is related and how it motivated this work.

Measures of Maximal Entropy for Suspension Flows Over the Full Shift

Tamara Kucherenko
The City College of New York, USA
Dan Thompson

We consider suspension flows with continuous roof function over the full shift on a finite alphabet. For any positive entropy subshift of finite type Y, we show there exists a roof function such that the measure(s) of maximal entropy for the suspension flow over the full shift are exactly the lifts of the measure(s) of maximal entropy for Y. In the case when Y is transitive, this gives a unique measure of maximal entropy for the flow which is not fully supported. If Y has more than one transitive component, all with the same entropy, this gives explicit examples of suspension flows over the full shift with multiple measures of maximal entropy. This contrasts with the case of a Hölder continuous roof function where it is well known the measure of maximal entropy is unique and fully supported.

Asymptotic Properties of Markov Operator Corresponding to Perturbed Non-Expanding Piecewise Linear Maps

Fumihiko Nakamura
Kitami Institute of Technology, Japan

We consider following perturbed systems, $x_{t+1} = S_{\alpha,\beta}(x_t) + \xi_t \pmod 1$ where $S_{\alpha,\beta}(x) = \alpha x + \beta \pmod 1$, $(0 < \alpha, \beta < 1)$, and $\{\xi_t\}$ are independent random variables each having same density g which is supported on $[0, \theta]$ with $\theta > 0$. The non-expanding piecewise linear map $S_{\alpha,\beta}$ is known as the Nagumo-Sato (NS) model, and it describes the simplified dynamics of a single neuron. It is known that the system $S_{\alpha,\beta}$ shows a periodic behavior of the trajectory for almost every (α, β). In this talk, we discuss asymptotic properties for the Markov operator corresponding to the perturbed NS model. Especially, we focus on the properties of "asymptotic periodicity" and "asymptotic stability". Then I will introduce our main result which tells us which the Markov operator corresponding to the perturbed system has either asymptotic periodicity or asymptotic stability for each (α, β) and θ.

Historic Behaviour for Nonautonomous Dynamical Systems

Yushi Nakano
Tokai University, Japan
Shin Kiriki, Teruhiko Soma

A point without time averages for a (nonautonomous) dynamical system is said to have historic behaviour. It is known: (i) There is a persistent class of dynamical systems such that the set of points with historic behaviour is of positive Lebesgue measure (Kiriki and Soma 2017); (ii) For any expanding maps, the set of points with historic behaviour is residual (Takens 2008); (iii) For any (nontrivial) nonautonomous dynamical system under independent and identically distributed (i.i.d.) noise, the set of points with historic behaviour is of zero Lebesgue measure (Araujo 2000). Based on the theorems, in this talk, we present the following two results about historic behaviour for nonautonomous dynamical systems. (1) For any nonautonomous expanding maps under ergodic noise (including i.i.d. noise), the set of points with historic behaviour is residual (arXiv:1510.00905). (2) There is a (nontrivial) nonautonomous dynamical system under a non-i.i.d. noise such that the set of points with historic behaviour is of positive Lebesgue measure (arXiv:1703.03163). The second result is a joint work with Kiriki and Soma (although it is independent of their theorem in 2017).

Stability of Hilbert Space Lyapunov Exponents

Anthony Quas
University of Victoria, Canada
Gary Froyland, Cecilia Gonzalez-Tokman

We consider the Lyapunov exponents of certain cocycles of operators on Hilbert space, and show that these exponents are continuous under a suitable family of perturbations of the cocycle.

Equidistribution of Holonomy in Homology Classes

Richard Sharp
Warwick University, England

For a compact group extension of an Anosov flow, we establish conditions under which the holonomies of periodic orbits are equidistributed when the orbits are restricted to a prescribed homology class. In particular, this applies to geodesic flows in negative curvature when the corresponding frame flow is ergodic.

Uniformly Perfect and Hereditarily Non Uniformly Perfect Sets in Dynamics

Rich Stankewitz
Ball State University, USA

We discuss uniformly perfect sets and hereditarily non uniformly perfect sets (compact sets for which no compact subset is uniformly perfect), as they arise in various dynamical settings (e.g., Julia sets of certain rational semigroups, Attractor sets of certain iterated function systems, and certain sets of well-approximable points).

Classification of Generic Random Holomorphic Dynamical Systems Associated with Analytic Families of Rational Maps

Hiroki Sumi
Kyoto University, Japan

We consider following perturbed systems, $x_{t+1} = S_{\alpha,\beta}(x_t) + \xi_t \pmod 1$ where $S_{\alpha,\beta}(x) = \alpha x + \beta \pmod 1$, $(0 < \alpha, \beta < 1)$, and $\{\xi_t\}$ are independent random variables each having same density g which is supported on $[0, \theta]$ with $\theta > 0$. The nonexpanding piecewise linear map $S_{\alpha,\beta}$ is known as the Nagumo-Sato (NS) model, and it describes the simplified dynamics of a single neuron. It is known that the system $S_{\alpha,\beta}$ shows a periodic behavior of the trajectory for almost every (α, β). In this talk, we discuss asymptotic properties for the Markov operator corresponding to the perturbed NS model. Especially, we focus on the properties of "asymptotic periodicity" and "asymptotic stability". Then I will introduce our main result which tells us which the Markov operator corresponding to the perturbed system has either asymptotic periodicity or asymptotic stability for each (α, β) and θ.

Topological Conditions for the Uniqueness of Sinai-Ruelle-Bowen Measures

Naoya Sumi
Kumamoto University, Japan

In this talk we consider C^2 diffeomorphisms f on an n-dimensional closed manifold M $(n \geq 1)$. We give some topological condition which guarantees that there exists at most one (f-invariant) hyperbolic ergodic Sinai-Ruelle-Bowen measure. In order to give a precise statement of our result, we denote as P all periodic points p whose stable and unstable sets are topological manifolds and have a topological transverse intersection at p. For $m \in \mathbb{N}$ we define as P_m the set of all $p \in P$ satisfying that the dimension of the stable set at p is equal to m. We assume that for $m \leq k$, $p \in P_m$ and $q \in P_k$, the unstable set at p and the stable set at q have a topological transverse intersection. Then there exists at most one hyperbolic ergodic SRB measure.

Large Deviation Principles for Countable Markov Shifts

Hiroki Takahasi
Keio University, Japan

We prove the Large Deviation Principles for a topologically mixing, one-sided topological Markov shift on a countably infinite number of alphabets which satisfies a strong combinatorial assumption called "the big images and pre-images property". More precisely, we assume the existence of a Gibbs measure in the sense of Bowen, and establish the level-2 Large Deviation Principles for the distribution of Birkhoff averages under the Gibbs measure, as well as that of weighted periodic points and iterated pre-images. As an illustration, we apply our results to the Gauss transformation and obtain a global limit theorem on the frequency of digits in the regular continued fraction expansion.

Convergence of the Gibbs Measures of Perturbed Graph Iterated Functions Systems with Degeneration

Haruyoshi Tanaka
Wakayama Medical University, Japan

We study a perturbed graph-directed iterated function system (GIFS) that some perturbed contraction maps converge to constant values. In our situation, the perturbed GIFS has a unique Gibbs measure $\mu(\epsilon, \cdot)$ associated with the dimension of the limit set for each $\epsilon > 0$ and on the other hand the unperturbed GIFS possesses several Gibbs measures $\mu_1, \mu_2, \ldots, \mu_m$. In particular, any limit point of $\mu(\epsilon, \cdot)$ in the sense of weak topology is expressed as a convex combination of $\mu_1, \mu_2, \ldots, \mu_m$. In this talk, under suitable conditions, we give a necessary and sufficient condition for convergence of $\mu(\epsilon, \cdot)$ using solutions of Bowen's equations and measure-theoretic entropies of Gibbs measures.

Asymptotic Counting in Conformal Dynamical Systems

Mariusz Urbanski
University of North Texas, USA
Mark Pollicott

I will consider the general setting of conformal graph directed Markov systems modeled by countable state symbolic subshifts of finite type. I will talk about two classes of such systems: attracting and parabolic. The latter being treated by means of the former. I will provide fairly complete asymptotic counting results for multipliers and diameters associated with preimages or periodic orbits ordered by a natural geometric weighting. These results will have direct

applications to a wide variety of examples, including the case of Apollonian Circle Packings, Apollonian Triangle, expanding and parabolic rational functions, Farey maps, continued fractions, Mannenville-Pomeau maps, Schottky groups, Fuchsian groups. Our approach is founded on spectral properties of complexified Ruelle–Perron–Frobenius operators and Tauberian theorems as used in classical problems of prime number theory.

Random Holomorphic Dynamics of Markov Systems

Takayuki Watanabe
Kyoto University, Japan
Hiroki Sumi

We consider the random holomorphic dynamical systems on the Riemann sphere whose choices of maps are related to Markov chain. Our motivation is to generalize the facts which hold in i.i.d. random holomorphic dynamical systems. Especially, we focus on the function $\mathbb{T}_\infty$ which represents the probability of tending to infinity. We show some sufficient conditions which make $\mathbb{T}_\infty$ continuous on the whole space and we characterize the Julia sets in terms of the function $\mathbb{T}_\infty$ under some assumptions. This is a joint work with Hiroki Sumi (Kyoto University).

Large Deviations for Systems with Non-Dense Ergodic Measures

Kenichiro Yamamoto
Nagaoka University of Technology, Japan

We introduce a weaker form of the specification property, called "one-way" specification property, which holds for a broad class of systems whose ergodic measures are not dense in the set of invariant measures, including Smale's horseshoe map, DA map and (symbolic spaces of) (-β)-transformations for Yrrap $\beta > 1$. As an application, we show that these three examples satisfy a level-2 large deviation principle with the normalized Lebesgue measure.

Discontinuity Versus Uniformly Ergodic Theorem

Zhe Zhou
Academy of Mathematics and Systems Science, CAS, Peoples Rep of China
Meirong Zhang, Zuohuan Zheng

The classical uniformly ergodic theorem requires the continuous transformation and the continuous observation. In this talk, we will push the boundary of uniformly ergodic theorems, and establish the corresponding result for discontinuous transformations and discontinuous observations. This is joint work with Meirong Zhang and Zuohuan Zheng.

Special Session 127: Dynamical Aspects of Diffusive Systems

Goro Akagi, Tohoku University, Japan
Eiji Yanagida, Tokyo Institute of Technology, Japan

The aim of this special session is to bring together experts in the area of PDEs and exchange recent results and ideas on the dynamics of various nonlinear diffusive systems. We are interested in qualitative behavior of solutions for reaction-diffusion equations, singular and degenerate diffusion equations, and related problems.

Convergence of Solutions to Fractional Cahn-Hilliard Systems

Goro Akagi
Tohoku University, Japan

This talk is concerned with the Cauchy-Dirichlet problem for fractional Cahn-Hilliard equations. We shall discuss global (in time) existence of weak solutions, characterization of parabolic smoothing effects (implying under proper condition eventual boundedness of trajectories), and convergence of each solution to a (single) equilibrium. In particular, to prove the convergence result, a variant of the so-called Lojasiewicz-Simon inequality is provided for the fractional Dirichlet Laplacian and (possibly) non-analytic (but C^1) nonlinearities. This talk is based on a joint work with G. Schimperna and A. Segatti (Pavia, IT).

Multiplicity of Ground States for the Scalar Curvature Equation: a Non Perturbative Result

Matteo Franca
Marche Polytechnic University (Ancona), Italy
F. Dalbono, A. Sfecci

In this talk we discuss the problem of existence and multiplicity of radial ground states with fast decay (GS for short) for

$$\Delta u + [1 + \epsilon k(|x|)]u^{\frac{n+2}{n-2}} = 0$$

where $x \in \mathbb{R}^n$, $n \geq 3$, $k \in C^1$, $k(|x|) \in [0, 1]$, $\epsilon > 0$ small. Nowadays several different conditions sufficient for the existence of GS are available in literature. Further, if k has a unique critical point and it is a maximum the GS is unique, see **[KYY]**. On the other side if the unique critical point is a minimum (and some other conditions are fulfilled) a large number of GS are found, if $\epsilon > 0$ is small enough, see **[CL]**. A similar result was obtained in **[FF]** replacing $[1+\epsilon k(|x|)]$ by a slowly varying function $k(|x|^\epsilon)$. Our purpose is to give a constructive argument which enable us to reprove the result in **[CL]** but giving an estimate on how small ϵ should be.

Singularities of Blowup Solutions for Heat Equation with a Nonlinear Boundary Condition

Junichi Harada
Akita University, Japan

We study the asymptotic behavior of blowup solutions for the heat equation with a nonlinear boundary condition on the half space. By the presence of nonlinearities on the boundary, solutions can blow up on the boundary. It is known that the singularity of blowup solutions along the direction vertical to the boundary are determined by unique self-similar solutions. A goal is to give a complete description of the singularity on the boundary. In this talk, we focus on derivation of the matched asymptotic expansion, which is a key part in the proof.

Asymptotic Behavior of Solutions of the Fast Diffusion Equation Near Its Extinction Time

Kin Ming Hui
Institute of Mathematics, Academia Sinica, Taiwan

Let $n \geq 3$, 00, I prove the existence and uniqueness (for $\beta \geq \frac{\rho_1}{n-2-nm}$) of radially symmetric singular solution $g_\lambda \in C^\infty(\mathbb{R}^n \setminus \{0\})$ of the elliptic equation $\Delta v^m + \alpha v + \beta x \cdot \nabla v = 0$, $v > 0$, in $\mathbb{R}^n \setminus \{0\}$, satisfying $\lim_{|x|\to 0} |x|^{\alpha/\beta} g_\lambda(x) = \lambda^{-\frac{\rho_1}{(1-m)\beta}}$. When β is sufficiently large, we prove the higher order asymptotic behaviour of radially symmetric solutions of the above elliptic equation as $|x| \to \infty$. I also obtain an inversion formula for the radially symmetric solution of the above equation. As a consequence I prove the extinction behaviour of the solution u of the fast diffusion equation $u_t = \Delta u^m$ in $\mathbb{R}^n \times (0, T)$ near the extinction time $T > 0$.

Convergence of a Threshold-Type Algorithm for Mean Curvature Flow

Katsuyuki Ishii
Kobe University, Japan

In this talk I would like to consider a threshold-type algorithm for curvature-dependent motions. The main result is the convergence of the algorithm to smooth/generalized mean curvature flow in the sense of the Hausdorff distance.

Diffusion for Biological Organisms in a Heterogeneous Environment

Yong-Jung Kim
KAIST, Korea
Beomjun Choi

The random dispersal of minute particles suspended in fluid is often mathematically modeled by a diffusion equation. The same diffusion equation is also taken to model the random migration of biological organisms and successfully provides fundamental insights into the role of migration in the context of spatial ecology. The purpose of this talk is to introduce a diffusion model based on the behavior of individual biological organisms instead of simply adopting a diffusion model for Brownian particles. We hope to obtain a better diffusion theory in this way and explain dispersal phenomena of biological organisms correctly.

Spreading Profile of Solutions for a Free Boundary Problem of a Nonlinear Diffusion Equation with a Positive Bistable Nonlinearity

Hiroshi Matsuzawa
National Institute of Technology, Numazu College, Japan
Yuki Kaneko, Yoshio Yamada

From the work of [Du-Lin, 2010], the propagation phenomena in free boundary problems of reaction-diffusion equations attract more and more attention of mathematicians. Among the various studies on the free boundary problem, recent work [Du-Matsuzawa-Zhou, 2014] showed that under certain condition, the profile of solution approaches a traveling wave solution associated with the free boundary problem. In this talk, I will give some recent study on propagation profiles of solutions for the free boundary problem of reaction-diffusion equation with some class of bistable nonlinearity.

The Variational Approach to Keller-Segel System

Yoshifumi Mimura
Nihon University, Japan

We introduce the Keller-Segel model for describing the aggregation phenomenon of certain microorganisms called slime molds, which have a characteristic property called chemotaxis. Chemotaxis is the motion toward higher concentration of a chemical substance. This kind of microorganism, when put in a nutrition-poor environment, produces a chemical substance that attracts other individuals within the same population. This leads to the formation of an aggregation which produces spores. In this way, the slime molds propagate next generation. From mathematical point of view, the aggregation phenomenon can be interpreted as the blow-up of the solution two simultaneous partial differential equations. In this talk, we show that the blow-up solution never exists if the mass of the slime molds is less than a certain value.

Near-Field Asymptotics for the Porous Medium Equation in Low-Dimensional Exterior Domains

Fernando Quiros
Universidad Autonoma de Madrid, Spain
Carmen Cortázar, Noemì Wolanski

Let $\mathcal{H} \subset \mathbb{R}^N$ be a non-empty bounded open set. We consider the porous medium equation in the complement of $\mathcal{H}$, with zero Dirichlet data on its boundary and nonnegative compactly supported integrable initial data. When $N = 1$, Kamin and Vázquez, in 1991, studied the large time behavior of solutions to this problem in the far-field scale, which is the adequate one to describe the movement of the free boundary. Gilding and Goncerzewicz, in 2007, performed an analogous study in dimension $N = 2$. Starting from their results in the far field, we study the large time behavior in the near field, in scales that evolve more slowly than the free boundary. In this way we get, in particular, the final profile and decay rate on compact sets. In contrast with the case of high dimensions, $N \geq 3$, in low dimensions the decay rate of solutions in the near field is not the same as in the far field.

Stability and Dynamics of Certain Reaction-Diffusion Equations with a Gradient Structure on Bounded and Unbounded Domains

Sinisa Slijepcevic
University of Zagreb, Croatia

We consider dynamics of certain reaction-diffusion equations, which can be interpreted as gradient flows on Banach spaces with a Riemannian structure. It has been shown by F. Otto for the porous media equation, and M. Liero and A. Mielke for diffusion equations with a reaction term, that there are examples of such systems where the entropy functional has positive definite Hessian, or equivalently which satisfy λ-convexity condition. We give some examples of such systems, and discuss implications to dynamics and stability on bounded and unbounded domains, including diffusive repair.

Compactly Supported Stationary States of the Degenerate Keller-Segel System in the Difusion-Dominated Regime

Yoshie Sugiyama
Osaka University, Japan
J. A. Carrillo

We show the existence of a unique global minimizer of free energy for all masses that are associated with a nonlinear diffusion type of the Keller-Segel system, but only in cases when the diffusion dominates over the attractive force of the chemo-attractant. We approximate the variational problem in the whole space as a minimization problem posed on bounded balls with large radii. We show that our stationary states have four different properties, they are unique up to translations of the balls center of mass, compactly supported, radially decreasing and smooth within the support of their respective global minimizer.

Nonlinear Fokker-Planck Equations with Reaction

Dmitry Vorotnikov
Universidade de Coimbra, Portugal

We interpret a class of nonlinear Fokker-Planck equations with reaction as gradient flows over the space of Radon measures equipped with the recently introduced HK-distance [1]-[3] and the spherical HK-distance [4]. We prove new isoperimetric-type functional inequalities, which allow us to control the relative entropy by its production. We establish the entropic exponential convergence of the trajectories of the flow to the equilibrium. Along with other applications, this result has an ecological interpretation as a trend to the ideal free distribution for a class of fitness-driven models of population dynamics.
Based on a joint work with S. Kondratyev.

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Special Session 128: Recent Advances in the Calculus of Variations and Elliptic PDE

Robin Neumayer, Northwestern University, USA
Connor Mooney, ETH Zurich, Switzerland

This special session aims to bring together experts and young researchers in the calculus of variations and elliptic PDE to exchange ideas and perspectives. Topics include isoperimetric problems, nonlocal equations, geometric PDE, and functional inequalities.

Stability of the Gaussian Isoperimetric Problem

Marco Barchiesi
University of Naples Federico II, Italy
Vesa Julin

I will present an analysis of the sets that minimize the gaussian perimeter plus the norm of the barycenter. These two terms are in competition, and in general the solutions are not the half-spaces. In fact we prove that for small masses the solutions are the complements of strips centered in the origin.

From the Free Boundary Condition for Hele-Shaw to the Fractional Parabolic

Hector Chang-Iara
Centro de Investigacion en Matematicas, Mexico
Nestor Guillen

We propose a method to determine the smoothness for the free boundary of sufficiently flat solutions of one phase Hele-Shaw problems. The novelty is the observation that under a flatness assumption the free boundary–represented by the hodograph transform of the solution–solves a nonlinear integro-differential equation. This nonlinear equation can be linearized to a (nonlocal) parabolic equation with bounded measurable coefficients, for which regularity estimates are available.

Non Local Gluing Methods for the Fractional Yamabe Problem with Singularities

Azahara Delatorre
University of Freiburg, Germany
W. Ao, H. Chan, M. del Pino, M. Fontelos, MdM Gonzalez, J. Wei

We construct some solutions for the fractional Yamabe problem which are singular along a prescribed set Σ. This is a problem which arises in conformal geometry when we try to find metrics that are conformal to the given one, have constant fractional curvature and are singular along the prescribed set. It is equivalent to finding positive and smooth solutions for

$$(-\Delta)^{\gamma} u = c_{n,\gamma} u^{\frac{n+2\gamma}{n-2\gamma}}, u > 0 \text{ in } \mathbb{R}^n \backslash \Sigma.$$

The fractional curvature, a generalization of the classical scalar curvature, is defined from the conformal fractional Laplacian, which is a non-local operator constructed on the conformal infinity of a conformally compact Einstein manifold. When the singular set Σ is composed of one point, some new tools for fractional order ODEs can be applied to show that a generalization of the usual Delaunay solves the fractional Yamabe problem with an isolated singularity at Σ. (Joint work with María del Mar González.) If the set Σ is a finite number of points, then, by using gluing methods, we will provide a solution for the fractional Yamabe problem with singularities at Σ. In order to preserve the non-locality of the problem, we need to glue infinitely many bubbles per point removed. This seems to be one of the first works in which a gluing method is successfully applied to a non-local problem. (Joint work with Weiwei Ao, María del Mar González and Juncheng Wei.) Finally, for a higher-dimension smooth submanifold Σ, we are able to adapt the classical gluing methods for the scalar curvature to the non-local setting. However, this requires us to develop new methods coming from conformal geometry and scattering theory for the study of non-local ODEs. Some examples which are worth mentioning are the construction of radial fast-decaying solutions (using a blow-up argument and a bifurcation method); the use conformal geometry to rewrite this non-local ODE, giving a hint of what a non-local phase-plane analysis should be; and the study of a fractional Schrödinger equation with a Hardy type critical potential. (Joint work with Weiwei Ao, Hardy Chan, Marcos Fontelos, María del Mar González and Juncheng Wei.)

Alexandrov Theorem Revisited

Matias Delgadino
Imperial College, Argentina
Francesco Maggi

We show that among sets of finite perimeter balls are the only volume-constrained critical points of the perimeter functional. To obtain the result, we revisit, in a low regularity setting, the proof of the Heintze-Karcher inequality.

The Geometry of the Free Boundary Near the Fixed Boundary Generated by a Fully Nonlinear Uniformly Elliptic Operator

Emanuel Indrei
Purdue University, USA

The dynamics of how the free boundary intersects the fixed boundary has been the object of study in the classical dam problem which is a mathematical model describing the filtration of water through a porous medium split into a wet and dry part. The case of the Laplacian is well-understood and the fully nonlinear uniformly elliptic case in two dimensions is joint work with Minne. This talk focuses on the problem in higher dimensions as well as the regularity of the free boundary.

Remark on a Sobolev Type Inequality in the Unit Ball

Norisuke Ioku
Ehime University, Japan

It is well known that the sharp constant of the Sobolev inequality in the whole space is attained by the Talenti function, but not in bounded domains since a scale invariance is breaking. In this talk, we introduce a scale invariant form of the Sobolev inequality in the unit ball, and show that the sharp constant is attained by a Talenti like function.

Free Singularities in Optimal Transport

Jun Kitagawa
Michigan State University, USA
Robert McCann

Regularity of solutions in the optimal transport problem require very rigid hypotheses (e.g., convexity of certain sets). In more general cases one can consider the question of partial regularity, i.e. in-depth analysis of the structure of singular sets. In this talk I will discuss finer structure of the set of "free singularities" which arise in an optimal transport problem from a connected set to a disconnected set. Such results are proven via a non-smooth implicit function theorem for convex functions, which is of independent interest. Time permitting, I will also discuss an application of this analysis to a stability result for optimal transport maps under perturbations of the target measure. This talk is based on joint work with Robert McCann.

Regularity Estimates for the Stochastic Homogenization of Elliptic Nondivergence Form Equations

Jessica Lin
McGill University, Canada
Scott Armstrong

I will present some regularity estimates related to the stochastic homogenization for nondivergence form elliptic equations. In a joint work with Scott Armstrong, we show that in the stochastic homogenization for linear uniformly elliptic equations in random media, solutions actually exhibit improved regularity properties in light of the homogenization process. In particular, we show that with extremely high probability, solutions of the random equation have almost the same regularity as solutions of the deterministic homogenized equation. This is a necessary ingredient in obtaining optimal error estimates for the stochastic homogenization of linear uniformly elliptic equations.

On Least Energy Solutions to a Higher-Order Obstacle Problem

Tatsuya Miura
The University of Tokyo, Japan

We discuss a higher-order variational problem with an obstacle constraint in a one-dimensional periodic setting. This problem arises from materials science and models thin elastic bodies adhering to non-flat solid substrates. In this talk we consider how physical parameters in the model affect the shapes of least energy solutions. We provide several conditions implying "flatness" of solutions in some sense and also give an example of a situation such that any least energy solution is "rougher" than the substrate.

Liftings of BV-Maps and Lower Semicontinuity

Filip Rindler
University of Warwick, England
Giles W. Shaw

Liftings and their associated Young measures are new tools to study the asymptotic behaviour of sequences of BV-maps under weak* convergence. Their main feature is that they allow to keep track of the precise shape of the jump path and as such are natural objects whenever different ways of approaching a jump need to be distinguished. While this tool has several promising applications, in this talk I will focus on its use to prove lower semicontinuity for linear-growth functionals that depend on the value of the argument function, u(x), besides its gradient. It is well known that in this situation the particular shape of jumps cannot be neglected. Using the theory of liftings, we can prove relaxation theorems under essentially optimal assumptions, generalizing a classical theorem by Fonseca & Müller (1993). The key idea is that liftings provide the right way of localizing the functional in the x and u variables simultaneously under weak* convergence. As a consequence, we are able to implement an optimal measure-theoretic blow up procedure.

Optimal Regularity for the Thin Obstacle Problem with Hoelder Coefficients

Angkana Rueland
Max-Planck Institute for Mathematics in the Sciences, Germany

In this talk I will present a regularity result for the thin obstacle problem, which is a free boundary value problem of obstacle type, where the obstacle is constrained to a co-dimension one set. More precisely, I will describe a linearization technique, which allows to deduce optimal regularity in the framework of only $C^{0,\alpha}$ Holder continuous, variable coefficients. The talk is based on joint work with Wenhui Shi.

The Singular Free Boundary in the Signorini Problem

Mariana Smit Vega Garcia
University of Washington, USA
Nicola Garofalo, Arshak Petrosyan

In this talk I will give an overview of the Signorini problem for a divergence form elliptic operator with Lipschitz coefficients, and I will describe a few methods used to tackle two fundamental questions: what is the optimal regularity of the solution, and what can be said about the singular free boundary in the case of zero thin obstacle. The proofs are based on Weiss and Monneau type monotonicity formulas. This is joint work with Nicola Garofalo and Arshak Petrosyan.

On Chen Submanifolds and the Chen Flow

Valentina-Mira Wheeler
University of Wollongong, Australia
Yann Bernard, Glen Wheeler

Chen's operator for a submanifold is the twice iterated Laplacian on the pullback bundle, sometimes known as the rough Laplacian in the literature. Chen's conjecture is that if Chen's operator applied to the immersion map vanishes, then the submanifold is minimal. In the last few years, work has progressed on the parabolic flow with velocity corresponding to Chen's operator applied to the immersion. This parabolic flow has an interesting variational characterisation, being between the biharmonic map heat flow and the Willmore flow. Algebraically, the Chen flow sits instead between surface diffusion and Willmore flow. Qualitatively its behaviour is much closer to the mean curvature flow. In particular, spheres shrink to points in finite time. In this talk we describe the Chen operator, Chen's conjecture, and some recent work on Chen's flow in two and four dimensions.

On the Isoperimetric Quotient Over Scalar-Flat Conformal Classes

Jingang Xiong
Beijing Normal University, Peoples Rep of China
Tianling Jin

Let (M, g) be a smooth compact Riemannian manifold of dimension n with smooth boundary ∂M. Suppose that (M, g) admits a scalar-flat conformal metric. We prove that the supremum of the isoperimetric quotient over the scalar-flat conformal class is strictly larger than the best constant of the isoperimetric inequality in the Euclidean space, and consequently is achieved, if either (i) $n \geq 12$ and ∂M has a non-umbilic point; or (ii) $n \geq 10$, ∂M is umbilic and the Weyl tensor does not vanish at some boundary point.

Special Session 130: Theoretical and Computational Analysis on Differential Equation Models

Shangbin Cui, Sun Yat-Sen University, Peoples Rep of China
Yunfeng Jia, Shaanxi Normal University, Peoples Rep of China
Bingtuan Li, University of Louisville, USA
Jianhua Wu, Shaanxi Normal University, Peoples Rep of China

This special session focuses on problems of mathematical models having direct or potential applications to the biological, physical, chemical sciences and engineering, including (but not limited to) epidemic models, tumor models, stage and size-structured population models, chemostat models, and other related differential equation models. The special interest is paid on the global existence and asymptotic behavior of time-dependent solutions, the spreading speed of traveling wave solutions, the existence and multiplicity of steady-state solutions and their stability, etc.

Analysis of Nonlinear Cell-Division Models

Meng Bai
Zhaoqing University, Peoples Rep of China

Some nonlinear cell-division models are discussed. The uniqueness and global existence of the positive solutions, the existence of steady solutions and the asymptotic behavior of solutions will be obtained by using the general theories of nonlinear equations in Banach spaces, the semigroups and the spectral analysis of positive semigroups.

Spreading Speeds and Linear Determinacy of Time Dependent Diffusive Cooperative/Competitive Systems

Xiongxiong Bao
Changan University, Peoples Rep of China
Wan-Tong Li, Wenxian Shen, Zhi-Cheng Wang

In this talk, I will discuss the spreading speeds and linear determinacy of diffusive cooperative/competitive systems with time recurrent dependence. First, the notion of spreading speed intervals for diffusive cooperative systems is introduced via the natural features of spreading speeds and some basic spreading properties are established. Next, some principal Lyapunov exponent and principal Floquent bundle theory for linear cooperative systems of ordinary differential equations is developed. In terms of the principal Lyapunov exponent and principal Floquent bundle theory, some upper and lower bounds for the spreading speed intervals for diffusive cooperative systems are then established. Under certain conditions, it is proved that a diffusive cooperative system has a single spreading speed and is linearly determinant.

Existence and Stability of Coexistence States in a Competition Unstirred Chemostat

Yuan Hailong
Shaanxi University of Science and Technology, Peoples Rep of China
Conghui Zhang, Yanling Li

In this paper, we consider the two similar competing species in a competition unstirred chemostat model with diffusion. The two competing species are assumed to be identical except for their maximal growth rates. In particular, we study the existence and stability of the coexistence states, and the semi-trivial equilibria or the unique coexistence state is the global attractor can be established under some suitable conditions. Our mathematical approach is based on Lyapunov-Schmidt reduction, the implicit function theory and spectral theory.

Properties and Numerical Simulations of Positive Solutions for a Variable-Territory Model

Hongling Jiang
Baoji University of Arts and Sciences, Peoples Rep of China
Jianhua Wu, Lijuan Wang

In this talk, we consider a variable-territory predator-prey model with Dirichlet boundary condition. We establish a necessary and sufficient condition for the existence of positive solutions to steady state. Furthermore, we also prove that the local bifurcation positive solutions are unconditional stable. By regular perturbation theorem, we investigate the convergence and stability of positive solutions when handling time m is large enough. At the end of this work, there are some numerical simulations and biological significance to check and complement our theoretical analysis results.

A Diffusive Predator-Prey Model in the Spatially Heterogeneous Environment: Effects of Predator Competition

Li Shanbing
Xidian University, Peoples Rep of China
Jianhua Wu

In this paper, we study a spatially heterogeneous predator-prey model where the interaction is governed by a Crowley-Martin type functional response. The corresponding stationary problem and related dynamical behavior are examined. It is found that the predator competition has beneficial effects on the survival of the prey when the predator growth rate is positive, moreover, when the predator competition is sufficiently strong, the model has at most one positive steady-state solution for any $\mu \in \mathbb{R}$, and it is globally asymptotically stable for any $\mu > 0$ (if it exists). Additionally, we show that when the prey growth rate is over a critical value, the two populations stabilize at a unique coexistence state if the predator growth rate is sufficiently strong.

Stationary Solutions of a Free Boundary Problem Modeling Growth of Angiogenesis Tumor with Inhibitor

Zejia Wang
Jiangxi Normal University, Peoples Rep of China
Xu Suzhen, Song Huijuan

In this talk, we consider a free boundary problem modeling the growth of angiogenesis tumor with inhibitor, in which the tumor aggressiveness is modeled by a parameter μ. The existences of radially symmetric stationary solution and symmetry-breaking stationary solution are established.

Existence and Asymptotic Profile of Endemic Equilibrium to a Diffusive Epidemic Model with Saturated Incidence Rate

Yane Wang
Shaanxi Normal University, Peoples Rep of China
Zhiguo Wang, Min Yuan

We study the existence and asymptotic profile of endemic equilibrium (EE) of a diffusive SIS epidemic model with saturated incidence rate. By introducing the basic reproduction number $\mathcal{R}_0$, the existence of EE is established when $\mathcal{R}_0 > 1$. The effect of diffusion rates and the saturated coefficient on asymptotic profile of EE is investigated. Our results indicate that when the diffusion rate of susceptible individuals is small and the total population N is below a certain level, or the saturated coefficient is large, the infected population dies out, while the two populations persist if at least one of the diffusion rates of the susceptible and infected individuals is large.

Positive Steady State Solutions of a Plant-Pollinator Model with Diffusion

Lijuan Wang
School of Mathematics and Information Science, Baoji University of Arts and Sciences, Rep of Cape Verde
Hongling Jiang

In this talk, a plant-pollinator population system with diusion is investigated, which is described by a cooperative model with B-D functional response. Using the Leray-Schauder degree theory, we discuss the existence of positive steady state solutions of the model. The result shows when the growth rate of plants is large and the death rate of pollinators is small, the plants and pollinators can coexist. By the regular perturbation theorem and monotone dynamical system theory, the uniqueness and stability of positive solutions have been studied. Especially, we show that the unique positive solution is a global attractor under some conditions. Furthermore, we present some numerical simulations, which is not only to check our theoretical results but also to supply some conjectures out of theoretical analysis.

On a Stochastic Coupled Kuramoto-Sivashinsky and Ginzburg-Landau-Type Model Driven by Multiplicative Noises for Marangoni Convection

Wei Wu
Qingdao Agricultural University, Peoples Rep of China

Marangoni convection refers to the motion of the fluid driven by the variation of surface tension caused by the variation of surface tension of a liquid with temperature or with the concentration of a surfactant, which has important applications in engineer fields. The coupled Kuramoto-Sivashinsky and Ginzburg-Landau-type $(KS-GL)$ model derived by Golovin *et al.* is one of important work for researching Marangoni convection. This model is different from other models for it aims at capturing important features and is simplified, more amenable to analysis. Recently, the mathematical analysis on the coupled $KS-GL$ system has been developed. Duan *et al.* proved existence and uniqueness of global solutions of the coupled $KS-GL$ system. Wu *et al.* proved global well-posedness of the stochastic coupled $KS-GL$ system driven by additive noises. In this talk, we present global well-posedness on the stochastic coupled $KS-GL$ system driven by multiplicative noises. Using the transformation of exponential functionals of Brownian motions, we turn the stochastic system into a random PDE system, then by applying the contraction mapping principle we prove the local well-posedness result and at last, we establish *priori* energy estimates which ensure the existence of global solutions.

Analysis of a Free Boundary Problem for Tumor Growth with Gibbs-Thomson Relation and Time Delays

Shihe Xu
Zhaoqing University, Peoples Rep of China
Meng Bai, Fangwei Zhang

In this paper we study a free boundary problem for tumor growth with Gibbs-Thomson relation and time delays. It is assumed that the process of proliferation is delayed compared with apoptosis. The delay represents the time taken for cells to undergo mitosis. By employing stability theory for functional differential equations, comparison principle and some meticulous mathematical analysis, we mainly study the asymptotic behavior of the solution, and prove that in the case c (the ratio of the diffusion time scale to the tumor doubling time scale) is sufficiently small, the volume of the tumor cannot expand unlimitedly. It will either disappear or evolve to one of two dormant states as $t \to \infty$. The results show that dynamical behavior of solutions of the model are similar to that of solutions for corresponding nonretarded problems under some conditions.

Special Session 131: Mean Field Games and Applications

Tonon Daniela, Université Paris Dauphine, France
Festa Adriano, INSA Rouen, France
Silva Francisco, Université de Limoges, France

Mean Field Game (MFG) theory is devoted to the analysis of differential games with a (very) large number of small players, i.e. players whose actions have a small influence on the overall system. At the limit, when the number of players goes to infinity, the equilibrium can be characterized in terms of a PDE system involving a Hamilton-Jacobi-Bellman equation and a Fokker-Planck equation. Connections with optimization problems, optimal transport, game theory, optimal control and fluid mechanics are important features of the problem. This theory is very lively at the moment and has several applications in economics, finance, social sciences and engineering. This session gathers young researchers working on open questions in MFGs.

Some Aspects of Mean-Field Type Modeling of Pedestrian Crowd Dynamics

Alexander Aurell
KTH, Stockholm, Sweden
Boualem Djehiche

This talk will review some recent results on a class of (anonymous) mean-field type control and game models for pedestrian crowds. The considered crowd models include a nonlocal aversion feature and arbitrarily but finitely many interacting crowds. The nonlocal aversion feature grants pedestrians a “personal space” where crowding is undesirable. The model is treated as a mean-field type game which is shown to be derived from a particle picture. Solutions to the mean-field type game are characterized via a Pontryagin-type Maximum Principle. The behavior of a crowd acting under nonlocal preferences is illustrated by numerical simulations. Extensions that include evacuation strategies and service planning, where anonymity is relaxed, and sticky boundary behavior will also be mentioned.

Proximal Methods for Stationary Mean Field Games with Local Couplings: Theory and Algorithms

Luis Briceno-Arias
U. Tecnica Federico Santa Maria, Chile
D. Kalise, F.J. Silva

We address the numerical approximation of Mean Field Games with local couplings. For power-like Hamiltonians, we consider both unconstrained and constrained stationary systems with density constraints in order to model hard congestion effects. For finite difference discretizations of the Mean Field Game system, we follow a variational approach. We prove that the aforementioned schemes can be obtained as the optimality system of suitably defined optimization problems. Next, assuming next that the coupling term is monotone, we study and compare several proximal type globally convergent first-order methods for solving the convex optimization problems. Each step of the proposed algorithms is easy computable, which leads to efficient implementations.

Concentration Phenomena in Mean Field Game Systems with Aggregation

Marco Cirant
Università di Padova, Italy
A. Cesaroni

I will present some results obtained in collaboration with A. Cesaroni (Padova) on stationary mean field game systems, that describe equilibrium configurations in a large population of rational individuals each of whom is subject to a coercive potential and an aggregation force. I will describe the variational framework for the analysis of these problems, and discuss concentration phenomena arising in the vanishing viscosity limit.

On Mean-Field Games with Constraints and Price Formation

Diogo Gomes
KAUST, Saudi Arabia
L. Nurbekyan and J. Saude

Here, we introduce a price-formation model for electricity markets where a large number of small players can store and trade electricity. Our model is a constrained mean-field game (MFG) where the price is a Lagrange multiplier for the supply vs. demand balance condition. Under mild conditions, we prove the uniqueness of the solution. Next, we examine some linear-quadratic models that have explicit solutions. Finally, we develop numerical methods and illustrate the behavior of the system numerically.

On Mean Field Games Models for Exhaustible Commodities Trade

Jameson Graber
Baylor University, USA
Charafeddine Mouzouni

In this paper we provide a rigorous justification of the mean field games models of exhaustible commodities production by solving the N-player game convergence problem. The model is based on recent works by Chan and Sircar as well as by Gueant, Lasry, and Lions. We show (i) the existence and uniqueness of

Nash equilibria for the N-player game, (ii) the existence and uniqueness of solutions to the mean field game, and (iii) that the mean field equilibrium is an "epsilon-equilibrium" to the N-player game.

Proximal Methods for Variational Mean Field Games: Computational Aspects

Dante Kalise
Imperial College London, England
Luis Briceno-Arias, Francisco J. Silva

In this talk we discuss the implementation of numerical schemes for the approximation of stationary Mean Field Games (MFG) with a variational structure. By casting the MFG as a PDE-constrained optimization problem, we apply a proximal method for its solution combined with a finite difference approximation of the state equation, leading to a fully discrete, first-order iterative scheme. The proposed scheme has several interesting features, such as robustness with respect to different viscosity values and modelling of congestion effects. Moreover, it can be easily implemented for time-dependent MFG. I this talk we will discuss computational aspects behind each of these features.

On Mean Field Models with Several Populations

Mathieu Lauriere
ORFE, Princeton University, USA
Alain Bensoussan, Tao Huang

In this talk, we compare four situations for two-population mean field models. We distinguish between cooperative or competitive equilibria (mean field type control problems and mean field games respectively), and between one or several objective functions. For each problem, we derive formally the PDE system characterizing the equilibria. As an illustration, we consider the case of linear-quadratic models and some models for crowd motion. Based on joint works with Alain Bensoussan and Tao Huang.

Sobolev Regularity for First Order Mean Field Games

Alpar R. Meszaros
UCLA, USA
Jameson Graber

In this talk we present some Sobolev estimates for weak solutions of first oder variational Mean Field Game systems with coupling terms that are local function of the density variable. Under some coercivity condition on the coupling, we obtain first order Sobolev estimates for the density variable, while under similar coercivity condition on the Hamiltonian we obtain second order Sobolev estimates for the value function. These results are valid both for stationary and time-dependent problems. In the latter case the estimates are fully global in time, thus we resolve a question which was left open in a recent paper of Prosinski and Santambrogio. Our methods apply to a large class of Hamiltonians and coupling functions.

Spatially Inhomogeneous Evolutionary Games

Marco Morandotti
Technische Universitaet Muenchen, Germany
L. Ambrosio, M. Fornasier, G. Savaré

We study an interaction model of a large population of players based on an evolutionary game, which describes the dynamical process of how the distribution of strategies changes in time according to their individual success. Differently from spatially homogeneous dynamical games, we assume that the population of players is distributed over a state space and that they are each endowed with probability distributions of pure strategies, which they draw at random to evolve their states. Simultaneously, the mixed strategies evolve according to a replicator dynamics, modeling the success of pure strategies according to a payoff functional. We establish existence, uniqueness, and stability of Lagrangian and Eulerian solutions of this dynamical game by using methods of ODE and optimal transport on Banach spaces.

One-Dimensional Non-Local First-Order Stationary Mean-Field Games with Congestion: a Fourier Approach

Levon Nurbekyan
KAUST, Saudi Arabia

Here, we study a one-dimensional non-local mean-field game model with congestion. When the kernel in the non-local coupling is a trigonometric polynomial, we reduce the problem to a finite-dimensional system. Furthermore, we treat the general case by approximating the kernel with trigonometric polynomials. Our technique is based on Fourier expansion methods.

Fully Nonlinear Mean-Field Games

Edgard Pimentel
PUC-Rio, Brazil

In this talk we examine fully nonlinear mean-field games. We start by deriving the system through a minimisation problem governed by a fully nonlinear elliptic operator. Then, we discuss gains of regularity and integrability for the solutions. An existence result concludes the talk.

Mean Field Models for Segregation Dynamics

Helene Ranetbauer
University of Vienna, Austria

In this talk we present different nonlinear PDE models describing the evolution of bidirectional and crossing pedestrian flows as well as segregation dynamics of multiple species. Starting with lattice based models, we derive the corresponding mean field PDE models using Taylor expansion. We discuss existence and stability of solutions and illustrate the dynamics with various numerical simulations.

On the Forward-Forward Mean Field Games

Marc Sedjro
AIMS–Tanzania, Tanzania
Diogo Gomes, Levon Nurbekyan

In this talk, we introduce several models of the so-called forward-forward Mean Field Games (MFGs). These problems arise in the study of numerical scheme to approximate stationary MFGs. We establish a link between these models and a class of hyperbolic conservation laws and certain nonlinear wave equations. We investigate the existence of solutions, existence of conserved quantities and examine long-time limit properties.

Finite Mean Field Games: Fictitious Play and Convergence Analysis

Francisco Jose Silva Alvarez
XLIM, DMI, Université de Limoges, France
Saeed Hadikhanloo

In this talk, based on an ongoing work with S. Hadikhanloo (Ecole Polytechnique and INRIA Saclay), we consider a class of finite state and discrete time Mean Field Games (MFGs) introduced by Gomes, Mohr and Rigao Souza in 2009. In this framework we first study an adaptation of the fictitious play procedure for continuous MFGs, introduced recently by Cardaliaguet and Hadikhanloo, and we prove the convergence to the solution of the finite MFG. In the second part of the talk, we consider a first order continuous MFG and an associated family of finite MFGs, parameterized by a finite time and space grid. We prove that, as the time and space steps tend to 0, the solutions of the finite MFGs converge to a solution of the continuous one.

Aggregation in Evolutionary Mean Field Games

Daniela Tonon
Paris Dauphine University, France
Marco Cirant

In this talk we consider time-dependent viscous Mean-Field Games systems in the case of local, decreasing and unbounded coupling. These systems arise in mean field game theory, and describe Nash equilibria of games with a large number of agents aiming at aggregation, i.e. at converging to a common state. From the PDE viewpoint, several issues are intrinsic in this framework, mainly caused by the lack of regularizing effects induced by increasing monotonicity of the coupling. Non-existence, non-uniqueness of solutions, non-smoothness, and concentration are likely to arise. Even more than in the competitive case, the assumptions on the Hamiltonian, the growth of the coupling and the dimension of the state space affect the qualitative behavior of the system. We prove the existence of weak solutions that are minimisers of an associated non-convex functional, by rephrasing the problem in a convex framework. Under additional assumptions involving the growth at infinity of the coupling, the Hamiltonian, and the space dimension, we show that such minimisers are indeed classical solutions by a blow-up argument and additional Sobolev regularity for the Fokker-Planck equation. These results are obtained in collaboration with Marco Cirant.

Special Session 132: Qualitative and Quantitative Techniques for Differential Equations arising in Economics, Finance and Natural Sciences

Rehana Naz, Lahore School of Economics, Pakistan
Imran Naeem, Lahore University of Management Sciences (LUMS), Pakistan
Rita Tracina, Università di Catania Viale A. Doria, Catania, Italy

The differential equations play a vital role in many disciplines from natural to social sciences. Most of physical laws in natural sciences are expressed in terms of differential equations. In this session we try to integrate analysis, models and methods in the scope of natural sciences as well as social sciences framework. The Economists study dynamical systems for sustainable Economic growth. Stochastic differential equations are the standard models for financial quantities important in financial market. Biologists (Epidemiologists) investigate the determinants of health-related states (including disease) using mathematical tools. Differential equations are mathematically studied from several different perspectives; this session will focus on the Qualitative and Quantitative techniques (including numerical methods) for ordinary differential equations, partial differential equations, fractional differential equations, difference equations, stochastic differential equations, integro-differential equations. Potential topics, of this session, include but are not limited to:

- Economic growth theory
- Optimal control
- Differential equations modeling natural and economic models
- Financial models e.g. Hamilton-Jacobi equation, Hamilton-Jacobi-Bellman equations, Option models, Black-Schole models
- Equivalence transformations
- Stability analysis
- Numerical techniques for special problems in modeling
- Symmetries, Differential Equations, and Applications
- Modeling and Math Biology
- Fluid Mechanics
- Reduction techniques and solutions and linearization
- Conserved quantities in natural phenomena

Some Singularly Perturbed Models in Ecology

Jacek Banasiak
University of Pretoria, So Africa

Many predator-prey systems lead to multiple scale models due to a significant difference of characteristic times of the vital processes of the involved species. This allows for an approximate reduction of the complexity of the models by singular perturbation techniques. We shall discuss some recent developments in the field and, in particular, the existence of canard solutions.

Global Dynamics of a Mathematical Model for the Possible Re-Emergence of Polio

Attila Dénes
Bolyai Institute, University of Szeged, Hungary
Làszlò Szèkely

Motivated by studies warning about a possible re-emergence of poliomyelitis in Europe, we analyse a compartmental model for the transmission of polio describing the possible effect of unvaccinated people arriving to a region with low vaccination coverage. We calculate the basic reproduction number, and determine the global dynamics of the system: we show that, depending on the parameters, one of the two equilibria is globally asymptotically stable. The main tools applied are Lyapunov functions and persistence theory. We illustrate the analytic results by numerical examples, which also suggest that in order to avoid the risk of polio re-emergence, vaccinating the immigrant population might result insufficient, and also the vaccination coverage of countries with low rates should be increased.

Analysis of the Criss-Cross Model of Tuberculosis

Urszula Forys
University of Warsaw, Poland
Marcin Choinski, Mariusz Bodzioch

On the basis of the simplest SI model we analyse a criss-cross model of tuberculosis for homless and non-homeless people. We apply this model to the data of Warmian-Masurian province of Poland.

Discrete Symmetries in Financial Instruments

Ebrahim Fredericks
University of Cape Town, So Africa
N. Ledwaba

We show how to compute the discrete symmetries for a given Black-Scholes (B-S) partial differential equation (PDE) with the aid of the full automorphism group of the Lie algebra associated to the standard B-S PDE. The paper determines the discrete symmetries using two methods. The first is by G. Silberberg which determines the full automorphism group by constructing the symmetry generators‘ centralizer and Lie algebra's radical. The other is by P. Hydon which is based on the observation that the adjoint action of any point symmetry of a partial differential equation is an automorphism of the PDE's Lie point symmetry algebra. Automorphisms are essential for constructing discrete symmetries of a given partial differential equation. How does one fit in this mathematical concept in the application of finance? The concept of arbitrage which in certain circumstances allows us to establish the precise relationship between prices and thence how to determine prices, underlies the theory of financial derivatives pricing and hedging. We use arbitrage together with the Black- Scholes model for asset price movements when trading derivative securities. Arbitrage is used to creating a portfolio and the discrete symmetries show how to create a portfolio. Gazizov and Ibragimov, computed the Lie point symmetries of the Black-Scholes PDE and found an infinite dimensional Lie algebra of infinitesimal symmetries generated by the operators.

A Stochastic Portfolio Optimization Model with Infinite Delay and Stochastic Volatility

Azmat Hussain
Lahore University of Management Sciences, Pakistan
Tao Pang

A portfolio optimization problem of the Merton's type with complete memory (infinite delay) over a finite time horizon is considered. In this model, the volatility of risky asset is considered to be stochastic. The goal is to choose the optimal investment and consumption controls to maximize the investor's expected total discounted utility. Using dynamic programming principle, the Hamilton-Jacobi-Bellman (HJB) equation is derived. Then, using the subsolution/supersolution method, we establish the existence result of classical solution to the HJB equation. Finally, we verify that the solution is equal to the value function, and derive the optimal investment and consumption controls.

On Markov Chain Methods for Multilevel Monte Carlo

Ajay Jasra
National University of Singapore, Singapore

This talk considers a novel approach to using Markov chain Monte Carlo (MCMC) in contexts where one may adopt multilevel (ML) and Multi-index (MI) Monte Carlo. The underlying problem is to approximate expectations w.r.t. an underlying probability measure that is associated to a continuum problem, such as a continuous-time stochastic process. It is then assumed that the associated probability measure can only be used (e.g. sampled) under a discretized approximation. In such scenarios, it is known that to achieve a target error, the computational effort can be reduced when using MLMC or MIMC, relative to exact sampling from the most accurate discretized probability. The ideas rely upon introducing hierarchies of the discretizations and assuming that less accurate approximations cost less to compute, one can introduce an appropriate collapsiing type sum expression for the target expectation. If a suitable coupling of the exact sampling of the probability measures in the hierarchy is achieved, then the reduction in cost is possible. This talk focuses on the case where such exact sampling is not possible. We show that given only access to MCMC kernels which are invariant to each discretized probability measure that such couplings are possible. We prove, under assumptions, that this coupled MCMC approach can reduce the cost to achieve a given error, relative to exact sampling in an ML context. Our approach is illustrated on several examples.

Identification of Large Jumps in Daily Share Prices of Stock Index Using a Jump Diffusion Model

Shuya Kanagawa
Tokyo City University, Japan

We investigate the daily share prices of the Nikkei 225 stock index to identify large jumps in daily share prices of the stock index using a jump diffusion model, which consists of the Black-Scholes model with stochastic volatility and a compound Poisson process. The volatility of the stock index is estimated by the historical volatility from the observation of daily share prices. We also refer to the number of daily share prices for historical volatility and show that the number is essential for the accuracy to identify large jumps.

Optimal Dosing Strategies in Radiotherapy

Adnan Khan
Lahore University of Management Sciences, Pakistan
Asgher Ali

Radiotherapy is an effective tool in the treatment of cancerous tumors. There are a variety of dosing regimens that can be used, from a large single dose or a few large doses (hypofractionation) to many smaller doses (hyperfractionation). The effectiveness of each strategy depends on a number of factors, including the tumor volume and the surrounding organs at risk. The most common tool for determining a time-dose relationship in radiotherapy is the Linear Quadratic (LQ) model. It has been shown that different mechanistic radiobiological models result in the LQ relationship for dose fractionation in some approximation.
In this study we determine the optimal radiation dosing strategy, using the full mechanistic models. We compare our results to those derived using the LQ formalism.

A Nonlocal Degenerate Parabolic Equation Arising in Game Theory

Johannes Lankeit
Paderborn University, Germany

We will consider the nonlocal degenerate parabolic equation

$$u_t = u\Delta u + u\int_\Omega |\nabla u|^2,$$

which arises in evolutionary game theory, and discuss recent results concerning existence of solutions, blow-up and long term behaviour. In particular, we will see how temporal asymptotics may depend on the decay of initial data in a counterintuitive way.

Some Gas-Like Models for Random Markets

Ricardo Lopez-Ruiz
University of Zaragoza, Spain

Some economic gas-like models for random conservative markets are addressed. In these models the agents trade by pairs bringing the system toward an statistical equilibrium, this is the asymptotic wealth distribution. The time evolution of these models are given by nonlinear functional mappings. These maps are nonlinear operators in the space of wealth distributions, which are shown to conserve the total and mean wealth of the economic system, and even an H-Theorem can be verified for some cases. Different asymptotic results for several models are presented. The decay to the exponential distribution is found in some of them and a transition to power-like distributions is sketched when a naive bank system is suggested. Simulations and implementations of these systems in different topologies are also presented.

Pattern Formation on Biological Evolving Surfaces: Modelling, Numerics and Applications

Anotida Madzvamuse
University of Sussex, England
Raquel Barreira, Charles M. Elliott

In this talk we propose models and a numerical method for pattern formation on evolving curved surfaces. We formulate reaction-diffusion equations on evolving surfaces using the material transport formula, surface gradients and diffusive conservation laws. The evolution of the surface is defined by a material surface velocity. The numerical method is based on the evolving surface finite element method (ESFEM). The key idea is based on the approximation of the continuous surface by a triangulated surface consisting of a union of triangles with vertices on the continuous surface. A finite element space of functions is then defined by taking the continuous functions on the triangulated surface which are linear affine on each simplex of the polygonal surface. To demonstrate the capability, flexibility, versatility and generality of our methodology we present results for uniform isotropic growth as well as anisotropic growth of the evolution surfaces and growth coupled to the solution of the reaction-diffusion system. The surface finite element method provides a robust numerical method for solving partial differential systems on continuously evolving domains and surfaces with numerous applications in developmental biology, tumour growth and cell movement and deformation.

Stability Index for the Characterization of Riddled Basin in a Coupled Dynamical System

Ummu Atiqah Mohd Roslan
University Malaysia Terengganu, Malaysia

We consider a coupled dynamical system with a Milnor attractor whose basin of attraction is riddled with the basin of a second attractor. We first study how the global geometry of the basin of attraction changes as we vary the parameter in the system. Secondly, we focus on the local geometry of the riddled basin of attraction. To characterize this riddled basin, we compute a stability index for the attractor in the system. Our numerical results show that for Lebesgue almost all points in the attractor, the index is positive for some parameter region where the riddled basin occurs.

Polar Differentiation Matrices and Applications

Marcela Molina Meyer
Universidad Carlos III, Spain
Frank Prieto Medina

The Laplace operator is used in mathematical models of growth and interactions between of species. In this talk we deduce the way of dealing, with no lifting, with Dirichlet, Neumann or Robin nonhomogeneous boundary conditions in a polar differentiation matrix for the Laplace operator in a disk.

First Integrals and Exact Solutions of Some Dynamical Systems

Imran Naeem
Lahore University of Management Sciences, Pakistan
B.U. Haq

This article investigates the first integrals and closed form solutions of some non-linear first order dynamical systems from diverse areas of applied mathematics. We introduce the notion of artificial Hamiltonian and we show that every first order system of ordinary differential equations (ODEs) can be written in the form of an artificial Hamiltonian system. One can also express the second order ODE or system of second order ODEs in the form of system of first order artificial Hamiltonian system. Then the partial Hamiltonian approach is employed to compute the partial Hamiltonian operators and the corresponding first integrals. The first integrals are utilized to construct the closed form solutions of laser photon model, duffing van-der pol oscillator and nonlinear optical oscillators under parameter restrictions. We show that how one can apply the existing partial Hamiltonian approach for nonstandard Hamiltonian systems. This study provides a new way of solving the dynamical systems of first order ODEs, second order ODE and second order systems of ODEs which are expressed into the artificial Hamiltonian system.

Exact Solutions Via Invariant Approach for Black-Scholes Model with Time-Dependent Parameters

Rehana Naz
Lahore School of Economics, Pakistan
A.G. Johnpillai

We analyze the Black-Scholes model with time-dependent parameters and it is governed by a parabolic partial differential equation (PDE). First, we compute the Lie symmetries of the Black-Scholes model with time-dependent parameters. It admits six plus infinite many Lie symmetries and thus it can be reduced to the classical heat equation. We utilize the invariant criteria for a scalar linear (1+1) parabolic PDE and obtain two sets of equivalence transformations. With the aid of these equivalence transformations, the Black-Scholes model with time-dependent parameters transforms to the classical heat equation. Moreover, the functional forms of the time-dependent parameters in the PDE are determined via this method. Then we utilize the equivalence transformations and known solutions of the heat equation to establish a number of exact solutions for the Black-Scholes model with time-dependent parameters.

Bifurcation Structures of a Cobweb Model with Memory and Competing Technologies

Nicolo Pecora
Catholic University, Italy

In this paper we study a simple model based on the cobweb demand-supply framework with costly innovators and free imitators. The evolutionary selection between technologies depends on a performance measure which is related to the degree of memory. The resulting dynamics is described by a two-dimensional map. The map has a fixed point which may lose stability either via supercritical Neimark-Sacker bifurcation or flip bifurcation and several multistability situations exist. We describe some sequences of global bifurcations involving attracting and repelling closed invariant curves. These bifurcations, characterized by the creation of homoclinic connections or homoclinic tangles, are described through several numerical simulations. Particular bifurcation phenomena are also observed when the parameters are selected inside a periodicity region. The analysis gives us the opportunity to discover a peculiar behavior occurring within such region. In fact we find that, unlike what usually takes place inside an Arnold tongue in a neighborhood of the Neimark-Sacker bifurcation curve, (i.e. a periodic orbit exists and the closed invariant curve is made by a saddle-node connection), an eventuality of multistability with a cycle and a closed invariant curve may arise. This leads us to conjecture the existence of further global bifurcations recurring within the periodicity region emanating from the Neimark-Sacker bifurcation curve, and the Arnold tongue seems to be a subset of a larger periodicity region.

Method of Integral Momenta for Gas-Dynamics-Like Systems

Olga Rozanova
Moscow State University, Russia

Total mass, energy and momentum are natural integral characteristics for many systems of PDEs describing meaningful models - from gas dynamics to liquid crystals, chemotaxis, self-gravitating gas, granulated media, economical mean field games, etc. All these equations have some common structure, which allows us to consider them as gas-dynamics-like systems. It is possible to introduce others integral quantities related to momentum of mass and to obtain a nonlinear ODE system of equalities and inequalities even in multidimensional case. Analysis of these systems is helpful for many purposes. It allows to obtain nonexistence result for the solutions to the Cauchy problem and free boundary problem, study the propagation of support of solution, and construct special classes of classical solution.

Solvability of Vertical Transmission and Cure of Vector-Borne Disease PDE Model

Lingeshwaran Shangerganesh
National Institute of Technology Goa, India
Manimaran Jayaraj

Vector-borne diseases are contagious diseases and it is spreaded by vectors. Vectors are living organisms can spread contagious diseases between from one host (carrier) to another. In this work, we discuss the problem of the nonlinear coupled reaction-diffusion equation, which incorporates the vector-borne diseases with vertical transmission and cure. The total population size can be divided into human hosts and vectors are denoted by N_1 and N_2. The host population can be divided into four subclasses represented by S, I, T, and R for the susceptible, infectious, under treatment and recovered classes. Thus, $N_1 = S + I + T + R$. The main aim of this work is to investigate the existence and uniqueness of weak solution of the above proposed model. Finally, numerical examples are given to illustrate the validity of theoretical results.

Discontinuous Coefficient Diffusion Models of Neurotransmitter Release for Independent Synaptic Currents

Jianzhong Su
University of Texas-Arlington, USA
Sat Byul Seo, Ege Kavalali

Synapses play a major role in neuron communications in the brain. The synapses act through a chemical process called synaptic fusion between pre-synaptic and post-synaptic terminals. In the paper, we develop a mathematical model in 3-D to emulate spontaneous and evoked neurotransmissions resulted from glutamate release within a single synapse. We propose numerical methods for solving piecewise continuous heat diffusion equation, estimate and verify its errors of second order accuracy. In order to identify the spatial relation between spontaneous and evoked glutamate releases, we consider quantitative factors, such as the size of synapses, inhomogeneity of diffusion coefficients, the geometry of synaptic cleft, and the release rate of neurotransmitter, that will affect postsynaptic currents. The computed results match well with existing experimental findings and provide a quantitative map of boundaries of physical constraints for having independent synaptic fusion events.

Parameter Estimated Standardized U-Statistics for Some Dependent Sequence

Hiroshi Takahashi
Tokyo Gakugei University, Japan

Gombay and Horvath obtained limit theorems for the maximum of standardized degenerate U-statistics when parameters are estimated. They considered the case where the observed data are independent. It is natural to consider dependent data because a stochastic model for a time series reflects past observations. In this talk, we show the theorems in the case of dependent data. We also give some examples concerning some finance models.

Simply Improved Averaging of Coupled Oscillators and Weakly Nonlinear Waves

Molei Tao
Georgia Tech, USA

In a system $\dot{X} = \Omega X + \epsilon F(X, t)$ where Ω is a matrix with all imaginary eigenvalues, the weak nonlinearity F can lead to highly nontrivial long time dynamics by interacting with fast oscillations generated by Ω. The method of averaging can characterize this interaction and provide an approximation of the long time effect of the nonlinearity. This talk will describe a simple way for improving the accuracy of 1st-order averaging, without having to resort to the much harder task of 2nd-order averaging. The efficacy of this improvement will also be demonstrated by examples, such as a new engineering device for wireless energy transfer, the Fermi-Pasta-Ulam problem, and a weakly-nonlinear advection-reaction PDE (note the method can be generalized to Ω being an anti-Hermitian operator). If time permits, numerical averaging will be discussed too.

Catch-To-Stock Dependence in Small Pelagic Fishery with Bounded Harvesting Effort

Olga Vasilieva
Universidad del Valle, Colombia
Erica Cruz-Rivera, Hector Ramirez

Empirical evidence and biological characteristics of pelagic fish suggest that, in contrast to traditional fishery models, marginal catch of pelagic species does not react in linear way to changes in stock level. In this presentation, we allow non-linearity in the harvesting function of Cobb-Douglas type and propose a variant of single-stock harvesting model with invariable “catch-to-effort” parameter and variable “catch-to-stock” parameter that measures the sensitivity of an additional catch yield to marginal changes in the fish stock level. Using the optimal control modeling framework, we provide an explicit form of sustainable fishing policy in terms of stationary stock that

continuously depends upon the "catch-to-stock" parameter and then focus on analysis of the equilibrium responses to changes in this parameter induced by external perturbations.

Population Dynamics of *Wolbachia* Invasion in Wild *Aedes Aegypti* Populations

Olga Vasilieva
Universidad del Valle, Colombia
Oscar E. Escobar-Lasso

Wolbachia-based biocontrol has recently emerged as a potential method for prevention and control of dengue and other vector-borne diseases. Major vector species, such as *Aedes aegypti* females, when deliberately infected with *Wolbachia* become less capable of getting viral infections and transmitting the virus to human hosts. *Wolbachia* is a bacterial symbiont that is maternally transmitted from one generation to another through insect's eggs and its spread among wild population is favored by a particular reproductive phenotype of cytoplasmic incompatibility. On the other hand, *Wolbachia* may reduce the host lifespan and fecundity; therefore, population dynamics of *Wolbachia* invasion usually exhibits very interesting features such as bistability and frequency-dependent Allee effect. In this presentation, we propose and qualitatively analyze an explicit sex-structured population model for *Wolbachia* invasion that describes an interaction of four mosquito compartments: *Wolbachia*-infected and wild males and females. Further, we introduce a simplified two-dimensional variant of the original four-dimensional model in order to obtain better visualization of the *Wolbachia* invasion dynamics, and to identify the minimum viable population size of wild mosquitoes. The latter is a threshold in frequency between wild and *Wolbachia*-infected mosquitoes below which *Wolbachia* gets established in local populations while wild mosquitoes become extinct.

Global Well-Posedness of Infectious Disease Models Without Life-Time Immunity: the Cases of Cholera and Avian Influenza

Kazuo Yamazaki
University of Rochester, USA

We study the systems of partial differential equations with diffusion that model the dynamics of infectious diseases without life-time immunity, in particular the cases of cholera and avian influenza. In both works, similarly to all others in the literature on various models of infectious diseases and more, it had to be assumed for a technical reason that the diffusivity coefficients of the susceptible, infected and recovered individuals, humans or birds, had to be identical in order to prove the existence of their unique solutions for all time. Considering that such uniform diffusivity strengths among the susceptible, infected and recovered hosts may not always be plausible in real world, we investigate the global well-posedness issue when such conditions are relaxed. In particular for the cholera model, we prove the global well-posedness with no condition on the diffusivity coefficients at all. For the avian influenza model, we prove the global well-posedness with no condition on the diffusivity coefficients if the spatial dimension is one, and under a partial condition that the diffusivity coefficients of the susceptible and the infected hosts are same otherwise.

Special Session 134: Recent Advances on Structure and Property-Preserving Numerical Approximations to PDEs

Qi Wang, University of South Carolina, USA
Yuezheng Gong, Nanjing University of Aeronautics and Astronautics, Peoples Rep of China
Jia Zhao, Utah State University, USA

Many partial differential equation models are based on certain physical laws or backgroud, such that the solutions intrinsically should satisfy the corresponding structure and properties, such as energy/mass conservation, energy dissipation and positiveness. It raises tremendous difficulties, and attention as well, to develop numerical schemes such that the numerical solutions preserve such structure and properties in the discrete level. In this session, we will gather experts in this field to discuss recent advances in developing structure and property preserving numerical schemes on approximating PDEs.

Second Order Fully-Discrete Energy Stable Methods on Staggered Grids for Hydrodynamic Phase Field Models of Binary Fluid Mixtures

Yuezheng Gong
Nanjing University of Aeronautics and Astronautics, Peoples Rep of China
Jia Zhao, Qi Wang

In this talk, we present fully-discrete, energy stable methods on a spatially staggered grid for a hydrodynamic phase field model of binary viscous fluid mixtures in a confined geometry subject to both physical and periodic boundary conditions. We apply the energy quadratization (EQ) strategy to develop a linear-implicit scheme. Then we extend it to a decoupled, linear scheme by introducing an intermediate velocity term, where the phase variable, velocity field and pressure can be solved sequentially. The two new fully discrete linear schemes are then shown to be unconditionally energy stable and the linear systems resulted from the schemes are proved uniquely solvable.

A Degenerate Cahn-Hilliard-Stokes-Darcy Model for Two-Phase Flow in Karst Geometry

Daozhi Han
Missouri University of Science and Technology, USA

In this talk we derive a degenerate Cahn-Hilliard-Stokes-Darcy model for coupled two-phase flows in conduit and in porous medium. In addition to an energy law, the model satisfies an entropy estimate which dictates the phase filed variable in its physical bound. We then present a bound-preserving fractional step method based on variational inequality for solving the model.

A Modified Cahn-Hilliard-Navier-Stokes Model with Interfacial Profile Correction Term

Yibao Li
Xian Jiaotong University, Peoples Rep of China
Jung-Il Choi, Junseok Kim

In this talk, we will introduce a new phase-field fluid model and computation with minimized Cahn-Hilliard (CH) dynamics. Using the CH equation, the internal structure of the interface layer is determined by explicit smoothing flow discontinuities. This method greatly simplifies gridding, discretization, and handling of topological changes. The original CH equation, however, has intrinsic dynamics such as interface length minimization, i.e., the motion by minus the Laplacian of the mean curvature. When the CH equation is applied to the modeling of multiphase fluid flows, we want to minimize its interface length minimization property. The surface tension formulation also requires the multiphase fluid interface to be a hyperbolic tangent profile. Typically, under the advection of flow, the interfacial transition is not a hyperbolic tangent profile, i.e., it is too compressed or sharpened. Even though the original CH dynamics conserves the total mass, the enclosed area obtained by its interface is not preserved. To overcome these shortcomings, we propose a modified CH equation with an interfacial profile correction term. Several numerical examples are presented to show the accuracy of the proposed method. The numerical results demonstrate that the proposed modified CH equation preserves the enclosed area better than the original CH equation.

An Energy Law Preserving Finite Element Scheme for Moving Contact Line Problems

Ping Lin
University of Dundee, Scotland

In this talk, we consider a phase field model of Cahn-Hilliard type for moving contact line problems governing the motion of isothermal multiphase incompressible fluids. The generalized Navier boundary condition proposed by Qian et al. is adopted. We discretize model equations using a continuous finite element method in space and a modified midpoint scheme in time. A discrete energy law which is a good approximation of the continuous energy law is derived for the scheme. Two kinds of immiscible fluids in a pipe and droplet displacement with a moving contact line under the effect of pressure driven shear flow are computed using a relatively coarse grid. We also derive the discrete energy law for the droplet displacement case, which is a slightly different problem due to the boundary condition. Accuracy and stability of the scheme are validated through some test computations.

Numerical Solutions for a Phase Field Model with Peng-Robinson Equation of State

Qiujin Peng
Renmin University of China, Peoples Rep of China

This work is concerned with mathematical modeling and numerical simulations of the steady state and the movements of complex fluids involved in oil exploitation practice. Capillary pressure caused by surface tension at the interface between every two adjacent different phases of the mixture is viewed as the leading force in oil recovery from fractured oil reservoirs. Therefore, the interface between contiguous phases has become a critical mathematical modeling aspect. The diffuse interface theory is our institutive choice. The density of the homogeneous part and the parameters of the gradient part of the Helmholtz free energy are provided by the widely used Peng-Robinson equation of state (EOS). The fourth-order parabolic equation is derived and solved numerically by several energy stable schemes to describe the evolution processes of one-component, two-phase substances. The theoretical analyses of these numerical schemes have been obtained to demonstrate their mass conservation, energy stability, unique solvability and convergence.

The SAV Approach for Gradient Flows: Error Analysis and Applications

Jie Shen
Purdue University and Xiamen University, USA

Recently, a new technique, the single auxiliary variable (SAV) approach, is proposed to deal with nonlinear terms in a large class of gradient flows. The technique is not restricted to specific forms of the nonlinear part of the free energy, it leads to linear and unconditionally energy stable second-order (or higher-order with weak stability conditions) schemes which only require solving decoupled linear equations with constant coefficients. Hence, these schemes are extremely efficient as well as accurate. We present a convergence and error analysis of the SAV approach ,and apply the SAV approach to deal with several challenging applications which can not be easily handled by existing approaches, and present convincing numerical results to show that the new schemes are not only much more efficient and easy to implement, but also can better capture the physical properties in these models.

Energy-Conserving Hamiltonian Boundary Value Methods for the Numerical Solution of the Korteweg-de Vries Equation

Yajuan Sun
AMSS, CAS, Peoples Rep of China
Luigi Brugnano, Gianmarco Gurioli

In this talk, the energy-preserving numerical methods for the Korteweg–de Vries equation are studied by combining HBVM (Hamiltonian Boundary Value method) in time and Fourier-Galerkin discretization in space. The efficient implementation of the methods for the resulting problem is considered. Also the numerical errors of solutions and conservative quantities are reported.

Structuring Preserving Numerical Approximations to Thermodynamically Consistent Models

Qi Wang
CSRC/Univ. of South Carolina, Peoples Rep of China
Jun Li, Jia Zhao, Xueping Zhao

Thermodynamically consistent models for any materials systems are the models that satisfy the conservation laws required by the physical systems and the thermodynamical laws for energy and entropy or equivalenetly by the Onsager principle for entropy production or energy dissipation. These models possess a special mathematical structure which can be exploited to yield energy stable schemes. This mathematical property of the models can also be exploited to produce positivity preserving schemes. Guided by the thermodynamical property, many nice numerical approximations can be developed to capture the physical properties of the underlying continuum models. We will present some strategies to render high order numerical schemes for such physical models.

Special Session 136: PDEs from Mathematical Physics and Geometry

Bongsuk Kwon, UNIST, Korea
Jinmyoung Seok, Kyonggi University, Korea

Mathematical physics and geometry are two fundamental sources of nonlinear PDEs. In this special session, we plan to bring active researchers together to discuss recent developments in the theory of PDEs arising from mathematical physics and geometry.

Effective Model for Heat Transfer in Channel with Rough Surface

Jaewook Ahn
Chung-Ang University, Korea
Jung-Il Choi, Kyungkeun Kang

We concern heat transfer by forced convection in a channel flow with rough boundary. In heat transfer system, the understanding of roughness-induced effects has been the subject of many studies because there are many practical applications. The problem is that resolving heat transfer motion over the rough surfaces with enforcing boundary condition on the rough geometry is impossible or inefficient in many situations of physical relevance. Therefore, developing an effective model reflecting the effects of the rough boundary is needed. In this talk, we introduce such an effective model in a channel. As a result, the criteria for the shape of the rough boundary on the heat transfer rate is established. Meanwhile, mathematical justifications for the effective model and wall law model are provided.

Analyticity of Navier-Stokes Equations

Hantaek Bae
Ulsan National Institute of Science and Technology, Korea

In this talk, I will present how to show analyticity of solutions to the inhomogeneous incompressible Navier-Stokes equations the barotropic compressible Navier-Stokes equations using the so-called Gevrey regularity.

A Kinetic Description for Sheath Formation

Sun-Ho Choi
Kyung Hee University, Korea
Bongsuk Kwon

Sheath formation is a kind of phase state of gas. Shortly, phase transition diagram is solid-liquid-gas-plasma-sheath. In this talk, I would like to present a kinetic description for sheath formation. The topic is a stationary Vlasov-Poisson system representing ion density on symmetric container with centered negative charged ball. Here, the stationary Vlasov-Poisson system is equivalent to the Gelfand equation with Neumann boundary conditions. The Neumann boundary conditions come from mass conservation and electric charge conservation. In order to prove the existence of the solution to the Gelfand equation, we first transform this problem into terminal value problem.

On the Energy Minimizing Solutions for the Lane-Emden System on Bounded Domain

Woocheol Choi
Incheon National University, Korea
Seunghyeok Kim

In this talk, we are concerned with the Lane-Emden system $-\Delta u = v^p$ and $-\Delta v = u^q$ posed on bounded domain with the Dirichlet zero boundary condition. For this system, if p and q satisfy

$$\frac{1}{p+1} + \frac{1}{q+1} = \frac{n}{n-2} - \epsilon$$

with very small $\epsilon > 0$, then it is believed that most of the solutions are usually spike-shaped and their asymptotic behavior can be described as it was done for the single equation $-\Delta u = u^{(n+2)/(n-2)-\epsilon}$. However, due to the technical difficulty, this type of analysis has been verified only for the energy-minimizing solutions by Guerra (08‘) with an additional assumption that $\min p, q \geq 1$ and Ω is convex, and he conjectured that the same asymptotic behavior would be true without the assumption $\min p, q \geq 1$. In our work, we settle this conjecture and also remove the convexity condition on the domain. This is based on a joint work with Seunghyeok Kim (KIAS).

Multipilicity of Solutions of the Self-Dual Einstein-Maxwell-Higgs Equation on Compact Surfaces

Jongmin Han
Kyung Hee University, Korea

In this talk, we introduce the self-dual Einstein-Maxwell-Higgs equation on compact surfaces. Under the suitable assumptions on string numbers, we prove the multiple existence of solutions.

Singularity Formation for the 2D Boussinesq System

Injee Jeong
KIAS, Korea
Tarek Elgindi

We consider the 2D Boussinesq system on sectors with angle less than π, and show that there exists Lipschitz continuous velocity field and density pair (u_0, ρ_0) which becomes singular in finite time. The initial data can be compactly supported and in particular the solution has finite energy. The proof consists of three parts: local well-posedness for the Boussinesq equation in critical spaces, the analysis of exactly scale-invariant solutions, and finally a cut-off argument. We also discuss implications of these results to the issue of singularity formation for the 3D Euler equations in the axi-symmetric geometry.

Remarks on Chern-Simons Gauged $O(3)$ Sigma Model in One Space Dimension

Guanghui Jin
National Center for Theoretical Sciences, Peoples Rep of China
Hyungjin Huh

In this talk, we find the stationary solutions of the Chern-Simons gauged $O(3)$ sigma model in one space dimension and study an initial value problem under the gauge condition $A_1 = 0$. We also study low regularity local well-posedness using bilinear estimates for wave-Sobolev space $\mathcal{H}^{1,\theta}$. As by-product, finite energy global solution is constructed by considering the conservation of energy.

Magnetic Field Induced Transition in Nematic Liquid Crystal Flows

Soojung Kim
Korea Institute for Advanced Study, Korea

In this talk, we study the dynamical instability of liquid crystals induced by applied magnetic fields, which is the so-called Freedericksz transition. The model under consideration is the simplified Ericksen-Leslie system for the hydrodynamic flow of a nematic liquid crystal with the magnetic effect. We show that if the magnetic field strength exceeds a threshold, any global solution converges exponentially to a unique nontrivial equilibrium. It is also proved that below the threshold, the equilibrium of the orientation angle is zero. This is based on joint work with Yuan Chen and Yong Yu.

On Elliptic-Hyperbolic Problem in One Space Dimension

Seonghak Kim
Kyungpook National University, Korea
Hyung Jun Choi

In this talk, I present a recent result on the initial-boundary value problem of a quasilinear elliptic-hyperbolic equation in one space dimension. A local-in-time weak solution to the problem is constructed by reformulating the problem as a partial differential inclusion and then appealing to the convex integration method through a sophisticated in-approximation scheme. In this way, the constructed solution exhibits a fine phase mixture in some space-time region on which the spatial derivative of the solution enters into the mixed elliptic-hyperbolic regime. I also add a numerical simulation through the FEM to show how such a solution can be well implemented.

The Existence of Vector Soluiton for Coupled Nonlinear Schrödinger Systems

Ohsang Kwon
Chungbuk National University, Korea
Jaeyoun Byeon, Jinmyoung Seok

In this talk, we consider coupled nonlinear Schrödinger systems. Using the variation method, we study the existence of its vector solution.

Existence of Non-Topological Solutions in the SU(3) Chern-Simons Model

Youngae Lee
NIMS, Korea
Chang-Shou Lin, Ting-Jung Kuo

We consider non-topological solutions of a nonlinear elliptic system problem derived from the SU(3) Chern-Simons models. The existence of non-topological solutions even for radial symmetric case has been a long standing open problem. Recently, Choe, Kim, and Lin showed the existence of radial symmetric non-topological solution when the vortex points collapse. However, the arguments in that paper cannot work for an arbitrary configuration of vortex points. In this talk, I introduce a new approach by using different scalings for different components of the system to construct a family of partial blowing up non-topological solutions. This talk is based on the joint work with Prof. Chang-Shou Lin and Prof. Ting-Jung Kuo.

Homogenization of Elliptic and Parabolic Soft Inclusions

Minha Yoo
National Institute for Mathematical Sciences, Korea
Ki-Ahm Lee

In this talk, we consider periodic Soft inclusion problems of elliptic and parabolic linear equation of non-divergence form. Usually, it is called Soft inclusion problems to find effective conductivity of composites consisting of a medium with non-conducting grains. Mathematically, non-conducting grains are described by union of disjoint holes with periodicity epsilon. For each epsilon, the unique current density function (the solution of epsilon problem) u-epsilon exists for a given boundary data. We note that, at the boundary of grains, the Neumann data of u-epsilon vanishes. Our main interest is to show the uniform convergence of u-epsilon and to find effective equation what the limit of u-epsilon satisfy.

Quantum BGK Model Near a Global Fermi-Dirac Distribution

Seok-Bae Yun
Sungkyunkwan University (SKKU), Korea
Gi-Chan Bae

In this talk, we consider the existence and asymptotic behavior of fermionic quantum BGK model, which is the relaxation model of the quantum Boltzmann equation for fermions, in the case when the initial data starts sufficiently close to a global Fermi-Dirac distribution. Two difficulties, among others, unobserved in the study of classical problems arise. First, the existence of the equilibrium parameters should be established through a set of nonlinear equations in each iteration step. Second, it is observed that the momentum weight imposed on the perturbation to derive the coercivity of the linearized relaxation operator should take a different form from that of the classical case.

Special Session 137: Analysis of Nonlinear Flows

Daneri Sara, FAU University Erlangen-Nürnberg, Germany

In this special session several topics in the analysis of fluids-dynamics will be covered: from Euler and Navier-Stokes equations to transport, porous media equations and several others, focusing on stability, existence, regularity or uniqueness isssues.

A Uniqueness Result for the Decomposition of Vector Fields in R^{D+1}

Stefano Bianchini
SISSA, Italy
Paolo Bonicatto

Given a vector field $\rho(1,\mathbf{b}) \in L^1_{\rm loc}(\mathbb{R}^+ \times \mathbb{R}^d, \mathbb{R}^{d+1})$ such that $\mathrm{div}_{t,x}(\rho(1,\mathbf{b}))$ is a measure, we consider the problem of uniqueness of the representation η of $\rho(1,\mathbf{b})\mathcal{L}^{d+1}$ as a superposition of characteristics $\gamma : (t^-_\gamma, t^+_\gamma) \to \mathbb{R}^d$, $\dot\gamma(t) = \underline{(}t, \gamma(t))$. We give conditions in terms of a local structure of the representation η on suitable sets in order to prove that there is a partition of $\mathbb{R}^{d+1}$ into disjoint trajectories $\wp_\mathfrak{a}$, $\mathfrak{a} \in \mathfrak{A}$, such that the PDE

$$\mathrm{div}_{t,x}\big(u\rho(1,\mathbf{b})\big) \in \mathcal{M}(\mathbb{R}^{d+1}), \qquad u \in L^\infty(\mathbb{R}^+ \times \mathbb{R}^d),$$

can be disintegrated into a family of ODEs along $\wp_\mathfrak{a}$ with measure r.h.s.. The decomposition $\wp_\mathfrak{a}$ is essentially unique. We finally show that $\mathbf{b} \in L^1_t(\mathrm{BV}_x)_{\rm loc}$ satisfies this local structural assumption and this yields, in particular, the renormalization property for nearly incompressible BV vector fields.

Untangling of Trajectories for Non-Smooth Vector Fields and Bressan's Compactness Conjecture

Paolo Bonicatto
University of Basel, Italy
S. Bianchini

Given $d \geq 1$, $T > 0$ and a vector field $\mathbf{b}\colon [0,T] \times \mathbb{R}^d \to \mathbb{R}^d$, we study the problem of uniqueness of weak solutions to the associated transport equation $\partial_t u + \mathbf{b} \cdot \nabla u = 0$ where $u\colon [0,T] \times \mathbb{R}^d \to \mathbb{R}$ is an unknown scalar function. In the classical setting, the method of characteristics is available and provides an explicit formula for the solution of the PDE, in terms of the flow of the vector field $\mathbf{b}$. However, when we drop regularity assumptions on the velocity field, uniqueness is in general lost. In the talk we will present an approach to the problem of uniqueness based on the concept of Lagrangian representation. This tool allows to represent a suitable class of vector fields as superposition of trajectories: we will then give local conditions to ensure that this representation induces a partition of the space-time made up of disjoint trajectories, along which the PDE can be disintegrated into a family of 1-dimensional equations. We will finally show that if $\mathbf{b}$ is locally of class BV in the space variable, the decomposition satisfies this local structural assumption: this yields in particular the renormalization property for nearly incompressible BV vector fields and thus gives a positive answer to the (weak) Bressan's Compactness Conjecture.

On the Convergence of Statistical Solutions

Anne Bronzi
University of Campinas, Brazil
Cecilia Mondaini, Ricardo Rosa

In this talk we will present an abstract framework for the theory of statistical solutions for general evolution equations. This theory extends the notion of statistical solutions initially developed for the 3D incompressible Navier-Stokes equations to other evolution equations that have global solutions which are not known to be unique. We will prove the convergence of statistical solutions of regularized evolution equations to statistical solutions of the original one. The applicability of the theory will be illustrated with the 2D inviscid limit, that is, the convergence of statistical solutions of the 2D Navier-Stokes to the statistical solutions of the 2D Euler equations.

PDE Analysis of a Class of Thermodynamically Compatible Viscoelastic Compressible and Incompressible Rate-Type Fluids with Stress-Diffusion

Miroslav Bulicek
Charles University, Czech Rep

We consider a system of pde's that governs the motion of non-Newtonian fluids described by a simplified viscoelastic rate-type model with a stress-diffusion term. The simplified model shares many qualitative features with more complex viscoelastic rate-type models that are frequently used in the modeling of fluids with complicated microstructure. As such, the simplified model provides important preliminary insight into the mathematical properties of these more complex and practically relevant models of non-Newtonian fluids. The simplified model that is analyzed from the mathematical perspective is shown to be thermodynamically consistent, and we extensively comment on the interplay between the thermodynamical background of the model and the mathematical analysis of the corresponding initial-boundary-value problem. We present results for incompressible as well as for compressible heat conducting fluids.

Longtime Behavior of Solutions to the 2D Keller-Segel Equation with Degenerate Diffusion

Jose Antonio Carrillo
Imperial College London, England

The Keller-Segel equation is a nonlocal PDE modeling the collective motion of cells attracted by a self-emitted chemical substance. When this equation is set up in 2D with a degenerate diffusion term, it is known that solutions exist globally in time, but their long-time behavior remains unclear. In a joint work with S. Hittmeir, B. Volzone and Y. Yao, we prove that all stationary solutions must be radially symmetric up to a translation, and use this to show convergence towards the stationary solution as the time goes to infinity.

Optimal Regularity for the Porous Medium Equation

Benjamin-Manuel Gess
MPI MIS Leipzig, Germany

We prove optimal regularity for solutions to porous media equations in Sobolev spaces, based on velocity averaging techniques. In particular, the obtained regularity is consistent with the optimal regularity in the linear limit. This improves previous results by Tadmor and Tao [Tadmor, Tao; CPAM, 2007].

On the Superposition Principle for Signed Measure-Valued Solutions of the Continuity Equations

Nikolay A. Gusev
Moscow Institute of Physics and Technology, Russia
P. Bonicatto

Let $b : \mathbb{R} \times \mathbb{R}^d \to \mathbb{R}^d$ be a bounded Borel vector field. Consider the continuity equation

$$\partial_t \mu_t + \operatorname{div}(b\mu_t) = 0$$

with respect to the measurable family $\{\mu_t\}_{t\in\mathbb{R}}$ of Borel measures on $\mathbb{R}^d$ (the equation is understodd in the sense of distributions). If the solution μ_t is non-negative, then the superposition principle holds: μ_t can be decomposed into measures concentrated on the integral curves of b. For smooth b this result follows from the method of characteristics, and in the general case it was established by L. Ambrosio. A partial extension of this result for signed measure valued solutions μ_t was obtained in a paper by L. Ambrosio and P. Bernard, where the following problem was proposed: does the superposition principle hold for signed measure-valued solutions in presence of unique flow of homeomorphisms solving the associated ordinary differential equation? We will present some related results (and counterexamples) obtained jointly with P. Bonicatto.

Regularity Estimates for Scalar Conservation Laws in One Space Dimension

Elio Marconi
University of Basel, Italy

We consider the scalar conservation law in one space dimension

$$u_t + f(u)_x = 0$$

and we study the regularizing effect that the non-linearity of the flux f has on the entropy solution u. More precisely, if the set $\{w : f''(w) \neq 0\}$ is dense, the regularity of the solution can be expressed in terms of BV^Φ spaces, where Φ depends on the non-linearity of f. If moreover the set $\{w : f''(w) = 0\}$ is finite, under the additional polynomial degeneracy condition at the inflection points, we prove that $f' \circ u(t) \in BV_{loc}(\mathbb{R})$ for every $t > 0$ and that this can be improved to $SBV_{loc}(\mathbb{R})$ regularity except an at most countable set of singular times.

On the Two-Dimensional Kuramoto-Sivashinsky Equation

Anna Mazzucato
Penn State University, USA
David Ambrose

I will discuss recent results concerning the Kuramoto-Sivashinky equation in two space dimensions with periodic boundary conditions. In particular, I will present a global existence result in the Wiener algebra, when growing modes are absent.

Non-Uniqueness for the Transport Equation with Sobolev Vector Fields

Stefano Modena
University of Leipzig, Germany
Làszlò Szèkelyhidi

We consider the linear transport equation $\partial_t \rho + u \cdot \nabla \rho = 0$, with unknown density ρ and given divergence-free vector field u, together with a given initial datum $\rho(x, 0) = \bar{\rho}(x)$. A celebrated result by DiPerna and Lions (1989) shows that if

$$\bar{\rho} \in L^p_x, \quad u \in L^1_t W^{1,\tilde{p}}_x$$

and

$$\frac{1}{p} + \frac{1}{\tilde{p}} \leq 1,$$

then the Cauchy problem admits a unique weak solution $\rho \in L^\infty_t L^p_x$. We show that the above condition is optimal: if

$$\frac{1}{p} + \frac{1}{\tilde{p}} > 1,$$

then there are divergence-free vector fields $u \in C_t W^{1,\tilde{p}}$ and initial data $\bar{\rho} \in L^p_x$ for which, despite the linearity of the equation, more than one weak solution in $C_t L^p_x$ exists. The result applies also to the transport-diffusion equation $\partial_t \rho + u \cdot \nabla \rho = \Delta \rho$.

Vortex Reconnection in the Three-Dimensional Navier-Stokes Equations

Daniel Peralta-Salas
ICMAT, Madrid, Spain

An important property of the 3D Euler equations is that the topology of the vortex structures of the fluid does not change in time as long as the solutions do not develop any singularities. To put it differently, the set of (say) vortex tubes and vortes lines of the fluid at time t is diffeomorphic to that of the initial vorticity, provided that the solution remains smooth up to this time. Of course, numerical simulations and experiments with real fluids have shown that the situation is completely different in the case of viscous fluids. In this talk I will show how vortex tubes and vortex lines, of arbitrarily complex topologies, are created and destroyed in smooth solutions to the 3D Navier-Stokes equations. This is joint work with Alberto Enciso and Renato Luca (Adv. Math. 309 (2017) 452-486).

A Quantiative Approach to the DiPerna–Lions Theory for Transport Equations

Christian Seis
University of Munster, Germany

In their celebrated theory of renormalized solutions, DiPerna and Lions (1989) establish well-posedness and stability properties for transport equations with Sobolev vector fields. In this talk, I present a new approach to well-posedness that is based on stability estimates for certain logarithmic Kantorovich–Rubinstein distances. The new approach recovers some of DiPerna's and Lions's old results. In addition, it allows for two new major applications that were inaccessible before: 1) We extend the theory to vector fields with L1 vorticities and present applications to the 2D Euler equation (joint work with G. Crippa, C. Nobili and S. Spirito). 2) We derive optimal estimates on the error of the numerical upwind scheme (joint work with A. Schlichting).

Special Session 138: Qualitative and Quantitative Properties of Quasilinear Elliptic and Parabolic Equations and Systems

Raul Manasevich, University of Chile, Chile
Marta Garcia-Huidobro, Pontifical Catholic University of Chile, Chile

In this session we want to gather together researchers working in existence, non existence and uniqueness of solutions of quasilinear elliptic and parabolic equations and systems in bounded or unbounded domains. Results showing connection with real world applications are welcome also.

Ground States of Elliptic Equations with Competition Between Power and Gradient Terms

Marie-Francoise Bidaut-Veron
University Francois Rabelais, Tours, France
Marta Garcia-Huidobro, Laurent Veron

Here we consider the nonnegative solutions of equations in a punctured ball $B(0, R) \setminus \{0\} \subset \mathbb{R}^N$ or in $\mathbb{R}^N$, of type

$$-\Delta u = u^p + M|\nabla u|^q$$

where $p, q > 1$ and $M \in \mathbb{R}$. We give new a priori estimates on the solutions and their gradient, and Liouville type results. We use Bernstein technique and Osserman's or Gidas-Spruck's type methods. The most interesting case is $q = 2p/(p + 1)$, where the equation is invariant by scaling. In the radial case, we give a precise description of all the solutions, improving the known results.

Large Time Behavior of Solutions of the Porous Medium Equation in Exterior Domains

Carmen Cortazar
U. Catolica de Chile, Chile
Fernando Quiros, Noemi Wolanski

We consider the porous medium equation in the complement of a bounded domain with zero Dirichlet data on its boundary and nonnegative compactly supported integrable initial data. Kamin and Vázquez, in 1991, studied the large time behavior of solutions of such problem in space dimension 1. Gilding and Goncerzewicz, in 2007, studied this same problem dimension 2. Using their results in the outer field we study the large time behavior of the solution in the near field scale, in particular in bounded sets of the domain. This a joint work with Fernando Quirós (Universidad Autonoma de Madrid, Spain) and Noemì Wolanski (Universidad de Buenos Aires, Argentina).

Construction of Solutions for Some Localized Nonlinear Schrödinger Equations

Matias Courdurier
Universidad Catolica de Chile, Chile
Olivier Bourget, Claudio Fernandez

For an N-body system of linear Schrödinger equation with space dependent interaction between particles, one would expect that the corresponding one body equation, arising as a mean field approximation, would have a space dependent nonlinearity. With such motivation we consider the following model of a nonlinear reduced Schrödinger equation with space dependent nonlinearity

$$-\varphi'' + V(x)h'(|\varphi|^2)\varphi = \lambda\varphi,$$

where $V(x) = -\chi_{[-1,1]}(x)$ is minus the characteristic function of the interval $[-1, 1]$ and where h' is any continuous strictly increasing function. In this talk, for any negative value of λ we present the construction and analysis of the infinitely many solutions of this equation, which are localized in space and hence correspond to bound-states of the associated time-dependent version of the equation.

Non Existence of Bound States for Time Dependent Evolution Equations

Claudio Fernandez
Universidad Catolica de Chile, Chile
M.A. Astaburuaga, O. Bourget, V. Cortes, M. Courdurier

We develop a method, based in the theory of positive commutators, for studying the existence of bound states for the propagator $(U(t, 0))$, at a fixed time t, associated to time dependent Schrödinger equation

$$i\,\varphi_t = H(t)\varphi.$$

We apply this method to shoe the non existence of solitons for the Hamiltonian of a quantum particle: $H(t) \equiv -\Delta + V(t)$, on $L^2(\mathbb{R}^n)$, in the so called repulsive case. We also show how to extend this result to other perturbations, including some nonlinear ones.

Some Results Concerning the Uniqueness of Sign Changing Bound State Solutions of a Weighted Semilinear Equation

Marta Garcia-Huidobro
Pontificia Universidad Catolica de Chile, Chile
C. Cortazar, P. Herreros

We consider radial solutions of a general elliptic equation involving a weighted Laplace operator with a subcritical nonlinearity. We establish the uniqueness of the higher radial bound state solutions

$$\operatorname{div}\big(\mathsf{a}\,\nabla u\big) + \mathsf{b}\, f(u) = 0\,, \qquad \lim_{|x|\to+\infty} u(x) = 0, \qquad (P)$$

where a and b are two positive, radial, smooth functions defined on $\mathbb{R}^d \setminus \{0\}$ and $f \in C(\mathbb{R})$ satisfying some growth conditions. We assume that the nonlinearity $f \in C(-\infty, \infty)$ is an odd function satisfying some convexity and growth conditions, and has one zero at $u_0 > 0$, is non positive and not identically 0 in $(0, u_0)$, positive in $[u_0, \infty)$, and is differentiable in $(0, \infty)$.

Multiplicity of Solutions for Some Elliptic Equations with a Gradient Term in the Nonlinearity

Ignacio Guerra
Universidad de Santiago, Chile

We consider the problem $-\Delta u = \lambda \frac{(1+|\nabla u|^q)}{(1-u)^p}$, in B, with $u = 0$ on ∂B, where B is the unit ball in $\mathbb{R}^N$, $p > 0$, $q \geq 0$ and $\lambda \geq 0$. The problem with $q = 0$ is well known. In fact, Joseph & Lundgren found that for a bounded range of dimensions there are infinitely many solutions for some $\lambda = \lambda_* > 0$. On the other hand, they also found that for large dimensions there exists λ^* such that there exists a unique solution for $\lambda \in (0, \lambda^*)$. In this talk, we present results of existence of solutions for this problem when $q > 0$. In this case, we found a rich structure of solutions depending on p, q and the dimension N. In addition, we study numerically the behaviour of solutions for related problems with a gradient term in the nonlinearity.

On Positive Solutions of the Lane-Emden Equation in the Plane

Nikola Kamburov
Pontificia Universidad Catolica de Chile, Chile
Boyan Sirakov (PUC-Rio)

We prove that positive solutions of the Lane-Emden equation in a two-dimensional smooth bounded domain are uniformly bounded for all large exponents. Recent work of De Marchis, Grossi, Ianni and Pacella provides a fairly complete asymptotic description of such solutions, under a certain integral bound. We show that this bound is always satisfied in star-shaped domains.

Some Nonlinear Systems of PDE Arising in Crime Modeling

Raul Manasevich
University of Chile, Chile

In this talk we present results for a nonlinear quasilinear system and a nonlinear parabolic system that come from the theory of repeat and near repeat in crime modeling. This is a rather new topic which is receiving increasing attention in many places. We will briefly review some of the important concepts that have originated something that we can call the beginning of a crime science. The subject has many different aspects, some of them give rise to semi-empirical results and some others to results coming from PDE and NLA.

Breathers and the Dynamics of Solutions to the KdV Type Equations

Claudio Munoz
University of Chile, Chile
Gustavo Ponce

In this paper our first aim is to identify a large class of non-linear functions f for which the IVP for the generalized Korteweg-de Vries equation does not have breathers or "small" breathers solutions. Also we prove that all small, uniformly in time $L^1 capH^1$ bounded solutions to KdV and related perturbations must converge to zero, as time goes to infinity, locally in an increasing-in-time region of space of order $t^{1/2}$ around any compact set in space. This set is included in the linearly dominated dispersive region $xllt$. Moreover, we prove this result independently of the well-known supercritical character of KdV scattering. In particular, no standing breather-like nor solitary wave structures exists in this particular regime.

Elliptic Equations with Hardy Potential and Gradient Nonlinearity

Tai Nguyen
Masaryk University, Czech Rep
Konstantinos Gkikas

In this talk, I will discuss the boundary value problem with measure data for equation (E) $-\Delta u - \frac{\mu}{\delta^2} u + g(|\nabla u|) = 0$ in a smooth bounded domain Ω, where μ is a parameter and δ denotes the distance function to $\partial\Omega$. I will show the existence and uniqueness result. I will also describe the isolated singularities of solutions.

Some Ideas on Inverse Problems for Water Waves

Jaime Ortega
Universidad de Chile, Chile
M.A. Fontelos, R. Lecaros, J.C. Lopez-Rios, G. Montecinos

The direct problem of water-wave equations is the problem of determining the surface, and its velocity potential, in time $T > 0$, for a given initial profile and velocity potential, where the profile of the bottom, the bathymetry, is known. In this talk, we present the inverse problem of recovering the shape of the solid bottom boundary of an inviscid, irrotational, incompressible fluid from measurements of a portion of the free surface. In particular, we will present some theoretical and numerical results for identifiability and numerical reconstruction of the bottom.

Nonlinear Elliptic Equations with Measure Valued Absorption Potential

Laurent Veron
University of Tours, France
Nicolas Saintier

We study the semilinear elliptic equation $-\Delta u + g(u)\sigma = \mu$ with Dirichlet boundary condition in a smooth bounded domain where σ is a nonnegative Radon measure, μ a Radon measure and g is an absorbing nonlinearity. We show that the problem is well posed if we assume that σ belongs to some Morrey class. Under this condition we give a general existence result for any bounded measure provided g satisfies a subcritical integral assumption. We study also the supercritical case when $g(r) = |r|^{q-1}r$, with $q > 1$ and μ satisfies an absolute continuity condition expressed in terms of some capacities involving σ

Existence Results for Nonlinear Boundary Value Problems with a Sign-Indefinite Weight

Fabio Zanolin
University of Udine, Italy

We present some recent results about existence and, in some cases, multiplicity of solutions to boundary value problems associated with the equation

$$u'' + a(t)g(u) = 0,$$

where $g : I \to \mathbb{R}_0^+$ is a (positive) function defined on an open interval I and $a : [0, T] \to \mathbb{R}$ is a sign-changing weight. We show some applications to the search of positive radially symmetric solutions for elliptic equations and to equations with singularities.

Special Session 139: Nonlinear Dynamics: Attractors, Patterns and Applications

Phillipo Lappicy, Universidade de São Paulo (USP), Brazil
Jia-Yuan Dai, Free University of Berlin, Germany
Chueh-Hsin Chang, Tunghai University, Taiwan

In this session we discuss several cutting-edge techniques to analyze nonlinear equations (such as ODEs, PDEs and delay differential equations) with the latest results on global attractors, pattern-formation, blow-up solutions and control theory. The main results have applications varying from general relativity to chemical reactions.

Non-Autonomous Morse-Smale Dynamical Systems

Alexandre Carvalho
University of Sao Paulo, Brazil
M. Bortolan, J. Langa and G. Raugel

In this lecture we present our recent results on structural stability of gradient Morse-Smale Dynamical Systems under non-autonomous perturbations. This is a joint work with G. Raugel (Paris XI), J. Langa (U. Sevilla) and M. Bortolan (UFSC-Brazil).

Heteroclinic Bifurcation of Three-Species Lotka-Volterra Competition-Diffusion Systems

Chueh-Hsin Chang
Tunghai University, Taiwan
Chiun-Chuan Chen

In population ecology, coexistence and exclusion are central problems for competiting species. Travelling wave solution can be partially used to answer the question about the competition behavior between species. In this talk, we consider the travelling waves of the 3-species Lotka-Volterra competition-diffusion systems. Such a travelling wave solution can be considered as a heteroclinic orbit of a vector field in R^6. Under suitable assumptions on the parameters of the equations, we apply bifurcation theories of heteroclinic orbits to show that a 3-species travelling wave can bifurcate from two 2-species waves which connect to a common equilibrium. The three components of the 3-species wave obtained are positive and have the profiles that one component connects a positive state to zero, one component connects zero to a positive state, and the third component is a pulse between the previous two with a long middle part close to a positive constant. This is a joint work with Professor Chiun-Chuan Chen.

Spectral Bounds Obtained by Reweighting Entries in Rows of a Matrix

Yen-Jen Cheng
National Chiao Tung University, Taiwan
Chih-Wen Weng

We establish a new approach to obtain sharp bounds of the spectral radius of a square matrix. The study of spectral radius has many applications, especially the stability of differential equations. The spectral radius is the largest absolute value of eigenvalues. The main idea of our approach is to reweight the entries such that their row sums do not change. With this approach, we can derive more sharp bounds then those in the literature in a systematic way.

A Comparison Between Estimates for the Fractal Dimension of Attractors

Arthur Cunha
University of São Paulo, Brazil

We describe methods to determine estimates for the fractal dimension of attractors in autonomous and non-autonomous dynamical systems in Banach and Hilbert spaces, these methods being strictly related with the process of covering of balls in spaces of infinity dimension. In the Banach case the strategy concerning the fractal dimension is based on approximations of the associated linearized system by finite dimensional spaces, while in the Hilbert one the treatment is by contraction of volumes. We present then comparisons between these methods in qualitative and quantitative aspects, with PDE's as potential applications.

Existence of Local Solutions of the Gowdy Spacetime on Three-Dimensional Tori

Jia-Yuan Dai
Free University of Berlin, Germany
Hannes Stuke

We consider a class of Gowdy spacetimes that reduces the Einstein's field equation to a system of two semilinear wave equations, by assuming a universe without matter, in which the gravitational wave fronts repeat in space and are mutually parallel. To prove the existence of local solutions of the system, in the literature only certain formal expansions near the Big Bang singularity are investigated. Nevertheless we aim at a rigorous proof. The main idea is to apply the Lyapunov-Schmidt reduction, for which we establish a functional setting that fits to the nonlinearity, and then solve the related small-divisor problem.

Unbounded Sturm Attractors for Quasilinear Equations

Phillipo Lappicy
Universidade de São Paulo, Brazil
Juliana Pimentel

We plan to construct explicitely the global attractors of quasilinear parabolic equations in one dimensional domain when solutions can grow-up, and hence there is a global attractor which is unbounded. In particular, we construct heteroclinic connections between bounded and/or unbounded hyperbolic equilibria.

Morse-Smale Transversality in Delay Equations with Monotone Feedback

Alejandro López Nieto
Free University Berlin, Germany

Morse-Smale transversality plays a crucial role in the description of the global attractors of a class of scalar parabolic partial differential equation on the circle. The crucial component necessary to prove transversality is a nodal property that this class of PDEs share with the technically completely unrelated family of scalar delay equations with monotone feedback. In the talk I will explore the way in which the nodal property can be used in order to prove Morse-Smale transversality for delay equations and I will present the array of possibilities that this unlocks for the study of global dynamics.

On Global Attractors for Dynamical Systems Without Natural Metrics

Junya Nishiguchi
Tohoku University, Japan

A global attractor is a notion for a topological semi-dynamical system whose phase space is a metric space. It should be noticed that the notion of a global attractor depends on the specific choice of a metric. In this talk, we "define" global attractors in the context of the "non-existence of natural metrics" of the phase space and study those properties. This includes a case where the phase space is a Frèchet space, which is motivated by differential equations with unbounded delay. We obtain sufficient conditions for the existence, which will be applied to such equations.

Limiting Grow-Up Behavior for a One-Parameter Family of Dissipative PDEs

Juliana Pimentel
Universidade Federal do ABC, Brazil
S. Bruschi, A.N. Carvalho

We provide a relation between the well known class of dissipative equations and the recently introduced class of slowly non-dissipative equations, in the setting of scalar reaction-diffusion equations. The latter type of equations is characterized by the existence of grow-up (i.e., infinite time blow-up) with absence of finite time blow-up. A particular small perturbation of an unbounded non-dissipative global attractor is considered, in such a way that the perturbed attractor is dissipative. Although the continuity of the family of attractors is verified in compact sets, our choice of perturbation produces a great change on the dynamics close to the infinity of the phase space. In other words, we prove that the limit of the compact attractors is not the unbounded attractor of the limiting equation.

Bioconvection Generated by Euglena Gracilis

Yuya Tokuta
Free University of Berlin, Germany

Microorganisms are known to form spatiotemporal patterns similar to those formed in the Rayleigh-Bénard model for thermal convection. Among such, Euglena gracilis form distinct patterns induced by phototaxes and sensitivity to the gradient of the light intensity. Mathematical analysis of patterns under stationary and oscillatory illumination conditions will be presented.

Sensitivity of Chemical Reaction Networks: Mathematical Approach and Techniques

Nicola Vassena
Free University of Berlin, Institut of Mathematics, Germany

We consider general systems of differential equations derived from chemical reaction networks,

$$\dot{x} = S\mathbf{r}(x).$$

Here, x is interpreted as the vector of the concentrations of chemicals, S is the stoichiometric matrix and $\mathbf{r}(x)$ is the vector of reaction functions, which we consider as **positive given parameters**. From an abstract network point of view: the vector x represents the vertices, the matrix S is the incidence matrix and the vector $\mathbf{r}(x)$ refers to the directed arrows. Sensitivity studies the response of equilibrium solutions to the perturbation of a single reaction function. In previous work, Fiedler, Matano, the author et al., were able to present systematic criteria, which distinguish zero response from nonzero response, for any other reaction function or metabolite concentration. Importantly, these results were only based on the network structure. Based on these works, we give here an overview of the results and techniques developed through this structural approach. In particular, we will focus on an extension of these results, which provides for the first time criteria for predicting the sign of any nonzero response, without requiring any additional input information.

Special Session 140: Classical and Geophysical Fluid Dynamics: Modeling, Analysis and Reduction

Mickael D. Chekroun, University of California, Los Angeles, USA
Honghu Liu, Virginia Tech, USA
Taylan Sengul, Marmara University, Turkey
Shouhong Wang, Indiana University Bloomington, USA

This special session aims to bring together researchers at all career stages to discuss recent advances as well as open problems in classical and geophysical fluid dynamics. A main focus will be on the development and usage of dynamical systems theories, in the broadest sense, to the understanding of dynamical behavior of the underlying governing equations. Topics that will be covered include modeling and mathematical analysis of turbulent flows in geophysical applications, state-of-the-art reduced order modeling techniques and parameterization techniques, the associated numerical implementations and usage in different contexts such as flow control, instabilities, bifurcation analysis and transition to turbulence, predictability and prediction, and uncertainty quantification.

Multiscale Stuart-Landau Emulators: Application to Wind-Driven Ocean Gyres

Mickael Chekroun
UCLA, USA
Dmitri Kondrashov, Pavel Berloff

The multiscale variability of the ocean circulation due to its nonlinear dynamics remains a big challenge for theoretical understanding and practical ocean modeling. This talk will present how the data-adaptive harmonic (DAH) decomposition and inverse stochastic modeling techniques introduced in (Chekroun and Kondrashov, (2017), *Chaos*, **27**), allow for reproducing with high fidelity the main statistical properties of multiscale variability in a coarse-grained eddy-resolving ocean flow. This fully-data-driven approach relies on extraction of frequency-ranked time-dependent coefficients describing the evolution of spatio-temporal DAH modes (DAHMs) in the oceanic flow data. In turn, the time series of these coefficients are efficiently modeled by a family of low-order stochastic differential equations (SDEs) stacked per frequency, involving a fixed set of predictor functions and a small number of model coefficients. These SDEs take the form of stochastic oscillators, identified as multilayer Stuart–Landau models (MSLMs), and their use is justified by relying on the theory of Ruelle–Pollicott resonances. The good modeling skills shown by the resulting DAH-MSLM emulators demonstrate the feasibility of using a network of stochastic oscillators for the modeling of geophysical turbulence. In a certain sense, the original quasiperiodic Landau view of turbulence, with the amendment of the inclusion of stochasticity, may be well suited to describe turbulence.

Resolving Large-Scale Geophysical Flows Over Unstructured Meshes

Qingshan Chen
Clemson University, USA
Lili Ju, Roger Temam

Unstructured meshes have been gaining popularity in recent years, because they are almost free of polar singularities, and remain highly scalable even at eddy resolving resolutions. However, to unleash the full potential of these meshes, new schemes are needed. The classical C-grid scheme, which is widely popular on structured meshes, has serious issues concerning the reconstruction of the tangential velocity component. This talk presents new numerical schemes based on an old idea, namely the collocated vorticity-divergence formulation (so-called Z-grid), for large-scale geophysical flows on unstructured centroidal Voronoi meshes. Using the finite-volume discretization technique, the schemes conserve the mass and the absolute vorticity locally, and the potential enstrophy globally. It is also shown that, in an area-averaged sense, the schemes reproduce the Lagrangian transport property for potential vorticity, which is fundamental to the understanding of the dynamics of large-scale geophysical flows. A major challenge of vorticity-divergence based numerical schemes is the specification of the boundary conditions for the PDEs. This project adopts a hybrid approach that combines explicit and implicit implementation of the boundary conditions on the streamfunction and velocity potential. This talk will go over the analytical and practical aspects of the schemes, and finish with some high-resolution numerical results.

Stochastic Marine Ice Sheet Variability

Henk Dijkstra
Utrecht University, Netherlands
T.E. Mulder, S. Baars, F.W. Wubs

It is well known that deterministic two-dimensional marine ice sheets can only be stable if the grounding line is positioned at a sufficiently steep, downward sloping bedrock. When bedrock conditions favour

instabilities, multiple stable ice sheet profiles may occur. Here, we employ continuation techniques to examine the sensitivity of a two-dimensional marine ice sheet to stochastic noise representing short time scale variability, either in the accumulation rate or in the sea level height. We find that in unique regimes, the position of the grounding line is most sensitive to noise in the accumulation rate and can explain excursions observed in field measurements. In the multiple equilibrium regime, there is a strong asymmetry in transition probabilities between the different ice sheet states, with a strong preference to switch to the branch with a steeper bedrock slope.

Adventures in Bayesian Statistical Inversion: Theory, Computation and Applications

Nathan Glatt-Holtz
Tulane University, USA

This talk will describe recent work on two different geophysical problems which we formulate as a Bayesian statistical inverse problem. The first problem concerns the estimation of a velocity field u from sparse observations of a scalar θ, e.g. concentration of a solute, passively advected and diffusing in the fluid medium. The second problem concerns the usage of historical records to provide improved understanding of a series of seismic events which produced Tsunamis in the Indonesian basin. Beyond the inherent interest for applications, these two problems raise theoretical questions of broader statistical, computational and mathematical interest. We will present results concerning consistency and describe some novel Markov Chain Monte Carlo (MCMC) algorithms to effectively sample from classes of infinite dimensional probability measures as arise in our examples.

On the Long-Time Stability of a Temporal Discretization Scheme for the Three Dimensional Viscous Primitive Equations

Chun-Hsiung Hsia
National Taiwan University, Taiwan
Ming-Cheng Shiue

In this talk, a semi-discretized Euler scheme to solve the three dimensional viscous primitive equations is studied. Based on suitable assumptions on the initial data and forcing terms, the long-time stability of the proposed scheme is proven by showing that the H1 norm (in space variables) of the solutions is bounded at each time step when the time step satisfies certain smallness condition.

On the Structure and Stability of the Atmospheric General Circulation

Chanh Kieu
Indiana University Bloomington, USA
Quan Wang

This study examines the dynamical mechanisms underlying the structure and stability of the atmospheric general circulation. Using the dynamical transition framework, it is proven that the atmospheric general circulation possesses a single-cell structure in the absence of the Earth's rotation as previously hypothesized. The large-scale circulation however bifurcates into a triple-cell structure in the presence of the Earth's rotation if the vertical temperature gradient and eddy coefficients satisfy a certain constraint. In particular, it is shown that the triple-cell structure as a result of the Earth's rotation is topologically stable, thus explaining the highly resilient structure of the Earth's atmospheric circulation despite various large-scale interactions and forcings.

Discrete-Time Approach to Stochastic Parameterization of Spatiotemporal Chaos

Kevin Lin
University of Arizona, USA
Fei Lu, Alexandre Chorin

Many dynamical systems of interest in science and engineering are too complex or computationally expensive to fully resolve, even though only a relatively small subset of the degrees of freedom are observable or of direct interest. In these situations, it is useful to have low-dimensional models that can predict the evolution of the variables of interest without reference to the remaining degrees of freedom, and reproduce their statistics at an acceptable cost. This talk concerns a discrete-time, parametric approach to the problem of constructing reduced models from data. I will discuss some of the theoretical and practical issues that arise, including the representation of memory and noise effects. The approach is illustrated using the Kuramoto-Sivashinsky PDE, a prototypical model of spatiotemporal chaos. Time-permitting, I will also discuss connections between this method and the Mori-Zwanzig formalism of nonequilibrium statistical mechanics. This is joint work with Alexandre Chorin and Fei Lu.

Stochastic Advection by Lie Transport and Location Uncertainty: a Common Ground for Uncertainty Quantification in Fluid Dynamics

Valentin Resseguier
Scalian, France
Baylor Fox-Kemper, Darryl D. Holm, Wei Pan

Despite the increasing accuracy of geophysical flows observations and numerical simulations, full resolution is far beyond our current technological capacities. Progress will occur as measurements and simulations are optimally combined using a quantification of the error in these two information sources. Uncertainty quantification remains a challenge in computational fluid dynamics. Thus, we propose to introduce stochasticity into the fluid dynamics equations. Part of the transport velocity – representing its unresolved small-scale component – is assumed to be random and uncorrelated in time. Two similar but independent approaches – location uncertainty (LU) and Stochastic Advection by Lie Transport (SALT) – have followed this path. Both models conserve essential physical invariants. Numerous methods exist to specify the small-scale velocity statistics and thereby fully parametrize these random models. After presenting the LU and the SALT formalisms, the talk will numerically compare two tuning-free parametrizations: a data-driven heterogeneous one and a self-adaptive homogeneous one. For a Surface Quasi-Geostrophic (SQG) flow, both parametrizations lead to similar and accurate uncertainty quantification. A non-stationary and heterogeneous modulation based on an important third-order moment – the energy flux – will also be discussed.

Hopf Bifurcations of Two Layer Western Boundary Currents

Taylan Sengul
Marmara University, Turkey
Kieu Chanh, Quan Wang

The talk will be on the dynamical transitions of the two-layer western boundary currents represented by the Munk profile. Firstly, we will demonstrate an upper bound on the intensity of the Munk profile below which the western boundary currents are locally nonlinearly stable. Next, we will describe the types of dynamical transitions from a pair of simple complex eigenvalues as well as from a double pair of complex conjugate eigenvalues. By careful numerical estimations, we will show that in both cases the transitions are of continuous type at the critical Reynolds number. Moreover, the transition from a pair of simple complex eigenvalue gives rise to a periodic circulation while the transition from a pair of double complex eigenvalues gives rise to quasi-periodic or even chaotic circulation patterns. Lastly, we will present a comparison between the transitions exhibited in one and two layer models.

Topological Phase Transitions and Applications to Geophysical Fluid Dynamics

Shouhong Wang
Indiana U, USA
Tian Ma

Phase transition is a universal phenomena of Nature. The central problem in statistical physics and in nonlinear sciences is on phase transitions. All phase transitions in Nature that we have encountered can be classified into the following two types: 1) dynamical phase transitions, and 2) topological phase transitions (TPTs), also called the pattern formation transitions. The notion of dynamic phase transition is applicable to all dissipative systems, including nonlinear dissipative systems in statistical physics, fluid dynamics, atmospheric and oceanic sciences, biological and chemical systems etc.
The systematic dynamic transition theory and its various applications are synthesized in [1]. TPTs are entirely different from dynamic phase transitions. Intuitively speaking, a TPT refers to the change of the topological structure in the physical space as certain system control parameter crosses a critical threshold. The notion of TPTs is originated from the pioneering work by J. Michael Kosterlitz and David J. Thouless, where they identified a completely new type of phase transitions in two-dimensional systems where topological defects play a crucial role.
With this work, they received 2016 Nobel prize in physics. In this talk, we review recent theories developed by the authors on both dynamic phase transitions and topological phase transitions. The theories are then applied to boundary layer and interior separation phenomena in geophysical fluid dynamics, including the wind-drivern circulation and tropical cyclones.

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Vanishing Viscosity Limit for Homogeneous Solutions of Stationary Incompressible Navier-Stokes Equations with Singular Rays

Xukai Yan
Georgia Institute of Technology, USA
Li Li, Yanyan Li

In this talk, we will first classify all (-1)-homogeneous axisymmetric no-swirl solutions of 3-d stationary incompressible NSE which are smooth on the unit sphere minus south and north poles. We will then present our results on the vanishing viscosity limit of sequences of (-1)-homogeneous axisymmetric no-swirl solutions. We will show that some sequences of solutions tend to solutions of Euler equations on the unit sphere, while for the other sequences of solutions boundary layer or interior layer behavior occur.

Special Session 141: Integrable Peakon Equations and Related Topics

Zhijun Qiao, University of Texas - Rio Grande Valley, USA
Tony Sheu, National Taibei University, Taiwan
Stephen Anco, Brock University, Canada

Recently, much progress have been made in integrable peakon equations both continuous and numerical. In our session, we will report some important development in peakon equations and integrable systems, including Lax pairs, multi-soliton solutions, spectral problems, weak solutions, multi-peakon solution, weak kink, kink-peakon interactional solutions, stability analysis, conservation laws, and Hamiltonian structures.

New Integrable Multi-Component Peakon Equations from a Modified AKNS Scheme

S.F. Anco
Brock Univ, Canada

The standard AKNS scheme for generating integrable evolution systems is modified to obtain integrable peakon systems. In the simplest case of 2x2 matrices, the modified scheme yields two large families of integrable multi-component peakon equations, together with their recursion operators, symmetries, conservation laws, and bi-Hamiltonian structure. These families includes the well-known Camassa-Holm equation, the modified Camassa-Holm (FORQ) equation, their 2-component generalizations, Qiao's equation involving arbitrary functions, and new peakon equations with nonlinearity coefficients that depend on the spatial variable x.

On Peakon, Toda Lattices and Associated Orthogonal Polynomials

Xiangke Chang
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Peoples Rep of China

A class of nonlinear integrable PDEs admit some special weak solutions called "peakons", which are characterised by ODE systems, namely peakon lattices. The celebrated Toda lattice was originally obtained as a simple model for describing a chain of particles with nearest neighbor exponential interaction. For some initial value problems, these lattices can be explicitly solved by use of inverse spectral method involving certain continued fractions, which also associate with some "orthogonality". In this talk, I will take Camassa-Holm (CH) peakon & Toda lattices & ordinary orthogonal polynomials (OPs), 2-component modified CH interlacing peakon & Kac-van Moerbeke lattices & symmetric OPs, Novikov peakon & B-Toda lattices & Partial skew OPs, Degasperis-Procesi peakon & C-Toda lattices & Cauchy Bi-OPs, as examples to illustrate these connections. Some of the results comes from my recent works with Xing-Biao Hu, Yi He, Shi-Hao Li, Jacek Szmigielski and Jun-Xiao Zhao.

Inverse Phaseless Scattering Uniqueness on the Line with Partial Information on Potential

Lung-Hui Chen
National Chung Cheng University, Taiwan

We study to determine the scattering source given certain knowledge of the potential scatterer. In one-dimensional problem, the scattering matrix consists of 2×2 entires of meromorphic functions. For the compactly-supported perturbation, we are able to quantitatively estimate the zeros and poles of each meromorphic entry. The size of potential support is connected to the zero density of scattered wave field. If we assume that the unknown part of potential is comparatively smaller than its known counterpart, then the unknown part is uniquely determined from the magnitude of transmission coefficient which is the magnitude of Fourier transform of certain traveling wave solution of Schrödinger equation.

Localized Waves

Yong Chen
East China Normal University, Peoples Rep of China

I introduce briefly the development of the localized waves, and then the major work about our team to the localized waves. It mainly contains four aspects to construct localized wave.

Multi-Soliton Solutions and the Cauchy Problem for a Two-Component Short Pulse System

Qiaoyi Hu
South China Agricultural University, Peoples Rep of China
Z. Zhaqilao, Zhijun Qiao

In this paper, we study multi-soliton solutions and the Cauchy problem for a two-component short pulse system. For the multi-soliton solutions, we first derive an N-fold Darboux transformation from the Lax pair of the two-component short pulse system, which is expressed in terms of the quasideterminant. Then by virtue of the N-fold Darboux transformation we obtain multi-loop and breather soliton solutions. In particular, one-, two-, three-loop soliton, and breather soliton solutions are discussed in detail with interesting dynamical interactions and shown through figures. For the Cauchy problem, we first prove the existence and uniqueness of a solution with an estimate of the analytic lifespan, and then investigate the continuity of the data-to-solution map in the space of an analytic function.

A Turning Point Approach for Q-Orthogonal Polynomials

Chun-Kong Law
National Sun Yat-Sen University, Taiwan
Mourad Ismail

We develop an intuitive approach to the Plancherel-Rotach asymptotics around the largest zero of a polynomial satisfying a linear functional equation. We first treat the toy problem of Hermite polynomials in order to explain the process by which we determine the correct Plancherel-Rotach asymptotics. Then we treat the Stieltjes-Wigert polynomials and some other q-orthogonal polynomials. Our approach does not use any refined properties of these orthogonal polynomials. We only use the second order operator whether it is differential, difference or q-difference. This is joint work with Mourad Ismail.

A New 3-Component Degasperis-Procesi Hierarchy

Nianhua Li
Huaqiao University, Peoples Rep of China

We study the bi-Hamiltonian property for the hierarchy of a 3-component Degasperis-Procesi (3-DP) equation. We show that Hamiltonian functionals of the hierarchy in negative direction are local, and in both directions are homogenous. Two different Liouville transformations for the equation are constructed via a reciprocal transformation. The first transformation shows that the associated equation is a reduced negative modified Yajima-Oikawa system. The second transformation shows that the associated system is a reduced negative generalized mKdV system and passes the Painlevé test, besides Hamiltonian structures of the 3-DP equation under this Liouville transformation are also discussed. Moreover, we consider a limit for the 3-DP equation.

Conserved Quantities and Reductions of a Limit System

Hongmin Li
Huaqiao University, Peoples Rep of China

We construct the infinite number of conserved quantities for a system, which can be viewed as a short-wave limit of a new three-component Degasperis-Procesi equation. Furthermore, we study the reductions of this system and find they are equivalent to some generalizations of the short pulse equation proposed by Hone, Novikov and Wang.

The Special Second Integral of the KdV

Yuqi Li
East China Normal University, Peoples Rep of China
Y. Chen, S.Y. Lou

The solution structure of the KdV equation is studied in light of the *special second integral*. The special second integral distinguishes itself from the general ones by that it is a more natural generalization of the first integral. Though the KdV have been studied so vastly and deeply, some completely new aspects of the second integrals of KdV are discovered. The complete understanding for the result is still missing.

Alice Bob Peakon Systems

Senyue Lou
Ningbo University, Peoples Rep of China

Some types of new Alice Bob peakon systems (ABPS) are proposed. Exact peakon solutions, Lax pairs, symmetries, conservation laws and bi-Hamiltonian structure for some ABPS are studied in detail.

High Order Peakon Models

Zhijun Qiao
Univ. of Texas Rio Grande Valley, USA
Quansheng Liu

In this talk, we will talk about some recent developments in integrable peakon systems, including the well-known CH, DP, FORQ/MCH, NE, and other models. Some high order peakon models will be reported first time.

A General Nonlocal Nonlinear Schrödinger Equation with Shifted Parity, Charge-Conjugate and Delayed Time Reversal

Xiaoyan Tang
East China Normal University, Peoples Rep of China
Zu-feng Liang

A general nonlocal nonlinear Schrödinger equation with shifted parity, charge-conjugate and delayed time reversal is derived from the nonlinear inviscid dissipative and equivalent barotropic vorticity equation in a β-plane. The modulational instability (MI) of the obtained system is studied, which reveals a number of possibilities for the MI regions due to the generalized dispersion relation that relates the frequency and wavenumber of the modulating perturbations. Exact periodic solutions in terms of Jacobi elliptic functions are obtained, which, in the limit of the modulus approaches unity, reduce to soliton, kink solutions and their linear superpositions. Representative profiles of different nonlinear wave excitations are displayed graphically. These solutions can be used to model different blocking events in climate disasters. As an illustration, a special approximate solution is given to describe a kind of two correlated dipole blocking events.

On the Dym-Type Hierarchy: Trigonal Curve and Quasi-Periodic Solutions

Lihua Wu
Huaqiao University, Peoples Rep of China

Resorting to the characteristic polynomial of Lax matrix for the Dym-type hierarchy, we define a trigonal curve, on which appropriate vector-valued Baker-Akhiezer function and meromorphic function are introduced. With the help of the theory of trigonal curve and three kinds of Abelian differentials, we obtain the explicit Riemann theta function representations of the meromorphic function, from which we get the quasi-periodic solutions for the entire Dym-type hierarchy.

Analysis on Some Two-Component Completely Integrable Systems with Peakon Solutions and Cubic Nonlinearity

Kai Yan
Huazhong University of Science and Technology, Peoples Rep of China
Zhaoyang Yin, Zhijun Qiao, Yufeng Zhang

In this talk, we first introduce some classes of two-component completely integrable systems with peakon solutions and cubic nonlinearity. Then the local well-posedness and the blow-up criterion will be established. Some new blow up strong solutions and asymptotic behavior will be studied as well. Finally, we will give some related open problems.

On Integrable Nonlocal Nonlinear Schrödinger Equation and Its Discrete Version

Zuonong Zhu
Shanghai Jiao Tong University, Peoples Rep of China

Very recently, Ablowitz and Musslimani introduced and investigated a class of reverse space, reverse time, and reverse space-time nonlocal nonlinear integrable equations. In this talk, we will report our results for integrable nonlocal nonlinear Schrödinger equation and its discrete version including their soliton solutions, relations with coupled Heisenberg equation in the spaces R^3 and R^{2+1} and their discrete versions. This is the joint work with Li-Yuan Ma.

Special Session 142: Differential Equation Based Modeling for Brain and Other Complex Bio-Systems

Jianzhong Su, University of Texas at Arlington, USA
Akif Ibraguimov, Texas Tech University, USA
Lixia Duan, North China University of Technology, Peoples Rep of China
Qingyun Wang, Beihang University, Peoples Rep of China

Many biological systems, such as neuronal systems, genomic systems, and immune systems, are featured by nonlinear and complex patterns in spatial and temporal dimensions. These phenomena carry significant biological information and regulate down-stream biological mechanisms. Understanding the mechanisms underlying such events by quantitative modeling represents a mathematical challenge of current interest. Yet all these systems share the similar dynamical system issues in ordinary/partial different equation such as bifurcation, stability, oscillations, stochastic noise as well as issues in determining model parameters from experimental data sets and computational errors of the models. This special session offers a forum to exchange the state of the art theoretical advances related to this promising area as well as computational tools. It will foster and encourage communication and interaction between researchers in these directions. The common themes include mathematical models and data analysis, theoretical analysis, computational and statistical methods of dynamical systems and differential equations for the bio-system focused models, as well as applications in brain research. The topics may include but are not restricted to:

- Dynamics and computation of neuronal systems; Modeling and dynamical analysis of biological neurons and neuronal networks; Generation, encoding and transduction of neuronal signals and patterns; Modeling and analysis of cognitive information processing mechanisms; Dynamic abnormality in neuronal systems due to diseases.
- Dynamics of immune systems; Modeling biomedical processes, including tumor growth, cardio-vascular diseases, infection, and healing, mediated by immunologic mechanisms; Analysis of mathematical models for dynamics features such as instabilities, bifurcations that provide insight into the nature of the underlying bio-physical mechanisms; Modeling wound healing and inflammatory responses, including cell to cell interactions, foreign body reactions and quantitative as well as qualitative comparison with experimental data.
- Data analysis and modeling of brain activities; Complexity theory applied to brain; Perception, learning and memory functions in brain; Computational evolutionary biology; Models, analysis and algorithms in Bioinformatics.

Neural Networks with Short Range and Long Range Connectivity

Yixin Guo
Drexel University, USA
Alex Onderdonk

We study a neural field model with both symmetric localized lateral inhibition and asymmetric long distance connections, motivated by the fact that neurons are connected by both short range and long range synapses. We develop a general method to convert the neural field model in the form of integro-differential equation to a delay differential equation. Then we solve the delay differential equation with a piecewise linear firing rate function using a combination of analytical and numerical approach. We show the existence of two coexisting traveling waves and the larger wave of the two is stable with the small one being unstable.

Dynamics of Tumor-CD4+-Cytokine-Host Cells Interactions with Treatments

Sophia Jang
Texas Tech University, USA
Xiaochuan Hu

Mathematical models of interactions between tumor cells, CD4+ T cells, cytokines, and host cells are proposed to investigate the role of CD4+ on tumor regression. Our results suggest that host cells along with the mechanism of production of CD4+ T cells play important roles in driving tumor dynamics. Cancer cells can be eradicated if the tumor has a small growth rate and is also not competitive. Treatments by either CD4+, cytokines, or a combination of the two are applied to study their effectiveness. It is concluded that doses of treatments along with the tumor size are critical in determining the fate of the tumor. Tumor cells can be eliminated completely if doses of treatments by cytokine are large. The treatments are in general more effective if the tumor size is smaller. Bistability is observed in all of the models with or without the treatment strategies indicating that there is a window of opportunity for clearing off the tumor cells

Neurotransmitters Release for Independent Synaptic Currents in a Single Synapse Modeled with Discontinuous Diffusion Coefficients

Sat Byul Seo
Kyungnam University, Korea
Jianzhong Su, Ege Kavalali

Networks of neurons and synapse play a key role of communication in the brain. Presynaptic terminals release neurotransmitters either in response to action potential or spontaneously independent of presynaptic activity. Spontaneous neurotransmission has an independent role in neuronal communication that is distinct from that of evoked release. However, the process of spontaneous neurotransmitter release is still unclear. We develop a mathematical model in 3-D to emulate spontaneous and evoked neurotransmissions resulted from glutamate release within a single synapse. We propose numerical methods for solving piecewise continuous heat diffusion equation, estimate and verify its errors of second order accuracy. In order to identify the spatial relation between spontaneous and evoked glutamate releases, we consider quantitative factors, such as the size of synapses, inhomogeneity of diffusion coefficients, the geometry of synaptic cleft, and the release rate of neurotransmitter, that will affect postsynaptic currents. We conclude quantitatively when a synaptic size is larger, the cleft space is less diffusive in the central area than the edge area, if the geometry synaptic cleft has a narrower gap in the center and if glutamate release is slower, then there is a better chance for independence of two modes of currents from spontaneous and evoked release. The computed results match well with existing experimental findings and provide a quantitative map of boundaries of physical constraints for having independent synaptic fusion events.

Brain Wave Dynamics by EEG Source Localization and Reconstruction

Jianzhong Su
University of Texas-Arlington, USA
Honghui Zhang, Hongguang Xi

Investigation of Soliton Model for Dynamics of Nerve Pulses

Michail Todorov
Technical University of Sofia, Bulgaria
Andrei Ludu

We present the original 1D equation for the shape $u(x,t)$ of a nerve pulse [1] in the moving frame as a Boussinesq-like equation

$$u_{tt} = \left(u - \frac{p}{2}u^2 + \frac{q}{3}u^3 - hu_{xx}\right)_{xx}$$

and study the influence of the nonlinear coefficients (p,q) on the solution, and also the influence of the h coefficient for the high-order diffusion term u_{xxxx}. We explore a large parametric set to study the elastic/inelastic interactions and the velocity/amplitude/phase shifts for localized pulses, solitary waves, critical velocities, newly born excitations, annihilation events, and the cross-modulation effect typical in other Boussinesq or vector Schrödinger equations. This equation admits the Hamiltonian system

$$u_t = q_{xx}$$
$$q_t = u - \frac{p}{2}u^2 + \frac{q}{3}u^3 - hu_{xx}.$$

We use the analytic 1-soliton solution obtained in [1] to pose a Cauchy problem for this Hamiltonian system. We find the energetic radiation contribution and check its consistency with conserved energy in the asymptotic solutions. We study the energy functional and understand its definiteness depending on (p,q,h) parameters magnitudes. This give us an information about the soliton stability.

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Modeling and Analysis of Focal Epilepsy with Epileptor Field Model

Honghui Zhang
Northwestern Polytechnical University, Peoples Rep of China
Pengcheng Xiao

We study the dynamical behaviors of coupled oscillators in different activity states with slow permittivity coupling in Epileptor model, including seizure like event (SLE), refractory status epilepticus (RSE), depolarization block (DB), and normal state. Our researches find that when one oscillator in normal states is coupled with any oscillator in SLE, RSE or DB states, both of these two oscillators can evolve into SLE states in the state of epileptiform synchronization or epileptiform anti-synchronization. Moreover, SLE can be suppressed while considering the fast electrical or chemical coupling. Additionally,

a reduced model is also given to study the seizure dynamics with more coupling numbers. Those results can help to understand the dynamical mechanism of the formation, propagation and termination of seizures in focal epilepsy.

Phase Synchronization Dynamics of Coupled Neurons with Coupling Phase in the Electromagnetic Field

Yong Zhao
Henan Polytechnic University, Peoples Rep of China

Synchronization phenomena are ubiquitous in many complex physical, biological and neuronal systems. Phase synchronization of coupled extended Hindmarsh-Rose neurons with magnetic and electrical couplings is discussed. It is found that the threshold for the coupling strength to reach phase synchronization is gradually smaller when the coupling phase is increased under the same stimulus current. Under the same coupling phase, the coupling strength to reach phase synchronization is almost increasing gradually with increasing the stimulus current, no matter in what state the neuron is. Our recent findings are significant and helpful for further understanding the collective behaviors of neuronal system including comprehensive physical mechanisms and information transmissions.

Special Session 143: Analytic and Numerical Approaches for Understanding Complex Systems

Chung-Min Lee, California State University Long Beach, USA
James von Brecht, California State University Long Beach, USA
Scott McCalla, Montana State University, USA

Many of the most exciting and difficult problems in applied mathematics stem from our explorations of systems with complex, intricate dynamics that involve numerous spatio-temporal scales, multiple processes, and/or a large number of different, highly interconnected system components. Researchers have spent decades working toward better understanding these systems using a wide variety of analytical and numerical approaches. We plan to bring together researchers with a range of different expertise to stimulate discussions and collaborations that will drive improvements in our understanding of complex systems.

Asymptotic Models for Water Waves

Ching-Hsiao Cheng
National Central University, Taiwan
Rafael Granero-Belinchon, Steve Shkoller, Jon Wilkening

The study of irrotational incompressible Euler equations has been a long tradition in the fluid community. When the free surface was taken into account, both the theoretical study and robust numerical schemes become very challenging, especially for the case of deep water (Euler equations on a fluid domain with infinite depth). In the numerical side, various methods used to compute the Dirichlet-to-Neumann map (which is highly related to the water wave equations) proposed by W. Craig et al (1993) and M.J. Ablowitz et al (2006, 2008) involve highly ill-conditioned intermediate calculations (while the difficulties can be overcome by implementing multiple-precision arithmetic). The boundary integral collocation method and the transformed field expansion method are then introduced to avoid catastrophic cancellation of digits in the intermediate results; however, carrying out those methods in the three-dimensional case seems difficult. Therefore, the search for good asymptotic models for water waves become appealing for it might provide models that can be easily implemented and at the same time provide accurate enough evolution of the free surface. In this talk, I will present how the Stokes expansions can be used to derive asymptotic models up to any order.

On the Mathematical Analysis of the Synchronization with Delays

Chun-Hsiung Hsia
National Taiwan University, Taiwan
Chang-Yeol Jung, Bongsuk Kwon, Sunghwan Moon, Yoshihiro Ueda

We investigate the partial synchronized collective behavior of the Kuramoto oscillators with delayed-time interaction and frustration effect. The phase synchronization of the identical case for N-oscillator case is in view.

A Numerical Study of Steklov Eigenvalue Problem Via Conformal Mapping

Chiu-Yen Kao
Claremont McKenna College, USA
Weaam Alhejaili

In this paper, a spectral method based on conformal mappings is proposed to solve Steklov eigenvalue problems and its related shape optimization problem in two dimensions. To apply spectral methods, we first reformulate the Steklov eigenvalue problem in the complex domain via conformal mappings. The eigenfunctions are expanded in Fourier series so the discretization leads to an eigenvalue problem for coefficients of Fourier series. For shape optimization problem, we use the gradient ascent approach to find the optimal domain which maximizes $k-$th Steklov eigenvalue with a fixed area for a given k. The coefficients of Fourier series of mapping functions from a unit circle to optimal domains are obtained for several different k.

A Variational Inequality Formulation for Transonic Compressible Steady Potential Flows

Eun Heui Kim
California State University Long Beach, USA
Yung-Sze Choi

We present a variational inequality formulation for a steady compressible potential flow. Its critical point satisfies the transonic equation and the associated jump conditions across its free boundary match the usual Rankine-Hugoniot jump conditions for a transonic shock. By means of example we validate our formulation, and establish the necessary and sufficient condition for the existence of a transonic shock.

Traveling Wave Solutions in a Model for Tumour Invasion with the Acid-Mediation Hypothesis

Robert Marangell
University of Sydney, Australia
P. Davis, P. van Heijster, R. Rodrigo

In this talk, I will discuss how Geometric Singular Perturbation Theory (GSPT) can be used to show the existence of travelling wave solutions in a Gatenby-Gawlinski model. In particular, I will show how GSPT can be used to show the existence of a slow travelling wave with an interstitial gap. Such a gap has been observed experimentally, and we provide a mathematical framework for its existence in terms of a dynamic transcritical bifurcation.

Nonlocal Interfacial Dynamics in Biological Systems

Scott Mccalla
Montana State University, USA
James von Brecht

Biological pattern formation has been extensively studied using reaction-diffusion and agent based models. In this talk we will discuss nonlocal pattern forming mechanisms in the context of bacterial colony formation and surface striping on animals with an emphasis on arrested fronts. This will lead to a novel nonlocal framework to understand the interfacial motion in biological systems. We will then use this approach to model an interesting bacterial phenomenon, and to understand simple microscopic requirements for flat stripe solutions to persist in nature.

A Multiscale Crowd Evacuation Model Capturing the Effect of Environment Awareness in the Presence of Fire and Smoke

Omar Richardson
Karlstad University, Sweden
Adrian Muntean

We study the evacuation dynamics of a crowd wanting to escape from a complex geometry in the presence of a fire as well as of a slowly spreading smoke curtain. The crowd we have in view is composed of two populations of individuals: one knowing the map of the building and one unaware of the given geometry, relying exclusively on potentially informed neighbors to identify a path of motion towards the exit. We aim at capturing the effect the knowledge of the environment has on the interaction between evacuees and their residence time in the presence of fire and evolving smoke. Our approach is genuinely multiscale – we employ a hybrid (continuum-discrete-stochastic) model that can distinguish between compressible and incompressible pedestrian flow regimes and allows for two-scale (micro and macro) pedestrian interactions. Simulations illustrate the expected qualitative behavior of the model. This is a joint work also with Andrei Jalba (TU Eindhoven, NL).

Ground State Patterns and Phase Transitions of Spin-1 Bose-Einstein Condensates Via Γ-Convergence Theory

Tien-Tsan Shieh
National Taiwan University, Taiwan
I-Liang Chern, Chiu-Fen Chou

We develop an analytic theory for the ground state patterns and their phase transitions for spin-1 Bose-Einstein condensates on a bounded domain in the presence of a uniform magnetic field. Within the Thomas-Fermi approximation, these ground state patterns are composed of four basic states: magnetic state, nematic state, two-component state and three-component state, separated by interfaces. A complete phase diagram of the ground state patterns are found analytically with different quadratic Zeeman energy q and total magnetization M for both ferromagnetic and antiferromagnetic systems. Using the Γ-convergence technique, it is found that the semi-classical limits of these ground states minimize an energy functional which consists of interior interface energy plus a boundary contact energy. As a consequence, the interface between two different basic states has constant mean curvature, and the contact angle between the interface and the boundary obeys Young's relation.

Convergence and Stability of the MAC Scheme for Stokes/Darcy Coupling Problems Based on Finite Difference Methods

Ming-Cheng Shiue
National Chiao Tung University, Taiwan
Ming-Chih Lai, Kian Chuan Ong

In this talk, to begin with, Stokes/Darcy coupling flows which arise from physical models such as biology, engineering and geophysical fluid dynamics are considered. In the literature, there are many numerical methods related to finite element methods applied to solve this coupling problem. Unlike finite element methods, due to that there is lack of natural variational formulation, in general, the analysis of the scheme based on finite difference methods becomes complicate. In this work, the MAC scheme for this coupling problem based on finite difference methods is used. Convergence and stability of the scheme will be presented. The second part will present the development of numerical schemes for Navier-Stokes and Darcy coupling problems based on projection methods. Numerical simulations demonstrate the results which match the case of only Navier-Stokes equations that also are computed using the projection method.

Special Session 144: Analytic Properties and Numerical Approximation of Differential Models Arising in Applications

Cecilia Cavaterra, Università degli Studi di Milano, Italy
Elisabetta Rocca, Università degli Studi di Pavia, Italy
Marita Thomas, Weierstrass Institute for Applied Analysis and Stochastics, Germany
Elena Bonetti, Università degli Studi di Milano, Italy

Advances in technology need to study complex systems of coupled equations of different nature. Therefore, the modeling, the analysis and the numerical simulation of these systems are topics of great interest not only for the applications to real-world problems but also for the challenge that such problems present from the mathematical viewpoint. The main goal of the proposed session is to provide an overview of the state of the art in the investigation of the properties of special materials and fluids, and the new emerging trends in non-smooth mechanics and fluid dynamics.

A Semidiscrete Numerical Method for the Gurtin-Pipkin Equation

Filippo Dell Oro
Politecnico di Milano, Italy
Olivier Goubet, Youcef Mammeri, Vittorino Pata

We introduce a new mathematical framework for the time discretization of evolution equations with memory. As a model, we focus on an abstract version of the equation

$$\partial_t u(t) - \int_0^\infty g(s)\Delta u(t-s)ds = 0$$

with Dirichlet boundary conditions, modeling hereditary heat conduction with Gurtin-Pipkin thermal law. Well-posedness and exponential stability of the discrete scheme are shown, as well as the convergence to the solutions of the continuous problem when the time-step parameter vanishes.

Recent Results on Some Nonlocal Diffuse-Interface Models for Incompressible Binary Fluids

Sergio Frigeri
Università Cattolica del Sacro Cuore, Brescia, Italy

In the talk we shall present the last results on some diffuse-interface models for flow and phase separation of binary fluids which are based on the coupling of the Navier-Stokes equations with the nonlocal Cahn-Hilliard equation. The nonlocal Cahn-Hilliard/Navier-Stokes system has been studied analytically in a series of papers (cf. [1, 5, 6, 7, 9, 3, 11]). The attention will be focused on the more recent results concerning the case where the two densities are different (weak solutions, cf.[2]), where the mobility is degenerate and the potential is singular (regularity and optimal control, cf. [4] and [10]), and where the two-phase fluid is non-Newtonian (weak solutions and uniqueness, cf. [8]).

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Some Recent Results on the Cahn-Hilliard-Hele-Shaw System

Andrea Giorgini
Politecnico di Milano, Italy

Many applications of multiphase flows in Engineering, Physics and Life Sciences involve the motion of fluids and thei mutual interplay at interfaces separating immiscible components. Complex mechanisms already appear in simple experiments, such as breakup and coalescence of drops, mixing in a driven cavity and thermocapillary flow, revealing remarkable similarities with global-impact issues, like tumor growth dynamics. Understanding and modelling the transition occurring when interfaces merge and reconnect is still a major challenge in Fluid dynamics. The Hele-Shaw flow is a paradigm to approximate fluid flows characterized by predominance of viscuos forces as opposed to the inertial forces. Such approximation is justified in the so-called Hele-Shaw cell, namely the fluids are confined between two flat plates separated by an infinitesimally small gap. The Cahn-Hilliard-Hele-Shaw (CHHS) system is a diffuse interface model describing the motion of two globally immiscible, incompressible and viscous fluids in a Hele-Shaw cell. This model couples a Darcy's law, which governs the volume-averaged fluid velocity, with a convective Cahn-Hilliard equation for the difference of the fluid concentrations (order parameter). In this talk I will present some recent results for the CHHS system with logarithmic potential, concerning uniqueness of physical weak solutions, the well-posedness of strong solutions as well as their further regularity properties.

Economic Growth with Heterogenous Space and Population: an Optimal Control Model of PDEs

Fausto Gozzi
Luiss University, Roma, Italy
Raouf Boucekkine, Giorgio Fabbri, Salvatore Federico

A series of pretty recent works have merged the spatial dimension with growth theory models using the framework coming from classical geographical economy. In such papers the underlying mathematical model is an optimal control problem where the state equation is a second order PDE. In this talk we briefly describe such models and their motivation. Then we provide the main results, obtained with the Dynamic Programming approach: - the existence of explicit solutions of the associated HJB equation; - the existence of optimal strategies in feedback form. We present some economic consequences of such results and discuss possible generalization of them when explicit solutions are not available.

Recent Results on the Nonlocal Cahn-Hilliard Equation

Maurizio Grasselli
Politecnico di Milano, Italy

The nonlocal Cahn-Hilliard equation is a conserved gradient flow generated by a Helmholtz free energy functional. This functional is given by a nonlocal and nonconvex bulk energy minus the total entropy, that is, a logarithmic function of the order parameter. The resulting equation is a second-order nonlocal and nonlinear evolution equation. We present some recent results on this equation when the mobility is either constant or degenerate.

Cahn-Hilliard Inpainting with Non-Smooth Potentials

Kei Fong Lam
The Chinese University of Hong Kong, Hong Kong
Harald Garcke, Vanessa Styles

Inpainting is the action of restoring missing or damaged details in images, and it is desirable to produce a result, which to the naked eye, does not distinguish where the inpainting has been performed. Out of the many existing approaches and algorithms proposed in the literature, we focus on the approach of Bertozzi, Esedoglu and Gillette that modifies the Cahn-Hilliard equation with a data fidelity term. This method seems to be simpler to implement and obtain good reconstruction much faster. We alter the potential term from a smooth quartic one to a non-smooth double-obstacle one and discuss the existence of weak solutions to both the time-dependent and stationary problems.

Capillary Surface in a Wedge

Fei-Tsen Liang
Academia Sinica, Taiwan

The distance of an almost-constant mean curvature boundary from a finite number of disjoint tangent balls with equal radii is quantitatively controlled by the oscillation of the mean curvature. We shall use this result to describe the geometry of capillary surfaces in a wedge.

Dimension Reduction in the Context of Structured Deformations

Marco Morandotti
Technische Universitaet Muenchen, Germany
G. Carita, J. Matias, D.R. Owen.

The theory of structured deformations shows good potential to deal with mechanical problems where multiple scales and fractures are present. Mathematically, it amounts to relaxing a given energy functional and to show also the relaxed one has an integral representation. In this seminar, I will focus on a problem for thin objects: the derivation of a 2D relaxed energy via dimension reduction from a 3D energy, incorporating structured deformations in the relaxation procedure. I will discuss the two-step relaxation (first dimension reduction, then structured deformations and vice-versa) and I will compare it with another result in which the two relaxation procedures are carried out simultaneously. An explicit example for purely interfacial initial energies will complete the presentation.

A Variational Approach of Damage in Rocks and Applications to Fracture Mechanics

Jaime Ortega
Universidad de Chile, Chile
R. Lecaros, G. Montecinos, J.H. Ortega, J. Ramirez

In this work, we present the model for the study of rock mechanics, where we introduce the concept of damage. following the ideas of J.J. Marigo et al., we consider a variational formulation for study the dynamics of the rocks damage and the fracture mechanics. We will discuss the optimality conditions of the problem and present some numerical results. Finally we will present some applications to underground mining.

Analytical Validation of the Young-Dupré Law for Epitaxially-Strained Thin Films

Paolo Piovano
University of Vienna, Austria
Elisa Davoli

A variational model for epitaxially-strained thin films is derived from the transition-layer and the sharp-interface models available in the literature. The regularity of energy-minimal film profiles is studied by establishing the internal-ball condition and by adopting some arguments from transmission problems. The possibility of different elastic properties between the film and the substrate is included in the analysis, as well as the surface tensions of all three involved interfaces: film/gas, substrate/gas, and film/substrate. The results relate to both the Stranski-Krastanow and the Volmer-Weber modes. Moreover, geometrical conditions are provided for the optimal wetting angle, i.e., the angle formed at the contact points between films and substrates. In particular, the Young-Dupré law is shown to hold, yielding what appears to be the first analytical validation of such law for a thin-film model in the context of Continuum Mechanics.

A Doubly-Nonlinear Cahn-Hilliard System with Nonlinear Viscosity

Luca Scarpa
University College London, England
Elena Bonetti, Pierluigi Colli, Giuseppe Tomassetti

We prove existence and uniqueness of solutions to a doubly-nonlinear Cahn-Hilliard system with nonlinear viscosity of the form

$$\begin{aligned} \partial_t u - \Delta\mu = 0 \qquad & \text{in } \Omega\times(0,T)\,, \\ \mu \in \varepsilon\partial_t u + \beta(\partial_t u) - \delta\Delta u + \psi'(u) - g \qquad & \text{in } \Omega\times(0,T)\,, \\ \mu = 0\,, \quad \partial_{\mathbf{n}} u = 0 \qquad & \text{in } \partial\Omega\times(0,T)\,, \\ u(0) = u_0 \qquad & \text{in } \Omega \end{aligned}$$

on a smooth bounded domain $\Omega \subseteq \mathbb{R}^3$, where ε, δ are positive parameters, β is a maximal monotone graph in $\mathbb{R}\times\mathbb{R}$, $\psi : (a,b) \to \mathbb{R}$ is a so-called double-well potential and g is a given source. Existence of solutions is proved by approximating the problem and passing to the limit through monotonicity and compactness arguments, under some growth assumptions either on β or on ψ. Moreover, the asymptotic behaviour of the solutions as $\delta \searrow 0$ and $\varepsilon \searrow 0$ (separately) is shown. This study is based on a joint work with Elena Bonetti (Università degli Studi di Milano), Pierluigi Colli (Università degli Studi di Pavia) and Giuseppe Tomassetti (Università degli Studi "Roma Tre").

A Model for Complex Fluids with Inertial Effects

Giulio Schimperna
University of Pavia, Italy
Gianluca Favre

We present some mathematical results on a PDE system describing the evolution of two-phase fluids in presence of inertial effects at the microscopic level. The model couples the Navier-Stokes equations for the macroscopic velocity u with a variant of the Allen-Cahn equation for the phase parameter ϕ where inertial effects are taken into account. As a consequence, we face the presence of of a second order *material* derivative of ϕ in the equation, which gives rise to severe mathematical difficulties. We discuss a number of analytical results for this model considering both the 2D and the 3D case. Due to the intrinsic difficulties of the problem, we can only show partial results whose validity may require simplifications of the equations or the presence of ad hoc regularizing terms.

On a Model for Dynamic Crack Growth

Rodica Toader
University of Udine, Italy

We consider a model of elastodynamics with brittle crack growth, based on energy-dissipation balance and a maximal dissipation condition. The results are obtained in collaboration with G. Dal Maso (SISSA, Italy) and C. Larsen (WPI, USA).

Special Session 145: Numerical Methods Involving Implicit or Non-Parametric Interfaces, and Point Clouds

Annabelle Collin, Enseirb-Matmeca, France
Julien Dambrine, Université de Poitiers, France
Catherine Kublik, University of Dayton, USA
Clair Poignard, Inria Bordeaux-Sud-Ouest, Institut de Mathématiques de Bordeaux, France

The goal of this session is to bring together researchers from all over the world who are working on numerical methods for solving PDEs, integral equations or simply doing computations on manifolds that are not defined through an explicit parameterization (e.g. immersed boundary methods using a level set function, sharp methods for interface problems or the closest point method). Such numerical methods are needed in applications where the interface is evolving in time and undergoing topological changes for instance, and where the solution of a PDE is needed on some manifold. The session is also motivated by the recent need to work with unstructured point sets that are either sampled from an underlying manifold or acquired through some device (e.g. 3D scanner). We hope that in bringing experts in this field together, we will produce fruitful discussions and new ideas.

Volume Reconstruction from Slices

Elie Bretin
ICJ, INSA de Lyon, France
F. Dayrens and Simon Masnou

Reconstructing a 3D volume from 2D slices is a classical problem in medical imaging. In a joint work with François Dayrens and Simon Masnou, we propose a variational model involving a geometric surface regularization term, typically the perimeter or the Willmore energy, and density constraints for the slices. We then explain how to approximate this optimization problem using a phase field model, for which, a Gamma convergence results can be established in the case of perimeter. We propose also an efficient numerical scheme, whose performances are illustrated on various examples involving various types of slices

Volumetric Variational Approach to Solve Partial Differential Equations Defined on Manifolds

Jay Chu
National Tsing Hua University, Taiwan
Richard Tsai

Partial differential equations on manifolds are important for many areas, such as materials science, fluid dynamics and biology. The computation of such problems can be costly and difficult when the manifolds have complicated structures. Traditional approaches require discretization on manifolds and projecting derivatives onto the tangent spaces of the manifolds. We introduce volumetric variational problems for solving such PDE's. We start with variational problems on manifolds and change them into extended problems in Eulerian formulation. The extended PDE's can be solved by many sophisticated numerical methods, such as finite element or finite difference methods. Based on special properties of the solutions, we design a special treatment for boundary conditions. By Fourier and Laplace transformation, we analysis the method and show that it is stable for elliptic and parabolic type of equations. However, it is not numerically stable for hyperbolic type equation. The instability can be fixed by modifying equations or reinitialization. Some numerical experiments are presented in the talk. This is a joint work with Richard Tsai.

Shape Optimization for the Wave-Making Resistance

Julien Dambrine
University of Poitiers, France
Evi Noviani, Morgan Pierre

The wave-making resistance problem consists in finding the an optimal shape for an obstacle immersed in an uniform free-surface flow, from the point of view of the drag induced by the generation of a wake. This is a classical problem in fluid mechanics, which has applications in the design of energy-efficient ships. Up to now, the few tentatives of optimisation of the obstacle relied on simplified representations of its shape, typically of small dimension, or relying on a thin body approximation. In our study, we aim to obtain optimal obstacles for the wave-making resistance in a much larger space of shapes. The numerical method we used relies on an implicit representation of the boundary of the obstacle which allows, for instance, topological changes without technical hindrances.

A Second-Order Immersed Boundary Method for the Numerical Simulation of Two-Dimensional Incompressible Viscous Flows Past Obstacles

Nicolas James
Université de Poitiers, France

We present a new second-order method, based on the MAC scheme on cartesian grids, for the numerical simulation of two-dimensional incompressible flows past obstacles. The time integration is achieved with a second-order (BDF) projection scheme. The discretization of the nonlinear terms, written in conservative form, is formulated in the context of finite volume methods. Finally, the resulting linear systems are solved by a direct method based on the capacitance matrix method. Accuracy and efficiency of the method are supported by numerical simulations of 2D flows past a cylinder at Reynolds number up to 9 500.

Numerical Reaction-Diffusion with Bulk-Surface Coupling

Colin Macdonald
Univ. of British Columbia, Canada
Steven J. Ruuth, Barry Merriman, Laurent Charette, Frederic Paquin-Lefebvre

The Closest Point Method is a set of mathematical principles and numerical techniques for solving PDEs posed on curved surfaces or other general domains. The method works by embedding the domain in a higher-dimensional space and solving the PDE in that space, using simple finite differences and interpolation. We describe this method for reaction-diffusion problems on surfaces, and show that it can be applied when surface processes are coupled to reaction-diffusion in a surrounding bulk. Example computations include point clouds, and some progress on biological applications.

Minimizing Movement Approach Using Level Set Functions for Evolving Spirals by Crystalline Eikonal-Curvature Flow

Takeshi Ohtsuka
Gunma University, Japan

In this talk we consider evolving spirals by crystalline eikonal-curvature flow. In particular, our focus is on the evolution of several spirals merging with each other. For describing such a situation, we have to introduce an implicit representation of spirals. On the other hand, Crystalline curvature is defined as the first variation of anisotropic arclength funcgtional with convex and piecewise linear energy density. Such an L^1 type regularizing problem with implicit formulation implies crucial difficulty caused by singularities of the energy density function. For this problem, we propose two energy minimizing approaches which are based on the algorithm using the signed distance function from the curve due to Chambolle in 2004. Generally, however, the signed distance function does not work well for spirals. Our proposed methods overcome this difficulty by constructing a locally signed distance function of spirals, or using just a level set functions instead of the distance function. Some numerical results with our proposed approaches are also presented. For numerical simulation of these algorithms, we introduce a split Bregman iteration for the energy functionals of our problems.

An RBF-FD Closest Point Method for Solving PDEs on Surfaces and Applications to PDEs on Moving Surfaces

Argyrios Petras
Basque Center for Applied Mathematics, Spain
Cecile Piret, Leevan Ling, Steven Ruuth

Partial differential equations (PDEs) on surfaces appear in many applications throughout the natural and applied sciences. The classical closest point method (Ruuth and Merriman, J. Comput. Phys. 227(3):1943-1961, [2008]) is an embedding method for solving PDEs on surfaces. Using a closest point representation of the surface, a constant-along-normal extension is employed to formulate the PDE in the embedded space, which can be solved numerically using standard finite difference schemes. We present a closest point method that uses finite difference schemes derived from radial basis functions (RBF-FD). When compared to the standard finite difference discretization of the original closest point method, the proposed method requires a smaller computational domain surrounding the surface, resulting in a decrease in the number of sampling points on the surface. In addition, higher-order schemes can easily be constructed by increasing the number of points in the RBF-FD stencil. Our method uses RBF centers on regular grid nodes, avoiding the ill-conditioning from point clustering on the surface. An implicit formulation that uses the least-squares method allows an easy and natural coupling with a grid based manifold evolution algorithm (Leung and Zhao, J. Comput. Phys. 228(8):2993-3024, [2009]). The method is tested in a number of applications on static and moving surfaces.

On Some Extensions of Thresholding Schemes

Karel Svadlenka
Kyoto University, Japan

In this talk I will present new results on several generalizations of the thresholding (BMO-type) algorithm for numerical approximation of curvature driven interface evolutions. In particular, I will focus on the computation of the hyperbolic mean curvature flow, anisotropic energies, multiphase problem and their combinations.

Trace Finite Element Methods for Partial Differential Equation on Evolving Surfaces

Xianmin Xu
Chinese Academy of Sciences, Peoples Rep of China
M. Olshanskii, C. Lehrenfeld

Many physical processes and biological phenomena could be modeled by partial differential equations on surfaces or manifolds. Recently, the method to solve surface PDEs has arisen much interests in the

community of numerical analysis. In this talk, I will introduce some trace finite element methods for convection-diffusion equations on evolving surfaces. The finite element space is a trace of a standard finite element space defined in the neighboring region of the surface. To deal with the evolving surface, an extension of the solution is done by fast marching method or a stabilization technique. The methods are based on a Eulerian framework and easily treat shape and topology changes of the surface. Numerical experiments and error estimates show the optimal convergence of the methods.

Special Session 146: Recent Developments in Stochastic Analysis, Stochastic Control and Related Fields

Chao Zhu, University of Wisconsin-Milwaukee, USA
Yu-Jui Huang, University of Colorado, USA

This session features new perspectives and methodologies toward stochastic processes, stochastic control and related fields. The specific topics to be presented in this session include asymptotic properties of certain novel regime-switching jump diffusion processes, optimal inventory control problems with a long-term average criterion, delay stochastic differential equations, and their applications in areas such as finance and risk management. This session will help to disseminate new research results, exchange ideas, and foster new collaborations.

Effective Dynamics of Nonlocal Stochastic Partial Differential Equations

Jinqiao Duan
Huazhong University of Science and Technology, Peoples Rep of China
Qiao Huang, Li Lin

The need to take stochastic effects into account for modeling complex systems has now become widely recognized. Stochastic partial differential equations arise naturally as mathematical models for multiscale systems under random influences. We consider macroscopic dynamics of microscopic systems described by stochastic partial differential equations. The speaker will present recent advances in deriving effective models for multiscale stochastic systems under non-Gaussian noise or nonlocal interactions, including homogenization reduction and slow manifold reduction techniques. The effectivity of the reduced systems is shown in the probabilistic sense of convergence.

Expected Exponential Utility Maximization of Insurers with a General Diffusion Factor Model: the Complete Market Case

Hiroaki Hata
Shizuoka University, Japan
Shuenn-Jyi Sheu, Li-Hsien Sun

In this talk, we consider the problem of optimal investment by an insurer. The insurer invests in a market consisting of a bank account and m risky assets. The mean returns and volatilities of the risky assets depend nonlinearly on economic factors that are formulated as the solutions of general stochastic differential equations. The wealth of the insurer is described by a Cramer–Lundberg process, and the insurer preferences are exponential. Adapting a dynamic programming approach, we derive Hamilton–Jacobi–Bellman (HJB) equation. And, we prove the unique solvability of HJB equation. In addition, the optimal strategy is obtained using the coupled forward and backward stochastic differential equations. Finally, proving the verification theorem, we construct the optimal strategy.

Optimal Equilibria for Time-Inconsistent Stopping Problems

Yu-Jui Huang
University of Colorado at Boulder, USA
Zhou Zhou

For time-inconsistent control/stopping problems, it is known that one should employ an equilibrium strategy, formulated in an intertemporal game between current and future selves. Such strategies, however, are not unique. This gives rise to two unsettled problems: (i) How do we find all equilibria? (ii) Among all equilibria, how do we select the appropriate one to use? For stopping problems under non-exponential discounting, we develop a new method, called the iterative approach, to resolve both (i) and (ii). First, we formulate equilibria as fixed points of an operator, which represents strategic reasoning that takes into account future selves' behavior. Under appropriate regularity conditions, every equilibrium can be found through a fixed-point iteration. When the state process is one-dimensional, we further establish the existence of an optimal equilibrium, which generates larger values than any other equilibrium does at all times. To the best of our knowledge, this is the first time a dominating subgame perfect Nash equilibrium is shown to exist in the literature of time-inconsistency. Our theory is illustrated explicitly in several real options models.

Investment and Reinsurance Non-Zero-Sum Games with Value-At-Risk Constraints

Zhuo Jin
University of Melbourne, Australia
Ning Wang, Nan Zhang, Linyi Qian

This work investigates a class of non-zero-sum stochastic differential investment and reinsurance game between two insurance companies. We assume that both insurers are allowed to purchase a proportional reinsurance and invest in risky and risk-free assets. When applying the generalized variance premium principle in determining reinsurance premium, the surplus process becomes quadratic in the retained proportion of claims. The optimization criterion of each insurer is to maximize his utility of the difference between his terminal surplus and that of his competitor. In addition, we incorporate dynamic VaR constraints to meet the capital requirement from regula-

tors. This game problem can be converted to solving a system of nonlinear equations when we assume both insurers are CARA agents. Finally, we use some numerical examples to illustrate the Nash Equilibrium strategy under different scenarios.

Portfolio Optimization in Presence of Proportional Transaction Costs and Regime-Switching

Ruihua Liu
University of Dayton, USA

This work extends the portfolio optimization problem via utility maximization in the presence of proportional transaction costs to regime-switching models. With regime-switching, the Hamilton-Jacobi-Bellman (HJB) equation becomes a system of m_0 coupled variational equalities where m_0 is the total number of regimes considered for the market. We consider a power utility function and establish important properties of the value function including the continuity in both time and state variables and the unique viscosity solution of the HJB equation. A numerical procedure is developed based on the formulation of the optimization in discrete time, and using an efficient discrete tree approximation of the underlying continuous time process.

Convergence of Nonlinear Filtering for Stochastic Dynamical Systems with Lévy Noises

Huijie Qiao
Southeast University, Peoples Rep of China

We consider the nonlinear filtering problem of multiscale non-Gaussian signal processes and observation processes with jumps. Firstly, we prove that the dimension for the signal system could be reduced. Secondly, convergence of the corresponding nonlinear filtering to the homogenized filtering is shown by weak convergence approach.

Portfolio Optimization with Delay Factor Models

Shuenn-Jyi Sheu
National Central University, Taiwan
Li-Hsien Sun, Zheng Zhang

We consider an optimal portfolio problem where the underlying risky assets are driven by the factor model with delay feature in order to describe the short term forecasting and the interaction with time delay among different financial markets. The delay phenomenon can be recognized as the integral type and the pointwise type. The optimal strategy is identified by maximizing the power utility. Due to the delay leading to the non-Markovian structure, the conventional PDE approaches are no longer applicable. Instead of using dynamic programming, we obtain the optimal strategy can be characterized by the solutions of a quadratic forward backward stochastic differential equations (QFBSDEs). The optimality is verified via the super-martingale argument. The existence and uniqueness of the solution to the QFBSDEs are established without requiring the assumption of complete market. In addition, we analyze three particular cases where the corresponding FBSDEs can be solved explicitly. The discussion is based on a joint work with Li-Hsien Sun and Zheng Zhang.

Equivalence of Two Definitions on Generalized Solutions of Dirichlet Problems Associated to Fractional Laplacian Operator

Qingshuo Song
City University of Hong Kong, Hong Kong

Probabilistic definition of the generalized solutions depends on fine topology of the associated stochastic process. In the context of PDE analysis, the generalized solution has been defined purely by viscosity superjet and subjet of differential operators, see for instance User's guide in the viscosity solution in 1992. We will show these two definitions are equivalent under very mild conditions. Together with Perron's method, the result can be used to prove solvability of HJB equations with Dirichlet boundary.

An Averaging Principle for Two-Time-Scale Functional Diffusions

Fuke Wu
Huazhong University of Science and Technology, Peoples Rep of China
George Yin

Dupire recently developed a functional Itô formula, which has changed the landscape of the study of stochastic functional equations and encouraged a reconsideration of many problems and applications. Delays are ubiquitous, pervasive, and entrenched in everyday life. Based on the new development, this work examines functional diffusions with two-time scales in which the slow-varying process includes path-dependent functionals and the fast-varying process is a rapidly-changing diffusion. The gene expression of biochemical reactions occurring in living cells in the introduction of this paper is such a motivating example. This paper establishes mixed functional Itô formulas and the corresponding martingale representation. Then it develops averaging and weak convergence methods. By treating the fast-varying process as a random noise, under appropriate conditions, it is shown that the slow-varying process converges weakly to a stochastic functional differential equation whose coefficients are averages of that of the original slow-varying process with respect to the invariant measure of the fast-varying process.

Representation of Adapted Solutions to Backward Stochastic Volterra Integral Equations

Jiongmin Yong
University of Central Florida, USA
Tianxiao Wang

For backward stochastic Volterra integral equations, under some mild conditions, the so-called adapted solutions or adapted M-solutions uniquely exist. However, satisfactory regularity of the solutions is difficult to obtain in general. Inspired by the decoupling idea of forward-backward stochastic differential equations, for a class of backward stochastic Volterra integral equations, a representation of adapted M-solutions is established by means of the so-called representation partial differential equations and (forward) stochastic differential equations.

Risk Sensitive Portfolio Optimization with Regime-Switching and Default Contagion

Xiang Yu
The Hong Kong Polytechnic University, Hong Kong
Lijun Bo, Huafu Liao

We study th risk-sensitive portfolio allocation in a regime-switching credit market with default contagion. The state space of the Markovian regime-switching process is assumed to be a countably infinite set. To characterize the value function of the risk sensitive stochastic control problem, we investigate the corresponding recursive infinite-dimensional nonlinear dynamical programming equations (DPEs) based on default states. We propose to work in the following procedure: Applying the theory of the monotone dynamical system, we first establish the existence and uniqueness of classical solutions to the recursive DPEs by a truncation argument in the finite state space. Moreover, the associated optimal feedback strategy is characterized by developing a rigorous verification theorem. Building upon results in the first stage, we construct a sequence of approximating risk sensitive control problems with finite state space and prove that the resulting smooth value functions will converge to the classical solution of the original system of DPEs. The construction and approximation of the optimal feedback strategy for the original problem are also discussed. Some numerical results are presented to illustrate our analytical conclusions.

Switching Between a Pair of Stocks: Trading Rules and More

Qing Zhang
University of Georgia, USA
Jingzhi Tie

This talk is about a stock trading rule involving two stocks. The trader may have a long position in either stock or in cash. She may also switch between them any time. Her objective is to trade over time to maximize an expected return. To treat the problem, we focus on the optimal trading control problem under a geometric Brownian motion model with regime switching. We use a two-state Markov chain to capture the general market modes. In particular, a single market cycle consisting of a bull market followed by a bear market is considered. We also impose a fixed percentage cost on each transaction. We focus on simple threshold type policies and study all possible combinations. We establish algebraic equations to characterize these threshold levels. We also present sufficient conditions that guarantee the optimality of these policies. Finally, some numerical examples will be provided to illustrate our results.

Global Closed-Form Approximation of Free Boundary for Optimal Investment Stopping Problems

Harry Zheng
Imperial College, England
Jingtang Ma, Jie Xing

We study a dynamic optimization problem with both stochastic control and optimal stopping in a finite time horizon. The problem can be converted into an equivalent free boundary problem of a fully nonlinear PDE. We give rigorous analysis of the dual control method for this problem with a class of utility functions, including power and non-HARA utilities. The dual problem is still a free boundary problem, but the governing PDE is linear. We analyse the asymptotic properties of the free boundary of the dual problem, construct a global closed-form approximation of the free boundary for the dual problem, and obtain the approximate formula for the dual value function, which in turn gives the approximate formulas for the primal value function, optimal control and optimal exercise boundary for the optimal investment stopping problem. Numerical examples show that the formulas are accurate and fast.

Bank Monitoring Incentives Under Moral Hazard and Adverse Selection

Chao Zhou
National University of Singapore, Singapore

In this paper, we extend the optimal securitisation model of Pagès and Possamaì and Pagès between an investor and a bank to a setting allowing both moral hazard and adverse selection. Following the recent approach to these problems of Cvitanič, Wan and Yang, we characterise explicitly and rigorously the so-called credible set of the continuation and temptation values of the bank, and obtain the value function of the investor as well as the optimal contracts through a recursive system of first-order variational inequalities with gradient constraints. We provide a detailed discussion of the properties of the optimal menu of contracts. This is a joint work with Nicolás Hernández Santibáñez and Dylan Possamaì.

A Weak Convergence Approach to Inventory Control Using a Long-Term Average Criterion

Chao Zhu
University of Wisconsin-Milwaukee, USA
K.L. Helmes, R.H. Stockbridge, C. Zhu

This work considers an optimal inventory control problem using a long-term average criterion. In absence of ordering, the inventory process is modeled by a one-dimensional diffusion on some interval of $(-\infty, \infty)$ with general drift and diffusion coefficients and boundary points that are consistent with the notion that demands tend to reduce the inventory level. Orders instantaneously increase the inventory level and incur both positive fixed and level dependent costs. In addition, state-dependent holding/backorder costs are incurred continuously. Examination of the steady state behavior of (s, S) policies leads to a two-dimensional nonlinear optimization problem for which a pair of optimizers establishes the levels for an optimal $(s*, S*)$ policy. Using average expected occupation and ordering measures and weak convergence arguments, weak conditions are given for the optimality of the (s_*, S_*) ordering policy in the general class of admissible policies. The analysis involves an auxiliary C^2 function that solves a particular system of linear equations and inequalities related to but different from the long-term average Hamilton-Jacobi-Bellman equation. This approach provides an analytical solution to the problem rather than a solution involving intricate analysis of the stochastic processes. The utility of these results is illustrated on drifted and geometric Brownian motion inventory models under conventional and non-conventional cost structures.

Consensus Control of Multi-Agent Systems with Noises and Time-Delays

Xiaofeng Zong
China University of Geosciences, Peoples Rep of China
Tao Li, Ji-Feng Zhang

Multi-agent systems in complex communication environments are inevitably subjected to communication delays and measurement noises. The delays and measurement noises are intertwined with the distributed structure of the multi-agent system, which significantly affects the convergence performance and brings substantial difficulties to the design of the control law. Based on probability theory and stochastic stability theory, this report will investigate the consensus control of the multi-agent systems with measurement noises and time-delays. The explicit consensus conditions related to the control gain and system parameters are deduced, where the system parameters include noise intensities, time-delay and communication graph.

Special Session 147: Structure Preserving Numerical Methods

Molei Tao, Georgia Institute of Technology, USA

Many systems in science and engineering are endowed with structures such as symplecticity, volume-preservation, invariant distribution, and/or monotonicity. Often, the numerical preservation of such structures results in improved long-term simulation accuracy, and even at a short scale it can lead to a better trade-off between accuracy and efficiency. This special session discusses progress in both recent and long-standing problems in structure preserving numerical methods.

Optimal Explicit Stabilized Integrator of Weak Order One for Stiff and Ergodic Stochastic Differential Equations

Ibrahim Almuslimani
University of Geneva, Switzerland
Assyr Abdulle, Gilles Vilmart

Explicit stabilized Runge-Kutta methods are efficient for solving stiff (deterministic or stochastic) differential equations in large dimensions. In this talk, we present a new explicit stabilized scheme of weak order one for stiff and ergodic stochastic differential equations (SDEs). In the absence of noise, the new method coincides with the classical deterministic stabilized scheme (or Chebyshev method) for diffusion dominated advection-diffusion problems and it inherits its optimal stability domain size, in contrast to known existing methods for mean-square stable stiff SDEs. In addition, the new method can be used to sample the invariant measure of a class of ergodic SDEs, and combined with postprocessing techniques of geometric numerical integration originally from the deterministic literature, it achieves a convergence rate of order two at a negligible overcost.

Runge-Kutta Semidiscretizations for Stochastic Maxwell Equations

Chuchu Chen
Chinese Academy of Sciences, Peoples Rep of China
Jialin Hong, Lihai Ji

In this talk we investigate the properties of stochastic Maxwell equations with additive noise, including regularity, symplecticity, and the involution laws of energy and divergence, etc. We propose a general class of stochastic Runge-Kutta methods in the temporal direction to discretize the stochastic Maxwell equations and show that under certain conditions on the coefficients the methods preserve symplecticity. We show that the mean-square convergence order of the semidiscrete scheme is 1 under appropriate assumptions.

Uncertainty Quantification of Numerical Errors in Geometric Integration Via Random Time Steps

Giacomo Garegnani
EPFL, Switzerland
Assyr Abdulle

We present a novel probabilistic integrator for ordinary differential equations (ODEs) which allows for uncertainty quantification of the numerical error [1]. In particular, we randomise the time steps and build a probability measure on the deterministic solution, which collapses to the true solution of the ODE with the same rate of convergence as the underlying deterministic scheme.
The intrinsic nature of the random perturbation implies that our probabilistic integrator conserves some geometric properties of the deterministic method it is built on, such as the conservation of first integrals or the symplecticity of the flow.
Finally, we present a procedure to incorporate our probabilistic solver into the frame of Bayesian inference inverse problems, showing how inaccurate posterior concentrations given by deterministic methods can be corrected by a probabilistic interpretation of the numerical solution.

References

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Finite Element Exterior Calculus for Parabolic Problems on Evolving Surfaces

Evan Gawlik
University of California, San Diego, USA
Michael Holst

Finite element exterior calculus provides a unified framework for analyzing the stability and convergence of mixed finite element discretizations of partial differential equations. Structure preservation plays a key role in this analysis, since it relies on the use of finite element spaces that form a subcomplex of the de Rham complex. The framework was originally developed for elliptic PDEs on open domains in Euclidean space, but recent efforts have extended it to time-dependent PDEs and PDEs on static surfaces. We extend the framework to another setting: parabolic PDEs posed on surfaces that evolve with time in a prescribed fashion. We prove a priori error estimates for numerical discretizations of such problems, taking into account variational crimes (discrepancies between the geometry of the exact and approximate surfaces) in the analysis.

Towards a Geometric Variational Discretization of Compressible Fluids

Francois Gay-Balmaz
CNRS Ecole Normale Superieure, France
Werner Bauer

We present a geometric variational discretization of compressible fluid dynamics. The numerical scheme is obtained by discretizing, in a structure preserving way, the Lie group formulation of fluid dynamics on diffeomorphism groups and the associated variational principles. Our framework applies to irregular mesh discretizations in 2D and 3D. It systematically extends work previously made for incompressible fluids to the compressible case. We consider in detail the numerical scheme on 2D irregular simplicial meshes and evaluate the behavior of the scheme for the rotating shallow water equations. While our focus is fluid mechanics, our approach is potentially useful for discretizing problems involving evolution equations on diffeomorphism groups.

Symplectic Parareal Schemes and Structure Preserving Parallel-In-Time Propagators

Frederic Legoll
Ecole des Ponts, France

Several numerical schemes have been proposed to speed-up the time propagation of evolution equations, among which the parareal algorithm. It is based on a decomposition of the time interval into subintervals, and on a predictor-corrector strategy, where the propagations over each subinterval for the corrector stage are concurrently performed on the different processors that are available. This method has been shown to perform well in many situations. The next step is to use it for problems with geometrical structure, such as Hamiltonian dynamics, where symplectic schemes are needed. Since the plain parareal iterations do not give rise to a symplectic scheme, several proposals have been put forward to correct for this drawback. After a short review of previous works, we present in this talk our new approach, together with theoretical and numerical evidence supporting it. Joint work with Y. Maday (Paris 6) and G. Turinici (Paris 9).

Construction of Gradient Flows in Metric Spaces Via BDF2

Simon Plazotta
Technical University Munich, Germany
Daniel Matthes

In this talk I will discuss the construction of λ-contractive gradient flows in abstract metric spaces by means of a semi discretization of second order in time. In the smooth setting, our scheme is simply a variational formulation of the BDF2 method; in the metric setting, it can be considered as the natural second order analogue of the Minimizing Movement or JKO scheme. In difference to the JKO method, our scheme does not necessarily decrease the energy of the discrete solution in each time step, but we can still prove a suitable *almost diminishing* property. It is well-know that in smooth situations, the BDF2 method converges to order τ^2. We prove that our variational scheme converges at least to order $\tau^{1/2}$ in the general non-smooth setting, provided a certain convexity hypothesis is satisfied. Specifically, that hypothesis is equivalent to λ-uniform convexity in the flat case, and is implied by λ-convexity along generalized geodesics in the L^2-Wasserstein case. In the special case of the Fokker-Planck equation seen as L^2-Wasserstein gradient flow an alternative approach can be used, which relies heavily on the differentiable structure of the underlying space. Here, we can prove a stronger convergence result, though without an explicit rate.

Nonlinear Stability of Symplectic Numerical Integration

Zaijiu Shang
Chinese Academy of Sciences, Peoples Rep of China
Lina Song

In this talk we will introduce a new approach to analyse the stability of symplectic numerical integration of Hamiltonian systems. We will illustrate the main idea by applying the symplectic Euler method to the normalized nonlinear oscillator and show at what extent a symplectic method can give a stable numerical simulation to the typical dynamics of Hamiltonian systems. The explanation is mainly based on the stability analysis of Hamiltonian systems and the backward analysis of numerical methods.

Numerical Simulation of Runaway Particles in Plasmas

Yajuan Sun
AMSS, CAS, Peoples Rep of China

One of the severe consequences of the plasma disruptions in tokamaks is the generations of the energetic particles. These particles accelerated by the electric field carry massive energy which has great danger to plasma facing components. Since the timescales of dynamics of runaway electrons span 11 orders of magnitude, the numerical simulation with long-term stability is needed. Based on geometric property inherited by the full orbit model of runaway particles we construct the numerical methods, and exhibit the numerical results.

Explicit High-Order Symplectic Integration of Arbitrary Hamiltonians

Molei Tao
Georgia Tech, USA

Symplectic integrators preserve the phase-space volume and have favorable performances in long time simulations. Methods for explicit symplectic integration have been extensively studied for separable Hamiltonians (i.e., H(q,p)=K(p)+V(q)), and they lead to both accuracy and efficiency. However, nonseparable Hamiltonians also model important problems. Unfortunately, implicit methods had been the only available symplectic approach with long-time accuracy for general nonseparable systems. This talk will construct explicit symplectic integrators for nonseparable systems. These new integrators are based on a mechanical restraint that binds two copies of phase space together, and they can be made arbitrarily high-order. Using backward error analysis, KAM theory, and some additional multiscale analysis, a pleasant error bound is established for integrable systems. Numerical evidence of statistical accuracy for non-integrable systems were also observed.

Variational Integrators for the Nonequilibrium Thermodynamics of Simple Systems

Hiroaki Yoshimura
Waseda University, Japan
Francois Gay-Balmaz

We propose variational integrators for the nonequilibrium thermodynamics focusing on the case of simple closed systems. The variational integrators are obtained by a variational discretization of the Lagrangian variational formulation of nonequilibrium thermodynamics, which is a natural extension of the variational discretization for Hamilton's principle in mechanics to include irreversible processes. First we show the continuous setting of a variational formulation of the nonequilibrium thermodynamics, in which we have a structure-preserving property of the flow of the evolution equations as we have the symplectic property associated with the Euler-Lagrange equations in Lagrangian mechanics. Then, we develop the discrete analog of the variational formulation of the nonequilibrium thermodynamics and show how the discrete flow of the numerical solution also has such a structure-preserving property. In particular, we discuss the regularity condition of the discrete evolution equations, which ensures the existence of the discrete flow of the system. We finally illustrate our theory by some numerical examples of simple closed systems.

Structure-Preserving Continuous-Stage Runge-Kutta-Nyström Methods

Jingjing Zhang
East China Jiaotong University, Peoples Rep of China
Wensheng Tang, Yajuan Sun

The second order ordinary differential equations are commonly encountered in various fields. Some of them can be reduced to the first order ordinary differential equations with the form of separable Hamiltonian systems. For such systems structure-preserving methods, for example, symplectic and symmetric methods are of importance. We present a sufficient condition for a continuous stage Runge-Kutta Nyström (RKN) method to be symplectic and symmetric. Based on Legendre polynomial expansion we show how to construct symplectic and symmetric RKN type method with a certain order in a simple way. Some numerical experiments are presented to show the efficiency of the newly obtained methods.

Special Session 148: Intersections in Probability and Nonlinear PDEs

Yu-Min Chung, University of North Carolina-Greensboro, USA
Nathan Glatt-Holtz, Tulane University, USA
Vincent R. Martinez, Tulane University, USA
Cecilia Mondaini, Texas A&M University, USA

Random effects are ubiquitous in the natural sciences, from quantum and fluid mechanics to biology and ecology. These fields provide a rich source of interesting and challenging problems which drive the development of theory at the intersection of stochastic analysis and partial differential equations. The heart of our session lies in the duality between "stochastic" and "deterministic" in the context of infinite-dimensional dynamical systems cast within a probabilistic framework. On the one hand, stochasticity can have a regularizing effect. For instance, by randomizing the initial data, one can improve low regularity well-posedness results for many dispersive equations, while in the context of dissipative systems such as the stochastically forced Navier-Stokes equations, one can establish the existence and uniqueness of ergodic, invariant measures through an emerging theory of hypo-ellipticity in infinite dimensions. On the other hand, as recent progress on the Karder-Parisi-Zhang equation has shown, stochastic versions of partial differential equations (PDEs) can present unique, but arrestingly difficult challenges to their well-posedness or qualitative theory of solutions. Indeed, many analogues of classical questions are at the forefront of research in the stochastic setting. This session promises to bring together a diverse group of leading researchers in probability, statistics, and the analysis of PDEs, particularly those working at their various interfaces.

Variational Approach to Closure of SPDEs: Markovian and Non-Markovian Parameterizations

Mickael Chekroun
UCLA, USA
Honghu Liu, James C. McWilliams, Shouhong Wang

Stochastic partial differential equations (SPDEs), with bilinear drift and driven by a degenerate additive noise, will be considered. For such equations, we will present new analytic formulas for Markovian as well as non-Markovian parameterizations of the scales lying beyond a cutoff wavenumber. The derivation of these formulas takes place within a variational approach relying on the theory of stochastic parameterizing manifolds whose main tools and concepts will be introduced. The relationships with the ergodic theory of SPDEs will be discussed and applications to closure in the context of "Burgulence" will be presented. The role of path-dependent, non-Markovian coefficients arising in the related closure systems will be also discussed.

MAP Estimators and Posterior Consistency for Bayesian Inverse Problems

Masoumeh Dashti
University of Sussex, England
S. Agapiou, M. Burger, T. Helin

We consider the inverse problem of recovering an unknown functional parameter from noisy and indirect observations. We adopt a Bayesian approach and, for some classes of prior measures, show that maximum a posteriori (MAP) estimates are characterized by the minimizers of a generalized Onsager-Machlup functional of the posterior. We also discuss some posterior consistency results. This is based on joint works with S. Agapiou, M. Burger and T. Helin.

Geometry of Turbulent Flows and the 3D Navier-Stokes Regularity Problem

Aseel Farhat
University of Virginia, USA

We describe several aspects of an analytic/geometric framework for the three-dimensional Navier-Stokes regularity problem, which is directly inspired by the morphology of the regions of intense vorticity/velocity gradients observed in computational simulations of three-dimensional turbulence.

Multilevel Sequential Monte Carlo Samplers

Kody Law
Oak Ridge National Laboratory, USA
Ajay Jasra

This talk will concern new algorithms for solving Bayesian inverse problems; in particular multilevel sequential Monte Carlo (SMC) samplers. Even when the underlying forward PDE model is linear for a fixed value of the parameter, the map from parameter to observation is often nonlinear. One cannot sample from the posterior distribution directly, but can only evaluate it, up to a normalizing constant. Therefore one must resort to computationally-intensive inference algorithms in order to construct estimators. Another difficulty which arises is that the PDE typically cannot be solved and needs to be approximated at finite resolution. The multilevel Monte Carlo method provides a way of optimally balancing discretization and sampling error on a hierarchy of approximation levels, such that cost is optimized. Recently this has been applied to computationally intensive inference. The resulting multilevel SMC samplers will be presented.

Galerkin Approximations of Nonlinear Delay Differential Equations with Or Without Noise

Honghu Liu
Virginia Tech, USA
Mickael D. Chekroun, Michael Ghil, Shouhong Wang

Delay differential equations (DDEs) are widely used in many applied fields to account for delayed responses of the modeled systems to either internal or external factors. In contrast to ordinary differential equations (ODEs), the phase space associated even with a scalar DDE is infinite-dimensional. Oftentimes, it is desirable to have low-dimensional ODE systems that capture qualitative features as well as approximate certain quantitative aspects of the DDE dynamics. In this talk, we present a new Galerkin scheme for general nonlinear DDEs, either deterministic or stochastic. The main new ingredient is the use of a type of polynomials that are orthogonal under an inner product with a point mass. Rigorous convergence results will be presented, and the efficiency of the approach will be numerically illustrated on DDE models arising from climate dynamics and mathematical biology. The presentation is based on joint work with Mickael D. Chekroun (UCLA), Michael Ghil (UCLA & ENS, France), and Shouhong Wang (IUB).

Anomalous Diffusion and the Generalized Langevin Equation

Hung Nguyen
Tulane University, USA
Nathan Glatt-Holtz, David Herzog, Scott McKinley

The Generalized Langevin Equation is commonly used to describe the velocity of microparticles in viscoelastic fluids. Formally, the Generalized Langevin Equation (GLE) is written

$$\begin{aligned} m\ddot{x}(t) &= -\gamma\dot{x}(t) - \Phi'(x(t)) \\ &\quad - \int_{-\infty}^{t} K(t-s)\dot{x}(s)ds + F(t) + \sqrt{2\gamma}\dot{W}(t), \end{aligned}$$

where $\Phi(x)$ is a non-linear potential well, $W(t)$ is a Brownian motion, and $F(t)$ is a stationary, mean zero and Gaussian process satisfying $E(F(t)F(s)) = K(t-s)$. Describing the long-term behavior of subdiffusive GLEs in non-linear potentials is a long-standing open problem. We will look at recent advances in establishing existence and uniqueness of a stationary distribution for an infinite-dimensional Markov representation of the GLE.

Obstacle Problems for Nonlocal Operators

Camelia Pop
University of Minnesota, USA
Donatella Danielli, Arshak Petrosyan

We prove existence, uniqueness, and regularity of viscosity solutions to the stationary and evolution obstacle problems defined by a class of nonlocal operators that are not stable-like and may have supercritical drift. We give sufficient conditions on the coefficients of the operator to obtain Hölder and Lipschitz continuous solutions. The class of nonlocal operators that we consider include non-Gaussian asset price models widely used in mathematical finance, such as Variance Gamma Processes and Regular Lèvy Processes of Exponential type. In this context, the viscosity solutions that we analyze coincide with the prices of perpetual and finite expiry American options.

Random Initial Conditions for Semi-Linear PDEs

Marco Romito
Università di Pisa, Italy
D. Blomker, G. Cannizzaro

We use semi–linear PDEs as a "proof of concept" to investigate the effect of random initial conditions for the existence of partial differential equations of evolution type. These ideas have been pioneered by Bourgain, and recently there have been a lot of activity, since the seminal papers by Burq and Tzvetkov. In this setting we wish to be able to answer a series of relevant questions for the subject, such as if and when a random initial condition turns out to be useful, if super–critical data are allowed, if renormalization is needed, if further ideas from the theory of singular stochastic PDEs can be borrowed, etc.

Well-Posedness for Stochastic Continuity Equations with Rough Coefficients

Samuel Samuel Punshon-Smith
University of Maryland, USA

According to the theory of Diperna/Lions, the continuity equation associated to a Sobolev (or BV) vector field has a unique weak solution in L^p. Under the addition of white in time stochastic perturbations to the characteristics of the continuity equation, it is known that uniqueness can be obtained under a relaxation of the regularity conditions. In this talk, we will consider the general stochastic continuity equation associated to an Itô diffusion with irregular drift and diffusion coefficients and discuss conditions under which the equation has a unique solution. Using the renormalization approach of DiPerna/Lions we will present another proof of uniqueness of solutions to the stochastic transport with additive noise and a drift in $L^q_t L^p_x$, satisfying the subcritical Ladyzhenskaya/Prodi/Serrin criterion $2/q + d/p < 1$.

Special Session 149: Analytic Approaches on Qualitative Properties of Solutions of PDE

Annamaria Barbagallo, Naples University, Federico II, Italy
Maria Alessandra Ragusa, University of Catania, Italy
Andrea Scapellato, University of Catania, Italy

Local and global regularity properties have been studied by many authors in the last thirty years. The speakers shows new regularity results for elliptic, parabolic equations and systems, assuming that the coefficients could be discontinuous.

The Cauchy-Dirichlet Problem for a Class of Hyperbolic Operators with Double Characteristics in Presence of Transition: Existence and Uniqueness Results

Annamaria Barbagallo
University of Naples Federico II, Italy
Vincenzo Esposito

The aim of the talk is to study the Cauchy-Dirichlet problem for the class of hyperbolic second order operators with double characteristics in the presence of transition. A priori local and global estimates for the solutions are obtained. Thanks to these estimates, existence and uniqueness results are established.

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The Equality Case in a Poincaré-Wirtinger Type Inequality

Barbara Brandolini
University of Naples Federico II, Italy

Let Ω be a convex, possibly unbounded, domain in $\mathbb{R}^2$ and denote by $\mu_1(\Omega)$ the first nontrivial Neumann eigenvalue of the Hermite operator in Ω. It is known that

$$\mu_1(\Omega) \geq 1. \tag{1}$$

The estimate is sharp since equality sign holds if Ω is any strip. Inequality (1) can be read as an optimal Poincaré-Wirtinger inequality for functions belonging to the weighted Sobolev space $H^1(\Omega, d\gamma_2)$, where γ_2 is the 2-dimensional Gaussian measure. We study the equality case and we show that $\mu_1(\Omega) = 1$ if and only if Ω is any strip.

Non-Zero Solutions for Elliptic Differential Problems

Giuseppina D'Agui
University of Messina, Italy

The aim of this talk is to point out a two non-zero critical point theorem for differentiable functionals defined in a Banach space. As an application, existence results of two positive solutions for elliptic differential problems, by requiring in particular a suitable condition on the nonlinearity, are presented.

Weinstock Inequality in Higher Dimensions

Vincenzo Ferone
Università di Napoli Federico II, Italy
Dorin Bucur, Carlo Nitsch, Cristina Trombetti

We prove that the Weinstock inequality for the first nonzero Steklov eigenvalue holds in $\mathbb{R}^n$, for $n \geq 3$, in the class of convex sets with prescribed surface area. The key result is a sharp isoperimetric inequality involving simultanously the surface area, the volume and the boundary momentum of convex sets. As a by product, we also obtain some isoperimetric inequalities for the first Wentzell eigenvalue.

Wavelet Method for PDEs on Besov Spaces

Emanuel Guariglia
University of Naples Federico II, Italy

This paper revises the PDE's theory through wavelet analysis. Taking into account the close link with the Besov spaces, some results are derived. Additionally, application to image processing with particular emphasis to image compression are given and discussed.

Effects of Some Estimates of Functionals in Calculus of Variations

Maria Alessandra Ragusa
Catania University-RUDN Univ, Moscow, Russia, Italy

Is showed a problem studied in cooperation with Professor Atsushi Tachikawa. We treat the regularity problem for minimizers of quadratic and non-quadratic growth functionals where the integrand is dependent on x, u, Du. We point out that about the dependence on the variable x is assumed only that $A(x, u, p)$ is in the class VMO, Vanishing Mean Oscillation class, as a function of x. Namely, the continuity of $A(x, u, p)$ with respect to x is not assumed. Are considered both partial and global regularity of the minimizer u.

Parabolic Equations on Morrey Spaces with Mixed Norm

Andrea Scapellato
University of Catania, Italy

Aim of the talk is to show some regularity results for solutions to linear partial differential equations of parabolic type in the framework of Morrey spaces with mixed norm. We use several estimates for some integral operators that appear in the representation formula both for the second order spatial derivatives and first order time derivative of a solution. It is worth pointing out that the coefficients of the principal part of the parabolic operator under consideration belong to the Sarason class of functions having vanishing mean oscillation and then they could be discontinuous.

Special Session 150: Eigenvalues of Elliptic Operators and Their Applications

Chiu-Yen Kao, Claremont McKenna College, USA
Pedro Antunes, University of Lisbon, Portugal

Since Lord Rayleigh conjectured that the disk should minimize the first Laplace-Dirichlet eigenvalue among all shapes of equal area more than a century ago, spectral study of elliptic operators has been active research topics with applications in various areas including mechanical vibration, optical resonator, photonic crystals, liquid sloshing, and population dynamics. This minisymposium features the latest progress on numerical and theoretical approaches for solving linear and nonlinear eigenvalue problems, eigenvalue optimization, and their applications.

Is It Possible to Tune a Drum?

Pedro Antunes
University of Lisbon, Portugal

It is well known that the sound produced by string instruments has a well defined pitch. Essentially, this is due to the fact that all the resonance frequencies of the string have integer ratio with the smallest eigenfrequency. However, it is enough to use Ashbaugh-Benguria bound for the ratio of the smallest two eigenfrequencies to conclude that it is impossible to build a drum with a uniform density membrane satisfying harmonic relations on the eigenfrequencies. On the other hand, it is known since the antiquity, that a drum can produce an almost harmonic sound by using different densities, for example adding a plaster to the membrane. This idea is applied in the construction of some Indian drums like the tabla or the mridangam. In this work we propose a density and shape optimization problem of finding a composite membrane that satisfy approximate harmonic relations of some eigenfrequencies. The problem is solved by a domain decomposition technique applied to the Method of Fundamental Solutions and Hadamard shape derivatives for the optimization of inner and outer boundaries. This method allows to present new configurations of membranes, for example a two-density membrane for which the first 21 eigenfrequencies have approximate five harmonic relations or a three-density membrane for which the first 45 eigenfrequencies have eight harmonic relations, both involving some multiple eigenfrequencies.

Shape Recognition Using the Eigenvalues of Elliptic Operators

Mohamed Ben Haj Rhouma
Qatar University, Qatar
Lotfi Hermi, M.A. Khabou

In this talk, we will review the latest developments regarding using the eigenvalues of linear operators for shape recognition. In particular, we will discuss what makes the properties that make the eigenvalues desirable and reliable feature vectors. We will also present the image data sets and the recognition rates obtained on them for different operators. In addition, we will present some numerical results showing evidence that there might be a one-to-one correspondence between the eigenvalues of operators and moment invariants which are standard tools used for shape recognition.

On Exact Pleijel's Constant for Some Domains

Vladimir Bobkov
University of West Bohemia, Czech Rep

Let λ_k and φ_k be the k-th eigenvalue and an associated eigenfunction, respectively, of the Dirichlet Laplacian on a bounded domain $\Omega \subset \mathbb{R}^2$. Denote by $\mu(\varphi_k)$ the number of nodal domains of φ_k. Courant's nodal domain theorem asserts that $\mu(\varphi_k) \leq k$ for any k. Pleijel obtained the following refinement of this fact:

$$Pl(\Omega) := \limsup_{k\to\infty} \frac{\mu(\varphi_k)}{k} \leq \frac{4}{j_{0,1}^2} = 0.69166\ldots$$

Here, $Pl(\Omega)$ is called Pleijel constant of Ω, and $j_{0,1}$ stands for the first zero of the Bessel function J_0. In the present talk, we discuss explicit expressions and values of the Pleijel constant $Pl(\Omega)$ for several domains Ω with separable geometries, such as a disk, annuli, and their sectors. Consideration of the case of annuli required the development of the theory of zeros of cross-products of Bessel functions, and revealed natural but open problems on multiplicity of corresponding eigenvalues. The talk is based on the preprints arXiv:1802.04357 and arXiv:1803.09972.

The Asymptotic Behavior of the Eigenvalues of a Heun Type Differential Operator Arising in Fluid Dynamics

Marina Chugunova
Claremont Graduate University, USA
Hans Volkmer

Eigenmode solutions are very important in stability analysis of dynamical systems. The set of eigenvalues of a non-self-adjoint differential operator originated from the linearization of some Cauchy problem is investigated. It is shown that the eigenvalues are purely imaginary, and that they are related to the eigenvalues of Heun's differential equation. These two results are used to derive the asymptotic behavior of the eigenvalues and to compute them numerically.

Extremal Spectral Gaps for Periodic Schrödinger Operators

Chiu-Yen Kao
Claremont McKenna College, USA
Braxton Osting

The spectrum of a Schrödinger operator with periodic potential generally consists of bands and gaps. In this paper, for fixed m, we consider the problem of maximizing the gap-to-midgap ratio for the m-th spectral gap over the class of potentials which have fixed periodicity and are pointwise bounded above and below. We prove that the potential maximizing the m-th gap-to-midgap ratio exists. In one dimension, we prove that the optimal potential attains the pointwise bounds almost everywhere in the domain and is a step-function attaining the imposed minimum and maximum values on exactly m intervals. Optimal potentials are computed numerically using a rearrangement algorithm and are observed to be periodic. In two dimensions, we develop an efficient rearrangement method for this problem based on a semi-definite formulation and apply it to study properties of extremal potentials. We show that, provided a geometric assumption about the maximizer holds, a lattice of disks maximizes the first gap-to-midgap ratio in the infinite contrast limit. Using an explicit parametrization of two-dimensional Bravais lattices, we also consider how the optimal value varies over all equal-volume lattices.

Optimal First Eigenvalue of P-Laplacian Operator

Seyyed Abbas Mohammadi
Yasouj University, Iran

An optimization problem related to the p-Laplacian operator is investigated. The aim is to determine a density function from a rearrangement class generated by a step function such that the principal eigenvalue is as small as possible. This optimization problem can be recasted into a shape optimization problem of finding a set of fixed measure such that the first eigenvalue will be minimal. At first some qualitative aspects of the minimizer set are obtained. Then, nearly optimal sets which are approximations of the minimizer for specific ranges of the parameter values are given. For those values of parameters, we show that these nearly optimal sets are in good agreement with the minimizers. In order to derive the optimal shape, a numerical algorithm is proposed and it is proved that the numerical procedure converges to a local optimizer. To demonstrate the efficiency and robustness of the algorithm, several numerical examples are provided. This is a joint work with Farid Bozorgnia.

Ground States of Spin-1 Bose-Einstein Condensates and Its Numerical Experiments

Tien-Tsan Shieh
National Taiwan University, Taiwan
I-Liang Chern

We develop an analytic theory for the ground state patterns and phase diagram of spin-1 Bose-Einstein condensates in a bounded domain. A complete phase diagram with explicit analytic formulae of Thomas-Fermi solutions on the parameter $q - M$ plane (q: quadratic Zeeman energy, M: total magnetization) for both ferromagnetic and antiferromagnetic systems and an entire characterization of ground-state patterns in the semi-classical regime will be present. We will also show our numerical experiments to compare with the theoretical results.

Optimization of Biharmonic Eigenvalue Problems of Vibrating Plates

Ying Wang
University of Oklahoma, USA
Chiu-Yen Kao

Biharmonic eigenvalue problems arise in the study of the mechanical vibration of plates. In this work, we discuss the minimization of the first eigenvalue of clamped plate, simply supported plate, and buckling of a plate. A rearrangement algorithm is proposed to find the optimal coefficient function based on the variational formula of the first eigenvalue. On various domains, such as square, circular and annular domains, the region where the optimal coefficient function takes the larger value may have different topologies. We show how the optimal configurations change with respect to plates with various domains.

Special Session 151: Nonlinear Elliptic and Parabolic Problems in Mathematical Physics and Related Topics

Soohyun Bae, Hanbat National University, Korea
Jann-Long Chern, National Central University, Taiwan
Jongmin Han, Kyung Hee University, Korea
Yoshitsugu Kabeya, Osaka Prefecture University, Japan

The aim of this session is to discuss recent progress in the study of nonlinear elliptic and parabolic PDEs arising from various sources of mathematical physic such as gauge field theories and fluid mechanics. This session bring together ideas from theoretical development in PDEs and its application to mathemarical physics.

The Quantitative Analysis of Solutions for a Coupled Equations Arsing from the Maxwell-Chern-Simons O(3) Sigma Model

Zhi-You Chen
National Changhua University of Education, Taiwan
Jann-Long Chern

In this talk, we consider the nonlinear equations arising from the self-dual Maxwell-Chern-Simons gauged $O(3)$ sigma model on (2+1)-dimensional Minkowski space $\mathbb{R}^{2,1}$ with the metric $\operatorname{diag}(1,-1,-1)$. We establish the asymptotic behavior of solutions corresponding to their flux and find the range of the flux for non-topological solutions. Moreover, we establish the sharp region of the flux for the non-topological solutions in one vortex point case.

Bubbling Solutions for the Gravitational $O(3)$ Model in $\mathbb{R}^2$

Nari Choi
Ewha Womans University, Korea
Jongmin Han

In this talk, we construct nontopological bubbling solutions of self-dual equation for the Maxwell gauged $O(3)$ sigma model coupled with gravity in $\mathbb{R}^2$. Our solutions blow up at some point of the anti-string, $\{p_j\}_{j=1}^{d_2}$ as the coupling parameter tends to zero and they are asymptotically radial near each blow-up point.

Bifurcation Analysis of the Modified Swift-Hohenberg Equation

Yuncherl Choi
Kwangwoon University, Korea
Taeyoung Ha, Jongmin Han

In this talk, we study dynamics bifurcation of the modified Swift-Hohenberg equation endowed with evenly periodic condition on an interval. Having the length of the periodicity as the bifurcation parameter λ, we prove that the trivial solution bifurcates to an attractor as λ crosses over a critical point. We also verify the structure of the bifurcated attractor by investigating the stability of singular points.

Entire Solutions for Superlinear Laplace Problems with Sign Changing Weights

Matteo Franca
Marche Polytechnic University (Ancona), Italy
A. Sfecci (Marche Polytechnic Uinversity, Italy)

In this talk we present some recent structure results concerning radial solutions for production/absorption/diffusion equations, i.e.:

$$\Delta u + k(|x|)u^{q-1} = 0$$

where $x \in \mathbb{R}^n$, $n > 2$, $q > 2$ and $k(|x|)$ changes sign.

Discrete N-Barrier Maximum Principle for a Lattice Dynamical System

Li-Chang Hung
National Taiwan University of Science and Technology, Taiwan
Chiun-Chuan Chen, Ting-Yang Hsiao

In the this talk, we show that an analogous N-barrier maximum principle remains true for lattice systems. This extends our previous results from continuous equations to discrete equations. In order to overcome the difficulty induced by a discretized version of the classical diffusion in the lattice systems, we propose a more delicate construction of the N-barrier which is appropriate for the proof of the N-barrier maximum principle for lattice systems.

Existence and Symmetric Properties of Solution to the Neumann Problem of Hardy-Sobolev Equation with Hardy Potential

Gyeongha Hwang
NCTS, Taiwan
Jann-Long Chern

We consider the nonlinear Neumann problem of Hardy-Sobolev equation with Hardy potential. Firstly, We establish existence of infinitely many positive radial solution which is singular at O under some condition. Secondly, we prove existence and regularity of a least energy solution. Lastly, we verify the symmetric property of a regular least energy solution.

Strauss's Radial Compactness and Its Application to Nonlinear Elliptic Problem with Variable Critical Exponent

Megumi Sano
Tokyo Institute of Technology, Japan
Masato Hashizume

We consider on compactness for the embedding from radial Sobolev spaces $W^{1,p}_{rad}(\mathbf{R}^N)$ to variable exponent Lebesgue spaces $L^{q(x)}(\mathbf{R}^N)$. In particular, we point out that the behavior of $q(x)$ at infinity plays an essential role on compactness. As an application we prove the existence of solutions of the quasi-linear elliptic equation with a variable critical exponent. This is a joint work with Masato Hashizume (Ehime University).

Self-Dual Condensate Solutions of O(3) Maxwell-Chern-Simons-Higgs Equations

Kyungwoo Song
Kyung Hee University, Korea

We consider the $O(3)$ Maxwell-Chern-Simons-Higgs equations on a two-dimensional flat torus arising from the $O(3)$ gauge field model. We study the existence of solutions of the equations via the topological degree theory.

Stationary Solitons of a Three-Wave Model Generated by Type II Second-Harmonic Generation in Quadratic Media

Yong-Li Tang
Feng Chia University, Taiwan
Zhi-You Chen

In this report, we consider an optical model generated by Type II Second Harmonic Generation (SHG) and prove the uniqueness of stationary standing wave solutions with behaviors tending to zero at infinity under certain conditions on parameters. In addition, we provide the same issues for the Dirichlet boundary value problems on the ball centered at the origin. A classification of solutions for radial case is also established.

2D Trudinger-Moser Inequality for Boltzmann-Poisson Equation Involving Probability Measure

Yohei Toyota
Osaka University, Japan
Takashi Suzuki

In this talk we study a functional associated with Boltzmann-Poisson equation involving probability measure, that is,

$$J_\lambda(v) = \frac{1}{2}\|\nabla v\|_2^2 - \lambda \int_{I_+} \log\left(\int_\Omega e^{\alpha v} dx\right)\mathcal{P}(d\alpha), \quad v \in H_0^1(\Omega)$$

where $\lambda > 0$ is a constant, $\Omega \subset \mathbb{R}^2$ is a smooth bounded domain and $\mathcal{P}(d\alpha)$ is a Borel probability measure on $I_+ = [0, 1]$. We show the boundedness of J_λ from below with the extremal case for λ when $\mathcal{P}(d\alpha)$ is continuous case and satisfies the suitable assumptions. To show this, we have to consider the behavior of minimizing sequence for the above functional near the blow up point. This work is supported by JSPS Grant-in-Aid for Scientific Research (A) 26247013.

Concentration Phenomenon in Stationary Solutions of a Spatially Heterogeneous Reaction-Diffusion Equation

Hiroko Yamamoto
Meiji University, Japan
Izumi Takagi

As a model of pattern formation in hydra, Gierer and Meinhardt proposed an activator-inhibitor system. Numerical simulations show that this system produces spiky patterns such that the distribution of solutions concentrates in a very narrow region around finitely many points. In the case of spatially uniform equations rigorous results have been obtained on the existence and stability of spiky patterns. In this talk we are interested in such concentration phenomena for a spatially heterogeneous reaction-diffusion equation. In particular, to study the effects of spatial heterogeneity on concentration points, we introduce a locator function composed only of the coefficients in the equation and prove that any concentration point must be a critical point of the locator function. Moreover, we construct a solution concentrating near a nondegenerate critical point of the locator function.

Special Session 153: Mathematical Foundations of Computing

Zhipeng Cai, Georgia State University, USA
Xiuzhen Cheng, The George Washington University, USA
Jiguo Yu, Qufu Normal University, Peoples Rep of China

Mathematics is one of the essential ingredients for computer science. For example, discrete mathematics is an indispensable component in theoretical computer science. This theoretical vein of computer science actually encompasses enormous mathematics branches, including computational geometry, computational number theory, quantum computation, algebra, etc. On the other hand, variety of computer science applications are heavily reliant on the mathematics substrate. Matrix algebra, statistic, graph theory, etc. have become inherent parts of many areas in computer science, such as machine learning, artificial intelligence, security and privacy aware computing, wireless communication, big data analytics, computer games and so on. It is indisputable that mathematics is a necessity for any computer science research. The purpose of this special session is to solicit talks with rigorous explorations of the cutting-edge techniques in computational mathematics and the integration of mathematics and computer science for various real-world applications.

Refined Complexity Analysis of Two Fundamental Problems in Data Quality

Zhipeng Cai
Georgia State University, USA
Dongjing Miao

In this talk, two critical problems, data quality evaluation and data repairing, will be investigated. First, a graph class called conflict graph will be introduced, which is a union of a finite number of given forests of complete multipartite graphs. It is demonstrated that vertex cover in conflict graphs can be utilized to evaluate data quality in database. It is interesting that conflict graphs can model many natural problems, such as in database applications. It is shown that this property is non-trivial if the number of forests of complete multipartite graphs is limited. Then the problem of vertex cover in conflict graphs will be introduced. For data repairing, view propagation in database will be introduced. It is shown that a repair of an inconsistent database is a maximal consistent subset. A comprehensive view of the computational complexity of conjunctive queries will be presented.

Identifying Traffic Vulnerabilities in Smart Home IoT

Xiuzhen Cheng
Shandong University, Peoples Rep of China

Smart home IoT devices have been more prevalent than ever before but the relevant security considerations fail to keep up with due to device and technology heterogeneity and resource constraints, making IoT systems susceptible to various attacks. In this talk, we discuss a novel mechanism to identify the vulnerabilities in the communication traffics of IoT devices for smart home systems. This approach takes one or more packet capture files as inputs to construct a traffic graph by passing the captured messages, identify the correlated subgraphs by examining the attribute-value pairs associated with each message, and quantify their vulnerabilities based on the sensitivity levels of different keywords. The effectiveness of the approach was validated in a small smart home system that can control a smart bulb LB100 via either the smartphone APP for LB100 or a Google Home speaker. The analysis on 58,714 messages captured within 15 minutes revealed 6 vulnerable correlated subgraphs, based on which 6 attack cases were implemented. These attacks can be easily reproduced by attackers with little knowledge of IoT. This study is interesting as it takes only the collected traffic files as inputs without requiring the knowledge of the device firmware while being able to identify new vulnerabilities. Future research on IoT traffic vulnerabilities will also be discussed.

Coverage Problem in Wireless Sensor Networks: a Geometric Perspective

Feng Li
Shandong University, Peoples Rep of China

Coverage problem is one of the fundamental issues in Wireless Sensor Networks (WSNs), since it demonstrates the sensing quality of WSNs. In this talk, we look at the coverage problem mainly from a geometric perspective. Specifically, we formulate the coverage problem as space tessellation, and design localized algorithms to drive the autonomous deployment of sensor nodes. Furthermore, we extend the paradigm of coverage to design a frequency allocation algorithm for multi-channel WSNs. In more details, we "tessellate" the sensor nodes in a unified virtual space consisting of sensor locations and frequencies, to enable the co-existence of ZigBee and WiFi with performance guaranteed in terms of both transmission throughput and quality.

Data Uploading Mechanism for Internet of Things with Energy Harvesting

Gaofei Sun
Changshu Institute of Technology, Peoples Rep of China

To facilitate uploading of sensing data with a tremendous and still growing number of devices in Internet of Things (IoT), is one of the most pressing tasks today. It is not sensible to equip each IoT device with cellular or other wide range access technologies, thus the network access management and sensing data fusion are necessary to solve the paradox of spectrum drain and endless user experience. In this framework, we considered a practical scenario where exist heterogeneous IoT devices with energy harvesting and limited short range access technologies, e. g. WiFi, BLE4.0, and their data can be uploaded through an access point (AP) in given area. The AP needs to schedule the data uploading of heterogeneous IoT devices, and conservatively satisfy the quality of experience (QoE) requirements, e. g. delay, sensing interval, data rate. First, we modeled the energy harvesting and sensing data of IoT devices by Markov chain and probability transfer matrix, and derived the expression of urgency function which can clearly distinguish the urgency of data transmission among devices. Secondly, an auction-based IoT devices data uploading mechanism is proposed, which satisfies the expected economic robustness with low communication overhead. Finally, we performed extensive simulations to verify the proposed data uploading scheme and algorithm. The simulation results indicate that the proposed scheme works well.

BKI: a Decentralized and Accountable Public-Key Infrastructure Based on Blockchain

Zhiguo Wan
Shandong University, Peoples Rep of China
Zhangshuang Guan

Traditional PKIs suffer from a well-known vulnerability due to bogus certificates issued by a compromised Certificate Authorities (CA). Several solutions like AKI and ARPKI have been proposed to address this vulnerability. However, they require complex interactions and synchronization among related entities, and they have not been validated with wide deployment. We propose an accountable, flexible and efficient decentralized PKI to achieve the same goal using the blockchain technology of Bitcoin, which has been proven to be secure and reliable. The proposed scheme, called BKI, realizes certificate issuance, update and revocation using specially designed transactions on a blockchain managed by multiple trusted maintainers. We also design a special Merkle Patricia Tree (MPT) to store the certificate status information, which implements highly efficient certificate status checking. BKI not only solves the well-known vulnerability in traditional PKIs, but also achieves accountability for certificate management. Moreover, the certificate status update interval of BKI can be in seconds, which makes the vulnerability window much shorter than AKI/ARPKI. In addition, BKI is also flexible since the number of required CAs to issue/revoke certificates is tunable for different applications. To prove security of BKI, we use the Tamarin prover to formalize a model for BKI and then security properties of BKI are proved with help of Tamarin. Finally, we implement BKI using smart contracts on Ethereum, and conduct comprehensive experiments to evaluate its performance.

Cooperative Jamming Based Communication Information Leakage Resolution in Vehicular Networks

Xiaoshuang Xing
Changshu Institute of Technology, Peoples Rep of China

Communication information leakage is a critical security problem to be solved in vehicular networks. Nowadays, each car is equipped with plenty of sensors and the traffic flow is quite dense in urban areas. Thus, quantity of protential communication interference exists aomong the cars and the sensors. Exploring these protental inference, we propose to solve the communication informaiton leakage problem in vehicular networks via using the cooperative jamming technology. We study cooperative jamming in vehicular networks from three main aspects: cooperative jamming modeling, cooperative jamming scheme design, and cooperative jamming performance validation.

Special Session 154: Analysis and Simulation of Equations for Multiscale Physics

Yuan Gao, HKUST, Hong Kong
Zhennan Zhou, Peking University, Peoples Rep of China

There is a growing interest in investigating multiscale differential equations originating from interdisciplinary research and progress in science. While multiscale phenomenon is ubiquitous in a wide spectrum of physical problems, conventional techniques from the analytical or the computational aspect may no longer stay adequate due to possible challenges from degeneracy, nonlocality, singular limiting behavior, etc. This special session will feature recent advances in understanding the mathematical nature of the multiscale models underlying various scientific background.

Dislocation Network Structures in 2D Bilayer System

Shuyang Dai
Wuhan University, Peoples Rep of China

We develop a multiscale continuum model to describe the interlayer defects in bilayer materials. The model incorporates both the anisotropy elasticity of each mono-layer in bilayer materials and the first-principle calculation informed interaction between two layers, i.e., the nonlinear atomistic potential energy between two layers. The equilibrium structures are obtained from the numerical simulations of the force balance differential equations. We apply this approach to determine the structure and energetics of twisted bilayer material. In tBLG, two distinct, modified Moire structures are observed. We also investigated the dislocation structure in heterogeneous bilayer material such as G/BN. Our model agrees well with the atomistic results. An analytical description is developed based on the obtained structural features.

A Robust Stochastic Galerkin Method for the Compressible Euler Equations with Uncertainty

Jingwei Hu
Purdue University, USA

It is known that the stochastic Galerkin method applied to hyperbolic systems such as the compressible Euler equations subject to random inputs may lead to an enlarged system which is not necessarily hyperbolic. In addition, such a method usually relies on the positivity of some macroscopic quantities (e.g. sound speed), which may break down when solution presents severe discontinuities. We introduce a stochastic Galerkin method for the compressible Euler equations based on a kinetic formulation. The method solves the Boltzmann equation efficiently for a large range of Knudsen numbers and reduces to an approximated (regularized) solver for the Euler equations when the Knudsen number is small. Furthermore, the method does not need to evaluate any macroscopic quantities nor require their values to be positive, hence is especially suited for problems involving discontinuities. Joint work with Shi Jin and Ruiwen Shu.

Optimization, Adaptation, and Initiation of Biological Transport Networks

Dan Hu
Shanghai Jiao Tong University, Peoples Rep of China

Blood vessel systems and leaf venations are typical biological transport networks. The energy consumption for such a system to perform its biological functions is determined by the network structure. In the first part of this talk, I will discuss the optimized structure of vessel networks, and show how the blood vessel system adapts itself to an optimized structure. Mathematical models are used to predict pruning vessels in the experiments of zebra fish. In the second part, I will discuss our recent modeling work on the initiation process of transport networks. Simulation results are used to illustrate how a tree-like structure is obtained from a continuum adaptation equation system, and how loops can exist in our model. Possible further application of this model will also be discussed.

Stability for Acoustic Wave in the Bounded Domain

Zhezhe Jiao
Northwestern Polytechnical University, Peoples Rep of China

We consider the linear wave equation with acoustic boundary conditions on a portion of the boundary and Dirichlet conditions on the rest of the boundary. Under some assumption on the memory kernel, we use resolvent estimates to show that the associated operator matrix generates a strongly continuous semigroup of contractions on a Hilbert space, and the semigroup is polynomially stable.

Volume Scattering by Sea Ice

Wenjia Jing
Tsinghua University, Peoples Rep of China
Olivier Pinaud

We consider the propagation and scattering of waves emitted from synthetic aperture radar located in the dry air over a vast layer of sea ice. The wave length is much smaller compared to the mean free path of the air but much larger than the correlation length of the medium inhomogeneities in the sea ice. By combining homogenization theory and kinetic limits techniques, we derive models for the scattered wave measured at the radar. Several inverse scattering problems will be considered, which concern the statistics of the inhomogeneities of the sea ice, the depth of the sea ice, etc. This talk is based on joint work with Olivier Pinaud.

Dislocation Climb Models from Atomistic Scheme to Dislocation Dynamics

Xiaohua Niu
Jimei University, Peoples Rep of China
Yang Xiang

We develop a mesoscopic dislocation dynamics model for vacancy-assisted dislocation climb by upscalings from a stochastic model on the atomistic scale. Our models incorporate microscopic mechanisms of (i) bulk diffusion of vacancies, (ii) vacancy exchange dynamics between bulk and dislocation core, (iii) vacancy pipe diffusion along the dislocation core, and (iv) vacancy attachment-detachment kinetics at jogs leading to the motion of jogs. Our mesoscopic model consists of the vacancy bulk diffusion equation and a dislocation climb velocity formula. The effects of these microscopic mechanisms are incorporated by a Robin boundary condition near the dislocations for the bulk diffusion equation and a new contribution in the dislocation climb velocity due to vacancy pipe diffusion driven by the stress variation along the dislocation. Our climb formulation is able to quantitatively describe the translation of prismatic loops at low temperatures when the bulk diffusion is negligible. Using this new formulation, we derive analytical formulas for the climb velocity of a straight edge dislocation and a prismatic circular loop. Our dislocation climb formulation can be implemented in dislocation dynamics simulations to incorporate all the above four microscopic mechanisms of dislocation climb. Simulations for the evolution, translation and coalescence of prismatic loops will be shown, which are in excellent agreement with available experimental and atomistic results.

Waves Near Resonance: from Fast Train Track to Moving Loads on Very Large Floating Structures

Zhan Wang
Chinese Academy of Sciences, Peoples Rep of China
Paul Milewski, Jean-Marc Vanden-Broeck

The problem of forced unsteady water waves under an elastic sheet is a model for waves under ice or under very large floating structures. Even though small-amplitude wavepacket solitary waves are not predicted to exist by standard perturbation analyses, we find large-amplitude solitary waves, and explore their crucial role in the forced problem of a moving load on the surface. This is meant to represent a model of the use of extended ice sheets as roads and aircraft runways. Some open problems on the theoretical side of this topic will also be mentioned in the end.

Analysis of Epitaxial Growth and Dislocation Models at Different Scales

Yang Xiang
Hong Kong University of Science and Technology, Hong Kong

We present some analysis results on the properties of defects in crystals using discrete and continuum models. We analyze the step bunching properties on epitaxial surfaces under elastic interactions, including the energy scaling law, the appearance of the bunch structure, and sharp bounds for bunch size. We also prove the convergence from the atomistic model to the Peierls-Nabarro model (a continuum model) for dislocations in crystals.

Towards a Mathematical Understanding of Surface Hopping Methods

Zhennan Zhou
Peking University, Peoples Rep of China
Jianfeng Lu

We develop a surface hopping algorithm based on frozen Gaussian approximation for semiclassical matrix Schrödinger equations, in the spirit of Tully's fewest switches surface hopping method. The algorithm is asymptotically derived from the Schrödinger equation with rigorous approximation error analysis. The resulting algorithm can be viewed as a path integral stochastic representation of the semiclassical matrix Schrödinger equations. Our results provide mathematical understanding to and shed new light on the important class of surface hopping methods in theoretical and computational chemistry.

Analysis and Computation of Topological Photonics

Yi Zhu
Tsinghua University, Peoples Rep of China

In this talk, I will introduce our recent results on the analysis and computation of the topological edge states in photonic graphene. Specifically, we study the propagation of electromagnetic waves governed by the two-dimensional Maxwell equations in honeycomb media. Thanks to the symmetries of the media, existence of Dirac points and corresponding Dirac dynamics are rigorously analyzed. Moreover, the introduction through small and slow variations of a domain wall across a line-defect gives rise to the bifurcation from Dirac points of highly robust (topologically protected) edge states. Via a rigorous multi-scale analysis, we give an explicit description (to leading order) of the edge states. Unfortunately, the multi-scale analysis only applies in the regime where the line defect is small and adiabatic. For large and non-adiabatic line defects, we propose a novel gradient recovery method based on Bloch theory for computation of such edge states. Compared to standard finite element methods, this method provides higher order accuracy with the help of gradient recovery technique. This higher accuracy is highly desired for constructing the full electromagnetic fields under propagation.

Special Session 155: Numerical Methods for Functional Equations

Qiumei Huang, Beijing University of Technology, Peoples Rep of China
Dongfang Li, Huazhong University of Science and Technology, Peoples Rep of China
Yin Yang, Xiangtan University, Peoples Rep of China

Functional equations are extensively appeared in nonlinear dynamics, biology, etc. and have a wide range of application in science and engineering. Numerical methods for functional equations have received considerable attention in recent years. In this special session, we aim to bring together the researchers to highlight the recent achievements in the related fields. The talks will focus on the delay differential equations, (functional) integral equations and (functional) integro-differential equations on the following topics: Stability theory, Asymptotic behavior, Oscillation theory, Numerical methods

A Recovery Based Linear Finite Element Method for Fourth Order Problems

Hongtao Chen
Xiamen University, Peoples Rep of China
Zhimin Zhang, Qingsong Zou

We analyze a gradient recovery based linear finite element method to solve some fourth order problems. Our method uses only C^0 element, which avoids complicated construction of C^1 elements and nonconforming elements. Optimal error bounds under various Sobolev norms are established. Moreover, after a post processing the recovered gradient is superconvergent to the exact one. Finally, some numerical experiments are presented to validate our theoretical findings.

Sparse Approximation for Data-Driven Polynomial Chaos Expansion and Their Applications in UQ

Ling Guo
Shanghai Normal University, Peoples Rep of China
Yongle Liu, Akil Narayan, Tao Zhou

In this talk, we will discuss collocation method via compressive sampling for recovering arbitrary Polynomial Chaos expansions (aPC). Our approach is motivated by the desire to use aPC to quantify uncertainty in models with random parameters. The aPC uses the statistical moments of the input random variables to establish the polynomial chaos expansion and can cope with arbitrary distributions with arbitrary probability measures. To identify the aPC expansion coefficients, we use the idea of Christoffel sparse approximation. We present theoretical analysis to motivate the algorithm. Numerical examples are also provided to show the efficiency of our method.

Superconvergence of Discontinuous Galerkin Solutions for Vanishing Delay Differential Equations

Qiumei Huang
Beijing University of Technology, Peoples Rep of China

In this report, we consider the discontinuous Galerkin method to solve vanishing delay differential equations under quasi-graded mesh. Global convergence and local superconvergence results are obtained. Based on local superconvergence results, several postprocessing techniques to accelerate the global convergence are proposed. The theoretical expectations are confirmed by numerical experiments.

Unconditionally Convergent $L1$-Galerkin FEMs for Nonlinear Time-Fractional Schrödinger Equations

Dongfang Li
Huazhong University of Science and Technology, Peoples Rep of China
Jilu Wang, Jiwei Zhang

In this paper, a linearized $L1$-Galerkin finite element method is proposed to solve the multi-dimensional nonlinear time-fractional Schrödinger equation. In terms of a temporal-spatial error splitting argument, we prove that the finite element approximations in L^2-norm and L^∞-norm are bounded without any time stepsize conditions. More importantly, by using a discrete fractional Gronwall type inequality, optimal error estimates of the numerical schemes are obtained unconditionally, while the classical analysis for multi-dimensional nonlinear fractional problems always required certain time-step restrictions dependent on the spatial mesh size. Numerical examples are given to illustrate our theoretical results.

Pseudospectral Methods for Computing the Multiple Solutions of the Schrödinger Equation

Zhaoxiang Li
Shanghai Normal University, Peoples Rep of China
Ji Lao, Zhongqing Wang

In this talk, we first consider multiple non-trivial solutions to the boundary value problem of Schrödinger equation on a square, by using the Liapunov-Schmidt reduction and symmetry-breaking bifurcation theory, combined with Legendre pseudospectral methods. Then, starting from the non-trivial solution branches of the corresponding nonlinear problem, we further obtain the whole positive solution branch with D_4 symmetry of the Schrödinger equation numerically by pseudo-arclength continuation algorithm. Next, we propose the extended systems, which can detect the fold and symmetry-breaking bifurcation points on the branch of the positive solutions with D_4 symmetry. We also compute the multiple positive solutions with various symmetries of the Schrödinger equation by the branch switching method based on the Liapunov-Schmidt reduction. Finally, the bifurcation diagrams are constructed, showing the symmetry/peak breaking phenomena of the Schrödinger equation. Numerical results demonstrate the effectiveness of these approaches.

One-Step Discretization for Index-1 Stochastic Delay Differential Algebraic Equations

Tingting Qin
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Chengjian Zhang

In this talk, we develop a class of general one-step discretization methods for solving the index-1 stochastic delay differential-algebraic equations. The existence and uniqueness theorem of strong solutions of index-1 equations is given. A strong convergence criterion of the methods is derived, which is applicable to a series of one-step stochastic numerical methods. Some specific numerical methods, such as the Euler-Maruyama method, stochastic θ-methods, split-step θ-methods are proposed, and their strong convergence results are given. Numerical experiments further illustrate the theoretical results.

Unconditional Energy Stability Analysis of 2-Order Implicit-Explicit LDG Method for the CH Equation

Huailing Song
Hunan University, Peoples Rep of China
Chi-Wang Shu

In this article, we present a second-order in time implicit-explicit (IMEX) local discontinuous Galerkin (LDG) method for computing the Cahn-Hilliard equation, which describes the phase separation phenomenon. It is well-known that the Cahn-Hilliard equation has a nonlinear stability property, i.e., the free-energy functional decreases with respect to time. The discretized Cahn-Hilliard system modeled by the IMEX LDG method can inherit the nonlinear stability of the continuous model. We apply a stabilization technique and prove unconditional energy stability of our scheme. Numerical experiments are performed to validate the analysis. Computational efficiency can be significantly enhanced by using this IMEX LDG method with a large time step.

The Asymptotic Expansions and Numerical Integration Methods to Nonlinear Singular Volterra Integral Equations of the Second Kind

Tongke Wang
Tianjin Normal University, Peoples Rep of China

The solutions of singular Volterra integral equations of the second kind usually behave like singular features about derivative at the left endpoint of the interval, which leads to obvious decreasing of computational accuracy when standard algorithms are used to solve these equations. This talk discusses the high accuracy computation to singular Volterra integral equations of the second kind. First, we derive the general Puiseux expansion of the solution at the singularity by Picard iteration and series decomposition, which is the accurate measurement of the singular type and singular degree of the solution. This asymptotic expansion can be used to approximate the solution when the variable is small, but the error increases rapidly as the variable becomes large. Second, we use some numerical integration methods to discretize the singular integral and derive the Euler-Maclaurin asymptotic expansion using the known Puiseux expansion of the solution. By accumulating some lower order error terms to the quadrature formulas, we can obtain high accuracy evaluations to the nonlinear Volterra integral equation. Finally, some examples are provided to demonstrate that the combination of the Puiseux expansion and the numerical integration can effectively solve nonlinear singular Volterra integral equations of the second kind.

The Hp Version Continuous Galerkin and Spectral Collocation Methods for Nonlinear Delay Differential Equations

Lijun Yi
Shanghai Normal University, Peoples Rep of China

In this talk we shall present the hp version continuous Galerkin and collocation methods for nonlinear delay differential equations with vanishing delays. We derive several a priori error estimates that are completely explicit with respect to the local time steps, the local polynomial degrees, and the local regularity of the exact solution. In particular, we show that the hp version continuous Galerkin scheme based on geometrically refined time steps and on linearly increasing approximation orders achieves exponential rates of convergence for solutions with start-up singularities. The theoretical results are illustrated by some numerical experiments.

Compact Θ-Method for the Generalized Delay Diffusion Equation

Qifeng Zhang
Zhejiang Sci-Tech University, Peoples Rep of China
Dinghua Xu, Yinghong Xu

The generalized diffusion equation with a delay has inherent complex nature because its analytical solutions are difficult to obtain. Therefore, one has to seek numerical methods, especially the high-order accurate ones, for their approximate solutions. In this talk, we have established the results of the numerical asymptotic stability and convergence of the compact θ-method for the generalized delay diffusion equation. In the end, a series of numerical tests on stability and convergence are carried out to support our theoretical results.

Modeling and Computation of Energy Efficiency Management with Emission Permits Trading

Shuhua Zhang
Tianjin University of Finance and Economics, Peoples Rep of China
Xinyu Wang

In this paper, we present an optimal feedback control model to deal with the problem of energy efficiency management. Especially, an emission permits trading scheme is considered in our model, in which the decision maker can trade the emission permits flexibly. We make use of the optimal control theory to derive a Hamilton-Jacobi-Bellman (HJB) equation satisfied by the value function, and then propose an upwind finite difference method to solve it. The stability of this method is demonstrated and the accuracy, as well as the usefulness, is shown by the numerical examples. The optimal management strategies, which maximize the discounted stream of the net revenue, together with the value functions, are obtained. The effects of the emission permits price and other parameters in the established model on the results have been also examined. We find that the influences of emission permits price on net revenue for the economic agents with different initial quotas are quite different. All the results demonstrate that the emission permits trading scheme plays an important role in the energy efficiency management.

Special Session 156: Dynamics, Control and Unpredictability in Physical and Biophysical Systems

Mattia T. Coccolo, Universidad Rey Juan Carlos, Spain
Jesus M. Seoane, Universidad Rey Juan Carlos, Spain
Miguel A.F. Sanjuan, Universidad Rey Juan Carlos, Spain

The main goal of this special session is to join together physicists, mathematicians and other scientists interested in new developments in the dynamics and unpredictability of dynamical systems, as well as methods of control of chaos, with special emphasis in the applications in physics and biology. The topics included, but not limited to, in this session, are: (1) Novel methods of controlling dynamical systems, (2) Uncertainty associated to fractal structures in phase space, (3) Unpredictability in Hamiltonian and dissipative systems, (4) Effects of time-delay in physical and biological systems, (5) Dynamics and physics of cancer, (6) Nonlinear dynamics methods applied to biological and physical systems.

A New Type of Resonance: the Bogdanov-Takens Resonance in Time-Delayed Systems

Mattia Coccolo
Universidad Rey Juan Carlos, Spain
Beibei Zhu, Miguel A.F. Sanjuan, J.M. Sanz-Serna

In this talk, we present our work on the oscillatory dynamics of a time–delayed dynamical system subjected to a periodic external forcing. We show that, for certain values of the time delay, the response can be greatly enhanced by a very small forcing amplitude. This phenomenon is related to the presence of a Bogdanov–Takens bifurcation and displays some analogies to other resonance phenomena, but also substantial differences.

Stochastic Activation in a Genetic Switch Model

Alvaro Correales
Stockholm University, Sweden
Joanna Tyrcha, John Hertz

Proteins are amazing biological entities. Among other important functions within cells, they can regulate the creation of other proteins and even of themselves. In the absence of noise, this autoregulation process can lead to bistability, but noise can induce transitions between the two states. We present a model in which a WKB formulation leads us to find the mean first jump time between these states, in total agreement with numerical simulations. In addition, the model exhibit bursting behavior in the limit of short mRNA lifetime.

Noise Interpretations in Insider Trading Modelling

Carlos Escudero
Universidad Autonoma de Madrid, Spain

A classical problem in the field of stochastic control is how to choose an optimal portfolio when a set of investment possibilities is open. When the investor is a dishonest trader who possesses privileged information on the future value of a stock, the problem cannot be modelled using the traditional stochastic framework of Itô integration. We will briefly discuss how to extend this framework to that of anticipating stochastic calculus in order to obtain a well-posed system of stochastic differential equations; we will also describe the potential paradoxes that may arise in this theoretical pathway.

Global Qualitative Analysis of Multi-Parameter Biomedical Dynamical Systems

Valery Gaiko
National Academy of Sciences of Belarus, Belarus

We carry out the global qualitative analysis of multi-parameter biomedical dynamical systems. First, using new bifurcational geometric methods, we solve Hilbert's Sixteenth Problem on the maximum number of limit cycles and their distribution for the 2D Holling-type quartic dynamical system and Kukles cubic-linear system. Then, applying a similar approach, we complete the strange attractor bifurcation scenario for the 3D Lorenz-type system connecting globally the homoclinic, period-doubling, Andronov-Shilnikov, and period-halving bifurcations of its limit cycles which is related to Smale's Fourteenth Problem.

Optimal Treatments in Cancer Immunotherapy Involving CD4+ T Cells

Sophia Jang
Texas Tech University, USA
Xiaochuan Hu

We apply optimal control theory to a model of interactions between cancer cells, CD4+ T cells, cytokines and host cells to devise best immunotherapies for treating cancer. The CD4+ T cells cannot kill cancer cells directly but use the cytokines produced to suppress tumor growth. The immunotherapy implemented is modeled as a control agent and it can be either transferring of CD4+ T cells, cytokines or both. We establish existence and uniqueness of the optimal control. The optimal treatment strategy is then solved numerically under different scenarios. Our numerical results provide best protocols in terms of strengths and timing of the treatments.

Basin Entropy: a Measure of the Final State Unpredictability and Applications to Some Physical Systems

Miguel Sanjuan
Universidad Rey Juan Carlos, Madrid, Spain

In nonlinear dynamics, basins of attraction link a given set of initial conditions to their corresponding final states. This notion appears in a broad range of applications where several outcomes are possible, which is a common situation in neuroscience, economy, astronomy, ecology, and many other disciplines. Depending on the nature of the basins, prediction can be difficult even in systems that evolve under deterministic rules. To address this issue, we introduce the concept of basin entropy, a measure to quantify this uncertainty. Its application is illustrated with several paradigmatic examples that allow us to identify the ingredients that hinder the prediction of the final state. The basin entropy provides an efficient method to probe the behavior of a system when different parameters are varied. These ideas have been applied to some physical systems such as experiments of chaotic scattering of cold atoms, models of shadows of binary black holes, and classical and relativistic chaotic scattering associated to the Hénon–Heiles Hamiltonian system in astrophysics. This is work in collaboration with Alvar Daza, Alexandre Wagemakers, Bertrand Georgeot, and David Guéry-Odelin.

Relativistic Effects in Chaotic Scattering

Jesus M. Seoane
University Rey Juan Carlos, Spain
Juan D. Bernal, Miguel A.F. Sanjuan

The phenomenon of chaotic scattering is very relevant in different fields of science and engineering. It has been mainly studied in the context of Newtonian mechanics, where the velocities of the particles are low in comparison with the speed of light. In this talk, we analyze global properties such as the escape time distribution and the decay law of the Henon-Heiles system in the context of special relativity. Our results show that the average escape time decreases with increasing values of the relativistic factor beta. As a matter of fact, we have found a crossover point for which the KAM islands in the phase space are destroyed when beta reaches a critical value. On the other hand, the study of the survival probability of particles in the scattering region shows an algebraic decay for values of beta below of that critical value, and this law becomes exponential for beta above to that value. Surprisingly, a scaling law between the exponent of the decay law and the beta factor is uncovered where a quadratic fitting between them is found. The results of our numerical simulations agree faithfully with our qualitative arguments. Besides, we compute the basin entropy and the fractal dimension of the set of singularities of the scattering function in function of beta. This is joint work with Miguel A.F. Sanjuan and Juan D. Bernal (Spain).

Special Session 157: Recent Trends in Stochastic Analysis and its Applications to Physics and Finance

Carlos Escudero, Universidad Autónoma de Madrid, Spain
Alvaro Correales, Stockholms Universitet, Sweden

Recent developments of stochastic analysis have pushed the theory to previously unknown horizons. Dealing with previously mysterious objects such as white noises, Wick products, non-adapted integrands or anticipating stochastic differential equations has become relatively common in the new developments that have taken place along recent years. Backward stochastic differential equations, forward-backward stochastic differential equations or stochastic partial differential equations with distribution-valued solutions are now part of the classical methodology within this field. The aim of this session is to highlight some of these recent and powerful advances of the theory and to put them in contact with potential applications. In particular, we will focus on their applications to finance, where the information flows can be very important to understand the evolution of wealth processes, and pose difficult technical questions too. Also to physics, since some recent developments, such as stochastic thermodynamics, need of stochastic descriptions that should allow for time reversibility.

Stochastic Thermodynamics for Complex Langevin Equations

Simone Borlenghi
KTH Royal Institute of Technology, Sweden
Stefano Iubini, Stefano Lepri, Jonas Fransson

We present a formulation of stochastic thermodynamics to describe transport phenomena in networks of nonlinear oscillators described by complex-valued Langevin equations, that account for coupling to different thermochemical baths. Dissipation is introduced via non- Hermitian terms in the Hamiltonian of the model. The stochastic thermodynamics formalism is applied to compute explicit expressions for the entropy production rates. We discuss in particular the nonequilibrium steady states of the network characterized by a constant production rate of entropy and flows of energy and particle currents. For some specific examples, a one-dimensional chain, a dimer, and a network of seven oscillators, numerical calculations are presented. The role of asymmetric coupling among the oscillators on the entropy production is illustrated. Possible applications to physical system and information processing with neural networks are also discussed.

Chemical Kinetics and the Imaginary Ito Interpretation

Alvaro Correales
Stockholm University, Sweden
Carlos Escudero (UAM), Mariya Ptashnyk (HWU)

The abstract chemical reaction

$$A + A \longrightarrow \emptyset$$

understood as a Markov chain in continuous time, has been studied in the physical literature for several years. It has been claimed that this reaction can be described by means of the stochastic differential equation

$$d\,\phi = -\phi^2\,dt + i\phi\,dW_t,$$

where i is the imaginary unit. This affirmation is, at least, intriguing, and has led to controversy and criticisms in the literature. The goal of this talk is to give evidence that such a description is indeed possible.

Some New Thoughts on the Old ItÔ Vs Stratonovich Dilemma and Its Resolution

Carlos Escudero
Universidad Autonoma de Madrid, Spain
Alvaro Correales

The Itô vs Stratonovich dilemma is a venerable topic within the field of applied stochastic analysis. It has generated many debates over several decades even after its broadly accepted resolution. We will reexamine this dilemma and one aspect of its resolution: the claim that one can easily pass from one noise interpretation to another by means of a simple transformation. We will show several examples of applied interest in which this is not the case; in fact, the long-time properties of the formally transformed models differ from those of the original ones.

Optimal Strategies

Alessandro Ferriero
UAM, Spain

We will present some results about optimal startegies for financial investments when a priori information on the value at a future point in time is available.

Radial Processes on RCD*(K,N)-Spaces

Kazuhiro Kuwae
Fukuoka University, Japan
Kazumasa Kuwada

$\mathsf{RCD}^*(K,N)$-spaces is a metric measure space generalizing Riemmanian manifolds with lower Ricci bound $K \in \mathbb{R}$ and an upper bound $N \in [1,+\infty[$. This class of spaces also contains the class of N-dimensional Alexandrov spaces, which was proved by Petrunin and Zhang-Zhu, and also contains the class of weighted Riemannian manifolds with Witten Laplacian and lower bound K of N-Bakry-Emery Ricci tensor. I will talk on new stochastic expression of radial process under the law for all starting point including the reference point appeared in the radial function provided the reference point fulfills a regularity condition depending on the geometric structure of the $\mathsf{RCD}^*(K,N)$-space. The expression of radial process is completely different from Kendall's expression (1987) including the local time on cut-locus without lower Ricci bound in the framework of Riemannian manifold. Our expression of radial process does not contain the local time on cut-locus. Instead of it, we extract a positive continuous additive functional, which can be thought of continuous additive functionals corresponding to the difference of Laplacians of radial functions between on the given space and on the model space. This is a joint work with Kazumasa Kuwada in Tohoku University.

W-Entropy Formulas and Langevin Deformation on Wasserstein Space Over Riemannian Manifolds

Xiangdong Li
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Peoples Rep of China
Songzi Li

Inspired by Perelman's seminal work on the entropy formula for the Ricci flow, we prove the W-entropy formula for the heat equation associated with the Witten Laplacian on n-dimensional complete Riemannian manifolds with the $CD(K,m)$-condition, and the W-entropy formula for the heat equation associated with the time dependent Witten Laplacian on n-dimensional compact manifolds equipped with a (K,m)-super Ricci flow, where $K \in \mathbb{R}$ and $m \in [n,\infty]$. Furthermore, we prove an analogue of the W-entropy formula for the geodesic flow on the Wasserstein space over Riemannian manifolds. Our result improves an important result due to Lott and Villani on the displacement convexity of the Boltzmann-Shannon entropy on Riemannian manifolds with nonnegative Ricci curvature. To better understand the similarity between above two W-entropy formulas, we introduce the Langevin deformation of geometric flows on the tangent bundle over the Wasserstein space and prove an extension of the W-entropy formula for the Langevin deformation. We also make a discussion on the W-entropy for the Ricci flow from the point of view of statistical mechanics and probability theory.

Determining the Role of Intrinsic Noise in Genetic Bistable Switches Using Minimum Action Principles

Ruben Perez-Carrasco
University College London / Crick, England
Pilar Guerrero, James Briscoe, Karen M. Page

Bistable switches are a common in many areas of physics and biology. In developing biological tissues, they are often controlled by gradients of secreted signalling molecules - morphogens -, providing a mechanism to convert a signalling gradient into stripes of gene expression that determine the arrangement of distinct cell types. In this talk I will present our work focusing on the role of intrinsic fluctuations in bistable switches that result from the stochastic nature of gene expression. To tackle this problem we make use of different techniques, using Gillespie simulations, Langevin equations and Minimum Action Path theory. The results reveal that noise induces a switching wave that propels the stripe boundary away from the morphogen source, eventually settling at a steady state different from the deterministic description. Additionally, the same formalism can be extended to understand transitions from oscillatory to steady state behaviour tackling situations such as neuronal activation spike trains.

Retrospective Approach for Age Structured Population Dynamics

Yuki Sughiyama
The University of Tokyo, Japan

Control of population growth is ubiquitous problem in many fields. In the context of medical treatment, we attempt to diminish the growing speed of a cell population composed of cancer cells or pathogens by using antibiotics or some special therapies. In terms of evolutionary biology, to survive in a fluctuating environment, cells maximize (optimize) their population growth by exploiting a risk hedge strategy for adaptation to the fluctuation. Recent development of experimental devices enables us to measure a big size lineage data that describes a growing cell population. In this study, by using these lineage data, we analyze a behavior of the population growth. Here, a structure of statistical mechanics using the large deviation theory plays an important role. As a results, we reveal that the population growth rate is given by the

Legendre transform of the large deviation function for the semi-Markov process that describes a stochastic switch of cell types in the time evolution. Furthermore, by using this structure, we show that responses of the population growth rate with respect to an environmental change can be evaluated by statistics on a retrospective lineage path.

Contributed Session 1: ODEs and Applications

Particle Swarm Optimization for the Numerical Solution of One-Dimensional Elliptic Boundary Value Problems

Jagdish Chand Bansal
South Asian University, India
Navnit Jha

Particle Swarm Optimization is one of the most popular and efficient swarm intelligence techniques for the global optimization. Due to stochastic nature of the solutions provided by particle swarm optimization, it is rarely applied to the numerical solution of differential-difference equations. Finding the analytical solutions to the one-dimensional mildly non-linear elliptic boundary value problem is a cumbersome process. Therefore, unconventional solution approaches may provide a better tool for the numerical solutions to this problem. In this paper, the problem of finding solution values after the three-point finite difference compact discritization of the one-dimensional elliptic equation is modeled as an optimization problem. Particle swarm optimization is then applied to find the solution values. The numerical experiments exhibit the applicability and efficiency of particle swarm optimization.

Regular Blocks and Conley Index of Isolated Invariante Continua in Surfaces

Hector Barge
Universidad Politecnica de Madrid, Spain

In this talk we shall be concerned with isolated invariant continua in surfaces using a topological point of view. We shall see that in this case the Conley index is determined in intrinsic terms by the knowledge of an initial section of the unstable manifold and the topology of the invariant set. This allows us to give a complete description of the Conley indices of isolated invariant continua in surfaces. From this description we are able to obtain interesting conclusions such as the topological classification of those isolated invariant continua in surfaces which do not have fixed points or the classification of isolated minimal sets

Boundary Controllability of Neutral Integrodifferential Impulsive Systems with Time Varying Delays in Banach Spaces

Radhakrishnan Bheeman
PSG College of Technology, India

In this paper, author establishes the boundary controllability results for neutral integrodifferential impulsive system with time varying delays in Banach Spaces. The sufficient condition of boundary controllability is proved under the evolution operator. The results are obtained by using the semigroup theory and the Schaefer fixed point theorem. An example is provided to illustrate the theory.

Variation of Parameters for a Differential Equation on Schwarzschild Spacetime

Petarpa Boonserm
Chulalongkorn University, Thailand
Tritos Ngampitipan

In general relativity, gravity is governed by the Einstein equations. In fact, the Einstein equations are so complicated that a qualitative and quantitative description of gravity cannot be made. However, under the condition of being perfect fluid spheres, the Einstein equations take a simpler form, and therefore, information about gravity can potentially be extracted. Such condition can be expressed as an ordinary differential equation. In this paper, we are interested in solving this differential equation using the method of the variation of parameters. The result shows that the Schwarzschild spacetime can be recovered by choosing an appropriate initial solution.

Space-Time Algebra for Multifluid Plasma Equations

Suleyman Demir
Anadolu University, Turkey
Murat Tanisli, Neslihan Sahin, M. Emre Kansu

The existence of correspondence between variables of electromagnetism and multifluid plasma permits to formulate a system of equations of multifluid plasma in a form analogous to electromagnetism governed by Maxwell equations. The mechanical forces subject to plasma are consistent with a fluid treatment. On the other hand, the spacetime algebra provides a practical and power tool for the deriving equations related to different areas of physics. In this work, stimulating from the theoretical analogy between the compressible fluid equations and multifluid plasma equations, the theorems and techniques of spacetime algebra are applied to the multifluid equations of a plasma in order to present a new formulation in a compact and elegant way. Moreover, the generalized plasma wave equation is presented.

Analysis of Some High-Order Compact Scheme on a Non-Uniform Mesh Network for One-Dimension Singularly Perturbed Elliptic Problems

Navnit Jha
South Asian University, India

A family of fourth and sixth-order compact difference scheme for solving one-dimension nonlinear singularly perturbed elliptic problems has been described. The proposed scheme is developed on non-uniformly spaced mesh-points and renders same order of accuracies both in uniform meshes and non-uniform meshes. The three-point finite difference discretization results in a tridiagonal Jacobian matrix and can be computed by means of Thomas algorithm. Numerical simulations with problems possessing boundary layer behavior have been presented to illustrate order and accuracies of the new scheme.

The Strong Average Shadowing Property for C^1-Generic Diffeomorphisms

Bowon Kang
Chungnam National University, Korea
Namjip Koo, Manseob Lee

K. Sakai (2000) proved that the C^1 interior of the set of all diffeomorphisms satisfying the average shadowing property on a two dimensional closed manifold is characterized as the set of all Anosov diffeomorphisms. In this talk we introduce the notion of strong average shadowing property(SASP) of diffeomorphisms on n-dimensional manifolds. Then we will discuss some relations between SASP and hyperbolicity of C^1-generic diffeomorphism in the direction of an extension of the Sakai's result.

Periodic Orbits in Nonlinear Wave Equations on Networks

Imene Khames
INSA Rouen Normandie, France
J.G. Caputo, A. Knippel, P. Panayotaros

We consider a discrete nonlinear wave equation in an arbitrary finite graph. It is the discrete ϕ^4 equation used to model coupled electromechnical oscillators. We show that inspecting the normal modes of the graph Laplacian, we can immediately identify which ones can be extended into nonlinear periodic orbits (generalizing work of Aoki, 2016). We first define monovalent, bivalent and trivalent nonlinear periodic orbits depending whether the components of the corresponding eigenvectors of the graph Laplacian are in $\{+1\}$, $\{-1,+1\}$ or $\{-1,0,+1\}$. Then, we perform a systematic linear stability (Floquet) analysis of these orbits. In particular, the linearized equations are decoupled for normal modes associated to eigenvectors without 0 (called soft nodes), these modes are the monovalent (Goldstone) and the bivalent orbits. We find that for chains the Goldstone mode is stable for a wide range of parameters while the bivalent mode is unstable. Nevertheless, the stability analysis for modes with soft nodes is more complicated since the linearized equations are coupled. Numerical results of some graphs show that trivalent periodic orbits that continue nondegenerate linear modes are unstable below an amplitude threshold; orbits continued from modes with frequency degeneracy are unstable.

Stability Properties for Impulsive Differential Equations of Fractional Order

Namjip Koo
Chungnam National University, Korea
Bowon Kang

In this talk we present some explicit solutions for impulsive linear fractional differential equations with impulses at fixed times, which provide a tool in deriving singular integral-sum inequalities. Then we discuss some results about the Mittag-Leffler stability for impulsive fractional differential equations by using impulsive fractional comparison principle and piecewise continuous functions of Lyapunov's method.

Networks Modeling by Systems of Ordinary Differential Equations

Felix Sadyrbaev
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Eduard Brokan

We consider systems of ordinary differential equations that appear in the theory of gene regulatory networks. These systems can be of arbitrary size but of definite structure that depends on the choice of regulatory matrices. The decisive role in behaviour of elements of such systems play attractors. The system in abbreviated form is

$$\frac{dx_i}{dt} = f(\sum w_{ij}x_j - \theta)v_g - x_iv_g - \eta,$$

where f is sigmoidal function, w_{ij} are entries of the regulatory matrix W, v_g is a parameter and η stands for stochastic behaviour. We neglect η and consider the system in extended form

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{1}{1+e^{-\mu_1(w_{11}x_1+w_{12}x_2+\ldots+w_{1n}x_n-\theta_1)}}v_1 - x_1v_1, \\ \dfrac{dx_2}{dt} = \dfrac{1}{1+e^{-\mu_2(w_{21}x_1+w_{22}x_2+\ldots+w_{2n}x_n-\theta_2)}}v_2 - x_2v_2, \\ \ldots \\ \dfrac{dx_n}{dt} = \dfrac{1}{1+e^{-\mu_n(w_{n1}x_1+w_{n2}x_2+\ldots+w_{nn}x_n-\theta_n)}}v_n - x_nv_n. \end{cases}$$

We study the structure of simple attractors that consist of a number of critical points for several choices of regulatory matrices W.

A Model for the Effects of Pollutants on Survival of Species

Saroj Kumar Sahani
South Asian University, India

We propose a delayed model for the effects of pollutants released in the environment on the species survival. We assumed that the species grows with logistic rate. It is very obvious that the uncontrolled emission of the pollutants have very adverse effects on the species and sometime it may lead to the extinction of the species. We, thought the long term dynamical model, will show how the pollutant addition term will affects the survival. We have simulated the system for its long term dynamical properties and establish the existence of non-trivial equilibrium point which will ascertain the survival of species in the polluted environment.

Existence of Solutions to Nonlocal Nonlinear Fractional Functional Integro-Differential Equations of Sobolev Type

Madhukant Sharma
Mahindra Ecole Centrale Hyderabad, India
Shruti Dubey

This article is concerned with the existence of solutions for a class of nonlinear nonlocal fractional functional integrodifferential equations of Sobolev type in a Banach space. We also render the criteria for global existence of solution and study the continuous dependence of solution on initial data. An application is given to illustrate the abstract results.

Global Stability for Coupled System Using Graph Theoretical Approach

Anuraj Singh
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The dynamics of a coupled system of differential equations is investigated. By combining results of graph theory, interesting results have been obtained. By using the method of constructing Lyapunov functions based on graph-theoretical approach for coupled systems, it is derived sufficient conditions under which the positive equilibrium of this model is unique and globally asymptotically stable if it exists. An exhaustive numerical simulation is done to substantiate the analytical findings.

Positive Solutions of a Nonlinear Fractional Differential Equation Boundary Value Problem

Yan Sun
Shanghai Normal University, Peoples Rep of China

In this article, we investigate sufficient conditions of the existence of at least three positive solutions for a class of nonlinear fractional differential equations with three point boundary value problems. Some new multiplicity and nonexistence results of positive solutions are established by making use of Leggett-Williams fixed point theorem on cones together with some comparison results. Finally, an example is presented to demonstrate the validity of the main results here.

Applications of the Generalized Miranda Theorem to Nonlocal Neumann Boundary Value Problems

Katarzyna Szymanska-Debowska
Lodz University of Technology, Poland

We study the nonlocal Neumann boundary value problem of the following form

$$u'' = f(t, u, u'), \quad u'(0) = 0, \quad u'(1) = \int_0^1 u'(s)dg(s),$$

where $f : [0,1] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ and $g = \operatorname{diag}(g_1, \ldots, g_n)$ with $g_i : [0,1] \to \mathbb{R}, i = 1, \ldots, n$. The problem was studied only in [1] and [3] so far. However, in both papers the function f is considered to be *bounded*. Here, using the generalized Miranda theorem (see [2]), we shall weaken the assumptions imposed upon the function f in the papers [1] and [3].

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Existence-Uniqueness and Exponential Estimate of Pathwise Solution of Retarded Stochastic Evolution Systems with Time Smooth Diffusion Coefficients

Weisong Zhou
Chongqing University of Posts and Telecommunications, Peoples Rep of China
Daoyi Xu

In this talk, we will investigate the existence-uniqueness and exponential estimate of the pathwise mild solution of retarded stochastic evolution systems driven by a Hilbert-valued Brownian motion. Firstly, the existence-uniqueness of the maximal local pathwise mild solution are given by the generalized local Lipschitz conditions, which extend a classical Pazy theorem on PDEs. We assume neither that the noise is given in additive form or that it is a very simple multiplicative noise, nor that the drift coefficient is global Lipschitz continuous. Secondly, the existence-uniqueness of the global pathwise mild solution are given by establishing an integral comparison principle, which extends the classical Wintner theorem on ODEs. Thirdly, an exponential estimate for the pathwise mild solution is obtained by constructing a delay integral inequality. Finally, the results obtained are applied to a retarded stochastic infinite system and a stochastic partial functional differential equation. Combining some known results, we can obtain a random attractor, whose condition overcomes the disadvantage in existing results that the exponential converging rate is restricted by the maximal admissible value for the time delay.

Contributed Session 2: PDEs and Applications

Finding Critical Domains of Quenching Set for Coupled Semilinear Parabolic Equations with a Localized Source

W.Y. Chan
Texas A&M University Texarkana, USA

Let Ω_1 and Ω_2 be two bounded domains in R^2, and $\partial\Omega_1$ and $\partial\Omega_2$ be their boundary, respectively. Suppose that u and v are functions of t, x, and y. Let (x_0, y_0) be a fixed point in Ω_1 and in Ω_2. In this presentation, we study the quenching set of the first initial-boundary value problem of quenching problems for the following coupled semilinear parabolic equations with a localized source:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{1}{1-v\left(x_0,y_0,t\right)} \text{ for } (x,y,t)\in\Omega_1\times(0,\infty),\\ \frac{\partial v}{\partial t} &= \frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{1}{1-u\left(x_0,y_0,t\right)} \text{ for } (x,y,t)\in\Omega_2\times(0,\infty),\end{aligned}$$

$$\begin{aligned}u\left(x,y,0\right) &= 0 \text{ for } (x,y)\in\bar{\Omega}_1 \text{ and}\\ v\left(x,y,0\right) &= 0 \text{ for } (x,y)\in\bar{\Omega}_2,\\ u\left(x,y,t\right) &= 0 \text{ for } t>0 \text{ and } (x,y)\in\partial\Omega_1 \text{ and}\\ v\left(x,y,t\right) &= 0 \text{ for } t>0 \text{ and } (x,y)\in\partial\Omega_2.\end{aligned}$$

Using a numerical method, we shall determine the approximated critical domains of the above problem for some shapes of Ω_1 and Ω_2.

Q(Telta)-Differential

Chou Hsin Chin
Chiao Tung University, Taiwan

Hypertranscendental q(telta)-differential is defined. Therefore the q(telta)-differential equations can be calculated.

A SIS Epidemic Reaction-Diffusion Model with Risk-Induced Dispersal

Wonhyung Choi
Korea University, Korea
Inkyung Ahn

This study examined a spatial SIS (susceptible-infected-susceptible) epidemic model where the dispersal of the infected individuals was non-linear. Spatial heterogeneity and dispersal of individuals are important factors that influence the persistence and eradication of an infectious disease. Based on spatial SIS reaction-diffusion models, diseases have been known to be eradicated if the mobility of the infected individuals is above a certain value. Additionally, an area is categorized as high- or low-risk if the infection rate average is higher or lower than the recovery rate average, respectively. In this study, we promoted a risk-induced dispersal (RID) method of the infected individuals i.e., the mobility of the infected individuals was high in high-risk areas and low in low-risk areas. We also examined the effect of RID on the spatial SIS model by defining the basic reproduction number ($\mathcal{R}_0$) in the spatial SIS reaction-diffusion model and investigating its stability i.e., $\mathcal{R}_0 1$ represented an endemic state. By comparing the $\mathcal{R}_0$ of these models, RID of infected individuals proved to be a better strategy than random dispersal in obtaining a disease-free state.

Existence of a Solution to Singular Semilinear Polyharmonic Equation with Exponential Nonlinearity

Gaurav Dwivedi
BITS Pilani, India

In this work, we establish the existence of a solution to the following polyharmonic problem:

$$(-\Delta)^m u + V(x)u = \frac{g(x)f(u)}{|x|^a} + h(x) \quad \text{in } \mathbb{R}^{2m},$$

where $m \geq 2$, $a \in [0,\ 2m)$, $V, g : \mathbb{R}^{2m} \to \mathbb{R}$ are continuous, $0 \not\equiv h \in H^{-m}(\mathbb{R}^{2m})$ and f behaves like $\exp(\alpha s^2)$ as $|s| \to \infty$. We also assume that the potential V is bounded below. We employ variational arguments with a version of Adams inequality to prove the existence of a solution when the perturbation h is suitably small.

On the Dynamics of Domain Wall Motion in Ferromagnetic Heterostructures Under the Influence of Interfacial Dzyaloshinskii-Moriya Interaction

Sharad Dwivedi
SRM Insitute of Science & Technology, Chennai, India

This work deals with the investigation of current-induced domain wall motion in ferromagnetic heterostructure in the steady-state and precessional dynamic regimes. To be precise, we consider the one-dimensional model of the generalized Landau-Lifshitz-Gilbert equation of micromagnetism that includes the effects of nonlinear dissipations, Rashba field, Spin-Hall effect (SHE), and interfacial Dzyaloshinskii-Moriya interaction (DMI). The derived results describe the domain wall velocity, threshold and Walker breakdown value of spin-polarized electric current in the steady-state regime, and the average domain wall velocity in the precessional dynamic regime. Finally, the interplay and effect of various parameters on the domain wall motion have been discussed analytically and numerically in both the dynamical regimes.

Nonexistence of Global Solutions to Some Quasilinear Wave Equations in Three Space Dimensions

Wei Han
North University of China, Peoples Rep of China

F. John studied the nonlinear wave equations in three space dimensions with the small initial values of a small parameter $\varepsilon > 0$, and he obtained the blow-up result of this problem and also got the precise estimates for the lifespan(the maximal existence time) of the solution to the problem. In this talk, we will extend John's result to the general kind of quasilinear wave equations. We will prove that for the compactly supported smooth initial values, the solution must blow up in finite time if the initial data are nonnegative and positive somewhere no matter how small the initial data are and also we give the sharp lifespan estimate of solutions for the problem.

Recent Progress on Schrödinger-Poisson System

Lirong Huang
Fujian Jiangxia University, Peoples Rep of China
Jianqing Chen, Eugenio Rocha

In this paper, we consider the existence of ground state and bound states as well as their bifurcation properties of the Schrödinger-Poisson system $-\Delta u + u + \phi u = |u|^4 u + \mu h(x) u$ in $\mathbb{R}^3$ and $-\Delta\phi = u^2$ in $\mathbb{R}^3$. Under suitable conditions on the function $h(x)$, we prove the existence of one ground state which must bifurcate from zero and the existence of another bound state which can not bifurcate from zero.

Stability of Travelling Waves for Stochastic Reaction-Diffusion Equations

Hermen Jan Hupkes
Leiden University, Netherlands
C.H.S. Hamster

We consider reaction-diffusion equations that are stochastically forced by a small multiplicative noise term. We show that spectrally stable travelling wave solutions to the deterministic system retain their orbital stability if the amplitude of the noise is sufficiently small. By applying a stochastic phase-shift together with a time-transform, we obtain a semilinear sPDE that describes the fluctuations from the primary wave. We subsequently develop a semigroup approach to handle the nonlinear stability question in a fashion that is closely related to modern deterministic methods.

Existence of Weak Solutions to the Time-Fractional Attraction-Repulsion Tumor Invasion System

Manimaran Jeyaraj
NIT Goa, India
L. Shangerganesh

Alzheimer's disease is a degenerate disease of the brain which robs the minds of our old population. Mathematical model of Alzheimer's disease introduced by Luca et.al (2003) and long time behavior of the solutions studied by Li et.al (2018). In this work, we consider the mathematical model with time-fractional derivative, which consists of three coupled system of nonlinear reaction-diffusion equations. Our main work is to establish the existence and uniqueness of the proposed mathematical model. The model can be described by the following mathematical equations representing of concentrations of Microglia u, chemoattractant v and chemoerpellent w.

$$\begin{cases} \partial_t^\alpha u = \nabla\cdot\ (A_1(x,t,\nabla u)) - \nabla\cdot(\chi u \nabla v) + \nabla\cdot(\xi u \nabla w), & \text{in } Q_T, \\ \partial_t^\alpha v = \nabla\cdot\ (A_2(x,t,\nabla v)) + au - bv, & \text{in } Q_T, \\ \partial_t^\alpha w = \nabla\cdot\ (A_3(x,t,\nabla w)) + cu - dw, & \text{in } Q_T, \\ \dfrac{\partial u}{\partial\eta} = \dfrac{\partial v}{\partial\eta} = \dfrac{\partial w}{\partial\eta} = 0, & \text{on } \Sigma_T, \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x),\ w(x,0) = w_0(x), & \text{in } \Omega \end{cases}$$

where $Q_T = \Omega\times(0,T)$, $\Sigma_T = \partial\Omega\times(0,T)$, Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$ and let $T > 0$. The parameter χ and ξ are chemoattractant coefficient which are positive and a, b, c and d are chemical proliferation and degradation rates. Further A_i, $i = 1, 2, 3$ are density dependent diffusion coefficients. Furthermore, here we used the fractional derivative in the sense of Caputo. We prove the existence and uniqueness of solutions using the Faedo-Galerkin approximation method and some compactness arguments.

Bio-Convective Nanofluid Flow Along a Horizontal Wavy Surface in Porous Media

Melusi Khumalo
University of South Africa, So Africa
F. Awad, M.P. Mkhatshwa

There are few theoretical studies on convective transport in nanofluids along wavy surfaces. This paper analyzes bio-convective flow of a nanofluid containing gyrotactic microorganisms over a horizontal wavy surface in a porous medium. The model for the nanofluid transport equations incorporates the effects of Brownian motion and thermophoresis. The wavy surface and the flow governing equations are first transformed into plane geometry case using suitable transformations. The governing partial differential equations are reduced into a set of coupled ordinary differential equations which are then solved numerically using the spectral relaxation method (SRM). The accuracy of the solutions has been determined by benchmarking the results against the spectral quasi-linearization method (SQLM). The effects of the governing parameters on the rate of heat, nanoparticle volume fraction and motile microorganisms transfer have been studied. Results indicate that the local Sherwood number and density number of motile microorganisms are increased as the Brownian motion parameter is increased, while the local Nusselt number is reduced. Moreover, the Nusselt number decreases as the thermophoresis parameter increases.

Deceleration of Mixing Rates in Discontinuous, One-Dimensional Mappings

Hannah Kreczak
University of Leeds, England
Rob Sturman, Mark C.T. Wilson

We present a study on the evolution of a passive scalar towards the uniform equilibrium distribution in an advection-diffusion problem where the advective transport is a composition of stretching and folding and cutting and shuffling. The linearity of the advection-diffusion problem means that the long-time evolution can be characterized by the eigenvalues and eigenmodes of the transfer operator. The rate of approach to equilibrium is shown to be dependent on the choice of permutation and changes non-monotonically with the diffusion coefficient for many of the permutations. This counter-intuitively predicts a deceleration in the asymptotic mixing rate with an increasing diffusivity rate. The global mixing rate in the limit of vanishing diffusivity approximates well the decay rates of the eigenmodes for small diffusivity, however this global mixing rate does not bound the rates for all values of the diffusion coefficient.

A Remark on the Existence of a Class of Stretched Magnetohydrodynamics Flow with Infinite Energy

Bataa Lkhagvasuren
Chonnam National University, Korea
Minkyu Kwak

We consider the existence of a class of special stretched solutions of $2D$ and $3D$ Magnetohydrodynamics equations in $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. The last components of the state variables have the form $u_n = x_n\gamma_1+\varphi_1$ and $b_n = x_n\gamma_2+\varphi_2$ and the functions $\gamma_1, \gamma_2, \varphi_1, \varphi_2$ and the first components of state variables depend on $x_1, ..., x_{n-1}$ and t for $n = 2, 3$. We investigate how the global solvability of the problem depends on the dimension.

Existence, Uniqueness and the Principal Eigenvalue for a Class of Quasilinear Singular Problems

Salvador Lopez Martinez
University of Granada, Spain
Jose Carmona, Tommaso Leonori, Pedro J. Martinez-Aparicio

Consider the family of singular elliptic problems

$$\begin{cases} -\Delta u = \lambda u + \mu(x)\dfrac{|\nabla u|^q}{u^{q-1}} + f(x) & \text{in } \Omega \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \qquad (P_\lambda)$$

in a bounded domain $\Omega \subset \mathbb{R}^N$ with smooth boundary, where $0 \leq \mu \in L^\infty(\Omega)$, $0 \leq f \in L^p(\Omega)$ with $p > \frac{N}{2}$, and 10, which is the *principal eigenvalue*, and the solution to (P_{λ^*}) is unique up to multiplication by positive constants. If $\inf_\Omega(f) > 0$, the set is $(-\infty, \lambda^*)$, and the solution to (P_λ) is unique. Finally, if $f \gneqq 0$ we prove existence for $\lambda\lambda^*$. Our results improve some contained in [1], where only the case $q = 2$ and $\mu \equiv cst$ is considered. The proofs are valid for purely quasilinear problems, in the sense that we avoid the Cole-Hopf change of variables. Our approach is based on a comparison principle that we also prove, an appropriate characterization of λ^* and an approximation and compactness argument.

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On the Regularity of Minimizers of Convex Variational Problems

Erika Maringova
Charles University, Czech Rep
Lisa Beck, Miroslav Bulíček, Bianca Stroffolini, Anna Verde

We study the existence and regularity of a minimizer to a wide class of convex, variational integrals having radial structure. For the linear growth setting, we sharply identify the class of problems which admit globally Lipschitz solution. Further, similar method is applied for superlinear growth. On the other hand, for a general BV minimizer, we study the regularity structure of its absolutely continuous and singular part. The result is not restricted to any geometrical assumption on the domain.

Averaged Fractional Control

Andrej Novak
University of Zagreb, Faculty of Science, Croatia
D. Mitroviċ, T. Uzunoviċ

In this talk, we generalize results concerning averaged controllability on fractional type equations: system of fractional ODE-s and the fractional diffusion equation.

On Weak Solvability of Fractional Models of Viscoelastic Fluid

Vladimir Orlov
Voronezh State University, Russia

The mathematical models of dynamics of viscoelastic fluids with constitutive relations containing fractional derivatives are under consideration. Fractional analogous of Voigt and anti-Zener models are under investigation. We establish the existence of weak solutions of the corresponding initial-boundary value problems. In the planar case the uniqueness of weak solutions is proved. For the proofs of the main results we approximate the problems under consideration by a sequence of regularized systems of Navier-Stokes type. The solvability of regularized systems and apriori estimates of their solutions allow to pass to the limit in the regularized systems and obtain the solvability of original problems. The theory of fractional powers of positive operators, fractional calculus and classical results on Navier-Stokes equations are used. This is a joint work with Victor Zvyagin.

Global Mild Solutions for a Nonautonomous/Impulsive 2D Navier-Stokes Equations

Ricardo Parreira da Silva
University of Brasilia, Brazil
E.M. Bonotto, J.G. Mesquita

The present paper deals with existence and uniqueness of global mild solutions for a 2D Navier-Stokes equations with impulses effects. Using the framework of nonautonomous dynamical systems, we extend previous results considering the 2D Navier-Stokes equations with impulse effects and allowing that the nonlinear terms are explicitly time-dependent. Additionally, we present suffcient conditions to obtain dissipativity (boundedness) for solutions starting in bounded sets.

The Global Attractor of a Multivalued Dynamical System Generated by a Two-Phase Heating Problem

Volker Reitmann
St. Petersburg State University, Russia
Dmitrii Zyryanov

In this talk we study the asymptotic behavior of solutions of Maxwells equations coupled with the heat equation for a phase change problem in 3-dimensional space arising in a microwave heating process. The two-phase changing process is accounted with the enthalpy operator. We consider an initial boundary value problem for this process. The uniqueness of weak solutions is not proved. In the talk it will be shown how to introduce a multivalued dynamical system based on weak solutions of this system. The existence of a global attractor for this multivalued dynamical system will be shown. Some numerical calculations for the temperature behavior are carried out.

Kinematical Conservation Laws: a Geometric Approach to Wave Propagation

Baskar Sambandam
Indian Insitute of Technology Bombay, India
Ram Murti, Phoolan Prasad

A pair of kinematical conservation laws (KCL) in a ray coordinate system are the basic equations governing the evolution of a moving curve in two space dimensions. KCL system is derived purely from a geometric perspective involving three unknown variables and hence is an under-determined system. If the moving curve is a wavefront of a smooth wave, then a closure relation can be obtained in the ray coordinate system from a transport equation that governs the amplitude of the wave along the wavefront. This transport equation is derived from a quasilinear hyperbolic system under a small amplitude perturbation (weakly nonlinear approximation) and a shortwave assumption. The resulting system, called the *weakly nonlinear ray theory* (WNLRT) system can be used to study the propagation of the wavefront in a given medium. This talk is intended to discuss the basic formulation of the WNLRT system, with a special emphasis on how the KCL system is related to the physical problem of weakly nonlinear wavefront propagation in inhomogeneous and moving media. A special focus will also be given to discuss the connection between the KCL based theory and the well-known method called the level set method for interface tracking.

On Quasilinear Viscous Approximations to Conservation Laws

Sivaji Ganesh Sista
Indian Institute of Technology Bombay, India
R. Mondal, S. Baskar

Convergence of quasilinear parabolic viscous approximations to an entropy solution (in the sense of Bardos-Le Roux-Nedelec) of a scalar conservation law posed on a bounded domain will be discussed. Differences and similarities with the well-known artificial viscous approximations will be highlighted.

Perturbation Theory of Bloch Eigenvalues and Homogenization

Vivek Tewary
Indian Institute of Technology Bombay, India
Sivaji Ganesh Sista

The spectrum of periodic operators in divergence form has a band structure, which is completely described by Bloch eigenvalues. We prove that the Bloch eigenvalues at a point are generically simple with respect to the coefficients of the periodic operator. We use this result to prove that a Bloch eigenvalue can be made simple for all parameter values through a perturbation of the operator. Further, a spectral edge can be made simple by a perturbation of coefficients of the operator. We apply these spectral tools to Bloch wave homogenization problems in the presence of multiplicity.

Cross-Diffusion in Population Dynamics

Ariane Trescases
CNRS, France
L. Desvillettes, Th. Lepoutre, A. Moussa

In population dynamics, reaction-cross diffusion systems model the evolution of the populations of competing species with a repulsive effect between individuals. For these strongly coupled, often nonlinear systems, a question as basic as the existence of solutions appears to be extremely complex. We introduce an approach based on the most recent extensions of duality lemmas and on entropy methods. We prove the existence of weak solutions in a general setting of reaction-cross diffusion systems, as well as some qualitative properties of the solutions.

On Bifurcation from Infinity to Fractional Laplace Equations

Jagmohan Tyagi
IIT Gandhinagar, India

We show that (λ_1, ∞) is a bifurcation point of

$$(-\Delta)^s u = \lambda u + f(\lambda,\, x,\, u) \text{ in } \Omega,$$

$u = 0$ in $\mathbb{R}^n \backslash \Omega$,

where λ_1 is the principal eigenvalue of

$$(-\Delta)^s v = \lambda v \in \Omega,$$

$v = 0 \in \mathbb{R}^n \backslash \Omega$.

An Inverse Problem for the Heat Equation on a Star Graph

Kim Tuan Vu
University of West Georgia, USA

Consider a heat equation on a simple star graph with three equal edges

$$(1)\begin{cases} \partial_t u^{(j)}(x,t) = \partial_x^2 u^{(j)}(x,t) - q^{(j)}(x) u^{(j)}(x,t) \\ \text{ for } 0 \le x \le a \text{ and } j = 1,2,3, \\ u^{(1)}(0,t) = u^{(2)}(0,t) = u^{(3)}(0,t) = 0 & (a) \\ u^{(1)}(a,t) = u^{(2)}(a,t) = u^{(3)}(a,t) & (b) \\ \partial_x u^{(1)}(a,t) + \partial_x u^{(2)}(a,t) + \partial_x u^{(3)}(a,t) = 0 & (c) \\ u^{(j)}(x,0) = f(x). \end{cases}$$

Equation (1) describes a heat process with internal sinks (or sources if $q^{(j)}(x) \le 0$) proportional to the temperature distributions $u^{(j)}(x,t)$. For simplicity we take equal edges in length, and the boundary condition (a) means that the three outer vertices are maintained at temperature zero. Condition (b) describes the fact that they have the same temperature at the common vertex $x = a$ while condition (c) is Kirchhoff's law, which says that the heat transferred through the node $x = a$ is conserved. The inverse problem is to recover the heat proportional coefficients $\left\{q^{(1)},\ q^{(2)}, q^{(3)}\right\}$ from readings of the heat flux at the vertices of the graph and of the temperature at the common vertex, which defines a map

$$f \to \left\{\partial_x u^{(j)}(0,t),\ \partial_x u^{(j)}(a,t),\ \ u^{(1)}(a,t) \quad \text{for } j = 1,2,3\right\}$$

for $0 < t < T \le \infty$. First we show that by a suitable choice of f we can determine all eigenvalues of the associated Dirichlet's Sturm-Liouville problem on the graph from the readings. Next we explain how to distinguish simple eigenvalues from multiple eigenvalues from observation. Then we can use sampling formula for Paley-Wiener functions to extract eigenvalues of Dirichlet's Sturm-Liouville problems on each edges. Finally we can recover the three $q^{(j)} \in L^2(0,a)$ uniquely from one reading only of $\left\{\ \partial_x u^{(j)}(0,t)\,,\ u^{(j)}(a,t)\,,\ \partial_x u^{(j)}(a,t)\right\}$, for $t \in (0,\ T)$.
This is a joint work with Dr. Amin Boumenir.

Blowup for the Compressible Euler Equations in R^N

Manwai Yuen
The Education University of Hong Kong, Hong Kong

The compressible Euler equations are fundamental models in the fluid dynamics. In this talk, we discuss the new blowup phenomena with the functional energy methods for the solutions of the Euler equations in R^N for the free boundary problems and non-vacuum states.

Solvability of Initial-Boundary Value Problem for Thermoviscoelastic Kelvin-Voigt Model

Andrei Zviagin
Voronezh State University, Russia

The initial-boundary value problem under consideration describes the weakly concentrated water polymer solutions motion. This mathematical model also is called Kelvin-Voigt model. This problem is considered with constitutive law which is frame indifferent, i.e. that do not change under the Galilean transformation. Also in this mathematical model the viscosity depends on a temperature, which leads to emergence of additional heat balance equation (it is a parabolic equation with nonsmooth coefficients and with right part from $L_1(0,T;L_1(\Omega))$. For the initial-boundary value problem under consideration the existence theorem of weak solutions is proved. For this the topological approximation approach for hydrodynamic problems is used. This research was supported by the Ministry of Education and Science of the Russian Federation (Grant 14.Z50.31.0037).

Mathematical Problem of Viscoelastic Media with Memory Motion

Victor Zvyagin
Voronezh State University, Russia

In the report the existence of weak solutions to the initial-boundary value problem for one viscoelastic model of Oldroyd's type fluid with memory along trajectories of the velocity field is obtained. This problem is solved on the base of a priori estimates and passing to the limit. Optimization problems for the feedback control systems for this model is also studied. The existence theorem for trajectory and global attractors for this system under consideration in the autonomous and non-autonomous cases is obtained. In the plane case, the initial-boundary value problems for a thermoelastic media for this model is shown to be nonlocally weakly solvable. The existence of weak solutions for a motion model of viscoelastic fluid with memory along trajectories of the velocity field on an infinite interval is proved. This research was supported by the Ministry of Education and Science of the Russian Federation (Grant 14.Z50.31.0037).

Contributed Session 3: Modeling, Math Biology and Math Finance

State Estimation for Uncertain Time-Varying SIR-SI Epidemiological Model of a Vector-Borne Disease

Maria Soledad Aronna
Escola de Matematica Aplicada (FGV), Brazil
Pierre-Alexandre Bliman

We consider an SIR-SI model for a vector-borne disease, subject to seasonal variations. The equations describe the evolution of the relative proportions of three classes in the host population: the susceptible S, the infective I and the recovered R, and of two corresponding classes S and I in the vector population. Assuming continuous measurement of the incidence rate (i.e. of the number of new infective individuals in the host population per unit time), we estimate the states of the system by constructing a class of interval observers. We provide asymptotic error bounds for the estimations. The method is based on the search for a common linear Lyapunov function for the monotone systems representing the evolution of the approximation errors.

The Dynamics of Genome Replication: an Inverse Problem Approach

Eduard Campillo-Funollet
University of Sussex, England

Genome replication is a crucial process in biology. It is a stochastic, nonlinear process, and yet it must occur in an accurate manner in order to guarantee the genome stability. The basic components of the process are well understood at the level of an individual cell, but many experimental results provide only information at a population level. We use several mathematical models to bridge the gap between the biological processes at the cell level and the experimental data at the population level. First, we derive a probabilistic model for the firing times of the replication origins, and extend it to obtain the probability distribution of the global replication profile. Furthermore, we use a discrete dynamic model to simulate the replication at the genome scale. Using a Bayesian approach, we fit the model parameters to experimental data on timing and polymerase usage. To illustrate the applicability of the model, we study the role of Rif1 in the replication timing of *S. Pombe*, and we study the effect at the population level, observed in the experiments, of events defined at the cell level—e.g. origin efficiency and replication front arrest.

A Systematic Iterative Method for Multi-Asset Constant Elasticity of Variance Spread Options

Po Hon Chau
The Chinese University of Hong Kong, Hong Kong
Chi Fai Lo

The paper proposes an efficient systematic iterative method for accurately pricing the multi-asset constant-elasticity-of-variance (CEV) spread options. The approach starts with an approximate solution proposed by Lo[1] and then improves its accuracy by computing the correction term in a systematic iterative manner. Numerical examples illustrate that only one iteration are sufficient enough to improve the percentage error up to an order of magnitude. In pricing n-asset options, n+1 integrals are required for every iteration. By using the Lie-Trotter splitting approximation, the dimensions of integration are reduced to two and thus the computation time is dramatically decreased. The correction by the proposed method is remarkably robust and efficient even in the deep-out-of-money and long maturities situation.

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A Cell-Based Model with Parameter Uncertainty Quantification Using Monte Carlo Simulations

Jiao Chen
Delft University of Technology, Netherlands
Daphne Weihs, Fred J. Vermolen

Cell amoeboid migration plays a detrimental role in cancer metastasis, where one cell could utilize deformation to pass or squeeze through obstacles driven by external stimulus and reach different body parts. Our model involves the deformation of the cell and nucleus during cancer metastasis. A generic signal attracts a cell. This is dealt with by Green's Fundamental solutions. The cell cytoskeleton is modelled by 30 springs between the cell membrane and nucleus surface, where the cell membrane and nucleus are divided by 30 nodal points. Moreover, the random walk is incorporated. To compute migration, an Implicit-Explicit (IMEX) time integration method is used. Furthermore, Monte Carlo simulations are carried out to investigate the uncertainties of parameters quantitatively. As we expected, our model phe-

nomenologically replicates cell migration behavior for 2D/ 3D cases in confined spaces, which is expected to predict the microenvironmental behavior of cells and the likelihood of cancer invasion.

Disease Transmission Models with a Demographic Allee Effect

Christine Craib
UNC Wilmington, USA

Incorporating density-dependent population dynamics into epidemiological models allows for the modeling of disease dynamics over longer periods of time. The models presented in this talk compartmentalize a varying population with respect to disease transmission, and describe the total population size with both logistic growth and density-dependent facilitation, or the Allee effect. Conditions are established for the existence and local stability of the equilibria, the thresholds are analyzed, and bifurcation analyses are performed. Numerical simulations are presented to demonstrate theoretical results and further explore model behavior. The biological significance of the results are then explained in terms of both disease and population growth dynamics.

Advanced Regression Models for Complex Regulatory Systems with Applications: CMARS Versus MARS

Ozlem Defterli
Cankaya University, Turkey
Ayse Ozmen

Mathematical models are important not only to provide a better understanding for the underlying dynamics of huge systems, whose components are highly correlated, but also enables us to interpret their future behavior and compare with other possible situations. In this respect, there is a crucial demand for efficient new approaches to infer such complex phenomena which may appear in real-world problems of the fields like system biology, medicine, engineering, finance, education, environment and so on. In such systems, the components appear in a high dimensional network structure where they are interconnected with each other and under the effect of some unknown external parameters needed to be identified. In this study, we implement conic multivariate adaptive regression spline (CMARS) technique on a real-world data from system biology for the inference of such complex regulatory networks. The performance of the model is investigated in comparison with the results obtained from multivariate adaptive regression spline (MARS) modeling approach.

Vector-Borne Disease Control: Dynamic Modeling of Personal Protection Strategies Limits the Diversity Amplification Effect

Bill Fagan
University of Maryland, USA
Jeff Demers, Sharon Bewick, Justin Calabrese

Personal protection measures, such as bed nets and repellents, are important tools for the suppression of vector-borne diseases like malaria and Zika. The ability of health agencies to distribute personal protection equipment, and encourage its use, plays an important role in the efficacy of disease management strategies. Recent modeling studies have discovered that a counterintuitive diversity-driven amplification in community-wide disease levels can occur when a population only partially adopts personal protection measures. In this way, partial adoption becomes detrimental to the larger goal of disease management. This amplification effect, however, may overestimate the negative impact of personal protection as a result of implicit, restrictive model assumptions regarding population compliance, access to personal protection measures, and the longevity of those measures. We establish a new modeling methodology for exploring control efforts in vector-borne disease systems featuring personal protection. This new method flexibly accounts for compliance, access, longevity, and control strategies by way of a flow between protected and unprotected populations rather than breaking the population into static protected and unprotected classes as has been done previously. Our new methodology yields large reductions in the severity and occurrence of amplification effects as compared to existing models.

A One-Dimensional Model of a False Aneurysm

Arpan Ghosh
Linkoping University, Sweden
Fredrik Berntsson, Matts Karlsson, Vladimir Kozlov, Sergey A. Nazarov

A false aneurysm is a hematoma, i.e. a collection of blood outside of a blood vessel that forms due to a hole in the wall of an artery. This represents a serious medical condition that needs to be monitored and, under certain conditions, treated urgently. In this work, a new one-dimensional model of a false aneurysm is proposed. The new model is based on a one-dimensional model of a curvilinear artery with varying width of the channel, previously presented by the authors and it takes into account the interaction between the hematoma and the surrounding muscle material as well. The model equations are derived using asymptotic analysis for the case of a simplified geometry. Even though the model is simple, it still supports a realistic behaviour for the system consist-

ing of the vessel and the hematoma. Using numerical simulations we illustrate the behaviour of the model. The simulations show that our model can reproduce realistic solutions.

Pricing Options with Heston Stochastic Volatility and Time-Dependent Parameters — Lie-Algebraic Approach

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Pricing options with Heston stochastic volatility is a challenging task because in Heston's stochastic volatility framework semi-analytical formulae for plain vanilla option prices can be derived but it is unfortunate that these formulae require the evaluation of logarithms with complex arguments during the involved inverse Fourier Tranform. Thus, most implementation of Heston's formulae suffers from an inherent numerical instability and are not robust for moderate to long dated maturities or strong mean reversion. Besides, if the model parameters are not constant or piecewise constant, no semi-analytical pricing formula is known. By exploiting the Lie symmetry of the pricing partial differential equation and applying the Wei-Norman theorem, we have succeeded to derive an accurate approximate analytical formula for pricing a vanilla call option with Heston stochastic volatility and time-dependent model parameters. In addition, the accuracy can be further improved in a systematic manner. Furthermore, the proposed approach can be easily extended to pricing constant-elasticity-of-variance options with Heston stochastic volatility and time-dependent parameters.

Backward Bifurcation in Measles Model with Logistic Growth and Vaccination

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Wirawan Chinviriyasit

In this paper, we formulate a measles model with logistic growth and vaccination for studying the phenomenon of backward bifurcation that would impact to measles transmission in Thailand. The proposed model has a locally asymptotically stable disease-free equilibrium whenever a certain epidemiological threshold, known as the basic reproduction number. It is found that the model exhibits the phenomenon of backward bifurcation where stable disease-free equilibrium of the model co-exists with a stable endemic equilibrium when the reproduction number is less than unity. The epidemiological consequence of this phenomenon is that the classical epidemiological requirement of making the reproduction number less than unity is no longer sufficient, although necessary, for effectively controlling the spread of measles in a community. Our results suggest that a very effective method to prevent the backward scenario is to concern the vaccine wane, controlling the efficacy of vaccines and to concern the optimal timing for the administration of vaccine, which covers the duration of vaccine-induced immunity to measles. Therefore, the results from this study suggest the critical information for decision makers and public health officials, who may have to deal with the prevention and control of an epidemic.

Predicting Rift Valley Fever Inter-Epidemic Activities and Outbreak Patterns: Insights from a Stochastic Host-Vector Model

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Rift Valley fever (RVF) outbreaks are recurrent, occurring at irregular intervals of up to 15 years at least in East Africa. Between outbreaks disease inter-epidemic activities exist and occur at low levels and are maintained by female Aedes mcintoshi mosquitoes which transmit the virus to their eggs leading to disease persistence during unfavourable seasons. Here we formulate and analyse a full stochastic host-vector model with two routes of transmission: vertical and horizontal. By applying branching process theory we establish novel relationships between the basic reproduction number, R_0, vertical transmission and the invasion and extinction probabilities. Optimum climatic conditions and presence of mosquitoes have not fully explained the irregular oscillatory behaviour of RVF outbreaks. Using our model without seasonality and applying van Kampen system-size expansion techniques, we provide an analytical expression for the spectrum of stochastic fluctuations, revealing how outbreaks multi-year periodicity varies with the vertical transmission. Our theory predicts complex fluctuations with a dominant period of 1 to 10 years which essentially depends on the efficiency of vertical transmission. Our predictions are then compared to temporal patterns of disease outbreaks in Tanzania, Kenya and South Africa. Our analyses show that interaction between nonlinearity, stochasticity and vertical transmission provides a simple but plausible explanation for the irregular oscillatory nature of RVF outbreaks. Therefore, we argue that while rainfall might be the major determinant for the onset and switch-off of an outbreak, the occurrence of a particular outbreak is also a result of a build up phenomena that is correlated to vertical transmission efficiency.

Kinetic Stability Analysis of ^{18}F-FDOPA in PET/CT Imaging for Parkinson's Disease

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Kinetic stability analysis is a useful method for illustration of the dynamic behaviour of metabolic biological systems. The ^{18}F-FDOPA administration is used to uptake in brain of the patient who has decreasing dopamine level. Then, Parkinson's Disease will be found with Positron Emission Tomography-Computed Tomography (PET/CT) scan. The purpose to study is to develop and analyze a kinetic model of ^{18}F-FDOPA, which is consequence to metabolic biological systems of this radioactivity. According to the Quasi-Steady-State Approximation (QSSA) demonstrates behaviour of the equilibrium point, which explains PET/CT data. Describing equations as follows: Let A(t), X(t), Y(t) and Z(t) be the concentration of FDOPA and FDA in blood or brain, including chemical reaction coefficients k_i, $\frac{dX}{dt} = K_1A(t) - (k_2 + k_3 + k_4)X(t)$, $\frac{dY}{dt} = k_3Z(t) - (k_5 + k_6)Y(t)$, $\frac{dZ}{dt} = k_5Y(t) - k_7Z(t)$, where A(t)=$\frac{k_2}{K_1+k_3}$. Our findings are expected to provide quantitative method for kinetic model analysis and reliability for evaluating of administrate and radiation dose.

Dynamics of a Delayed Predator-Prey Model with Saturated SIS Epidemic on the Prey Population

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A predator-prey model with disease in the prey population is considered. The transmission of disease in the prey population is of type II functional response, while the incidence rate between the whole prey population and the predator population is Lotka-Volterra type. The delay term arises on the increase of the predator population due to interaction with both prey, may it be infected or not. The stability of the equilibrium points and the existence of Hopf-Bifurcation are analysed. Lastly, numerical examples are done to illustrate the theoretic results.

Eigensolutions for a Model of Vertical Gene Transfer of Plasmids

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We consider a model for vertical gene transfer of genetic elements in bacteria called plasmids. The model incorporates non-constant cell division and death, plasmid reproduction, and segregation which is modeled by an integral operator describing the fraction of plasmids the respective daughter receives from the mother. The dynamics of the system is determined by plasmid reproduction which increases the number of plasmids in the population and cell division which decreases the number of plasmids per cell. We obtain a growth-fragmentation equation for the bacterial population structured by the number of plasmids. The model equation is a hyperbolic partial differential equation with an integral term. As we are interested in the long-term distribution of plasmids we consider the associated eigenproblem and prove existence of eigensolutions along the lines of [1] extending the results to models with non-constant cell death rates. In order to obtain stability results, we analyze the spectrum of the differential operator given by the model equation. We find a real dominating eigenvalue. The corresponding eigenfunction attracts all solutions.

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Modeling the Growth of Blood Vessels

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The growth of blood vessels (angiogenesis) in a living tissue is a complex multiscale process that can be either physiological, being essential for organ development and repair, or pathological, since its unbalance can lead to various disorders and foster cancer proliferation. Angiogenesis is a response to lack of oxygen. Hypoxic cells produce growth factors that prompt the generation of new capillary sprouts from a nearby vessel. Thus, the vessel network expands, transporting oxygen and nutrients to the surrounding tissue. A hybrid tip cell model for tumor-induced angiogenesis is presented, in which trajectories of vessel tips are the capillaries advancing towards the tumor, driven by the gradient of a growth factor. Vessel extension follows from stochastic differential equations coupled to reaction-diffusion equations for the involved substances concentration. Branching of new tips and anastomosis (destruction of tips merging with existing capillaries) are also modeled. Numerical simulations of the model are carried out and a deterministic description of the active tips density is derived from ensemble averages over many replicas of the angiogenic process. It is shown that such density forms a stable soliton-like wave whose dynamics (location, velocity, size) can be characterized by a system of collective coordinates.

Sensitivity of Chemical Reaction Networks: Model, Results and Computational Issues

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Sensitivity studies the network response to the perturbation of a reaction rate or of a metabolite concentration, at steady state. Biological experiments in this direction have been performed by Ishii et al., among others. In these experiments they studied the response of metabolic networks to the knock-out of certain targeted reactions. The response is given as the increase, decrease or zero response, of experimentally accessible concentrations of metabolites or reaction fluxes in the system. Our mathematical model translates this problem into a perturbation analysis of a system of differential equations derived from chemical reaction network. We summarize here the results obtained in this direction by Fiedler, Matano, the author et al., with a focus on computational issues.

Modeling the Time to Importation of Zika Virus at a Global Scale

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Hiroshi Nishiura

Background: Since the 2015 Zika virus (ZIKV) epidemic in Brazil, ZIKV has spread worldwide and has remained to be global public health concern. ZIKV infection is a vector-borne disease and is transmitted by Aedes species of mosquito. In the case of epidemic in Brazil, dangue was first suspected but its possibility was excluded based on molecular and serological diagnostic method. The effective distance has been known that it can reliably predict the arrival time of infectious diseases including ZIKV.
Objectives: To estimate the actual arrival time of ZIKV importation which is the time at which the first imported case arrived.
Methods/Principal findings: In addition to utilize the hazard function (inverse function of the effective distance), we employed the distribution of reporting delay. In order to consider the possible factors which can cause the delay, we categorize the countries into five according to GDP, and parameters for gamma distribution were estimated for each category. Data on arrival times of ZIKV importation was collected from publicly available data sources. We devised a simple model to reconstruct the geographic transmission dynamics and here we report the preliminary results from our exercise.

Kinetic Model of ^{99m}Tc-ECD Absorption in Brain for Epilepsy Patients and Stability Analysis

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Epilepsy is a neurological disorder caused by electrical abnormalities in brain, causing seizures or unusual behavior. The Single-Photon Emission Computed Tomography (SPECT) has dramatically changed the method for epilepsy evaluation. In this study, we developed a model of ^{99m}Tc-ECD absorption in brain, depending on the acquiring time of patients with epilepsy. Collection of epilepsy data from ^{99m}Tc-ECD samples was then adjusted for kinetic parameters and numerical methods were used to find the solution of the kinetic model to compare data from SPECT imaging, specifications, and relative error to get the appropriate parameters for patients with epilepsy. The epileptic patients had absorption around 27.9084 MBq/ml. From SPECT imaging, the essential parameter of ^{99m}Tc-ECD absorption model are $k_{inject} = 0.067 \pm 0.03204$, $K_1 = 0.30 \pm 0.08944$, $k_2 = 0.233 \pm 0.02582$, $k_3 = 0.517 \pm 0.02582$, $k_4 = 0$ and $k_{out} = 0.005 \pm 0.00412$. These parameters could also describe accurately the ^{99m}Tc-ECD absorption of epileptic data. Furthermore the stability analysis of this model illustrates ^{99m}Tc-ECD behavior.

Contributed Session 4: Control and Optimization

Optimal Control Problems for Stress Tensor in Plastic Plane Medium

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This paper joins some concepts from Mechanics, Partial Differential Equations and Control Theory in order to solve bi-time optimization problems related to stress tensor in plastic deformations. The main goal is to analyze some optimal control problems constrained by the equilibrium equations of the stress tensor in perfect plastic plane medium. As consequence of this approach, a natural split of the constraints arises, leading to integrability conditions and changing a classical variational problem into an optimal control one. The final outcomes confirm all the expectations related to the physical features of plastic deformations phenomenon.

Numerical Study of Hamilton-Jacobi-Bellman Equation for Time-Optimal Trajectory Generation of Dubins Vehicle

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Jing-Sin Liu

This article mainly applies Hamilton-Jacobi-Bellman (HJB) equation for collision-avoidance path planning for Dubins vehicles in the static obstacles workspace. The Dubins vehicles are simple robot cars which can only move ahead and with a given minimum turning radius bound.

In 1957, Dubins was a pioneer to investigate the time-minimal 2D path for such vehicles [1]. For the aspect of obstacle-avoidance task, there are many strategies (like APF, RRT, ...etc.) [10] to efficiently avoid obstacles in the workspace. However, it is not easy to consider both collision-avoidance task and time-optimal requirement for the Dubins vehicle at the same time. Takei and Tsai in [2] provide detailed discussions by using numerical methods for solving HJB equation for path planning. Mathematically, HJB equation is a first-order nonlinear PDE which does not typically exist a classical solution. However, Crandall and Lions in their famous papers [3, 4] provide the existence and uniqueness proof of viscosity solution to HJB equation and any monotone, differenced-form, consistent numerical Hamiltonian would lead the numerical solution converge to true viscosity solution. Godunov numerical Hamiltonian and semi-Lagrange scheme are two typically methods which satisfy the conditions of convergence [2, 5]. To implement those schemes, fast sweeping method is an efficient method for numerical computation and was first proposed by Zhao [6]. In this paper, we demonstrate Godunov numerical Hamiltonian and semi-Lagrange scheme with fast sweeping method for solving HJB equation. Unlike the HJB equation considered by Takei et. al. [2], we consider a slightly modified equation by Kruzkov transform to avoid unboundedness in the initial condition to assure the existence of viscosity solution and computational feasibility.

Once the solution is obtained, the characteristic line from any given initial pose gives the optimal control [9] and thus is the time-optimal collision-avoidance path, namely the Dubins path. In this paper, we verify the accuracy of both numerical methods with analytical solution provided by Dubins [1]. Moreover, Shiller et. al. in [7] proposed an algorithm from the knowledge of the structures of shortest path for selecting a near-optimal path by the pseudo-return function. The main advantage of this algorithm is it is efficient in computation and lowly depends on the number of obstacles in the workspace.

The paper also compares some features like the level sets of each value function, the path results and the total lengths of paths. In conclusion, the paper presents the feasibility of Dubins path planning and collision-avoidance task through the HJB equation.

Numerical methods are also applied for solving the HJB equation. From the simulations in the paper, the semi-Lagrange scheme performs better than Godunov scheme. It is because the semi-Lagrange scheme is more able to deal with discontinuous part of the solution [2]. Once, the numerical solution is obtained, the optimal control is given by the characteristic line equation.

It would be used for optimal collision-free path planning. However, the future work of the paper is to apply or investigate the high order accuracy schemes such as limiters, WENO method for high order accurate solution and apply the results to the practical UAV (Unmanned Aerial Vehicle) flight experiments for 3D path planning.

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Approximate Controllability of a Nonlocal Quasilinear Differential Equations with Repeated Deviating Arguments

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Duranta Chutia

In this article, we prove the sufficient conditions for the approximate controllability of a nonlocal quasilinear differential equations with repeated deviating arguments in a Hilbert space. The existence of mild solution is established by the Sobolevskǐ-Tanabe theory of parabolic equations, fractional powers of operators. We show the approximate controllability of the equation. An example is also discussed for the established results.

The Properties of the Solutions for the Incompressible Flows on an Exterior Domain

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In this talk, we want to see the properties of the smooth solutions $\mathbf{u}$ of the incompressible flows on an exterior domain Ω of R^2. Specially, when the vorticity $\omega = \nabla \times \mathbf{u}$ has a bounded support, with suitable conditions we will show that there exists a constant $C(p, q)$ such that

$$\| \mathbf{u} \|_{L^p(\Omega)} \leq \ C \| \mathbf{u} \|_{L^q(\Omega)}$$

for $1 < p \leq q \leq \infty$.

Optimal Control Strategies for the Resurgence of Vaccine Preventable Diseases in Thailand

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The purpose of this research is to study the allocation of limited vaccine to get the optimal control strategies for the resurgence of vaccine preventable diseases in Thailand. Using data of movement patterns of migrant workers (Cambodia, Myanmar and Laos) and the patients with diphtheria data in Thailand in 2015, we determined proportion of vaccine allocation for three migrant populations who should receive the vaccine. From the mathematical models and an optimization method with 100% doses of vaccine, the optimal vaccine allocation for migrant workers from Myanmar, Laos and Cambodia are 64.5%, 35.56% and 0% respectively. Sensitivity analysis was performed with attentive to vaccine efficacy. We also found that vaccine-preventable epidemics could further decrease by 50% among migrants and Thais, taking into account the effects of population movement.

Optimal Control on a Discrete Time Model for Tuberculosis

John Sebastian Simon
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Rodolfo Po III

An optimal control problem is studied in a discrete time model describing the dynamics of Tuberculosis transmission in which treatment in both latency and infectious period is considered. Maximising the susceptible class at the final observational time, and minimising the cost induced from the controls are aimed. The controls are characterised through Pontriyagin's Optimality Principle, and are solved numerically, together with the state equations, using forward-backward sweep method.

Optimal Feedback Control for 3D Bingham Fluid Motion Model

Mikhail Turbin
Voronezh State University, Russia
Victor Zvyagin

The problem of control of fluid motion by external forces often arises in practical applications. Usually, the control is chosen from some given (finite) set. However, in papers of V.G. Zvyagin, V.V. Obukhovskii, M.V. Turbin it were considered problems with external forces (control), depending on the velocity of the fluid (such problems are called problems with feedback). This approach allows to select the control more precisely, since in this case the control is chosen not from the finite set of available controls, but belongs to the image of some multivalued map (it must be bounded, upper semicontinuous, weakly closed and has non-empty, compact and convex values). The solution of control problem for the considered fluid model is the pair (v, f), where v is the velocity of the fluid, and f is the control. Moreover, f belongs to the image of some multivalued map depending on the velocity v of the fluid. Due to the fact that there can be many such pairs, it naturally arises the concept of an optimal solution which gives a minimum to a given quality functional. We consider the feedback control problem for the mathematical Bingham model of fluid motion in the 3D case with periodic conditions on spatial variables.

Control of Mobile Robots on Time Scales

Özkan Öztürk
Giresun University, Turkey
Elvan Akìn, Hacì M. Gùzey

We deal with the control of unmanned mobile robots which are modeled by three-dimensional systems of first order dynamic equations. Our goal is to show the asymptotic stability of the zero solution. And we end up with that the results in the continuous case are improved by proposing different controllers.

Contributed Session 5: Scientific Computation and Numerical Algorithms

A Direct Quadrature Method for Auto-Convolution Volterra Integral Equations

Chengming Huang
Huazhong University of Science and Technology, Peoples Rep of China
Min Li

In this talk, we are concerned with the numerical solution of auto-convolution Volterra integral equations. A composite quadrature method based on linear barycentric rational interpolation is introduced. The method is easy to be implemented because only a linear equation needs to be solved in each time step. Collocation method is used as the starting procedure. The boundedness and convergence of the numerical solution are studied in detail. Some numerical experiments are carried out to confirm the theoretical results.

Constrained Data Visualization Using A-Fractal Rational Function

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This article is devoted to the development of a constructive approach to constrained interpolation problems from hidden variable fractal interpolation function (HFIF). We identify a family of $\mathbb{R}^2$-valued continuous fractal functions with a prescribed continuous function $\mathbf{f} : I \to \mathbb{R}^2$ using a finite sequence of base functions instead of a single base function, where $\mathbb{R}^2$ is endowed with the l^1-norm (taxicab norm). As an application of the developed theory, we construct an A-fractal function $\mathbf{f}[\mathbf{A}]$ by a fractal perturbation of a traditional rational cubic trigonometric interpolant and investigate its shape preserving aspects. In particular, we obtain $\mathcal{C}^1$-rational cubic trigonometric spline HFIF (RCTSHFIF) for shape preserving univariate interpolation involving shape parameters for the first time in the literature. The shape preserving interpolation scheme developed herein generalizes and enriches its traditional nonrecursive counterpart and its fractal extension. These interpolants can reflect the nature of self-referentiality or non-self-referentiality of the original data defining function depending on the values of the parameters. With suitably chosen numerical examples, the effectiveness of the shape preserving interpolation scheme are illustrated.

Complex and Quaternion Maps of Blaschke Products

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Blaschke Mapping is motivated by W. Blaschke, who published an article entitled as KINEMATICS AND QUATERNIONS in 1958. In this presentation, we study the complex and quaternion mapping, which as W. Blaschke indicated that the mathematical efforts will flourish the development of geometry of motion or kinematics in physics with the use of the quaternions that L. Euler introduced to that end in 1748 and coined by W.R. Hamilton in 1843. Blaschke products is a mathematical generalization of Lorentz formalism, and hereby we define the functional sets in momentum space. The applications of this effort can be in various scientific fields, such as astronomy, high energy physics, as well as physical statistics. The set defined in unit cubic will form separated subsets in crystal-like structures after given steps of functional iterations. In the linear regions, defined as functional order of 1 and 2, we observed that a set in spherical-shaped structure transforms into subsets in a simple-cubic-crystal structure, while for functional order of 3 and greater, more complicated transformations and structures exist depending on the parameter space. We finally compare the analog and difference of complex mapping versus quaternion mapping in conjunction with the Blaschke products.

On a Stable Numerical Scheme for Magnetic Nanoparticles in Incompressible Fluid

Patrick Weiss
University Erlangen-Nuremberg, Germany

We are concerned with suspensions of magnetized nanoparticles. A fully practical numerical scheme is proposed for a thermodynamically consistent pde-model which couples hydrodynamic variables, particle density, magnetization and magnetic field to each other. In particular, the effect of the demagnetizing field is embedded. The scheme's design admits an energy/entropy-estimate mimicking the estimates in the continuous setting. The core concepts for stability estimates will be explained, leading to discrete existence results via appropriate fixed-point arguments. A proof of concept for numerics in 2D will be presented, too.

A Smooth Fictitious Domain / Multiresolution Method for Elliptic Equations on General Domains

Ping Yin
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We propose a smooth fictitious domain / multiresolution method for enhancing the accuracy order in solving second order elliptic partial differential equations on general bivariate domains. We prove the existence and uniqueness of the solution of a corresponding discrete problem and a so-called interior error estimate which justifies the improved accuracy order. Numerical experiments are conducted on a Cassini oval.

Contributed Session 6: Bifurcation and Chaotic Dynamics

Global Limit Cycle Bifurcations of Multi-Parameter Dynamical Systems

Valery Gaiko
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We carry out the global bifurcation analysis of multi-parameter dynamical systems. To control all their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all their rotation parameters. It can be done by means of the development of new bifurcational geometric methods based on Erugin's global qualitative investigation approach and Perko's planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity). Applying these methods, we study global limit cycle bifurcations for the Holling-type quartic, Kukles cubic-linear and Lorenz-type quadratic dynamical systems.

Lorenz-Like Attractors in a Nonholonomic Model of Celtic Stone

Aleksandr Gonchenko
Lobachevsky State University of Nizhni Novgorod, Russia

We consider nonholonomic models of Celtic stone moving along the plane. We study regular and chaotic dynamics in these models. In particular, we show that chaotic dynamics is very diverse here: it can be representes not only by strange attractors (spiral, torus chaos ones) but also the mixed dynamics and nearly conservative chaos. Moreover, for certain types of Celtic stone (possessing certain geometrical and physical properties) strange Lorenz-like attractors were found in their nonholonomic models. In this talk we observe the corresponding results related to scenarios of appearance and break-down of such strange attractors.

Nonautonomous Period Doubling Border Collision Bifurcation

Volker Reitmann
St. Petersburg State University, Russia
Anastasia Maltseva

In our talk we consider nonautonomous difference equations with piecewise smooth right part and with deterministic and random forcing. The cardiac conduction system is an important source of such equations. For such nonautonomous difference equations with deterministic or random forcing we construct parameter-dependent cocycles. The existence of a period doubling border collision bifurcation (deterministic and random) is shown for such systems.

Unboundedness of Solutions in Parameter Dependent Second Order Differential Equations

Felix Sadyrbaev
Institute Mathematics, University of Latvia, Latvia
Alexander Baryshnikov

Linear and nonlinear second order ordinary differential equations are considered with coefficients depending on parameters. The unboundedness of solutions is studied. The stability diagrams for the Meissner's equation of the form

$$x'' + (\delta + Ap(t))x = 0, \tag{1}$$

where $p(t)$ is the piece-wise constant function switching from 1 to -1 periodically, are obtained (A and δ are treated as parameters). The unboundedness of solutions of nonlinear equations of the form

$$x'' + a(t)x + b(t)x^{2n+1} = 0, \tag{2}$$

where $a(t)$ and $b(t)$ are piece-wise constant coefficients, is studied also. The values of coefficients and moments of switching between them are treated as parameters.

Conservative Lotka-Volterra Systems: Hamiltonian Structure and Dynamics

Xiaohua Zhao
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Lotka-Volterra system is the system with the following form:

$$\frac{dx_j}{dt} = x_j(\epsilon_i + \sum_{k=1}^{n} a_{jk}x_k), \quad j = 1, \cdots, n.$$

The system is called conservative if there exists a positive diagonal matrix D such that DA is skew-symmetric, $DA^T = -AD$. In this talk, we will present our new results on Hamiltonian structure and dynamical behaviors, including periodic solutions, invariant tori and chaos, of conservative Lotka-Volterra systems.

Poster Session

A Standby Host Sharing Two Software with Repair Partial Failure and Group Replacement at Complete Failure of the System

Idris Ahmed
Sule Lamido University, Nigeria
Ibrahim Yusuf

We study a series system consisting of two host sharing two software. The system is subjected to two types of failures. Type I failure is minor in which the system is imperfectly repaired. Type II failure is major failure in which the system is replaced. We developed the explicit expression for mean time to system failure (MTSF), steady-state availability, busy period and profit function using kolmogorov equations method.

New Derived Systems of Hide's Coupled Dynamo Model

Ali Allahem
Qassim University, Saudi Arabia

In this conference, we aim to present the results of a preliminary analytical and numerical study of new derived system of single and double coupled self-exciting Faraday disk homopolar dynamos by Hide et al. Also, well-known systems including, Rikitake and Bullard systems have been reached by using Tikhonov theorem and eliminating method.

State Estimation for Uncertain Time-Varying SIR-SI Epidemiological Model of a Vector-Borne Disease

Maria Soledad Aronna
Escola de Matematica Aplicada (FGV), Brazil
Pierre-Alexandre Bliman

We consider an SIR-SI model for a vector-borne disease, subject to seasonal variations. The equations describe the evolution of the relative proportions of three classes in the host population: the susceptible S, the infective I and the recovered R, and of two corresponding classes S and I in the vector population. Assuming continuous measurement of the incidence rate (i.e. of the number of new infective individuals in the host population per unit time), we estimate the states of the system by constructing a class of interval observers. We provide asymptotic error bounds for the estimations. The method is based on the search for a common linear Lyapunov function for the monotone systems representing the evolution of the approximation errors.

Particle Swarm Optimization for the Numerical Solution of One-Dimensional Elliptic Boundary ValueProblems

Jagdish Chand Bansal
South Asian University, India
Navnit Jha

Particle Swarm Optimization is one of the most popular and efficient swarm intelligence techniques for the global optimization. Due to stochastic nature of the solutions provided by particle swarm optimization, it is rarely applied to the numerical solution of differential-difference equations. Finding the analytical solutions to the one-dimensional mildly non-linear elliptic boundary value problem is a cumbersome process. Therefore, unconventional solution approaches may provide a better tool for the numerical solutions to this problem. In this paper, the problem of finding solution values after the three-point finite difference compact discritization of the one-dimensional elliptic equation is modeled as an optimization problem. Particle swarm optimization is then applied to find the solution values. The numerical experiments exhibit the applicability and efficiency of particle swarm optimization.

Regular Blocks and Conley Index of Isolated Invariante Continua in Surfaces

Hector Barge
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In this talk we shall be concerned with isolated invariant continua in surfaces using a topological point of view. We shall see that in this case the Conley index is determined in intrinsic terms by the knowledge of an initial section of the unstable manifold and the topology of the invariant set. This allows us to give a complete description of the Conley indices of isolated invariant continua in surfaces. From this description we are able to obtain interesting conclusions such as the topological classification of those isolated invariant continua in surfaces which do not have fixed points or the classification of isolated minimal sets

Outflow Boundary Condition Leading to Minimal Energy Dissipation for an Incompressible Flow

Michal Bathory
Charles University, Czech Rep

A method for determining the boundary condition on artificial boundaries is presented. This method is formulated as an optimization problem for the energy dissipation. We show that this functional attains its minimum on the set of solutions to the stationary Stokes (Navier-Stokes) system with partially unspecified boundary condition. Thus, this method gives rise to a physically reasonable implicit boundary condition which ensures the existence of a solution to the system. In particular, for the Stokes system, this implicit condition implies certain modification of the "do-nothing" boundary condition. For the Navier-Stokes system, we foreshadow the "correct" explicit boundary condition.

Modeling the Impact of Dam Removal on the Formosan Landlocked Salmon in the Context of Climate Change

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Hao-Yen Chang, Chyng-Shyan Tzeng, Hsing-Juh Lin

Dam removal is analyzed as a conservation strategy for the Formosan landlocked salmon Oncorhynchus masou formosanus, a critically endangered species whose last refuge is the Wuling basin in central Taiwan. In a previous study, a stochastic age-structured simulation model was developed and used to assess the effectiveness of removing four dams in increasing salmon abundance in Kaoshan Stream in the context of climate change. Three check dams remain intact in Chichiawan and one of its tributaries. In this study, we recalibrate the model for these regions and simulate the removal of each dam. Model analysis suggests that the combined effect of dam removal and climate change decreases the effectiveness of dam removal while increasing the negative impact of climate change on abundance. The model predicts that removing the dam in Chichiawan Stream has the largest potential for increasing the 2035 abundance, but only under narrow conditions of climate change and effectiveness of dam removal. The potential benefit from removing one of the tributary dams is smaller, but the conditions for reaching closer to its potential are less restrictive. This type of analysis is useful for dam removal management decisions regarding habitat restoration.

Optimal Control Problems for Stress Tensor in Plastic Plane Medium

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University Politehnica of Bucharest, Romania
Simona Dinu

This paper joins some concepts from Mechanics, Partial Differential Equations and Control Theory in order to solve bi-time optimization problems related to stress tensor in plastic deformations. The main goal is to analyze some optimal control problems constrained by the equilibrium equations of the stress tensor in perfect plastic plane medium. As consequence of this approach, a natural split of the constraints arises, leading to integrability conditions and changing a classical variational problem into an optimal control one. The final outcomes confirm all the expectations related to the physical features of plastic deformations phenomenon.

Boundary Controllability of Neutral Integrodifferential Impulsive Systems with Time Varying Delays in Banach Spaces

Radhakrishnan Bheeman
PSG College of Technology, India
Nil

In this paper, author establishes the boundary controllability results for neutral integrodifferential impulsive system with time varying delays in Banach Spaces. The sufficient condition of boundary controllability is proved under the evolution operator. The results are obtained by using the semigroup theory and the Schaefer fixed point theorem. An example is provided to illustrate the theory.

Variation of Parameters for a Differential Equation on Schwarzschild Spacetime

Petarpa Boonserm
Chulalongkorn University, Thailand
Tritos Ngampitipan

In general relativity, gravity is governed by the Einstein equations. In fact, the Einstein equations are so complicated that a qualitative and quantitative description of gravity cannot be made. However, under the condition of being perfect fluid spheres, the Einstein equations take a simpler form, and therefore, information about gravity can potentially be extracted. Such condition can be expressed as an ordinary differential equation. In this paper, we are interested in solving this differential equation using the method of the variation of parameters. The result shows that the Schwarzschild spacetime can be recovered by choosing an appropriate initial solution.

The Dynamics of Genome Replication: an Inverse Problem Approach

Eduard Campillo-Funollet
University of Sussex, England

Genome replication is a crucial process in biology. It is a stochastic, nonlinear process, and yet it must occur in an accurate manner in order to guarantee the genome stability. The basic components of the process are well understood at the level of an individual cell, but many experimental results provide only information at a population level.

We use several mathematical models to bridge the gap between the biological processes at the cell level and the experimental data at the population level. First, we derive a probabilistic model for the firing times of the replication origins, and extend it to obtain the probability distribution of the global replication profile. Furthermore, we use a discrete dynamic model to simulate the replication at the genome scale. Using a Bayesian approach, we fit the model parameters to experimental data on timing and polymerase usage. To illustrate the applicability of the model, we study the role of Rif1 in the replication timing of *S. Pombe*, and we study the effect at the population level, observed in the experiments, of events defined at the cell level—e.g. origin efficiency and replication front arrest.

Finding Critical Domains of Quenching Set for Coupled Semilinear Parabolic Equations with a Localized Source

W.Y. Chan
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Let Ω_1 and Ω_2 be two bounded domains in R^2, and $\partial\Omega_1$ and $\partial\Omega_2$ be their boundary, respectively. Suppose that u and v are functions of t, x, and y. Let (x_0, y_0) be a fixed point in Ω_1 and in Ω_2. In this presentation, we study the quenching set of the firstinitial-boundary value problem of quenching problems for the following coupled semilinear parabolic equations with a localized source:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{1 - v(x_0, y_0, t)} \text{ for } (x, y, t) \in \Omega_1 \times (0, \infty), \\ \frac{\partial v}{\partial t} &= \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{1 - u(x_0, y_0, t)} \text{ for } (x, y, t) \in \Omega_2 \times (0, \infty), \end{aligned}$$

$$\begin{aligned} u(x, y, 0) &= 0 \text{ for } (x, y) \in \bar{\Omega}_1 \text{ and} \\ v(x, y, 0) &= 0 \text{ for } (x, y) \in \bar{\Omega}_2, \\ u(x, y, t) &= 0 \text{ for } t > 0 \text{ and } (x, y) \in \partial\Omega_1 \text{ and} \\ v(x, y, t) &= 0 \text{ for } t > 0 \text{ and } (x, y) \in \partial\Omega_2. \end{aligned}$$

Using a numerical method, we shall determine the approximated critical domains of the above problem for some shapes of Ω_1 and Ω_2.

A Systematic Iterative Method for Multi-Asset Constant Elasticity of Variance Spread Options

Po Hon Chau
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Chi Fai Lo

The paper proposes an efficient systematic iterative method for accurately pricing the multi-asset constant-elasticity-of-variance (CEV) spread options. The approach starts with an approximate solution proposed by Lo[1] and then improves its accuracy by computing the correction term in a systematic iterative manner. Numerical examples illustrate that only one iteration are sufficient enough to improve the percentage error up to an order of magnitude.
In pricing n-asset options, n+1 integrals are required for every iteration. By using the Lie-Trotter splitting approximation, the dimensions of integration are reduced to two and thus the computation time is dramatically decreased. The correction by the proposed method is remarkably robust and efficient even in the deep-out-of-money and long maturities situation.

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A Cell-Based Model with Parameter Uncertainty Quantification Using Monte Carlo Simulations

Jiao Chen
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Daphne Weihs, Fred J. Vermolen

Cell amoeboid migration plays a detrimental role in cancer metastasis, where one cell could utilize deformation to pass or squeeze through obstacles driven by external stimulus and reach different body parts. Our model involves the deformation of the cell and nucleus during cancer metastasis. A generic signal attracts a cell. This is dealt with by Green's Fundamental solutions. The cell cytoskeleton is modelled by 30 springs between the cell membrane and nucleus surface, where the cell membrane and nucleus are divided by 30 nodal points. Moreover, the random walk is incorporated. To compute migration, an Implicit-Explicit (IMEX) time integration method is used. Furthermore, Monte Carlo simulations are carried out to investigate the uncertainties of parameters quantitatively.
As we expected, our model phenomenologically replicates cell migration behavior for 2D/ 3D cases in confined spaces, which is expected to predict the microenvironmental behavior of cells and the likelihood of cancer invasion.

Q(Telta)-Differential

Chou Hsin Chin
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Hypertranscendental q(telta)-differential is defined. Therefore the q(telta)-differential equations can be calculated.

A SIS Epidemic Reaction-Diffusion Model with Risk-Induced Dispersal

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Inkyung Ahn

This study examined a spatial SIS (susceptible-infected-susceptible) epidemic model where the dispersal of the infected individuals was non-linear. Spatial heterogeneity and dispersal of individuals are important factors that influence the persistence and eradication of an infectious disease. Based on spatial SIS reaction-diffusion models, diseases have been known to be eradicated if the mobility of the infected individuals is above a certain value. Additionally, an area is categorized as high- or low-risk if the infection rate average is higher or lower than the recovery rate average, respectively. In this study, we promoted a risk-induced dispersal (RID) method of the infected individuals i.e., the mobility of the infected individuals was high in high-risk areas and low in low-risk areas. We also examined the effect of RID on the spatial SIS model by defining the basic reproduction number (
$mathcal R_0$) in the spatial SIS reaction-diffusion model and investigating its stability i.e.,
$mathcal R_0 1$ represented an endemic state. By comparing the
$mathcal R_0$ of these models, RID of infected individuals proved to be a better strategy than random dispersal in obtaining a disease-free state.

Numerical Study of Hamilton-Jacobi-Bellman Equation for Time-Optimal Trajectory Generation of Dubins Vehicle

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Jing-Sin Liu

This article mainly applies Hamilton-Jacobi-Bellman (HJB) equation for collision-avoidance path planning for Dubins vehicles in the static obstacles workspace. The Dubins vehicles are simple robot cars which can only move ahead and with a given minimum turning radius bound.
In 1957, Dubins was a pioneer to investigate the time-minimal 2D path for such vehicles [1]. For the aspect of obstacle-avoidance task, there are many strategies (like APF, RRT, ...etc.) [10] to efficiently avoid obstacles in the workspace. However, it is not easy to consider both collision-avoidance task and time-optimal requirement for the Dubins vehicle at the same time. Takei and Tsai in [2] provide detailed discussions by using numerical methods for solving HJB equation for path planning.
Mathematically, HJB equation is a first-order non-linear PDE which does not typically exist a classical solution. However, Crandall and Lions in their famous papers [3, 4] provide the existence and uniqueness proof of viscosity solution to HJB equation and any monotone, differenced-form, consistent numerical Hamiltonian would lead the numerical solution converge to true viscosity solution. Godunov numerical Hamiltonian and semi-Lagrange scheme are two typically methods which satisfy the conditions of convergence [2, 5]. To implement those schemes, fast sweeping method is an efficient method for numerical computation and was first proposed by Zhao [6]. In this paper, we demonstrate Godunov numerical Hamiltonian and semi-Lagrange scheme with fast sweeping method for solving HJB equation. Unlike the HJB equation considered by Takei et. al. [2], we consider a slightly modified equation by Kruzkov transform to avoid unboundedness in the initial condition to assure the existence of viscosity solution and computational feasibility.
Once the solution is obtained, the characteristic line from any given initial pose gives the optimal control [9] and thus is the time-optimal collision-avoidance path, namely the Dubins path.
In this paper, we verify the accuracy of both numerical methods with analytical solution provided by Dubins [1].
Moreover, Shiller et. al. in [7] proposed an algorithm from the knowledge of the structures of shortest path for selecting a near-optimal path by the pseudo-return function. The main advantage of this algorithm is it is efficient in computation and lowly depends on the number of obstacles in the workspace. The paper also compares some features like the level sets of each value function, the path results and the total lengths of paths.

In conclusion, the paper presents the feasibility of Dubins path planning and collision-avoidance task through the HJB equation. Numerical methods are also applied for solving the HJB equation. From the simulations in the paper, the semi-Lagrange scheme performs better than Godunov scheme. It is because the semi-Lagrange scheme is more able to deal with discontinuous part of the solution [2]. Once, the numerical solution is obtained, the optimal control is given by the characteristic line equation. It would be used for optimal collision-free path planning. However, the future work of the paper is to apply or investigate the high order accuracy schemes such as limiters, WENO method for high order accurate solution and apply the results to the practical UAV (Unmanned Aerial Vehicle) flight experiments for 3D path planning.

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Disease Transmission Models with a Demographic Allee Effect

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Incorporating density-dependent population dynamics into epidemiological models allows for the modeling of disease dynamics over longer periods of time. The models presented in this talk compartmentalize a varying population with respect to disease transmission, and describe the total population size with both logistic growth and density-dependent facilitation, or the Allee effect. Conditions are established for the existence and local stability of the equilibria, the thresholds are analyzed, and bifurcation analyses are performed. Numerical simulations are presented to demonstrate theoretical results and further explore model behavior. The biological significance of the results are then explained in terms of both disease and population growth dynamics.

Modified Method of Fundamental Solutions for Heat Conduction Problems

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Surbhi Arora

In this study, we made an attempt to extend the modified method proposed by Rostamian and Shahrezaee, to a steady state anisotropic problem. The extension is applied to the two dimensional space and its performance is tested with the help of numerical experiments set on both convex and non-convex domains and the results obtained are analyzed. The modified technique overcomes the problem of fictitious boundary arising in standard MFS. The numerical experiments conducted, in combination with regularization, help us to establish the accuracy of the new scheme for both exact and noisy data.

Advanced Regression Models for Complex Regulatory Systems with Applications: CMARS Versus MARS

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Cankaya University, Turkey
Ayse Ozmen

Mathematical models are important not only to provide a better understanding for the underlying dynamics of huge systems, whose components are highly correlated, but also enables us to interpret their future behavior and compare with other possible situations. In this respect, there is a crucial demand for efficient new approaches to infer such complex phenomena which may appear in real-world problems of the fields like system biology, medicine, engineering, finance, education, environment and so on. In such systems, the components appear in a high dimensional network structure where they are interconnected with each other and under the effect of some unknown external parameters needed to be identified. In this study, we implement conic multivariate adaptive regression spline (CMARS) technique on a real-world data from system biology for the inference of such complex regulatory networks. The performance of the model is investigated in comparison with the results obtained from multivariate adaptive regression spline (MARS) modeling approach.

Space-Time Algebra for Multifluid Plasma Equations

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Murat Tanisli, Neslihan Sahin, M. Emre Kansu

The existence of correspondence between variables of electromagnetism and multifluid plasma permits to formulate a system of equations of multifluid plasma in a form analogous to electromagnetism governed by Maxwell equations. The mechanical forces subject to plasma are consistent with a fluid treatment. On the other hand, the spacetime algebra provides a practical and power tool for the deriving equations related to different areas of physics. In this work, stimulating from the theoretical analogy between the compressible fluid equations and multifluid plasma equations, the theorems and techniques of spacetime algebra are applied to the multifluid equations of a plasma in order to present a new formulation in a compact and elegant way. Moreover, the generalized plasma wave equation is presented.

Admissible Spaces for a First Order Equation with Delayed Argument

Lea Dorel
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N. Chernyavskaya, L. Shuster

We consider the equation

$$-y'(x) + q(x)y(x - \varphi(x)) = f(x), \quad x \in \mathbb{R}, \qquad (1)$$

where $f \in C(\mathbb{R})$ and

$$0 \le \varphi \in C^{\mathrm{loc}}(\mathbb{R}), \quad 1 \le q \in C^{\mathrm{loc}}(\mathbb{R}). \qquad (2)$$

Here $C^{\mathrm{loc}}(\mathbb{R})$ is the set of functions continuous in every point of the number axis $\mathbb{R}$. By a solution of (1) we mean any function y, continuously differentiable everywhere in $\mathbb{R}$, which satisfies the equation (1) for all $x \in \mathbb{R}$. We show that under certain conditions on the functions φ and q in addition to (2), the equation (1) has a unique solution y, satisfying the inequality

$$||y'||_{C(\mathbb{R})} + ||qy||_{C(\mathbb{R})} \le c||f||_{C(\mathbb{R})}$$

where the constant $c \in (0, \infty)$ does not depend on the choice of $f \in C(\mathbb{R})$.

Inverse Problems for Scalar Conservation Laws: a Bayesian Approach

Duc Lam Duong
University of Sussex, England
Masoumeh Dashti

We consider a Bayesian inverse problem for scalar conservation laws where one has to reconstruct initial data or flux function from measurements of solutions. Some questions related to how the shocks are travelling and how it effects our measuring will also be discussed.

Existence of a Solution to Singular Semilinear Polyharmonic Equation with Exponential Nonlinearity

Gaurav Dwivedi
BITS Pilani, India

In this work, we establish the existence of a solution to the following polyharmonic problem:

$$(-\Delta)^m u + V(x)u = \frac{g(x)f(u)}{|x|^a} + h(x) \quad \text{in } \mathbb{R}^{2m},$$

where $m \ge 2$, $a \in [0,\ 2m)$, $V, g : \mathbb{R}^{2m} \to \mathbb{R}$ are continuous, $0 \not\equiv h \in H^{-m}(\mathbb{R}^{2m})$ and f behaves like $\exp(\alpha s^2)$ as $|s| \to$
$infty$. We also assume that the potential V is bounded below. We employ variational arguments with a version of Adams inequality to prove the existence of a solution when the perturbation h is suitably small.

On the Dynamics of Domain Wall Motion in Ferromagnetic Heterostructures Under the Influence of Interfacial Dzyaloshinskii-Moriya Interaction

Sharad Dwivedi
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This work deals with the investigation of current-induced domain wall motion in ferromagnetic heterostructure in the steady-state and precessional dynamic regimes. To be precise, we consider the one-dimensional model of the generalized Landau-Lifshitz-Gilbert equation of micromagnetism that includes the effects of nonlinear dissipations, Rashba field, Spin-Hall effect (SHE), and interfacial Dzyaloshinskii-Moriya interaction (DMI). The derived results describe the domain wall velocity, threshold and Walker breakdown value of spin-polarized electric current in the steady-state regime, and the average domain wall velocity in the precessional dynamic regime. Finally, the interplay and effect of various parameters on the domain wall motion have been discussed analytically and numerically in both the dynamical regimes.

On Methods for Asymptotic Analysis of Probabilistic Cellular Automata

Kazushige Endo
Waseda University, Japan

When we analyze asymptotic behavior of probabilistic cellular automata, we have an assumption called reduction relation which reduce occurrence probability of long length patterns into that of short length patterns. Utilizing the reduction relation and equilibrium equations of the occurrence probability of patterns, we can derive a fundamental diagram which shows relationship between mean flux and density of conserved quantities as asymptotic behavior of probabilistic cellular automata. However, we only confirm existence of the reduction relation with numerical experiments. In the talk, we discuss existence of the reduction relation mathematically and introduce a method for asymptotic analysis without the relation utilizing methods for stochastic processes such as transition probability matrix.

Vector-Borne Disease Control: Dynamic Modeling of Personal Protection Strategies Limits the Diversity Amplification Effect

Bill Fagan
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Jeff Demers, Sharon Bewick, Justin Calabrese

Personal protection measures, such as bed nets and repellents, are important tools for the suppression of vector-borne diseases like malaria and Zika. The ability of health agencies to distribute personal protection equipment, and encourage its use, plays an important role in the efficacy of disease management strategies. Recent modeling studies have discovered that a counterintuitive diversity-driven amplification in community-wide disease levels can occur when a population only partially adopts personal protection measures. In this way, partial adoption becomes detrimental to the larger goal of disease management. This amplification effect, however, may overestimate the negative impact of personal protection as a result of implicit, restrictive model assumptions regarding population compliance, access to personal protection measures, and the longevity of those measures. We establish a new modeling methodology for exploring control efforts in vector-borne disease systems featuring personal protection. This new method flexibly accounts for compliance, access, longevity, and control strategies by way of a flow between protected and unprotected populations rather than breaking the population into static protected and unprotected classes as has been done previously. Our new methodology yields large reductions in the severity and occurrence of amplification effects as compared to existing models.

Global Limit Cycle Bifurcations of Multi-Parameter Dynamical Systems

Valery Gaiko
National Academy of Sciences of Belarus, Belarus

We carry out the global bifurcation analysis of multi-parameter dynamical systems. To control all their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all their rotation parameters. It can be done by means of the development of new bifurcational geometric methods based on Erugin's global qualitative investigation approach and Perko's planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity). Applying these methods, we study global limit cycle bifurcations for the Holling-type quartic, Kukles cubic-linear and Lorenz-type quadratic dynamical systems.

Non-Existence of Stable Solutions for Weighted P-Laplace Equation

Prashanta Garain
Indian Institute of Technology Kanpur, India
Kaushik Bal

We provide sufficient conditions on $w \in L^1_{loc}(\mathbb{R}^N)$ such that the weighted p-Laplace equation

$$-\operatorname{div}\left(w(x)|\nabla u|^{p-2}\nabla u\right) = f(u) \text{ in } \mathbb{R}^N$$

does not admit any stable $C^{1,\zeta}_{loc}$ solution in $\mathbb{R}^N$ where $f(x)$ is either $-x^{-\delta}$ or e^x.

A One-Dimensional Model of a False Aneurysm

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Fredrik Berntsson, Matts Karlsson, Vladimir Kozlov, Sergey A. Nazarov

A false aneurysm is a hematoma, i.e. a collection of blood outside of a blood vessel that forms due to a hole in the wall of an artery. This represents a serious medical condition that needs to be monitored and, under certain conditions, treated urgently. In this work, a new one-dimensional model of a false aneurysm is proposed. The new model is based on a one-dimensional model of a curvilinear artery with varying width of the channel, previously presented by the authors and it takes into account the interaction between the hematoma and the surrounding muscle material as well. The model equations are derived using asymptotic analysis for the case of a simplified geometry. Even though the model is simple, it still supports a realistic behaviour for the system consisting of the vessel and the hematoma. Using numerical simulations we illustrate the behaviour of the model. The simulations show that our model can reproduce realistic solutions.

Lorenz-Like Attractors in a Nonholonomic Model of Celtic Stone

Aleksandr Gonchenko
Lobachevsky State University of Nizhni Novgorod, Russia

We consider nonholonomic models of Celtic stone moving along the plane. We study regular and chaotic dynamics in these models. In particular, we show that chaotic dynamics is very diverse here: it can be representes not only by strange attractors (spiral, torus chaos ones) but also the mixed dynamics and nearly conservative chaos . Moreover, for certain types of Celtic stone (possessing certain geometrical and physical properties) strange Lorenz-like attractors were found in their nonholonomic models. In this talk we observe the corresponding results related to scenarios of appearance and break-down of such strange attractors.

Approximate Controllability of a Nonlocal Quasilinear Differential Equations with Repeated Deviating Arguments

Rajib Haloi
Tezpur University India, India
Duranta Chutia

In this article, we prove the sufficient conditions for the approximate controllability of a nonlocal quasilinear differential equations with repeated deviating arguments in a Hilbert space. The existence of mild solution is established by the Sobolevsk
ui-Tanabe theory of parabolic equations, fractional powers of op- erators. We show the approximate controllability of the equation. An example is also discussed for the established results.

Nonexistence of Global Solutions to Some Quasilinear Wave Equations in Three Space Dimensions

Wei Han
North University of China, Peoples Rep of China

F. John studied the nonlinear wave equations in three space dimensions with the small initial values of a small parameter
varepsilon > 0, and he obtained the blow-up result of this problem and also got the precise estimates for the lifespan(the maximal existence time) of the solution to the problem. In this talk, we will extend John's result to the general kind of quasilinear wave equations. We will prove that for the compactly supported smooth initial values, the solution must blow up in finite time if the initial data are nonnegative and positive somewhere no matter how small the initial data are. and also we give the sharp lifespan estimate of solutions for the problem.

Multiple Positive Solutions for Semilinear Elliptic Problems Involving Two Critical Hardy-Sobolev Exponents and a Concave Term

Tsing-San Hsu
Chang Gung University, Taiwan
Huei-Li Lin

In this paper, we study a semilinear elliptic equation, which involves doubly critical Hardy-Sobolev exponents, a Hardy-type term and a concave term. By means of variational methods, the multiplicity of positive solutions to the problem is obtained.

A Direct Quadrature Method for Auto-Convolution Volterra Integral Equations

Chengming Huang
Huazhong University of Science and Technology, Peoples Rep of China
Min Li

In this talk, we are concerned with the numerical solution of auto-convolution Volterra integral equations. A composite quadrature method based on linear barycentric rational interpolation is introduced. The method is easy to be implemented because only a linear equation needs to be solved in each time step. Collocation method is used as the starting procedure. The boundedness and convergence of the numerical solution are studied in detail. Some numerical experiments are carried out to confirm the theoretical results.

Recent Progress on Schrödinger-Poisson System

Lirong Huang
Fujian Jiangxia University, Peoples Rep of China
Jianqing Chen, Eugenio Rocha

In this paper, we consider the existence of ground state and bound states as well as their bifurcation properties of the Schrödinger-Poisson system $-\Delta u+u+\phi u=|u|^4u+\mu h(x)u$ in $\mathbb{R}^3$ and $-\Delta\phi=u^2$ in $\mathbb{R}^3$. Under suitable conditions on the function $h(x)$, we prove the existence of one ground state which must bifurcate from zero and the existence of another bound state which can not bifurcate from zero.

Stability of Travelling Waves for Stochastic Reaction-Diffusion Equations

Hermen Jan Hupkes
Leiden University, Netherlands
C.H.S. Hamster

We consider reaction-diffusion equations that are stochastically forced by a small multiplicative noise term. We show that spectrally stable travelling wave solutions to the deterministic system retain their orbital stability if the amplitude of the noise is sufficiently small.
By applying a stochastic phase-shift together with a time-transform, we obtain a semilinear sPDE that describes the fluctuations from the primary wave. We subsequently develop a semigroup approach to handle the nonlinear stability question in a fashion that is closely related to modern deterministic methods.

Existence of Weak Solutions to the Time-Fractional Attraction-Repulsion Tumor Invasion System

Manimaran Jeyaraj
NIT Goa, India
Shangerganesh L

Alzheimer's disease is a degenerate disease of the brain which robs the minds of our old population. Mathematical model of Alzheimer's disease introduced by Luca et.al (2003) and long time behavior of the solutions studied by Li et.al (2018). In this work, we consider the mathematical model with time-fractional derivative, which consists of three coupled system of nonlinear reaction-diffusion equations. Our main work is to establish the existence and uniqueness of the proposed mathematical model. The model can be described by the following mathematical equations representing of concentrations of Microglia u, chemoattractant v and chemoerpellent w.

$$\begin{cases} \partial_t^\alpha u=\nabla\cdot\ (A_1(x,t,\nabla u))-\nabla\cdot(\chi u\nabla v)+\nabla\cdot(\xi u\nabla w), & \text{in } Q_T, \\ \partial_t^\alpha v=\nabla\cdot\ (A_2(x,t,\nabla v))+au-bv, & \text{in } Q_T, \\ \partial_t^\alpha w=\nabla\cdot\ (A_3(x,t,\nabla w))+cu-dw, & \text{in } Q_T, \\ \dfrac{\partial u}{\partial\eta}=\dfrac{\partial v}{\partial\eta}=\dfrac{\partial w}{\partial\eta}=0, & \text{on } \Sigma_T, \\ u(x,0)=u_0(x),\ v(x,0)=v_0(x),\ w(x,0)=w_0(x), & \text{in } \Omega \end{cases}$$

where $Q_T=\Omega\times(0,T),\ \Sigma_T=\partial\Omega\times(0,T),\ \Omega$ is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$ and let $T>0$. The parameter χ and ξ are chemoattractant coefficient which are positive and a,b,c and d are chemical proliferation and degradation rates. Further $A_i,\ i=1,2,3$ are density dependent diffusion coefficients. Furthermore, here we used the fractional derivative in the sense of Caputo. We prove the existence and uniqueness of solutions using the Faedo-Galerkin approximation method and some compactness arguments.

Analysis of Some High-Order Compact Scheme on a Non-Uniform Mesh Network for One-Dimension Singularly Perturbed Elliptic Problems

Navnit Jha
South Asian University, India

A family of fourth and sixth-order compact difference scheme for solving one-dimension nonlinear singularly perturbed elliptic problems has been described. The proposed scheme is developed on non-uniformly spaced mesh-points and renders same order of accuracies both in uniform meshes and non-uniform meshes. The three-point finite difference discretization results in a tridiagonal jacobian matrix and can be computed by means of Thomas algorithm. Numerical simulations with problems possessing boundary layer behavior have been presented to illustrate order and accuracies of the new scheme.

Approximate Controllability of Semilinear Nonlocal Fractional Differential Systems Via an Approximating Method

Shaochun Ji
Huaiyin Institute of Technology, Peoples Rep of China

We study the control system governed by a class of abstract nonlocal fractional differential equations. By using the fractional calculus and approximating technique, we give the approximate problem of the control system and get the compactness of approximate solutions set. Then new sufficient conditions for the approximate controllability of the control system are obtained when the compactness conditions or Lipschitz conditions for the nonlocal function are not required.

The Strong Average Shadowing Property for C^1- Generic Diffeomorphisms

Bowon Kang
Chungnam National University, Korea
Namjip Koo, Manseob Lee

K. Sakai (2000) proved that the C^1 interior of the set of all diffeomorphisms satisfying the average shadowing property on a two dimensional closed manifold is characterized as the set of all Anosov diffeomorphisms. In this talk we introduce the notion of strong average shadowing property(SASP) of diffeomorphisms on n-dimensional manifolds. Then we will discuss some relations between SASP and hyperbolicity of C^1-generic diffeomorphism in the direction of an extension of the Sakai's result.

Constrained Data Visualization Using A-Fractal Rational Function

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This article is devoted to the development of a constructive approach to constrained interpolation problems from hidden variable fractal interpolation function (HFIF). We identify a family of $\mathbb{R}^2$-valued continuous fractal functions with a prescribed continuous function $\mathbf{f} : I \to \mathbb{R}^2$ using a finite sequence of base functions instead of a single base function, where $\mathbb{R}^2$ is endowed with the l^1-norm (taxicab norm). As an application of the developed theory, we construct an A-fractal function $\mathbf{f}[\mathbf{A}]$ by a fractal perturbation of a traditional rational cubic trigonometric interpolant and investigate its shape preserving aspects. In particular, we obtain $\mathcal{C}^1$-rational cubic trigonometric spline HFIF (RCTSHFIF) for shape preserving univariate interpolation involving shape parameters for the first time in the literature. The shape preserving interpolation scheme developed herein generalizes and enriches its traditional nonrecursive counterpart and its fractal extension. These interpolants can reflect the nature of self-referentiality or non-self-referentiality of the original data defining function depending on the values of the parameters. With suitably chosen numerical examples, the effectiveness of the shape preserving interpolation scheme are illustrated.

Periodic Orbits in Nonlinear Wave Equations on Networks

Imene Khames
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J.G. Caputo, A. Knippel, P. Panayotaros

We consider a discrete nonlinear wave equation in an arbitrary finite graph. It is the discrete ϕ^4 equation used to model coupled electromechnical oscillators. We show that inspecting the normal modes of the graph Laplacian, we can immediately identify which ones can be extended into nonlinear periodic orbits (generalizing work of Aoki, 2016). We first define monovalent, bivalent and trivalent nonlinear periodic orbits depending whether the components of the corresponding eigenvectors of the graph Laplacian are in $\{+1\}$, $\{-1, +1\}$ or $\{-1, 0, +1\}$. Then, we perform a systematic linear stability (Floquet) analysis of these orbits. In particular, the linearized equations are decoupled for normal modes associated to eigenvectors without 0 (called soft nodes), these modes are the monovalent (Goldstone) and the bivalent orbits. We find that for chains the Goldstone mode is stable for a wide range of parameters while the bivalent mode is unstable. Nevertheless, the stability analysis for modes with soft nodes is more complicated since the linearized equations are coupled. Numerical results of some graphs show that trivalent periodic orbits that continue nondegenerate linear modes are unstable below an amplitude threshold; orbits continued from modes with frequency degeneracy are unstable.

Bio-Convective Nanofluid Flow Along a Horizontal Wavy Surface in Porous Media

Melusi Khumalo
University of South Africa, So Africa
F. Awad, M.P. Mkhatshwa

There are few theoretical studies on convective transport in nanofluids along wavy surfaces. This paper analyzes bio-convective flow of a nanofluid containing gyrotactic microorganisms over a horizontal wavy surface in a porous medium. The model for the nanofluid transport equations incorporates the effects of Brownian motion and thermophoresis. The wavy surface and the flow governing equations are first transformed into plane geometry case using suitable transformations. The governing partial differential equations are reduced into a set of coupled ordinary differential equations which are then solved numerically using the spectral relaxation method (SRM). The accuracy of the solutions has been determined by benchmarking the results against the spectral quasi-

linearization method (SQLM). The effects of the governing parameters on the rate of heat, nanoparticle volume fraction and motile microorganisms transfer have been studied. Results indicate that the local Sherwood number and density number of motile microorganisms are increased as the Brownian motion parameter is increased, while the local Nusselt number is reduced. Moreover, the Nusselt number decreases as the thermophoresis parameter increases.

The Analysis of Time-Delay Reservoir Computing Focusing on Dynamical Properties of Delay Differential Equations

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The University of Tokyo, Japan
Akihiko Akao, Sho Shirasaka, Kiyoshi Kotani, Yasuhiko Jimbo

Reservoir Computing (RC) is a machine-learning paradigm that is capable to process empirical time-series data. This paradigm is based on a neural network with a fixed hidden layer owning a high-dimensional state space, called a reservoir. Reservoirs including time-delays are considered to be good candidates for practical applications because they make hardware realization of the high-dimensional reservoirs simple. Performance of the well-trained RCs depends both on dynamical properties of attractors of the reservoirs and tasks they solve. Therefore, in the conventional RCs, there arise task-wise optimization problems of the reservoirs, which have been solved based on trial and error approaches. This means that it is necessary to switch reservoirs for the optimization of the performance for each task. In this study, we compared and investigated the time-delay RCs method for various tasks, focusing on the dynamical properties of reservoirs and using multiple time-delay nodes. Specifically, the performance in various tasks was measured using reservoirs having different dynamic characteristics arranged in parallel. These findings pave the way to a more versatile and robust time-delay RCs.

Stability Properties for Impulsive Differential Equations of Fractional Order

Namjip Koo
Chungnam National University, Korea
Bowon Kang

In this talk we present some explicit solutions for impulsive linear fractional differential equations with impulses at fixed times, which provide a tool in deriving singular integral-sum inequalities. Then we discuss some results about the Mittag-Leffler stability for impulsive fractional differential equations by using impulsive fractional comparison principle and piecewise continuous functions of Lyapunov's method.

Deceleration of Mixing Rates in Discontinuous, One-Dimensional Mappings

Hannah Kreczak
University of Leeds, England
Rob Sturman, Mark C.T. Wilson

We present a study on the evolution of a passive scalar towards the uniform equilibrium distribution in an advection-diffusion problem where the advective transport is a composition of stretching and folding and cutting and shuffling. The linearity of the advection-diffusion problem means that the long-time evolution can be characterized by the eigenvalues and eigenmodes of the transfer operator. The rate of approach to equilibrium is shown to be dependent on the choice of permutation and changes non-monotonically with the diffusion coefficient for many of the permutations. This counter-intuitively predicts a deceleration in the asymptotic mixing rate with an increasing diffusivity rate. The global mixing rate in the limit of vanishing diffusivity approximates well the decay rates of the eigenmodes for small diffusivity, however this global mixing rate does not bound the rates for all values of the diffusion coefficient.

Mathematical Modeling of Coupled Transport Phenomena in Fractured Rock Masses

Lukas Krupicka
Czech Technical University, Czech Rep

This contribution deals with a system of nonlinear differential equations arising from the coupled water and heat flow through a partially saturated porous media. The existence of a global weak solution of the problem on an arbitrary interval of time is proved by means of discretization in time, deriving suitable a-priori estimates and concluding that the solutions of steady problems converge to the solution of the original problem.

A Study of Chaotic Behavior of Logistic Map in Mann Orbit and an Improved Chaos-Based Traffic Control Model

Ashish Kumar
Southeast University, Peoples Rep of China
Jinde Cao, Renu Chugh

With a vital role of discrete chaos, standard logistic map has found a celebrated place in the dynamics of chaos theory and in various applications of science, such as a discrete traffic flow model, image encryption in cryptography, secure communication, and weather forecasting. Traditionally, this discrete chaos is controlled by one parameter λ using Picard orbit, a one-step feedback procedure. This article presents a one-step forward, applying Mann

orbit (superior orbit) the chaotic properties such as period-doubling, period-3 window, and Lyapunov exponent of the standard logistic map are investigated. The results are illustrated analytically and experimentally followed by concluding remarks and a few counter examples. Due to the extra degree of freedom in parameter λ, the map provides improved chaotic properties that increase the performance of dynamical phenomena. Moreover, this study describes an improved chaos-based discrete traffic control model. Surprisingly, added new parameter α in Mann orbit works as control variable that increase the stability performance of the traffic model.

Global Linearization and Fiber Bundle Structure of Invariant Manifolds

Matthew Kvalheim
University of Michigan, USA
Jaap Eldering, Shai Revzen

We study properties of the global (center-)stable manifold of a compact normally attracting invariant manifold (NAIM) for a flow, the special case of a normally hyperbolic invariant manifold (NHIM) with empty unstable bundle. As a slight generalization, we allow the NAIM to be inflowing invariant, which means that the boundary of the NAIM might be nonempty, but the vector field points inwards there. We extend a classical result of Pugh and Shub for the boundaryless case, showing that the flow near an inflowing NAIM is "linearizable" or topologically conjugate to its linearization at the NAIM. Moreover, we give conditions ensuring the existence of a C^k conjugacy. We also show that a C^k locally linearizable inflowing NAIM is automatically globally C^k linearizable, with the conjugacy defined on the entire global (center)-stable manifold. We finally show that under weaker k-center bunching conditions, the global stable foliation has a C^k disk bundle structure, which can be interpreted as a weak form of our global linearization results. We apply our results to geometric singular perturbation theory by extending the domain of the Fenichel Normal Form to the entire global stable manifold, and under additional nonresonance assumptions we derive a smooth global linear normal form.

Generation Mechanism of Rogue Waves for the Discrete Nonlinear Schrödinger Equation

Min Li
North China Electric Power University, Peoples Rep of China
Tao Xu

In this poster, we study the generation mechanism of rogue waves for the discrete nonlinear Schrödinger (DNLS) equation from the viewpoint of structural discontinuities. By the Darboux transformation (DT), we derive the analytical breather solutions of the DNLS equation on a plane-wave background. Then, based on the explicit expressions of group and phase velocities, we give the parameteric conditions for the occurence of velocity jumps, which are exactly the conditions for deriving the rogue-wave solution via the generalized DT. Furthermore, we verify that the first-order rogue wave can be obtained from the breather solutions at the velocity-jumping point. This work might be helpful for understanding the generation of rogue waves in the DNLS model.

Some Advances in Study of Heat Balance Integral Method

Feng Ling
Zhaoqing University, Peoples Rep of China

The heat balance integral method (HBIM) originally proposed by Goodman is an effective approach of approximate analytical solutions for the diffusion equations. HBIM has fascinated many researchers in the past a couple of decades since it can be applied with relatively ease to heat conduction problems with phase change. The current work addresses some advances in development of HBIM for a variety of phase change problems, including refined integral method, alternative refined integral method, combined integral method, convergence of the refined solution, the general method for choice of the approximating function, and the improvement of the accuracy. Some challenges related to the application of the developed approximate methods were also presented.

Pricing Options with Heston Stochastic Volatility and Time-Dependent Parameters — Lie-Algebraic Approach

Chi Hei Christopher Liu
The Chinese University of Hong Kong, Hong Kong

Pricing options with Heston stochastic volatility is a challenging task because in Heston's stochastic volatility framework semi-analytical formulae for plain vanilla option prices can be derived but it is unfortunate that these formulae require the evaluation of logarithms with complex arguments during the involved inverse Fourier Tranform. Thus, most implementation of Heston's formulae suffers from an inherent numerical instability and are not robust for moderate to long dated maturities or strong mean reversion. Besides, if the model parameters are not constant or piecewise constant, no semi-analytical pricing formula is known.
By exploiting the Lie symmetry of the pricing partial differential equation and applying the Wei-Norman theorem, we have succeeded to derive an accurate approximate analytical formula for pricing a vanilla call option with Heston stochastic volatility and time-dependent model parameters. In addition, the accuracy can be further improved in a systematic manner. Furthermore, the proposed approach can be easily extended to pricing constant-elasticity-of-variance options with Heston stochastic volatility and time-dependent parameters.

A Remark on the Existence of a Class of Stretched Magnetohydrodynamics Flow with Infinite Energy

Bataa Lkhagvasuren
Chonnam National University, Korea
Minkyu Kwak

We consider the existence of a class of special stretched solutions of $2D$ and $3D$ Magnetohydrodynamics equations in $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. The last components of the state variables have the form $u_n = x_n\gamma_1 + \varphi_1$ and $b_n = x_n\gamma_2 + \varphi_2$ and the functions $\gamma_1, \gamma_2, \varphi_1, \varphi_2$ and the first components of state variables depend on $x_1, ..., x_{n-1}$ and t for $n = 2, 3$. We investigate how the global solvability of the problem depends on the dimension.

Existence, Uniqueness and the Principal Eigenvalue for a Class of Quasilinear Singular Problems

Salvador Lopez Martinez
University of Granada, Spain
Jose Carmona, Tommaso Leonori, Pedro J. Martinez-Aparicio

Consider the family of singular elliptic problems

$$\begin{cases} -\Delta u = \lambda u + \mu(x)\dfrac{|\nabla u|^q}{u^{q-1}} + f(x) & \text{in } \Omega \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \qquad (P_\lambda)$$

in a bounded domain $\Omega \subset \mathbb{R}^N$ with smooth boundary, where $0 \le \mu \in L^\infty(\Omega)$, $0 \le f \in L^p(\Omega)$ with $p > \frac{N}{2}$, and 10, which is the *principal eigenvalue*, and the solution to (P_{λ^*}) is unique up to multiplication by positive constants. If $\inf_\Omega(f) > 0$, the set is $(-\infty, \lambda^*)$, and the solution to (P_λ) is unique. Finally, if $f \gneqq 0$ we prove existence for $\lambda\lambda^*$.
Our results improve some contained in [1], where only the case $q = 2$ and $\mu \equiv cst$ is considered. The proofs are valid for purely quasilinear problems, in the sense that we avoid the Cole-Hopf change of variables. Our approach is based on a comparison principle that we also prove, an appropriate characterization of λ^* and an approximation and compactness argument.

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Homgenization of Time Dependent Random Flow

Junlong Lyu
The University of Hong Kong, Hong Kong
Zhiwen Zhang

We consider the problem of computing the effective diffusivity of the passive tracer model when the velocity field is a time dependent incompressible random field. We also investigate the existence of the residual diffusivity. Namely we want to find the limit situation when the molecular diffusivity is small, and the flow are dominated by the velocity field. We design efficient numerical integrators to solve the target model problems and obtain some numerical results that show the existence of the convection enhanced behaviours for the flow.

Some Properties of a Differential Sequence Based Upon the Kummer-Schwarz Equation

Adhir Maharaj
Durban University of Technology, So Africa
Andriopoulos & Leach PGL

A recursion operator for the Kummer-Schwarz equation is determined according to the standard definition introduced by Olver. This leads to a sequence with properties less inviting than of a different sequence constructed by a particular generator as was the case for the Riccati equation . We examine the elements of this sequence in terms of the usual properties to be investigated – symmetries, singularity properties, integrability, alternate sequence.

On the Regularity of Minimizers of Convex Variational Problems

Erika Maringova
Charles University, Czech Rep
Lisa Beck, Miroslav Bulíček, Bianca Stroffolini, Anna Verde

We study the existence and regularity of a minimizer to a wide class of convex, variational integrals having radial structure. For the linear growth setting, we sharply identify the class of problems which admit globally Lipschitz solution. Further, similar method is applied for superlinear growth. On the other hand, for a general BV minimizer, we study the regularity structure of its absolutely continuous and singular part. The result is not restricted to any geometrical assumption on the domain.

Bifurcation and Stability Analysis of a Holling Type II Predator-Prey Model with Omnivores

Sambath Muniyagounder
Periyar University, India
C. Gokila

This paper consider a predator prey model with omnivore population and Holling type II response. First, we have studied the boundedness of the system. The local and global stability of the equilibrium is investigated by analyzing the eigenvalues and constructing the appropriate Lyapunov functions respectively. The persistence of positive equilibrium is also discussed. The existence of Hopf bifurcation is investigated by analyzing the distribution of eigenvalues at the positive equilibrium point. By using the normal form theory and explicit formula which determine the direction of bifurcating periodic solutions are derived. Some numerical simulations are carrying out, to check our theoretical results.

Complex and Quaternion Maps of Blaschke Products

David Ni
Direxion Technolgy, Taiwan

Blaschke Mapping is motivated by W. Blaschke, who published an article entitled as KINEMATICS AND QUATERNIONS in 1958. In this presentation, we study the complex and quaternion mapping, which as W. Blaschke indicated that the mathematical efforts will flourish the development of geometry of motion or kinematics in physics with the use of the quaternions that L. EULER introduced to that end in 1748 and coined by W.R. Hamilton in 1843. Blaschke products is a mathematical generalization of Lorentz formalism, and hereby we define the functional sets in momentum space. The applications of this effort can be in various scientific fields, such as astronomy, high energy physics, as well as physical statistics.
The set defined in unit cubic will form separated subsets in crystal-like structures after given steps of functional iterations. In the linear regions, defined as functional order of 1 and 2, we observed that a set in spherical-shaped structure transforms into subsets in a simple-cubic-crystal structure, while for functional order of 3 and greater, more complicated transformations and structures exist depending on the parameter space. We finally compare the analog and difference of complex mapping versus quaternion mapping in conjunction with the Blaschke products.

Averaged Fractional Control

Andrej Novak
University of Zagreb, Faculty of Science, Croatia
D. Mitrovic, T. Uzunovic

In this talk, we generalize results concerning averaged controllability on fractional type equations: system of fractional ODE-s and the fractional diffusion equation.

Backward Bifurcation in Measles Model with Logistic Growth and Vaccination

Kadkanok Nudee
King Mongkut's University of Technology Thonburi, Thailand
Wirawan Chinviriyasit

In this paper, we formulate a measles model with logistic growth and vaccination for studying the phenomenon of backward bifurcation that would impact to measles transmission in Thailand. The proposed model has a locally asymptotically stable disease-free equilibrium whenever a certain epidemiological threshold, known as the basic reproduction number. It is found that the model exhibits the phenomenon of backward bifurcation where stable disease-free equilibrium of the model co-exists with a stable endemic equilibrium when the reproduction number is less than unity. The epidemiological consequence of this phenomenon is that the classical epidemiological requirement of making the reproduction number less than unity is no longer sufficient, although necessary, for effectively controlling the spread of measles in a community. Our results suggest that a very effective method to prevent the backward scenario is to concern the vaccine wane, controlling the efficacy of vaccines and to concern the optimal timing for the administration of vaccine, which covers the duration of vaccine-induced immunity to measles. Therefore, the results from this study suggest the critical information for decision makers and public health officials, who may have to deal with the prevention and control of an epidemic.

On Weak Solvability of Fractional Models of Viscoelastic Fluid

Vladimir Orlov
Voronezh State University, Russia

The mathematical models of dynamics of viscoelastic fluids with constitutive relations containing fractional derivatives are under consideration. Fractional analogous of Voigt and anti-Zener models are under investigation. We establish the existence of weak solutions of the corresponding initial-boundary value problems. In the planar case the uniqueness of weak solutions is proved. For the proofs of the main results we approximate the problems under consideration by a sequence of regularized systems of Navier-Stokes type. The solvability of regularized systems and apriori estimates of their solutions allow to pass to the limit in the regularized systems and obtain the solvability of original problems. The theory of fractional powers of positive operators, fractional calculus and classical results on Navier-Stokes equations are used. This is a joint work with Victor Zvyagin.

From Kinetic Theory of Multicellular Systems to Hyperbolic Tissue Equations: Asymptotic Limits and Computing

Nisrine Outada
UCA University, Morocco
Nicolas Vauchelet, Thami Akrid, Mohamed Khaladi

This work deals with the analysis of the asymptotic limit toward the derivation of macroscopic equations for a class of equations modeling complex multicellular systems by methods of the kinetic theory. After having chosen an appropriate scaling of time and space, a Chapman-Enskog expansion is combined with a closed, by minimization, technique to derive hyperbolic models at the macroscopic level. The resulting macroscopic equations show how the macroscopic tissue behavior can be described by hyperbolic systems which seem the most natural in this context. We propose also an asymptotic-preserving well-balanced scheme for the one-dimensional hyperbolic model, in the two dimensional case, we consider a time splitting method between the conservative part and the source term where the conservative equation is approximated by the Lax-Friedrichs scheme.

On Damped Navier-Stokes Equation with Slip Boundary Conditions

Subha Pal
Tezpur University India, India
Rajib Haloi

In this article, we prove the solution to the damped Navier–Stokes equation with Navier boundary condition in a bounded domain Ω in $\mathbb{R}^3$ with smooth boundary. The existence of the solution is global with the damped term $\vartheta|u|^{\beta-1}u, \vartheta > 0$. The uniqueness of solutions with Navier boundary condition is also proved. This extends existing results in literature.

Locomotion of a Single-Flagellated Bacterium

Yunyoung Park
Chung-Ang University, Korea
Yongsam Kim, Sookkyung Lim

Single-flagellated bacteria propel themselves by rotating a flagellar motor, translating rotation to the filament through a compliant hook, and subsequently driving the rotation of the flagellum. The flagellar motor alternates the direction of rotation between counterclockwise and clockwise, and this leads to the forward and backward directed swimming. Such bacteria can change the course of swimming as the hook experiences its buckling caused by the change of bending rigidity. In this paper, we present a comprehensive model of a monotrichous bacterium as a free swimmer in a viscous fluid. We describe a cell body as a rigid body using the penalty method and a flagellum as an elastic rod using the Kirchhoff rod theory. The hydrodynamic interaction of the bacterium is described by the regularized Stokes formulation. Our model of a single-flagellated micro-organism is able to mimic the swimming pattern that is well matched with the experimental observation. Furthermore, we find the critical thresholds of the rotational frequency of the motor and the bending modulus of the hook for the buckling instability, and investigate the dependence of the buckling angle and the reorientation of the swimming cell after buckling on the physical and geometrical parameters of the model.

Global Mild Solutions for a Nonautonomous/Impulsive 2D Navier-Stokes Equations

Ricardo Parreira da Silva
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E.M. Bonotto, J.G. Mesquita

The present paper deals with existence and uniqueness of global mild solutions for a 2D Navier-Stokes equations with impulses effects. Using the framework of nonautonomous dynamical systems, we extend previous results considering the 2D Navier-Stokes equations with impulse effects and allowing that the nonlinear terms are explicitly time-dependent. Additionally, we present suffcient conditions to obtain dissipativity (boundedness) for solutions starting in bounded sets.

Predicting Rift Valley Fever Inter-Epidemic Activities and Outbreak Patterns: Insights from a Stochastic Host-Vector Model

Sansao Pedro
Universidade Eduardo Mondlane, Mozambique
Shirley Abelman, Henri E.Z. Tonnang

Rift Valley fever (RVF) outbreaks are recurrent, occurring at irregular intervals of up to 15 years at least in East Africa. Between outbreaks disease inter-epidemic activities exist and occur at low levels and are maintained by female Aedes mcintoshi mosquitoes which transmit the virus to their eggs leading to disease persistence during unfavourable seasons. Here we formulate and analyse a full stochastic host-vector model with two routes of transmission: vertical and horizontal. By applying branching process theory we establish novel relationships between the basic reproduction number, R_0, vertical transmission and the invasion and extinction probabilities. Optimum climatic conditions and presence of mosquitoes have not fully explained the irregular oscillatory behaviour of RVF outbreaks. Using our model without seasonality and applying

van Kampen system-size expansion techniques, we provide an analytical expression for the spectrum of stochastic fluctuations, revealing how outbreaks multi-year periodicity varies with the vertical transmission. Our theory predicts complex fluctuations with a dominant period of 1 to 10 years which essentially depends on the efficiency of vertical transmission. Our predictions are then compared to temporal patterns of disease outbreaks in Tanzania, Kenya and South Africa. Our analyses show that interaction between nonlinearity, stochasticity and vertical transmission provides a simple but plausible explanation for the irregular oscillatory nature of RVF outbreaks. Therefore, we argue that while rainfall might be the major determinant for the onset and switch-off of an outbreak, the occurrence of a particular outbreak is also a result of a build up phenomena that is correlated to vertical transmission efficiency.

Nonautonomous Period Doubling Border Collision Bifurcation

Volker Reitmann
St. Petersburg State University, Russia
Anastasia Maltseva

In our talk we consider nonautonomous difference equations with piecewise smooth right part and with deterministic and random forcing. The cardiac conduction system is an important source of such equations. For such nonautonomous difference equations with deterministic or random forcing we construct parameter-dependent cocycles. The existence of a period doubling border collision bifurcation (deterministic and random) is shown for such systems.

The Global Attractor of a Multivalued Dynamical System Generated by a Two-Phase Heating Problem

Volker Reitmann
St. Petersburg State University, Russia
Dmitrii Zyryanov

In this talk we study the asymptotic behavior of solutions of Maxwells equations coupled with the heat equation for a phase change problem in 3-dimensional space arising in a microwave heating process. The two-phase changing process is accounted with the enthalpy operator.
We consider an initial boundary value problem for this process. The uniqueness of weak solutions is not proved. In the talk it will be shown how to introduce a multivalued dynamical system based on weak solutions of this system. The existence of a global attractor for this multivalued dynamical system will be shown. Some numerical calculations for the temperature behavior are carried out.

The Properties of the Solutions for the Incompressible Flows on an Exterior Domain

Jaiok Roh
Hallym University, Korea

In this talk, we want to see the properties of the smooth solutions $\mathbf{u}$ of the incompressible flows on an exterior domain Ω of R^2. Specially, when the vorticity $\omega = \nabla \times \mathbf{u}$ has a bounded support, with suitable conditions we will show that there exists a constant $C(p,q)$ such that

$$\| \mathbf{u} \|_{L^p(\Omega)} \leq \; C \, \| \mathbf{u} \|_{L^q(\Omega)}$$

for $1 < p \leq q \leq \infty$.

Networks Modeling by Systems of Ordinary Differential Equations

Felix Sadyrbaev
Institute Mathematics, University of Latvia, Latvia
Eduard Brokan

We consider systems of ordinary differential equations that appear in the theory of gene regulatory networks. These systems can be of arbitrary size but of definite structure that depends on the choice of regulatory matrices. The decisive role in behaviour of elements of such systems play attractors. The system in abbreviated form is

$$\frac{dx_i}{dt} = f(\sum w_{ij}x_j - \theta)v_g - x_i v_g - \eta,$$

where f is sigmoidal function, w_{ij} are entries of the regulatory matrix W, v_g is a parameter and η stands for stochastic behaviour. We neglect η and consider the system in extended form

$$\begin{cases} \frac{dx_1}{dt} = \frac{1}{1+e^{-\mu_1(w_{11}x_1+w_{12}x_2+\ldots+w_{1n}x_n-\theta_1)}}v_1 - x_1v_1, \\ \frac{dx_2}{dt} = \frac{1}{1+e^{-\mu_2(w_{21}x_1+w_{22}x_2+\ldots+w_{2n}x_n-\theta_2)}}v_2 - x_2v_2, \\ \ldots \\ \frac{dx_n}{dt} = \frac{1}{1+e^{-\mu_n(w_{n1}x_1+w_{n2}x_2+\ldots+w_{nn}x_n-\theta_n)}}v_n - x_nv_n. \end{cases}$$

We study the structure of simple attractors that consist of a number of critical points for several choices of regulatory matrices W.

Unboundedness of Solutions in Parameter Dependent Second Order Differential Equations

Felix Sadyrbaev
Institute Mathematics, University of Latvia, Latvia
Alexander Baryshnikov

Linear and nonlinear second order ordinary differential equations are considered with coefficients depending on parameters. The unboundedness of solutions is studied. The stability diagrams for the Meissner's equation of the form

$$x'' + (\delta + Ap(t))x = 0,$$

where $p(t)$ is the piece-wise constant function switching from 1 to -1 periodically, are obtained (A and δ are treated as parameters). The unboundedness of solutions of nonlinear equations of the form

$$x'' + a(t)x + b(t)x^{2n+1} = 0,$$

where $a(t)$ and $b(t)$ are piece-wise constant coefficients, is studied also. The values of coefficients and moments of switching between them are treated as parameters.

A Model for the Effects of Pollutants on Survival of Species

Saroj Kumar Sahani
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We propose a delayed model for the effects of pollutants released in the environment on the species survival. We assumed that the species grows with logistic rate. It is very obvious that the uncontrolled emission of the pollutants have very adverse effects on the species and sometime it may lead to the extinction of the species. We, thought the long term dynamical model, will show how the pollutant addition term will affects the survival. We have simulated the system for its long term dynamical properties and establish the existence of non-trivial equilibrium point which will ascertain the survival of species in the polluted environment.

Kinematical Conservation Laws: a Geometric Approach to Wave Propagation

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A pair of kinematical conservation laws (KCL) in a ray coordinate system are the basic equations governing the evolution of a moving curve in two space dimensions. KCL system is derived purely from a geometric perspective involving three unknown variables and hence is an under-determined system. If the moving curve is a wavefront of a smooth wave, then a closure relation can be obtained in the ray coordinate system from a transport equation that governs the amplitude of the wave along the wavefront. This transport equation is derived from a quasilinear hyperbolic system under a small amplitude perturbation (weakly nonlinear approximation) and a short-wave assumption. The resulting system, called the *weakly nonlinear ray theory* (WNLRT) system can be used to study the propagation of the wavefront in a given medium. This talk is intended to discuss the basic formulation of the WNLRT system, with a special emphasis on how the KCL system is related to the physical problem of weakly nonlinear wavefront propagation in inhomogeneous and moving media. A special focus will also be given to discuss the connection between the KCL based theory and the well-known method called the level set method for interface tracking.

Kinetic Stability Analysis of 18F-FDOPA in PET/CT Imaging for Parkinson's Disease

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Kinetic stability analysis is a useful method for illustration of the dynamic behaviour of metabolic biological systems. The ^{18}F-FDOPA administration is used to uptake in brain of the patient who has decreasing dopamine level. Then, Parkinson's Disease will be found with Positron Emission Tomography-Computed Tomography (PET/CT) scan. The purpose to study is to develop and analyze a kinetic model of ^{18}F-FDOPA, which is consequence to metabolic biological systems of this radioactivity. According to the Quasi-Steady-State Approximation (QSSA) demonstrates behaviour of the equilibrium point, which explains PET/CT data. Describing equations as follows : Let A(t), X(t), Y(t) and Z(t) be the concentration of FDOPA and FDA in blood or brain, including chemical reaction coefficients k_i, $\frac{dX}{dt} = K_1 A(t) - (k_2 + k_3 + k_4)X(t)$, $\frac{dY}{dt} = k_3 Z(t) - (k_5 + k_6)Y(t)$, $\frac{dZ}{dt} = k_5 Y(t) - k_7 Z(t)$, where A(t)=$\frac{k_2}{K_1+k_3}$. Our findings are expected to provide quantitative method for kinetic model analysis and reliability for evaluating of administrate and radiation dose.

Existence of Solutions to Nonlocal Nonlinear Fractional Functional Integro-Differential Equations of Sobolev Type

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This article is concerned with the existence of solutions for a class of nonlinear nonlocal fractional functional integrodifferential equations of Sobolev type in a Banach space. We also render the criteria for global existence of solution and study the continuous dependence of solution on initial data. An application is given to illustrate the abstract results.

Dynamics of a Delayed Predator-Prey Model with Saturated SIS Epidemic on the Prey Population

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A predator-prey model with disease in the prey population is considered. The transmission of disease in the prey population is of type II functional response, while the incidence rate between the whole prey population and the predator population is Lotka-Volterra type. The delay term arises on the increase of the predator population due to interaction with both prey, may it be infected or not. The stability of the equilibrium points and the existence of Hopf-Bifurcation are analysed. Lastly, numerical examples are done to illustrate the theoretic results.

Optimal Control on a Discrete Time Model for Tuberculosis

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Rodolfo Po III

An optimal control problem is studied in a discrete time model describing the dynamics of Tuberculosis transmission in which treatment in both latency and infectious period is considered. Maximising the susceptible class at the final observational time, and minimising the cost induced from the controls are aimed. The controls are characterised through Pontriyagin's Optimality Principle, and are solved numerically, together with the state equations, using forward-backward sweep method.

Global Stability for Coupled System Using Graph Theoretical Approach

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The dynamics of a coupled system of differential equations is investigated. By combining results of graph theory, interesting results have been obtained. By using the method of constructing Lyapunov functions based on graph-theoretical approach for coupled systems, it is derived sufficient conditions under which the positive equilibrium of this model is unique and globally asymptotically stable if it exists. An exhaustive numerical simulation is done to substantiate the analytical findings.

On Quasilinear Viscous Approximations to Conservation Laws

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Convergence of quasilinear parabolic viscous approximations to an entropy solution (in the sense of Bardos-Le Roux-Nedelec) of a scalar conservation law posed on a bounded domain will be discussed. Differences and similarities with the well-known artificial viscous approximations will be highlighted.

Eigensolutions for a Model of Vertical Gene Transfer of Plasmids

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We consider a model for vertical gene transfer of genetic elements in bacteria called plasmids. The model incorporates non-constant cell division and death, plasmid reproduction, and segregation which is modeled by an integral operator describing the fraction of plasmids the respective daughter receives from the mother. The dynamics of the system is determined by plasmid reproduction which increases the number of plasmids in the population and cell division which decreases the number of plasmids per cell. We obtain a growth-fragmentation equation for the bacterial population structured by the number of plasmids. The model equation is a hyperbolic partial differential equation with an integral term. As we are interested in the long-term distribution of plasmids we consider the associated eigenproblem and prove existence of eigensolutions along the lines of [1] extending the results to models with non-constant cell death rates. In order to obtain stability results, we analyze the spectrum of the differential operator given by the model equation. We find a real dominating eigenvalue. The corresponding eigenfunction attracts all solutions.

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Stability Analysis of Switched Singular Stochastic Linear Systems

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This topic is devoted to study the stability of switched singular stochastic linear systems with both stable and unstable subsystems. By using the method of multiple Lyapunov functions and the notion of average dwell time, we provide some sufficient conditions for the exponential mean-square stability of switched singular stochastic systems in terms of a proper switching rule and the linear matrix inequalities. An example is given to illustrate the effectiveness of the obtained results

Positive Solutions of a Nonlinear Fractional Differential Equation Boundary Value Problem

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In this article, we investigate sufficient conditions of the existence of at least three positive solutions for a class of nonlinear fractional differential equations with three point boundary value problems. Some new multiplicity and nonexistence results of positive solutions are established by making use of Leggett-Williams fixed point theorem on cones together with some comparison results. Finally, an example is presented to demonstrate the validity of the main results here.

Applications of the Generalized Miranda Theorem to Nonlocal Neumann Boundary Value Problems

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We study the nonlocal Neumann boundary value problem of the following form

$$u'' = f(t, u, u'), \quad u'(0) = 0, \quad u'(1) = \int_0^1 u'(s)dg(s),$$

where $f : [0,1] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ and $g = \mathrm{diag}(g_1, \dots, g_n)$ with $g_i : [0,1] \to \mathbb{R}$, $i = 1, \dots, n$. The problem was studied only in [1] and [3] so far. However, in both papers the function f is considered to be *bounded*. Here, using the generalized Miranda theorem (see [2]), we shall weaken the assumptions imposed upon the function f in the papers [1] and [3].

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Modeling the Growth of Blood Vessels

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The growth of blood vessels (angiogenesis) in a living tissue is a complex multiscale process that can be either physiological, being essential for organ development and repair, or pathological, since its unbalance can lead to various disorders and foster cancer proliferation. Angiogenesis is a response to lack of oxygen. Hypoxic cells produce growth factors that prompt the generation of new capillary sprouts from a nearby vessel. Thus, the vessel network expands, transporting oxygen and nutrients to the surrounding tissue. A hybrid tip cell model for tumor-induced angiogenesis is presented, in which trajectories of vessel tips are the capillaries advancing towards the tumor, driven by the gradient of a growth factor. Vessel extension follows from stochastic differential equations coupled to reaction-diffusion equations for the involved substances concentration. Branching of new tips and anastomosis (destruction of tips merging with existing capillaries) are also modeled. Numerical simulations of the model are carried out and a deterministic description of the active tips density is derived from ensemble averages over many replicas of the angiogenic process. It is shown that such density forms a stable soliton-like wave whose dynamics (location, velocity, size) can be characterized by a system of collective coordinates.

Perturbation Theory of Bloch Eigenvalues and Homogenization

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The spectrum of periodic operators in divergence form has a band structure, which is completely described by Bloch eigenvalues. We prove that the Bloch eigenvalues at a point are generically simple with respect to the coefficients of the periodic operator. We use this result to prove that a Bloch eigenvalue can be made simple for all parameter values through a perturbation of the operator. Further, a spectral edge can be made simple by a perturbation of coefficients of the operator. We apply these spectral tools to Bloch wave homogenization problems in the presence of multiplicity.

Cross-Diffusion in Population Dynamics

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In Population dynamics, reaction-cross diffusion systems model the evolution of the populations of competing species with a repulsive effect between individuals. For these strongly coupled, often nonlinear systems, a question as basic as the existence of solutions appears to be extremely complex. We introduce an approach based on the most recent extensions of duality lemmas and on entropy methods. We prove the existence of weak solutions in a general setting of reaction-cross diffusion systems, as well as some qualitative properties of the solutions.

Optimal Feedback Control for 3D Bingham Fluid Motion Model

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Victor Zvyagin

The problem of control of fluid motion by external forces often arises in practical applications. Usually, the control is chosen from some given (finite) set. However, in papers of V.G. Zvyagin, V.V. Obukhovskii, M.V. Turbin it were considered problems with external forces (control), depending on the velocity of the fluid (such problems are called problems with feedback). This approach allows to select the control more precisely, since in this case the control is chosen not from the finite set of available controls, but belongs to the image of some multivalued map (it must be bounded, upper semicontinuous, weakly closed and has non-empty, compact and convex values). The solution of control problem for the considered fluid model is the pair (v, f), where v is the velocity of the fluid, and f is the control. Moreover, f belongs to the image of some multivalued map depending on the velocity v of the fluid. Due to the fact that there can be many such pairs, it naturally arises the concept of an optimal solution which gives a minimum to a given quality functional. We consider the feedback control problem for the mathematical Bingham model of fluid motion in the 3D case with periodic conditions on spatial variables.

On Bifurcation from Infinity to Fractional Laplace Equations

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We show that (λ_1, ∞) is a bifurcation point of

$$(-\Delta)^s u = \lambda u + f(\lambda,\, x,\, u) \text{ in } \Omega, u = 0 \text{ in } \quad \mathbb{R}^n \backslash \Omega,$$

where λ_1 is the principal eigenvalue of $(-\Delta)^s v = \lambda v$ in Ω, $v = 0$ in $\mathbb{R}^n \backslash \Omega$.

Sensitivity of Chemical Reaction Networks: Model, Results and Computational Issues

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Sensitivity studies the network response to the perturbation of a reaction rate or of a metabolite concentration, at steady state. Biological experiments in this direction have been performed by Ishii et al., among others. In these experiments they studied the response of metabolic networks to the knock-out of certain targeted reactions. The response is given as the increase, decrease or zero response, of experimentally accessible concentrations of metabolites or reaction fluxes in the system. Our mathematical model translates this problem into a perturbation analysis of a system of differential equations derived from chemical reaction network. We summarize here the results obtained in this direction by Fiedler, Matano, the author et al., with a focus on computational issues.

Consider a Heat Equation on a Simple Star Graph with Three Equal Edges

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$$(1) \begin{cases} \partial_t u^{(j)}(x,t) = \partial_x^2 u^{(j)}(x,t) - q^{(j)}(x) u^{(j)}(x,t) \\ \quad \text{for } 0 \le x \le a \text{ and } j = 1,2,3, \\ u^{(1)}(0,t) = u^{(2)}(0,t) = u^{(3)}(0,t) = 0 & (a) \\ u^{(1)}(a,t) = u^{(2)}(a,t) = u^{(3)}(a,t) & (b) \\ \partial_x u^{(1)}(a,t) + \partial_x u^{(2)}(a,t) + \partial_x u^{(3)}(a,t) = 0 & (c) \\ u^{(j)}(x,0) = f(x). \end{cases}$$

Equation (1) describes a heat process with internal sinks (or sources if $q^{(j)}(x) \le 0$) proportional to the temperature distributions $u^{(j)}(x,t)$. For simplicity we take equal edges in length, and the boundary condition (a) means that the three outer vertices are maintained at temperature zero. Condition (b) describes the fact that they have the same temperature at the common vertex $x = a$ while condition (c) is Kirchhoff's law, which says that the heat transferred through the node $x = a$ is conserved. The inverse problem is to recover the heat proportional coefficients $\left\{q^{(1)},\ q^{(2)}, q^{(3)}\right\}$ from readings of the heat flux at the vertices of the graph and of the temperature at the common vertex, which defines a map

$$f \to \left\{\partial_x u^{(j)}(0,t),\ \partial_x u^{(j)}(a,t),\ \ u^{(1)}(a,t) \quad \text{for } j = 1,2,3\right\}$$

for $0 < t < T \leq \infty$. First we show that by a suitable choice of f we can determine all eigenvalues of the associated Dirichlet's Sturm-Liouville problem on the graph from the readings. Next we explain how to distinguish simple eigenvalues from multiple eigenvalues from observation. Then we can use sampling formula for Paley-Wiener functions to extract eigenvalues of Dirichlet's Sturm-Liouville problems on each edges. Finally we can recover the three $q^{(j)} \in L^2(0,a)$ uniquely from one reading only of $\left\{ \partial_x u^{(j)}(0,t),\ u^{(j)}(a,t),\ \partial_x u^{(j)}(a,t) \right\}$, for $t \in (0,\ T)$.
This is a joint work with Dr. Amin Boumenir.

Non-Floquet Invariant Tori in Reversible Systems

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In this paper we obtain a theorem about the persistence of non-floquet invariant tori of analytic reversible systems by an improved KAM iteration. This theorem can be applied to solve the persistence problem of completely hyperbolic-type degenerate invariant tori fora class of reversible system.

Structure Preserving Scheme for Stochastic Systems with Applications in Computing Effective Diffusivity

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Zhang Zhiwen, Jack Xin

In this poster we propose a symplectic scheme in calculating effective diffusivities in molecular diffusion process. Then via backward error analysis, we prove that such error of such scheme in calculating D^E does not depents on time asymtotically.

On a Stable Numerical Scheme for Magnetic Nanoparticles in Incompressible Fluid

Patrick Weiss
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We are concerned with suspensions of magnetized nanoparticles. A fully practical numerical scheme is proposed for a thermodynamically consistent pde-model which couples hydrodynamic variables, particle density, magnetization and magnetic field to each other. In particular, the effect of the demagnetizing field is embedded. The scheme's design admits an energy/entropy-estimate mimicking the estimates in the continuous setting. The core concepts for stability estimates will be explained, leading to discrete existence results via appropriate fixed-point arguments. A proof of concept for numerics in 2D will be presented, too.

Modeling the Time to Importation of Zika Virus at a Global Scale

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Background: Since the 2015 Zika virus (ZIKV) epidemic in Brazil, ZIKV has spread worldwide and has remained to be global public health concern. ZIKV infection is a vector-borne disease and is transmitted by Aedes species of mosquito. In the case of epidemic in Brazil, dangue was first suspected but its possibility was excluded based on molecular and serological diagnostic method. The effective distance has been known that it can reliably predict the arrival time of infectious diseases including ZIKV.

Objectives: To estimate the actual arrival time of ZIKV importation which is the time at which the first imported case arrived.

Methods/Principal findings: In addition to utilize the hazard function (inverse function of the effective distance), we employed the distribution of reporting delay. In order to consider the possible factors which can cause the delay, we categorize the countries into five according to GDP, and parameters for gamma distribution were estimated for each category. Data on arrival times of ZIKV importation was collected from publicly available data sources. We devised a simple model to reconstruct the geographic transmission dynamics and here we report the preliminary results from our exercise.

Kinetic Model of 99mTc-ECD Absorption in Brain for Epilepsy Patients and Stability Analysis

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Epilepsy is a neurological disorder caused by electrical abnormalities in brain, causing seizures or unusual behavior. The Single-Photon Emission Computed Tomography (SPECT) has dramatically changed the method for epilepsy evaluation. In this study, we developed a model of ^{99m}Tc-ECD absorption in brain, depending on the acquiring time of patients with epilepsy. Collection of epilepsy data from ^{99m}Tc-ECD samples was then adjusted for kinetic parameters and numerical methods were used to find the solution of the kinetic model to compare data from SPECT imaging, specifications, and relative error to get the appropriate parameters for patients with epilepsy. The epileptic patients had absorption around 27.9084 MBq/ml. From SPECT imaging, the essential parameter of ^{99m}Tc-ECD absorption model are $k_{inject} = 0.067 \pm 0.03204$, $K_1 = 0.30 \pm 0.08944$, $k_2 = 0.233 \pm 0.02582$, $k_3 = 0.517 \pm 0.02582$, $k_4 = 0$

and $k_{out} = 0.005 \pm 0.00412$. These parameters could also describe accurately the ^{99m}Tc-ECD absorption of epileptic data. Furthermore the stability analysis of this model illustrates ^{99m}Tc-ECD behavior.

Onset of Thermal Instability of a Non-Newtonian Fluid in a Two-Layer System

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The onset of thermal convection in a two-layer system composed of a fluid layer overlying a porous medium saturated with a non-Newtonian fluid heated from below is studied in the paper. The neutral curves are observed to be bi-modal in the porous-fluid system. The ratio of the depth of the fluid layer to that of the porous medium is found to be a significant parameter to dominate the instability patterns of the two-layer system. For the non-Newtonian fluids, the oscillatory convection is also investigated as well as the stationary convection. The effects of different parameters on both of convection are investigated in detail.

A Smooth Fictitious Domain/Multiresolution Method for Elliptic Equations on General Domains

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We propose a smooth fictitious domain/multiresolution method for enhancing the accuracy order in solving second order elliptic partial differential equations on general bivariate domains. We prove the existence and uniqueness of the solution of a corresponding discrete problem and a so-called interior error estimate which justifies the improved accuracy order. Numerical experiments are conducted on a Cassini oval.

Blowup for the Compressible Euler Equations in R^N

Manwai Yuen
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The compressible Euler equations are fundamental models in the fluid dynamics. In this talk, we discuss the new blowup phenomena with the functional energy methods for the solutions of the Euler equations in R^N for the free boundary problems and non-vacuum states.

Conservative Lotka-Voterra Systems:Hamiltonian Structure and Dynamics

Xiaohua Zhao
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Lotka-Volterra system is the system with the following form:

$$\frac{dx_j}{dt} = x_j(\epsilon_i + \sum_{k=1}^{n} a_{jk}x_k), \quad j = 1, \cdots, n.$$

The system is called conservative if there exists a positive diagonal matrix D such that DA is skew-symmetric, $DA^T = -AD$. In this talk, we will present our new results on Hamiltonian structure and dynamical behaviors, including periodic solutions,invariant tori and chaos, of conservative Lotka-Volterra systems.

Existence-Uniqueness and Exponential Estimate of Pathwise Solution of Retarded Stochastic Evolution Systems with Time Smooth Diffusion Coefficients

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Daoyi Xu

In this talk, we will investigate the existence-uniqueness and exponential estimate of the pathwise mild solution of retarded stochastic evolution systems driven by a Hilbert-valued Brownian motion. Firstly, the existence-uniqueness of the maximal local pathwise mild solution are given by the generalized local Lipschitz conditions, which extend a classical Pazy theorem on PDEs. We assume neither that the noise is given in additive form or that it is a very simple multiplicative noise, nor that the drift coefficient is global Lipschitz continuous. Secondly, the existence-uniqueness of the global pathwise mild solution are given by establishing an integral comparison principle, which extends the classical Wintner theorem on ODEs. Thirdly, an exponential estimate for the pathwise mild solution is obtained by constructing a delay integral inequality. Finally, the results obtained are applied to a retarded stochastic infinite system and a stochastic partial functional differential equation. Combining some known results, we can obtain a random attractor, whose condition overcomes the disadvantage in existing results that the exponential converging rate is restricted by the maximal admissible value for the time delay.

Solvability of Initial-Boundary Value Problem for Thermoviscoelastic Kelvin-Voigt Model

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The initial-boundary value problem under consideration describes the weakly concentrated water polymer solutions motion. This mathematical model also is called Kelvin-Voigt model. This problem is considered with constitutive law which is frame indifferent, i.e. that do not change under the Galilean transformation. Also in this mathematical model the viscosity depends on a temperature, which leads to emergence of additional heat balance equation (it is a parabolic equation with nonsmooth coefficients and with right part from $L_1(0, T; L_1(\Omega))$. For the initial-boundary value problem under consideration the existence theorem of weak solutions is proved. For this the topological approximation approach for hydrodynamic problems is used.

This research was supported by the Ministry of Education and Science of the Russian Federation (Grant 14.Z50.31.0037).

Mathematical Problem of Viscoelastic Media with Memory Motion

Victor Zvyagin
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In the report the existence of weak solutions to the initial-boundary value problem for one viscoelastic model of Oldroyd's type fluid with memory along trajectories of the velocity field is obtained. This problem is solved on the base of a priori estimates and passing to the limit.

Optimization problems for the feedback control systems for this model is also studied.

The existence theorem for trajectory and global attractors for this system under consideration in the autonomous and non-autonomous cases is obtained.

In the plane case, the initial-boundary value problems for a thermoelastic media for this model is shown to be nonlocally weakly solvable.

The existence of weak solutions for a motion model of viscoelastic fluid with memory along trajectories of the velocity field on an infinite interval is proved.

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Control of Mobile Robots on Time Scales

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We deal with the control of unmanned mobile robots which are modeled by three-dimensional systems of first order dynamic equations. Our goal is to show the asymptotic stability of the zero solution. And we end up with that the results in the continuous case are improved by proposing different controllers.

List of Contributors